PART III: Basic Survey Observations

3.1. Horizontal Distance

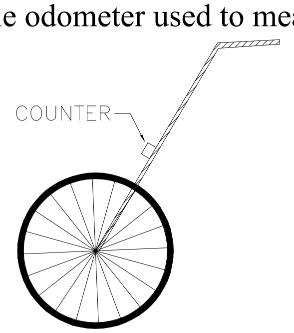
One of the most basic surveying operations is the measurement of distance. It is essential that distances be referenced to the same plane such that subsequent measurements can be compared to the original. In the case of distances, measurements are always referenced to the horizontal plane, even if the actual measurements were measured in some other plane.

3.1.1. Odometer, Mileage Recorder, Pacing, etc.

Among the less precise means of measuring distance is pacing, although it can be effectively used for rough estimates of length. The problem with pacing is that the length of a pace varies with slope and fatigue of the individual that is doing the pacing. It is very clear that as a person becomes more tired, the length of their step decreases. In addition, when the terrain is sloping, the length of the pace will also differ. For example, when going down a hill, a person has a tendency to shorten their pace to keep balance. When using pacing as a means of estimating distance, the individual must use a normal gait. If they are not careful, there is a tendency to lengthen the step slightly thereby creating more error. As a rule of thumb, a pace is only accurate to about one foot over short distances where the terrain is flat and the pacer is walking at their normal pace.

To determine the length of a pace, set out two points on level terrain about 60 meters apart. Then walk the distance back and forth a couple of times counting the number of paces between the points. Then simply divide the distance between the points by the number of steps to arrive at the distance per step. When using this value, adjust for slope and fatigue.

Figure 3.1
Simple odometer used to measure distance.

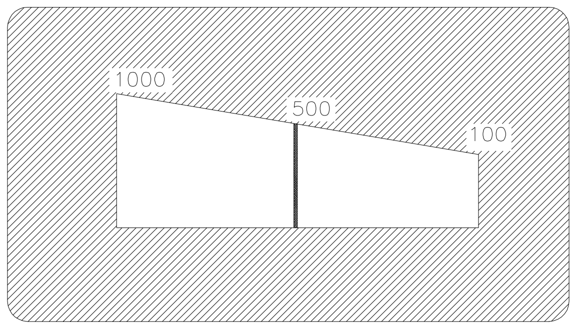


Another approximate means of measuring distance is by an odometer. This instrument consists of a wheel (Figure 3.1) that counts the revolutions between the two points. Knowing the diameter of the wheel, the distance can be measured better than pacing. There are errors associated with the use of the odometer. First, the distance measured is along the ground therefore any dip or hump that exists along the length of the line will be measured. This means that the actual length of the distance will be too long. Second, if the person operating the odometer walks too fast then there is a tendency for the wheel to bounce and not maintain proper contact with the ground surface. Nonetheless, the odometer can be used very effectively for rough distance measurement. One finds their use frequently on construction projects where contractors use these distances for estimating the amount of material, such as concrete, used on a particular project.

Another form of the odometer is found in our vehicles. Generally these odometers are subdivided into tenths of a mile or kilometer. The vehicle odometer provides an effective tool in marking mileage to a control point, as an example, and is often found in control monument descriptions. These distances are very crude but do allow the surveyor to navigate to the location of the control point quickly. Once at the site, better witnesses are used in the recovery process.

There are numerous other methods of determining distance at the level of accuracy of pacing. The most common is the rangefinder. This tool can be a very simple card (see Figure 3.2) or an actual viewing instrument that is used to gauge distance based on the stadia principle. If the height of an object is known then that height is fit into the window of the rangefinder to determine the actual distance. The basic principle is that an object becomes smaller with increasing distance from the observer and that this change in size is a linear function of the distance to the object. Again, like pacing and the odometer, this distance is very crude and should only be used for planning purposes.

Figure 3.2
Example rangefinder card to estimate distance.



Another very useful tool for the surveyor to determine distance is using the subtense bar. The principles are based on basic trigonometry (Figure 3.3). Here, a theodolite is set over one point while the subtense bar is centered over the point at the other end of the line. The subtense bar consists of a fixed base, usually 2 meters, with targets attached to either end of the bar. The angle between the two targets is measured. The distance is found using the relationship:

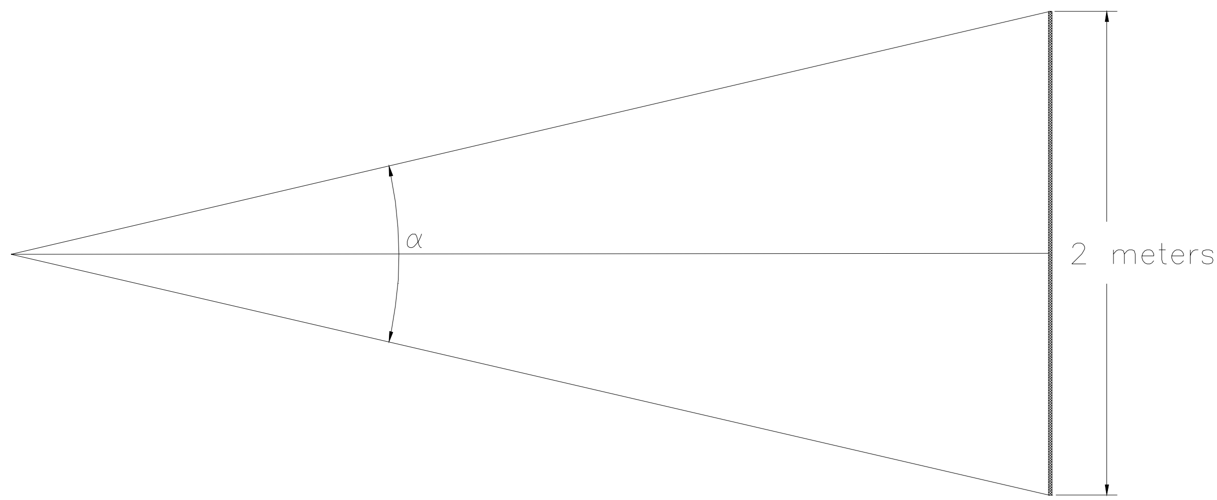
$$d = \frac{b}{2\,\tan\left(\frac{\alpha}{2}\right)}$$

where: d is the distance,
b is the base of the subtense bar, and
α is the angle measured with the theodolite.

3.1.2. Taping and Systematic Corrections

Taping is one of the most common methods of determining distance. Tapes come in all types and forms, from cloth tapes for less precise measurements of distance to invar steel tapes for the most precise distance measurements. Cloth tapes are often used for measurements that do not need a high

Figure 3.3
Basic principles of subtense bar.



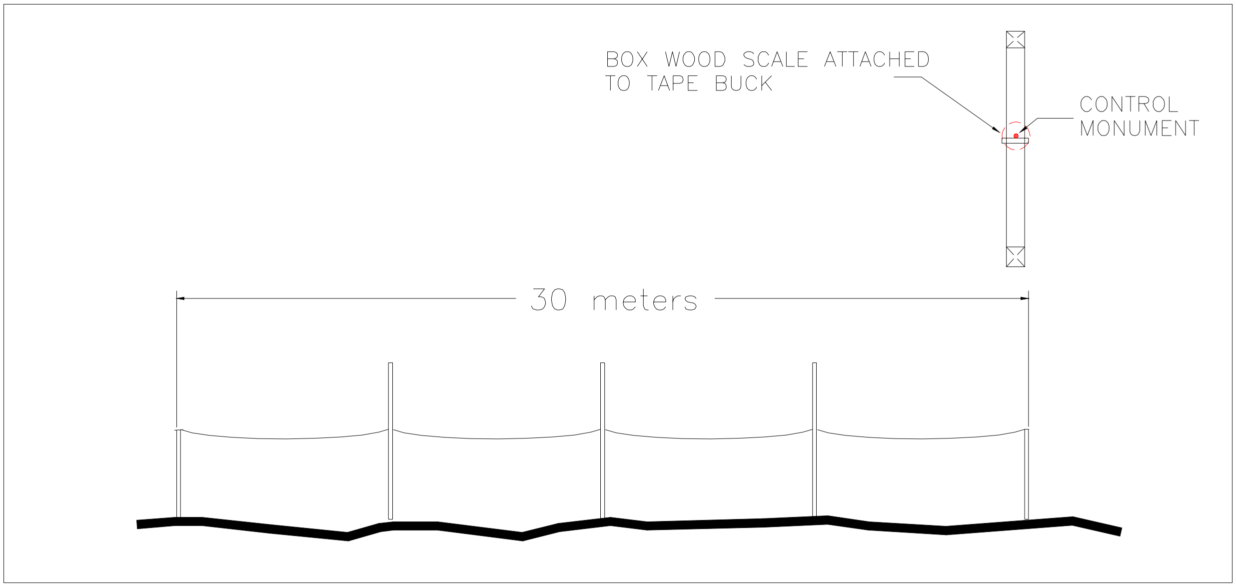
degree of precision. They are susceptible to dimensional change due to tension and the elements. Newer cloth tapes have steel threads interwoven in the cloth that helps maintain their dimensional stability better than regular cloth tapes. These kinds of tapes are frequently used to measure the size of objects and distances to witnesses for control monumentation.

Invar tapes are used for high precision taping because they have a very low coefficient of thermal expansion. The measurement of high accuracy distance by taping is an elaborate process as shown in Figure 3.4. The process begins by placing taping bucks over the points. A simple approach would be to use 2"x4" lumber where the horizontal cross-member is placed over the point. Then, a scale, such as a box-wood scale, is clamped vertically over the point. The scale is not set to zero over the point. Instead, the scale is read by projecting the point vertically by use of a theodolite. Then the distance between the two points is measured.

The measurement process involves two individuals at either end who hold the tape in place. One has a tension handle from which the proper tension is applied to the tape. Also at each end are the "contact" individuals who read the 0-m and 30-m ends of the tape on the appropriate scales. Interspersed along the length of the tape are additional personnel who will keep the tape suspended to eliminate the effects of sag. The rear contact person signals when to make the scale readings. This procedure is continued along the entire length of the line.

Most taping procedures do not require this level of care. In fact, for precise distance measurements, most surveyors will use electronic distance measurement (EDM) equipment (see section III.1.C). Nonetheless, taping does require both skill in the measurement process and care to ensure that systematic errors are removed from the final distance.

Figure 3.4
Precise taping process.



One of the most common systematic errors that needs to be applied to taping is called the standard length correction. This error is the difference between the nominal length of the tape and the calibrated length. Because of inherent manufacturing defects, the distance between the two ends of the tape may not be the exact distance as shown on the tape. From calibration, the true length of the tape can be determined. The correction is generally shown as:

$$C_\ell = l_S - 30m$$

where: C_ℓ = the standard length correction,
l_S = the actual length of the tape determined from calibration, and
30 m = the nominal tape length for a 30-meter tape.

This correction is then added to each tape length along the line. It can also be applied to the total length of the line by multiplying this correction by the total number of tape lengths along the line.

The next major error in taping is due to temperature. When a tape is manufactured, its nominal length is correct at a standard temperature, usually 20º C. When the temperature of the tape differs from this standard temperature then the tape is susceptible to dimensional change. When the tape is too hot, it expands and when it is too cold it contracts. Thus, the distance measured will not be correct. The temperature correction is defined, for metric tapes, as

$$C_t = \alpha l\left(t - t_S\right)$$

where: C_t = the correction for temperature,
α = the coefficient of thermal expansion, 0.0000116 per 1° C,
t = the temperature of the tape, and
t_S = the standardized temperature.

Like the standard length correction, the temperature correction can be applied individually to each tape length or to the total length of the line by multiplying C_t by the total number of tape lengths along the line.

When the tape is suspended at its ends during the taping process, the center of the tape will sag, forming a catenary curve. To alleviate this error, one can hold the tape suspended along the length of the tape as was shown in Figure 3.4. Generally, though, the tape is not suspended throughout and an error ensues. The correction for sag is defined as

$$C_S = -\frac{W^2 l}{24 p^2}$$

where: C_S = the correction due to sag,
W = the weight of that portion of the tape that is suspended,
l = the length of that portion of the tape that was suspended, and
p = the applied tension to the tape.

This correction can also be shown in a slightly different form with the weight of the tape per meter (w) being used.

$$C_S = -\frac{w^2 l^3}{24 p^2}$$

Note that the correction due to sag will always result in a distance that is too long.

The next correction that may need to be applied to taping is the tension correction. This is used when the applied tension of the tape is different than the standard tension. This correction is:

$$C_P = \frac{(p - p_S)\, l}{AE}$$

where: C_P = the correction due to tension,
p = the applied tension to the tape,
p_S = the standard tension,
l = length of the tape
A = the cross-sectional area of the tape, and
E = Young's modulus of elasticity.

Young's modulus of elasticity is usually taken as 2.1 x 10^6 kg/cm^2 (kilograms per square centimeter).

The cross-sectional area will depend on the tape with heavy 30-m tapes having a value of around 0.058 cm^2 while light-weight 30-m tapes having a value of A of about 0.019 cm^2.

The final major error source in taping is the slope correction. This is applied to distance measurements that were made along the slope of the ground. This error always results in a distance that is too long. The correction is based upon the known information, either the vertical angle or the difference in height. If a correction factor is to be applied, for vertical angles it is presented as:

$$C_h = s(\cos\alpha - 1)$$

where: C_h = the correction for slope,
s = distance measured along the slope, and
α = the vertical angle.

If the difference in height (h) between the two points is known, then the correction term is given expressed as:

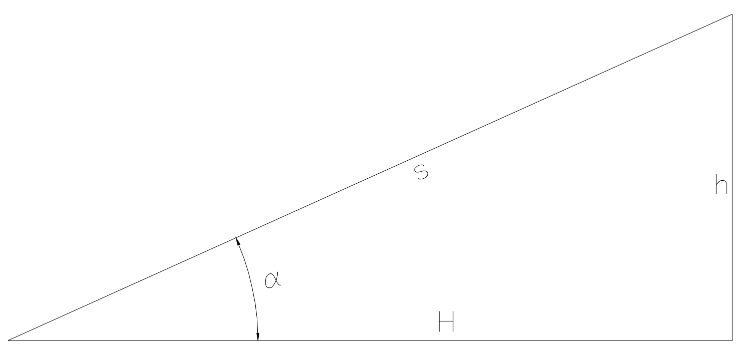
$$C_h = \sqrt{s^2 - h^2} - s$$

Sometimes this correction term is expressed in terms of the binomial theorem.

$$C_h = -\frac{h^2}{2s} - \frac{h^4}{8s^3} - \frac{h^6}{16s^5} - \frac{5h^8}{128s^7}$$

Generally, only the first term is used in this last equation, although additional terms may be necessary if the slope is great and the accuracy of the survey is high. While a correction term can be applied to the slope measurements, it is often easier to compute the horizontal distance (H) directly using either a trigonometric function or the Pythagorean Theorem (Figure 3.5). Thus, if the vertical angle is known, then the horizontal distance is computed using the cosine function

Figure 3.5
Geometry for correcting slope measurements to the horizontal.



$$H = s\cos\alpha$$

whereas if the difference in elevation between the two stations is known then the Pythagorean

Theorem is employed.

$$H = \sqrt{s^2 - h^2}$$

3.1.3. Electronic Distance Measurement

By far the easiest and most common method of measuring distances is with electronic distance measuring (EDM) equipment. EDM come in all different forms but are generally classified by their form of electronic signal. Electro-optical instruments employ infrared light or lasers while electromagnetic use microwave transmissions. The most common is the electro-magnetic. EDM can also be classified by their range. Short-range EDM measure distances up to about 2 kilometers. Intermediate-range instrument have a range up to about 10 kilometers while long-range EDM can measure distances (depending on the type) up to 50 kilometers. Short-range EDM are usually lighter and more compact and are favored by many surveyors. EDM accuracy is expressed in two terms: a constant part and a proportional part. The constant error is present in all measurements, no matter what the length and it does not vary. The proportional part is a function of the distance measured. Thus, an example accuracy might be expressed as: ± (5 mm + 5 ppm) where ppm represents parts per million. If a distance of 1,000 meters was measured (assume that there are no other errors in this measurement), then the uncertainty in the distance due to the instrument alone is ± 1 cm.

When using EDM, it is critical that the total system be calibrated periodically. This includes not only the instrument itself but also the tribrach used to center the EDM over the point and the reflector that is used with that particular EDM. If multiple prisms are used, then they should be calibrated separately. There are a number of EDM comparison ranges located throughout the state from which distance comparisons can be made. The procedures for performing these comparisons are shown in **Appendix D**.

3.1.4. Basic Principles

In general, the EDM computes distance by measuring the time the signal takes to go from the transmitter to the receiver. Generally, both the transmitter and the receiver are located in the same instrument. In this case, the actual distance measured by the EDM is twice the length of the line. The transmitter sends out a modulated signal with a known frequency (f) and wavelength, λ (Figure 3.6). The basic equation for determining the wavelength is

$$\lambda = \frac{V}{f}$$

where V is the velocity of light through the atmosphere. If the instrument emits a signal at a predefined frequency and wavelength, then the velocity can be measured. From basic principles of physics, one can recognize that the velocity will be affected by the atmospheric conditions.

Figure 3.6
Modulated light wavelength emitted from an EDM.

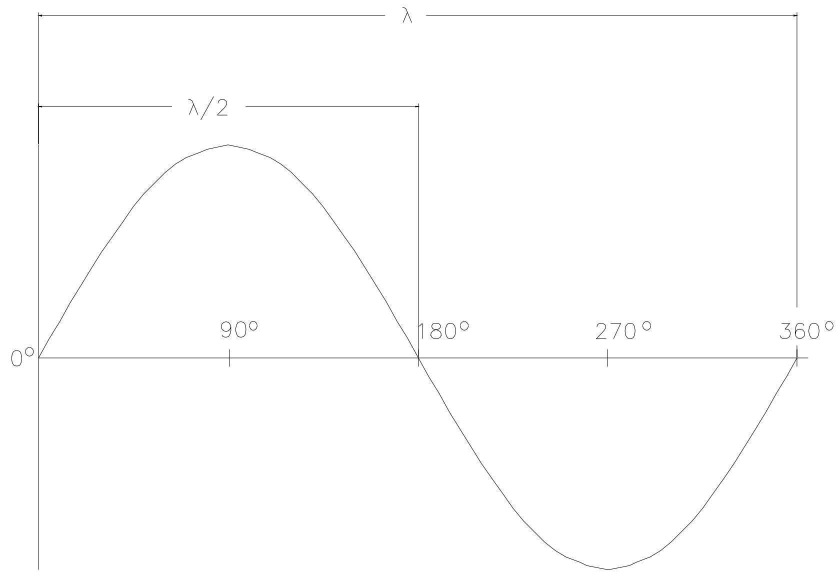
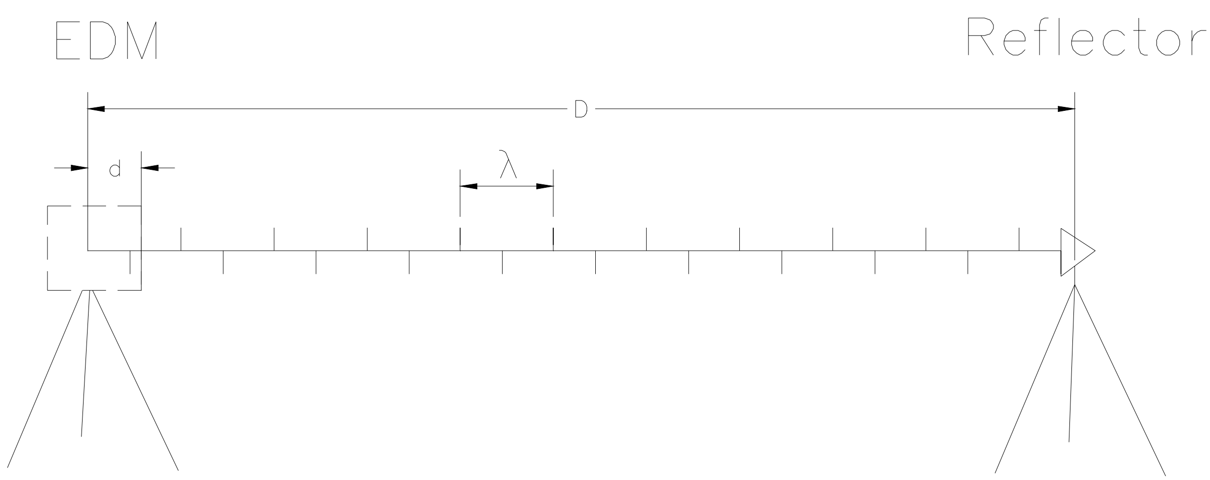


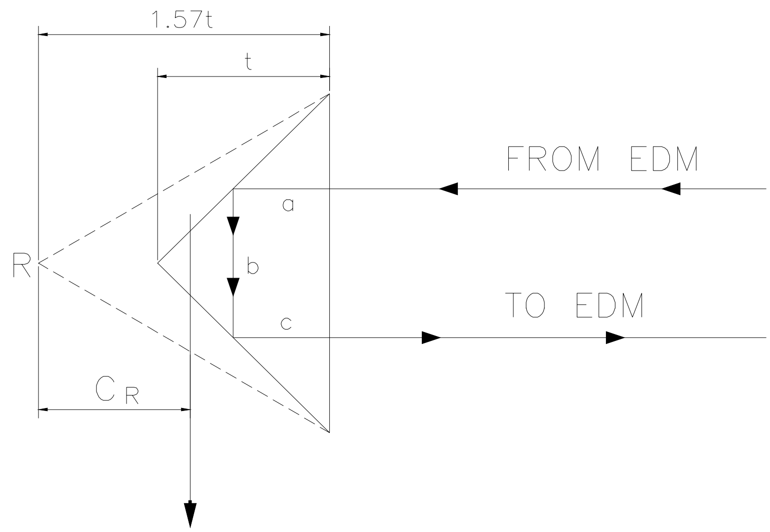
Figure 3.7
Basic principles of EDM measurement.



The EDM sends out a signal of a defined wavelength towards the prism (reflector) where it is reflected back to the EDM instrument (Figure 3.7). The unknown in the measurements is the number of full wavelengths it takes to travel the length of the line and back. This is solved by using multiple wavelengths to resolve this ambiguity. At the receiver, the phase of the incoming signal is measured with respect to the transmitted wave and the phase offset is determined. Then, knowing the length of the wavelength (λ), the number of integer wavelengths along the line (m), and the phase delay (d), the distance between the two ends of the line can be determined using

$$D = \frac{1}{2}\left(m\lambda + d\right)$$

Figure 3.8
Reflector constant



One of the considerations that needs to be accounted for in EDM measurements is the prism constant. As the light wave enters the prism, it is slowed down because of the refractive index of the glass. Since the processor onboard the EDM does not recognize this, it assumes that the light ray is traveling through the atmosphere. The distance the light wave travels within the prism is (Figure 3.8)

$$a + b + c = 2t$$

where t is the thickness of the prism from the front face to the corner of the cube. The EDM processor assumes that the light ray actually traveled to a point R which is the effective corner of the cube. Therefore, the distance that the EDM thinks the light wave traveled through the prism is actually 1.57 x 2t. The distance between R and the location of the plumb line from which the prism is centered over the point is the prism constant, C_R. Care must be taken with this prism constant. For example, some instruments that are sold with a particular prism already have the offset inserted within their processor and this may be displayed on the display panel as having no reflector offset. If another prism is used that has, as an example, an offset of zero, then the measured distance may be in error. The best practice is to use the correct prism with the correct instrument and to treat any other prism carefully.

3.1.5. Correction for Ray Path

There are two corrections that the EDM ray path may need to be taken into account. The first is called the first velocity correction, or commonly referred to as the atmospheric correction. This correction is necessary whenever the EDM ray passed through the atmosphere which has a different refractive index than the modeled atmosphere within the EDM processor. This correction is instrument dependent and can be applied by either dialing in the atmospheric conditions (temperature and pressure) into the instrument and letting its processor use those values to correct the measured distance or the instrument operator can input the parts per million (ppm) correction into the instrument directly. Normally this value is found through a nomogram supplied with the instrument which shows the correction given the atmospheric temperature and pressure. In addition to these two methods, one can also apply no correction into the instrument directly and then use the atmospheric equation model most manufacturers provide with the instrument.

The second correction to the ray path also involves the atmosphere and is referred to as the second velocity correction. This correction is necessary to compensate for the different layers of atmosphere that the ray path may be traveling through. Since temperature and pressure readings are taken only at the ends of the line and an average value used, there is no account for the existence of other atmospheric layers along the ray path. The correction for the second velocity is given as:

$$K^{//} = -\left(k - k^2\right) \frac{d^{/3}}{12R^2}$$

where: $K^{//}$ = the second velocity correction,
k = the coefficient of refraction
$d^{/}$ = the distance displayed on the instrument, and
R = the mean radius of curvature along the line.

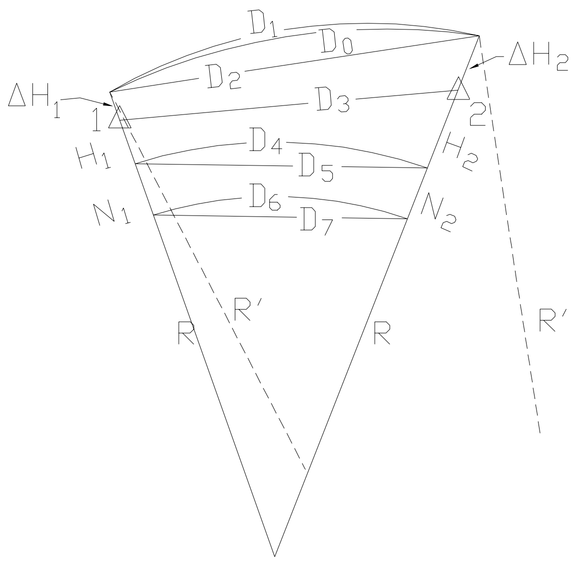
The corrected ray path is then determined by adding these two correction to the distance as shown on the instrument display. The effects of the second velocity correction are more significant with microwave instruments than electro-optical instruments. Moreover, the effects are more important for longer lines than shorter lines.

3.1.6. Reduction Using Elevation Differences

Before any adjustments can be performed on the survey data, distances must be reduced to their appropriate plane. Figure 3.9 shows the geometric relationships between the different types of distance that can be reported[2]. The distances are defined as follows:

[2]Fronczek, Charles J., 1977. "Use of Calibration Base Line", NOAA Technical Memorandum NOS NGS-10, Rockville, MD.

Figure 3.9
Geometric relationship between different forms of surveying measurements.



D_0 = Observed distance corrected for the first velocity correction, instrument and mirror constant, etc.
D_1 = Observed distance corrected for first velocity correction
D_2 = Chord distance at the elevation of the instrument
D_3 = Mark-to-mark chord distance at the station elevations
D_4 = Geoidal distance (also sometimes called the sea level distance)
D_5 = Chord distance at either the geoid or sea level surface
D_6 = Ellipsoid or geodetic distance
D_7 = Chord distance on the ellipsoid

The mathematical reductions are given in Fronczek (1977) and are summarized here. The chord distance at the instrument elevations (D_2) is found from

$$D_2 = 2R'\sin\left(\frac{D_1}{2}\,R'\right)$$

where R' is radius of the arc distance of the measured line and it is defined as $R' = \frac{R}{k}$ where k is the coefficient of refraction. The radius of curvature, R, can be determined using the relationship

$$R = \frac{N}{1 + e'^2\cos^2\varphi\,\cos^2\alpha}$$

where e' is the second eccentricity of the ellipse which is found from $e'^2 = \frac{a^2 - b^2}{b^2}$ with a and

b being the semi-major and semi-minor axes of the ellipse respectively.
The value N is the radius of curvature in the prime vertical and it can be determined from

$$N = \frac{a^2}{b\left(1 + e'^2\cos^2\varphi\right)^{\frac{1}{2}}}$$

The chord distance at sea level is computed as

$$D_5 = \left[\frac{D_2^2 - \Delta H^2}{\left(1 + \frac{H'_1}{R}\right)\left(1 + \frac{H'_2}{R}\right)}\right]^{\frac{1}{2}}$$

where: $H'_1 = H_1 + \Delta H_1$

$H'_2 = H_2 + \Delta H_2$

$\Delta H = H'_1 - H'_2$

The corresponding chord distance along the geoid or sea level is computed using the next formula.

$$D_4 = 2R\sin^{-1}\frac{D_5}{2R}$$

To determine the mark-to-mark distance, use

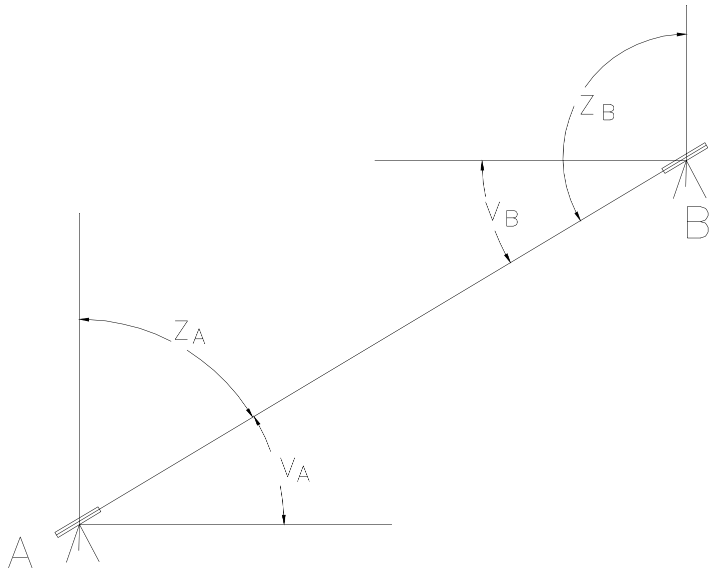
$$D_3 = \left[D_5^2\left(1 + \frac{H_1}{R}\right)\left(1 + \frac{H_2}{R}\right) + \left(H_1 - H_2\right)^2\right]^{\frac{1}{2}}$$

The chord distance at the ellipsoid is computed from

$$D_7 = \left[\frac{D_2^2 - \Delta h^2}{\left(1 + \frac{h_1}{R}\right)\left(1 + \frac{h_2}{R}\right)}\right]^{\frac{1}{2}}$$

where: $h_1 = H'_1 + N_1$

Figure 3.10
Reciprocal vertical (or zenith) angles.



$$h_2 = H'_2 - N_2$$

$$\Delta h = h_1 - h_2$$

Finally, the geodetic distance is found as

$$D_6 = \sin^{-1}\frac{D_7}{2R}$$

3.1.7. Reciprocal Vertical Angles

While the reduction of EDM distances has been presented when elevation differences are known, it is common to observe the vertical angles in the field at the point and use these angles in the reduction of measured distances. Generally, it is the zenith angle that is actually measured (Figure 3.10). This is the complement to the vertical angle. Vertical angles are very susceptible to the effects of refraction therefore they are weaker than the use of elevation differences. Nonetheless, acceptable results can be used when reciprocal vertical angles are measured. This means that the angle is measured at both ends of the line and an average value used for subsequent computations. It is also preferable if the reciprocal vertical angles were measured simultaneously because this ensures that the same atmospheric conditions exist for both sets of observations.

To perform the reduction, most of the relationships that were presented in the previous section can be employed. The major difference is in the slope reduction. Designating D_H as the horizontal distance at the mean elevation of the two stations, this distance can be found from

$$D_H = D_0 \sin z_A - \frac{D_0^2 \,(2 - k)}{4R} \sin 2z_A$$

where z is the zenith angle measured at point A. For increased accuracy, the average zenith angle should be employed. Then, to determine the chord distance on the geoid, a correction factor can be computed using

$$D_5 = D_H - \frac{H_1}{R} D_H$$

3.2. Vertical Distance

Distance measurements can also be made in the vertical direction. These distances are often referred to as elevations or differences in elevations. The determination of height is generally carried out by leveling, although vertical distances can also be determined from the global positioning system and by measuring the vertical angle and distance to the object.

3.2.1. Definitions

The height system most commonly used in reporting elevations is referred to as orthometric heights. Gravity will form a series of surfaces, each which are perpendicular to the direction of gravity, that form equipotential surfaces (Figure 3.11). Along these surfaces, there is a constant gravity potential. One can note these surfaces are not parallel. It is also evident that the distance from the reference

Figure 3.11
Equipotential surfaces are not parallel to each other or the reference surface.

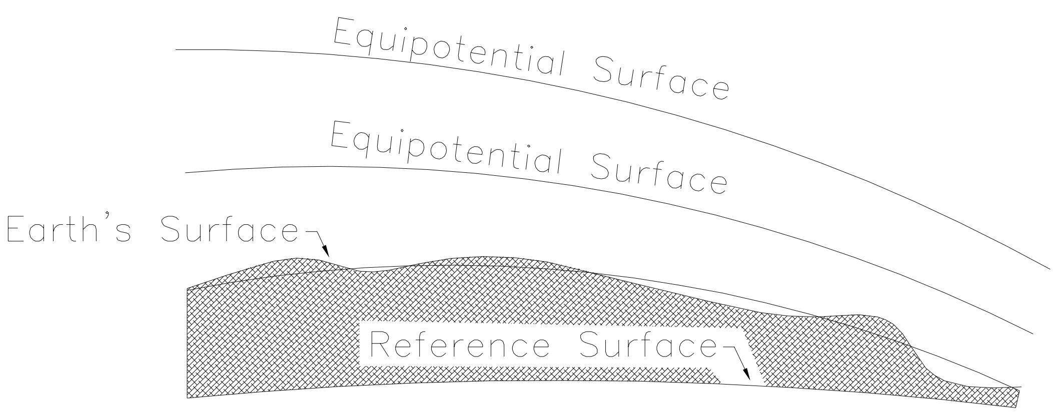
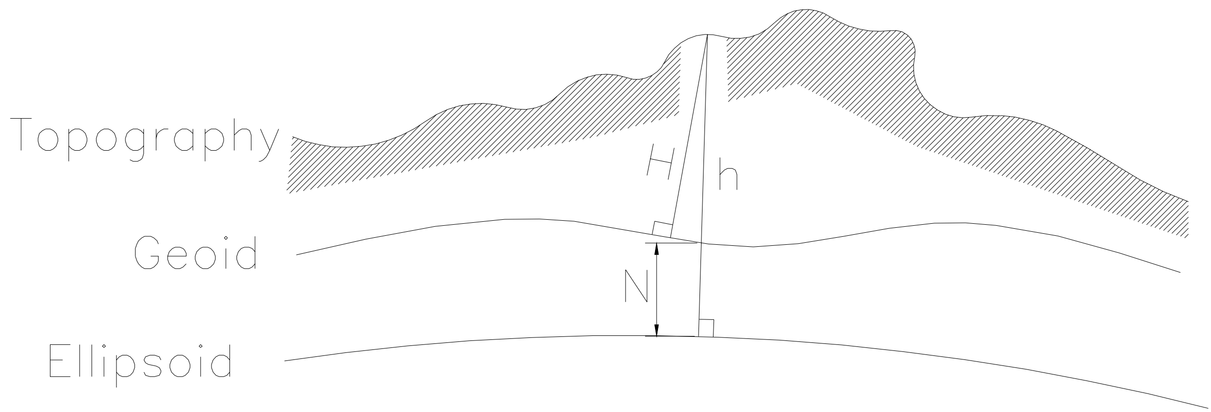


Figure 3.12
Relationship between the ellipsoid and orthometric height.



surface will vary along these equipotential surfaces, increasing as the line heads towards the equator. Since leveling depends upon the direction of gravity, discrepancies will occur in the height measurements.

The geoid is a special reference surface from which all orthometric heights are referenced. It is that equipotential surface that closely approximates the figure of the earth. Sometimes the geoid is envisioned as that surface that follows the mean sea level over the entire world and is thus referred to as the mean sea level surface.

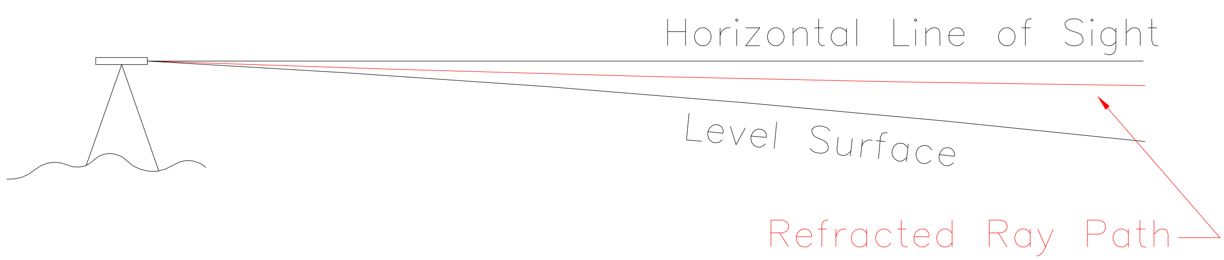
Ellipsoid height is the height of a point above the ellipsoid. It is measured along a line that is perpendicular to the ellipsoid that passes through the point (Figure 3.12). The ellipsoid height is designated as h whereas the orthometric height is depicted as H. The difference between the geoid and the ellipsoid is called the geoid height or geoid undulation. It is the difference between the geoid and the ellipsoid. Hence, $N = h - H$. The significance of this relationship is that if the ellipsoid height were measured, as from GPS, and if the geoid undulation is known or can be computed, then the orthometric height can be easily calculated.

3.2.2. Curvature and Refraction

We have already seen that the surface of the earth is not a plane but is curved. Yet, the level line from the level defines a plane that is tangent to the curve at the instrument. The effect is that the reading on the level rod will be too high by the amount that the horizontal line of sight deviates from the level surface.

Since the light ray is traveling through the atmosphere, it also bends according to the law of

Figure 3.13
Effects of curvature and refraction on leveling.



refraction. The effect of refraction is to bend the light ray towards the earth. Thus, it tends to counter, to some extent, the effects of earth curvature. The combined effect of curvature and refraction are expressed as

$$(c + r) = 0.0675 K^2 \ meters$$

where (c + r) is the combined effect of curvature and refraction and K is the distance in kilometers. The effects of curvature and refraction can be reduced if the backsight and foresight distances are the same. These errors are generally insignificant for most leveling operations but do affect precise leveling surveys.

3.2.3. Orthometric Correction

As it was pointed out in **Part 3.2.1**, the equipotential surfaces (level surfaces) are not parallel and thus the difference between these surfaces will vary with latitude. This means that level lines run between two points in a north and south direction will yield two different height differences. Therefore, it is important for precise surveys to account for the convergence of the level surfaces. This is called the orthometric correction. It is given as:

$$C_{orthometric} = -0.005288 \left(\sin 2\varphi\right) h \left(\Delta\varphi\right) \rho$$

where: φ = the latitude of the beginning of the line,
h = the elevation of the first point,
$\Delta\varphi$ = the difference in latitude between the two point in minutes, and

ρ = the conversion of the units for $\Delta\varphi$ into radian measure (arc 1' = 0.000290).

The sign for $\Delta\varphi$ is positive when the level line runs from the south to the north (towards the pole) and negative otherwise.

3.2.4. Methods of Leveling

Like all survey operations, there are a number of different methods in which elevations can be measured. The most common is the direct method whereby a level is used to measure the difference in height between points by observing on a level staff. Indirect methods of leveling include reciprocal, trigonometric, and gravimetric leveling and the use of GPS for height determination.

3.2.4.1. Direct

The direct methods of determining elevation involves the use of a level where the differences in height between points is observed directly using a level staff or rod. There are four basic methods of determining the elevation using this method: single wire, three wire, automatic (using a bar code), and use of a wedge reticle.

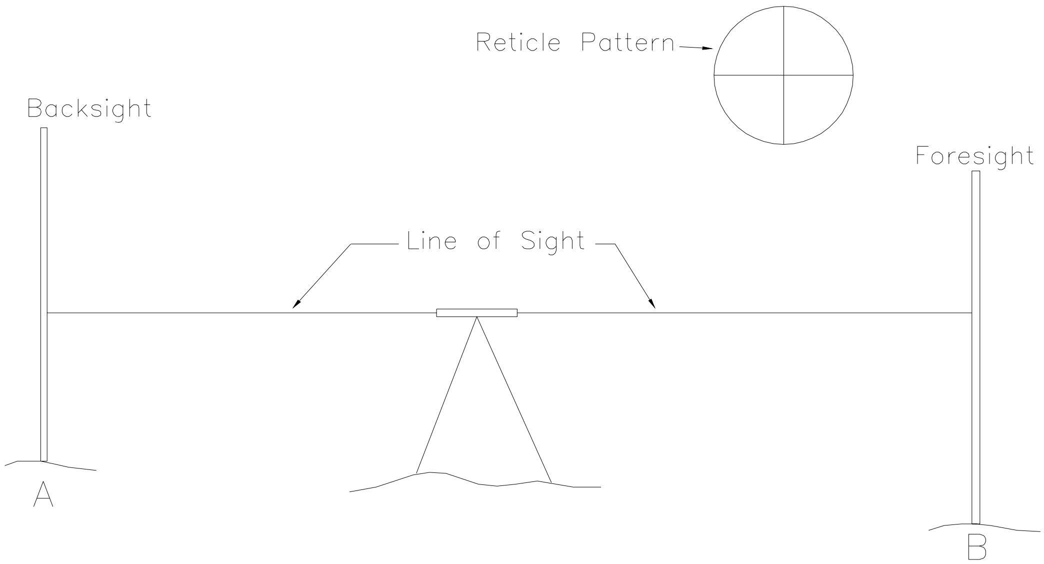
3.2.4.1.1. Single Wire

Most of the leveling performed by surveyors involves setting up the level between two points and reading the level rod where the center horizontal cross-wire intersects the level rod. An example of a simple reticle pattern where only one horizontal cross-wire is shown along with the leveling configuration in Figure 3.14. The instrument is set up between two points and leveled as always. Once the instrument is leveled, the center of the objective lens and the center of the reticle pattern will define a level line. Leveling begins by observing the reading on a level rod over a point whose elevation is either known or assumed. This is the backsight. Next, the level rod is moved to a second point and the level is pointed to that rod and a foresight reading is taken on that rod. For precise leveling, two rods with invar steel ribbons are used. The difference in height between the foresight and backsight is the difference in readings on those two points.

$$\Delta H_{AB} = R_{FS} - R_{BS}$$

One of the problems with the use of a single wire in leveling is that there are no internal checks on the observations. If a blunder in reading occurred, there is no way to find this out unless the level rod was read again before the instrument was moved to its next position. Therefore, careful observational procedures must be employed to ensure that blunders, such as misreading the level rod, do not occur.

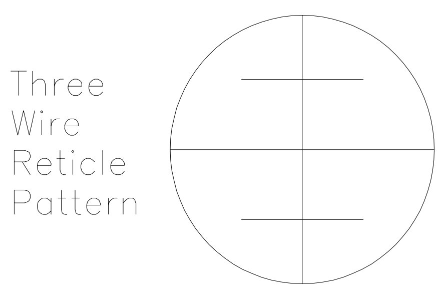
Figure 3.14
Single reticule pattern with basic leveling procedure.



3.2.4.1.2. Three Wire

For precise leveling, other techniques are employed. One of these is called three-wire leveling. Here, the level reticle pattern contains three horizontal lines (Figure 3.15). Level rod readings are taken on all three wires on the reticle. The three readings are checked for accuracy in that the differences between the middle and both the top and bottom cross-wires (half intercepts) should be about the same. Moreover, the difference between the top and bottom should be about twice the half intercepts. Once these checks are acceptable, the rod reading is the average of the three readings. This approach to leveling minimizes the possibility of blunders in reading the level rod. Another advantage of this procedure is that the distance to the level rod can be found using the stadia principles. This is very helpful in subsequent data reduction when determining whether the sum of the backsight and foresight distances were equal throughout the level loop. The disadvantage is that three-wire leveling triples the number of observations that need to be taken and requires additional processing.

Figure 3.15
Three-wire reticle pattern.



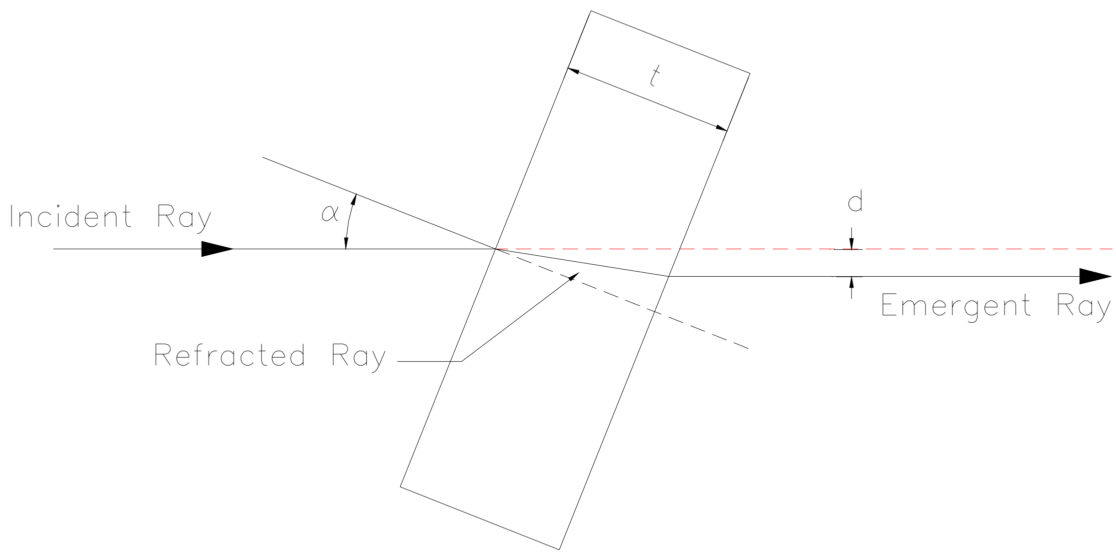
3.2.4.1.3. Automatic (Bar Code)

The newest level in use today is the digital, self-leveling level that is used with an associated bar code level rod. The level is very similar to the optical automatic level except that a beam splitter is inserted along the optical path of the instrument. The image from the object space is sent to both the eyepiece and to a CCD (charge coupled device) where the bar-coded image is captured and processed in the determination of the horizontal line of sight. Because of the design of the level rod, distance to the rod can also be determined. One of the advantages of the digital level is that it minimizes the possibility of blunders in reading the rod. In addition, the data can be automatically recorded into a data collector for subsequent processing.

3.2.4.1.4. Use of Wedge Reticle

Another method of performing precise leveling is using the wedge reticle with an optical micrometer on the level. The principle of this level is simple. When light passes through a lens in a direction other than perpendicular to the surface of that lens, then the light ray is bent (Figure 3.16). If the lens is flat and both surfaces are parallel to each other, then when the light ray emerges from the lens it is bent again such that the emergent light ray is parallel to the incident light ray. The amount of displacement can be determined and this vertical shift is added to the rod reading for a more refined measurement. The reticle pattern consists of a wedge (Figure 3.17) which appears to move up and down until the wedge is coincident with one of the markings on the level rod. The rod reading is taken and the amount of lateral displacement from the optical wedge is also recorded and added to the rod level reading. This allows for a finer reading on the rod staff.

Figure 3.16
Optical wedge geometry.



3.2.4.2. Reciprocal Leveling

Sometimes it is impossible to keep sight distances within the limits recommended for leveling. A perfect example is when crossing a river or ravine. In this case, reciprocal leveling must be employed (Figure 3.18). The instrument is first set up at point A and the backsight is taken on point 1. Next, take a foresight reading on point 2. The instrument is moved to B and the process is repeated. The

Figure 3.17
Wedge reticle pattern.

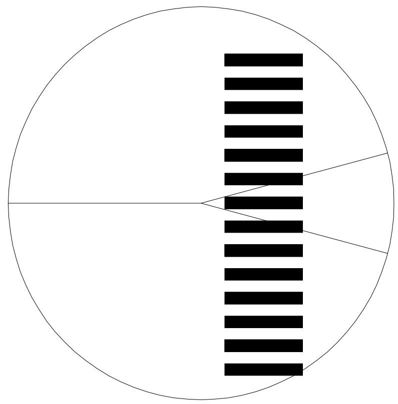
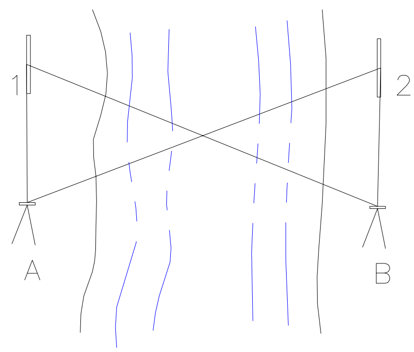


Figure 3.18
Reciprocal leveling.



average difference between the two sets of readings are then used. This approach has a number of hazards and should not be used unless prior approval is given from the Design Survey Manager.

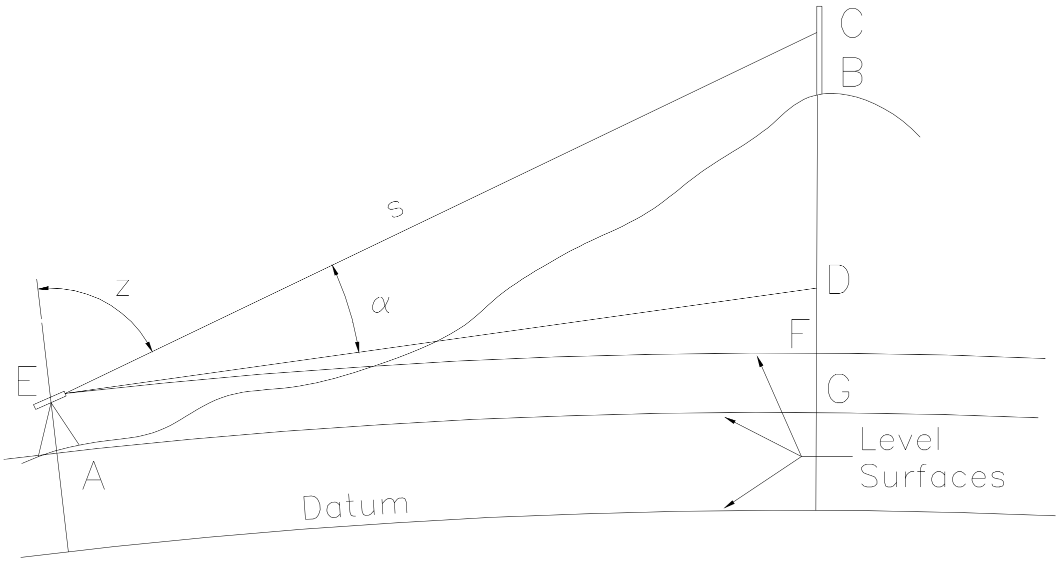
3.2.4.3. Trigonometric Leveling

Trigonometric leveling involves the measurement of either the zenith angle (z) or vertical angle (α) and usually the slope distance from the instrument to a target at the other point. With most total stations, the measurement of the zenith angle and the slope distance can be processed on-board the instrument. Output could be, for example, the elevation of point B. The difference in height between points A and B is given as:

$$\Delta H_{AB} = D_{AE} + D_{DF} + D_{EC}\cos z - D_{BC}$$

$$\Delta H_{AB} = D_{AE} + D_{DF} + D_{EC}\sin\alpha - D_{BC}$$

Figure 3.19
Trigonometric leveling.



3.2.4.4. Gravimetric

Elevations can also be derived from gravimetric measurements. Using a gravimeter, the value of gravity at a station can be calculated based on the principle that the magnitude of the gravity vector decreases with elevation. This can be correlated with differences in elevation (see 3.2.A. for a discussion). One of the weaknesses in using gravity observations lies in the fact that the level surfaces are not parallel to each other.

3.2.5. Level Rods

Level rods come in all shapes and configurations. Level rods for precision surveys consist of an invar ribbon upon which the graduations are marked. This kind of ribbon has a low coefficient of thermal expansion making the level rods more stable than conventional level rods. For precise surveys, the rods must be calibrated and maintained properly. A current calibration report must be provided to the Design Survey Manager for leveling projects that comply with NGS standards and specifications.

3.3. Direction

Survey lines are vectors and, as such, requires two values to define them. The first is distance which was discussed earlier. The second is a direction. Directions tell the reader where the line is heading.

3.3.1. Meridians

A meridian is a reference line that is used for determining direction. Ideally, it is a line whose direction is either due north or due south. There are several methods of determining a meridian.

3.3.1.1. True (Astronomic)

The true or astronomic meridian is a line that is found using astronomical observations. It is called the true meridian because it is a reproducible value since it remains unchanged. The direction of this line is true north-south.

3.3.1.2. Grid Meridian

When projecting a spherical surface onto a flat surface, one of the meridians within the project is designated as the north direction. On a grid, a Cartesian coordinate system is employed where the X and Y axes are perpendicular to each other. Because of this kind of geometry, the meridional lines will be drawn parallel to the Y-axis and will not converge at the poles, like true meridians will do.

3.3.1.3. Magnetic

The magnetic meridian defines the north direction in terms of the compass heading. The magnetic north pole does not coincide with the true north pole and is susceptible to considerable variation. Because of magnetic declination, magnetic directions should not be used for MDOT surveys unless those directions are used for rough estimates of direction such as when determining the direction of witness ties.

3.3.1.4. Assumed

The least favorable means of defining the direction of a line is by using an assumed direction. Generally this is used when surveys of plats and adjacent parcels give a bearing or azimuth of a line that will be used in the existing survey. Otherwise, assumed bearings should never be used. They cannot be reproduced independently in the field.

3.3.2. Bearings and Azimuths

There are two methods of delineating directions: bearings and azimuths (see Figure 3.20). A bearing is an angle measured either from the north or south to the east or west. Bearing can never exceed 90°. Bearings can be measured from the right or left and can point to either direction as well. Bearings will contain two parts: the angular value, and the direction to which the line is facing. An azimuth is an angle measured in a clockwise direction from north to the line. It has a range up to 360°.

Azimuths (α) can be converted into bearings (β) by applying the geometry shown in Figure 3.20. The conversion is shown in Table 3.1. In a similar fashion, bearings can also be transformed into azimuths using the relationships shown in Table 3.1 and geometry from Figure 3.20.

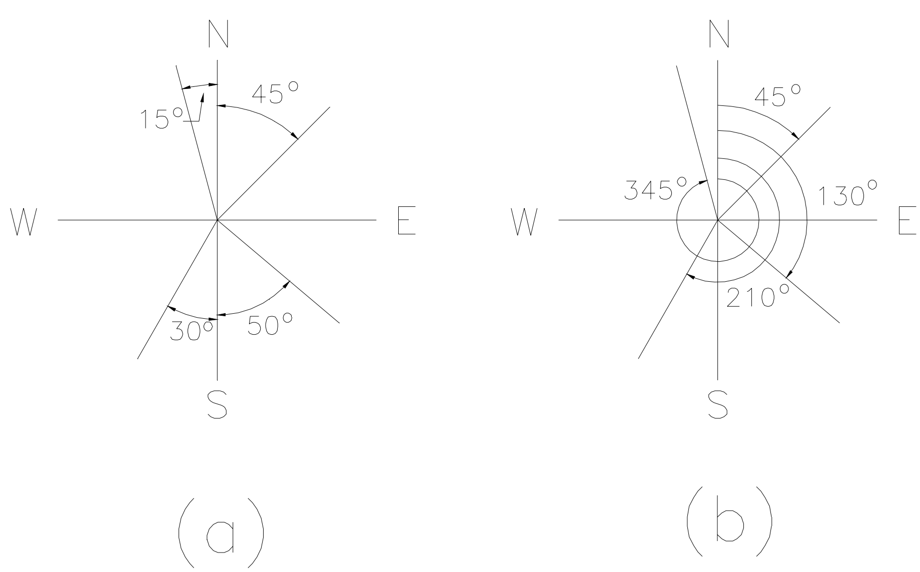
Table 3.1
Relationships between azimuths and bearings.

Quadrant	Azimuth to Bearing Conversion	Bearing to Azimuth Conversion
NE	$\beta = \alpha$	$\alpha = \beta$
SE	$\beta = 180° - \alpha$	$\alpha = 180° - \beta$
SW	$\beta = \alpha - 180°$	$\alpha = 180° + \beta$
NW	$\beta = 360° - \alpha$	$\alpha = 360° - \beta$

3.3.3. Measuring Horizontal Angles by Repetition

Accurate measurements of angles require that redundancy be incorporated into the measurement. This means repeating the measurement of angles to reduce the possibility of the occurrence of blunders. Horizontal angles are measured by repetition by accumulating the circle reading on the horizontal circle of the theodolite/transit. The instrument sights on the backsight and a value of 0° is input into the horizontal circle. Then the instrument is turned, by unlocking the upper motion, to the foresight and the angle is read. By releasing only the lower clamp, the instrument can be pointed back at the backsight with the initial angle value still on the horizontal circle. Sight on the

Figure 3.20
Relationship between bearings and azimuths



backsight again and then release the upper motion and center on the foresight again. The angle on the horizontal circle should represent twice the measured angle. This process is continued for the desired number of repetitions. The final angle is then divided by the number of times the angle was measured. It is also important to use double centering when measuring the angles to account for any instrumental problems and biases. Double centering is the process of inverting the telescope tube such that half of the observations are taken in the direct or normal position (sometimes called face left) and half in the reverse or inverted position (also called face right).

Directional theodolites do not have an upper motion to lock the horizontal circle to the alidade when measuring angles. In this case the observations are broken down into sets. Each set consists of a backsight and foresight(s) measured in both the direct and reverse positions. Table 3.2 shows an example with 4 sets using one foresight, although multiple foresights can be incorporated into the measurement scheme. Station A is the backsight and station C is the foresight.

To reduce these notes, the direct and reverse readings are averaged as

$$Mean = \frac{D + R - 180^o}{2}$$

If the reverse reading is less than the direct reading, add 360° to the reverse reading before taking the mean. The reduced mean is found by forcing the backsight reading to 0°. When doing this, it is also necessary to reduce the foresight(s) by the exact same amount for each set. The angle is the reduced mean reading. Then simply take the arithmetic mean of the set angles to obtain the mean angle. In this case the mean is 56° 17' 20"

Different instruments may have slightly different reduction forms. It is also important, as shown in Table 3.2, that the horizontal circle be advanced by some value during the observation. The desire is to use different parts of the horizontal circle during the measurement process. To determine how much to advance the circle, use the ratio 180°/n, where n is the number of sets. It is also important to advance the micrometer scale as well between sets.

3.3.4. Measuring Vertical Angles by Repetition

Unlike horizontal angles, vertical angles (this also applies to zenith angles) cannot be accumulated on the vertical circle. Instead, the vertical angles should be read the required number of times in both the direct and reverse directions. Then the average of the measured values can be used for subsequent processing of the survey.

3.4. Collimation (C-factor)

Collimation exists when the line of sight is not truly horizontal when the instrument is accurately leveled. This error can be eliminated by keeping the foresight and backsight distance equal since the collimation error is linear and a function of the distance from the instrument.

Table 3.2
Sample reduction of directional theodolite notes.

SET	STA	D/R	READING	MEAN	REDUCED MEAN	SET ANGLE
1	A	D	0° 00' 15"	0° 00' 18"	0° 00' 00"	
		R	180° 00' 21"			
	B	D	56° 17' 32"	56° 17' 35"	56° 17' 17"	56° 17' 17"
		R	236° 17' 38"			
2	A	D	45° 02' 26"	45° 02' 28"	0° 00' 00"	
		R	225° 02' 30"			
	B	D	101° 19' 51"	101° 19' 50"	56° 17' 22"	56° 17' 22"
		R	281° 19' 49"			
3	A	D	90° 04' 42"	90° 04' 53"	0° 00' 00"	
		R	270° 05' 04"			
	B	D	146° 22' 13"	146° 22' 11"	56° 17' 18"	56° 17' 18"
		R	326° 22' 09"			
4	A	D	135° 07' 58"	135° 08' 01"	0° 00' 00"	
		R	315° 08' 04"			
	B	D	191° 25' 22"	191° 25' 25"	56° 17 24"'	56° 17 24"'
		R	371° 25' 28"			

3.4.1. Determination

The determination of the correction and performing the peg test to obtain the C-factor for the level are described in **Appendix D**.

3.4.2. Application

The C-factor is applied to elevation differences in elevations between two stations based on the difference in the accumulated distances between the backsights and foresights. The corrected difference in height is shown as:

$$\Delta H_{correct} = \Delta H_{observed} + C\left(\sum BS_{dist} - \sum FS_{dist}\right)$$

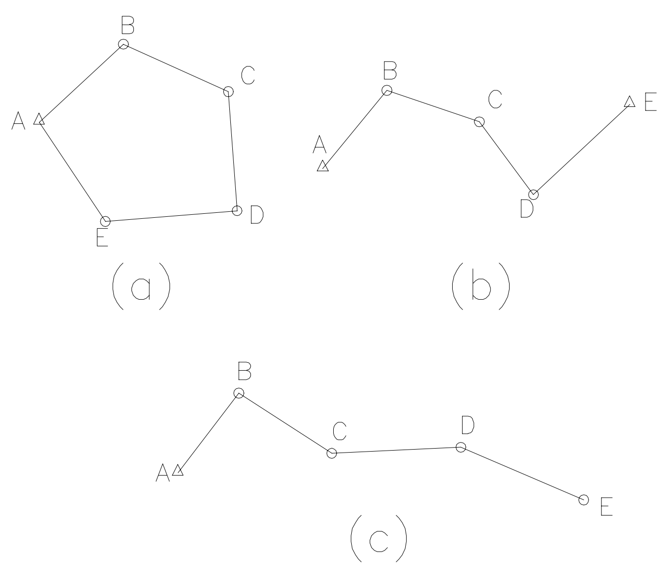
If the distances between the backsights and foresights were equal, then the effects of the collimation error are not applicable.

3.5. Traverse

A traverse is a connected set of lines with directions and distances (refer to Figure 3.21). A closed traverse is one that starts and ends at a known point or on an assumed point provided that the traverses closes on the original position. An open traverse can begin at a known or unknown point and ends at an unknown point. There are no checks on the computations when an open traverse is executed between points.

Figure 3.21 shows three examples of traverses where the triangle indicates a known control point and the circle delineates an unknown traverse point. In Figure 3.21 (a), the traverse is called a closed-loop

Figure 3.21
Examples of closed and open traverses.



traverse since it terminates at the same station that it started with (point A in this example). Figure 3.21 (b) is also a closed traverse, but not a closed-loop traverse, provided that the control points, A and E in this example, are in the same coordinate system. By inversing between the two control points, the traverse is closed and a check on the field measurements can be made. Finally, Figure 3.21 (c) shows an open traverse because the terminus point, E, is not known. Traverses for MDOT projects must be closed traverses unless prior approval is given by the Design Survey Manager.

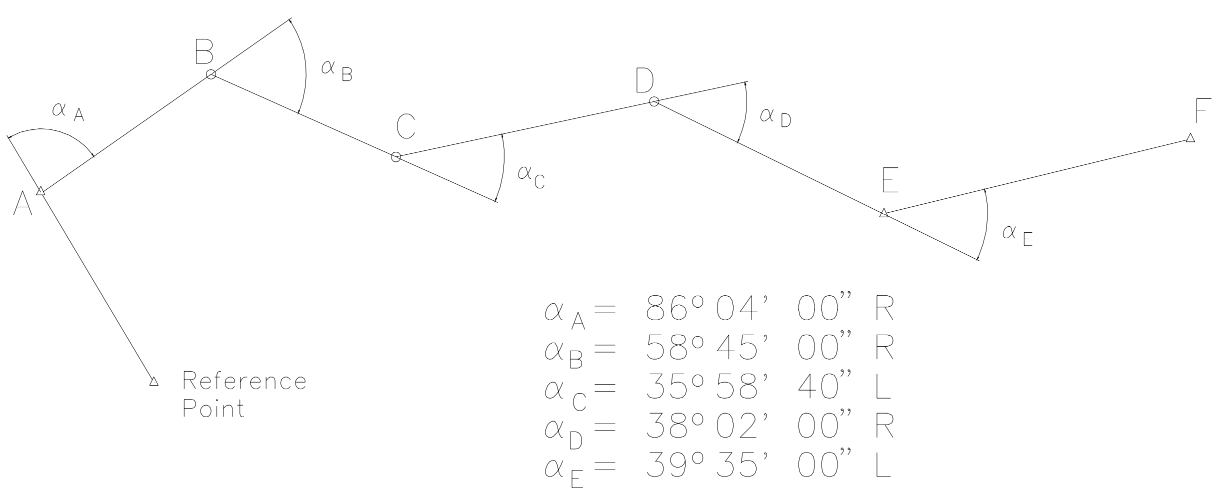
3.5.1. Purpose

The purpose of a traverse is to determine the location of points along a survey with respect to each other. Traverses are performed to locate or establish boundaries and for establishing control for topographic, construction, and photogrammetric control surveys.

3.5.2. Types of Traverse

The types of traverse are dependent upon the method of angle measurement. For example, a deflection angle traverse is one whose angle are all deflection angles (Figure 3.22). The main advantages of the deflection angle traverse is that the computation of azimuths is very easy. Moreover, deflection angles are used in circular curves which has advantages in route surveys. The disadvantage is that the angle can never be greater than 180°. Therefore, it is essential to note whether the angle is to the right or left. For a closed deflection angle traverse, one can perform a check on the angles to see if they meet the specifications for the project. The general form is given as:

Figure 3.22
Example of a deflection angle traverse.



$$Az_{origin} + \sum_{i=1}^{n} \angle_R - \sum_{i=1}^{n} \angle_L - Az_{close} - 360^o = 0$$

where the azimuth at the origin is the forward azimuth and the azimuth at the close is the azimuth of the closing line.

In the traverse in Figure 3.22, assume that the azimuth of the line from A to the Reference Point is 148° 50' 30" and the azimuth of the closing line E to F is 76° 08' 20". Then, the angular error can be found as:

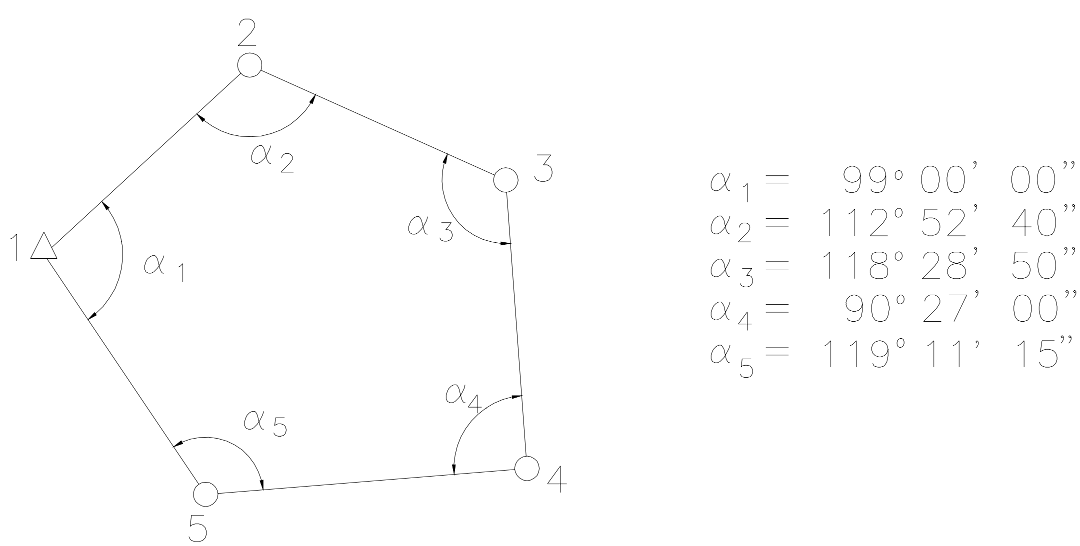
328°50'30" + (86°04'00" + 58°45'00" + 38°02'00") - (35°58'40" + 39°35'00") - 76°08'20" - 360° = -30"

Thus, there is a 30" error in the angles. The corrected angles can be found by dividing this number by the number of angles (5) and then subtracting this error per angle from each of the angles. Treat the angles to the left as negative numbers.

An interior angle traverse is one where all of the internal angles in the polygon have been measured (Figure 3.23). The geometric relationship that exists between the angles is that the sum of the interior angles must equal 180° (n - 2) where n is the number of angles measured within the traverse. This can be written as

$$(n - 2)\ 180^o - 360^o = 0$$

Figure 3.23
Example of an interior angle traverse.



In the traverse shown in Figure 3.23, the error is -15". Again, by dividing by 5, the error per angle is -3" which can be subtracted from each of the angles to arrive at the adjusted angular values.

3.5.3. Traverse in Two Dimensions

The conventional approach is to perform the traversing on a two-dimensional, horizontal plane. Horizontal angles and distances along the traverse sides are the only values measured. There is no regard for the vertical component. This is how the examples above have been presented.

3.5.4. Traverse in Three Dimensions

With the onset of GPS, there is a renewed interest in three-dimensional traversing. GPS measures in a 3-D mode yielding X, Y, and Z coordinates (or some derivative). For a conventional traverse to be placed in a three dimensional coordinate framework, the vertical component must be measured. This normally implies that the vertical angle/zenith distance be measured at each point along the traverse. With total stations, this process is easy to perform since a 3-D mode can be programmed into the processor and data recorded automatically within the data recorder.

When traversing in 3-D, most surveyors will employ a local coordinate system. This is shown in Figure 3.24. Unfortunately, GPS measures in an earth-centered, earth-fixed (ECEF) coordinate framework as shown in Figure 3.25. It is essential that the local coordinate system be transformed into the ECEF coordinate system (see **Appendix H**).

Figure 3.24
Local Coordinate System.

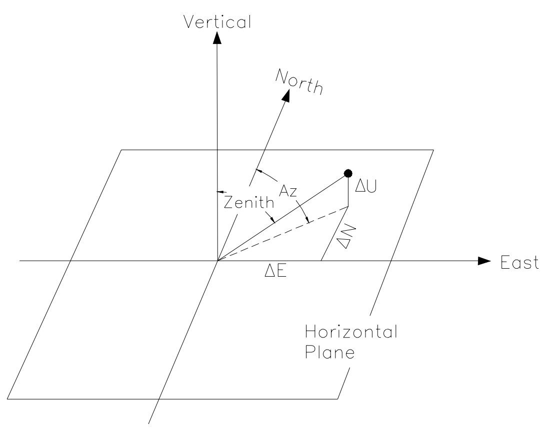
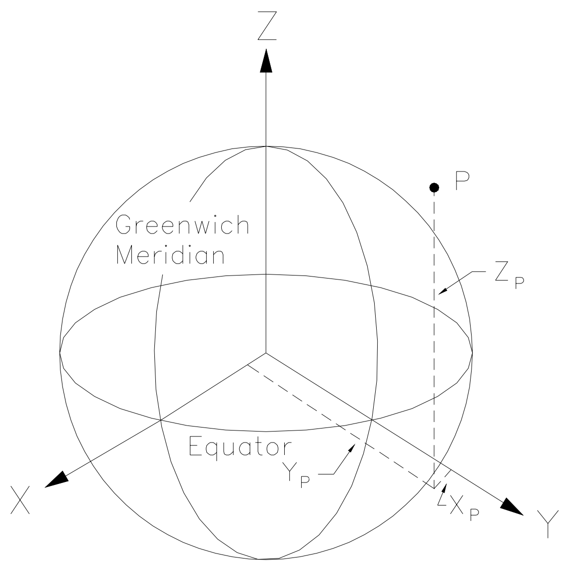


Figure 3.25
Earth-Centered, Earth-Fixed Coordinate System.



3.6. Intersection and Resection

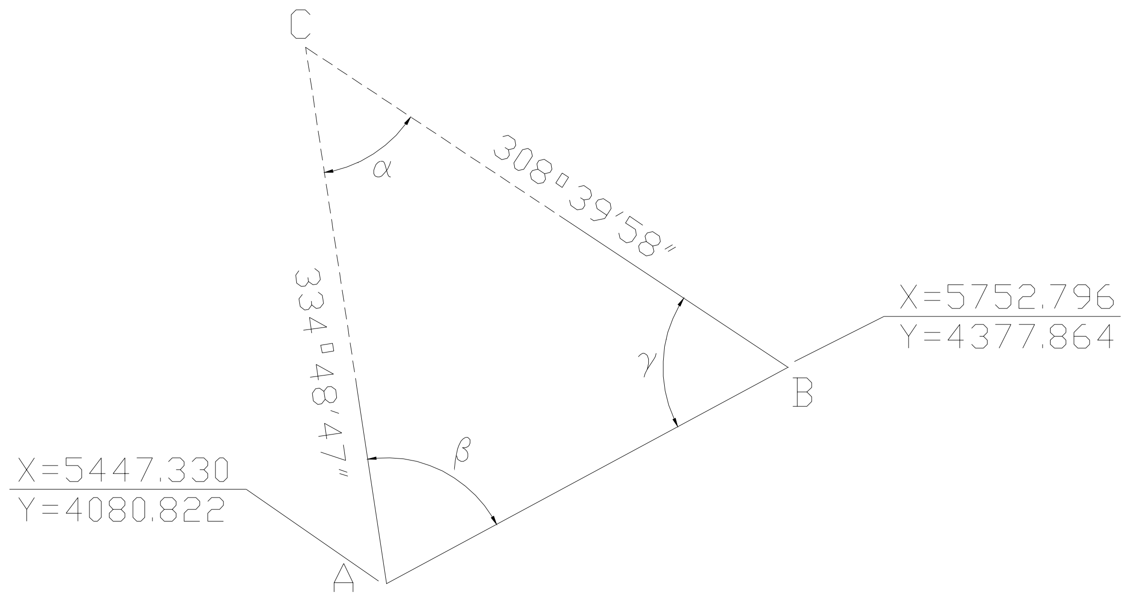
It is not uncommon that a point of interest in a survey is not observed directly. Instead, the location of the point can be found indirectly using intersection and/or resection principles. These locations are referred to as derived locations. Because they are derived coordinates, care must be taken to ensure that the results meet the specifications of the survey. In this case, the geometry is the most important aspect of the coordinate computations. A geometry with "skinny" or small angles will create a weakness in the solution.

3.6.1. Intersection

The simple line-line intersection problem involves the determination of the coordinates of a point without having to compute both the directions and lengths of the lines to the unknown point in the field. The problem breaks down into a simple triangle and as such, there must be at least three elements of that triangle known. One of these elements must be a distance to provide scale for the triangle, The method of solving this problem is one of using simple geometry. For example, Figure 3.26 shows a situation where the coordinates of two points (A and B) and the azimuths from both points to the unknown point are known.

By simply solving for the individual elements of the triangle, the coordinates of point C can be found. First, the angle at C (α) can be found as

Figure 3.26
Example Line-Line Intersection Problem.



$$\alpha = Az_{C-A} - Az_{C-B}$$
$$= 154^o 48' 47'' - 128^o 39' 58''$$
$$= 26^o 08' 49''$$

The azimuth from A to B can be determined using the arctangent function.

$$Az_{A-B} = \tan^{-1}\left[\frac{X_B - X_A}{Y_B - Y_A}\right] = 45^o\ 48'\ 04''$$

The corresponding distance is found using the Pythagorean theorem.

$$D_{A-B} = \sqrt{(X_B - X_C)^2 + (Y_B - Y_C)^2} = 426.079$$

The angle at A can also be found by finding the difference in the azimuths of lines AB and AC.

$$\beta = Az_{A-B} - Az_{A-C} = 70^o\ 59''\ 17''$$

The distance from B to C is found using the sine law.

$$D_{B-C} = \frac{D_{A-B}}{\sin\ \alpha} \sin\ \beta = 914.136$$

Now, the coordinates of C can be computed from point B.

$$X_C = X_B + D_{B-C} \sin\ Az_{B-C} = 5039.038$$

$$Y_C = Y_B + D_{B-C} \cos\ Az_{B-C} = 4948.999$$

As a check, compute the coordinates of C from point A. An alternative method of solving this problem is to use the base-line method. In the example above, the angles β and γ are computed from the azimuths, like shown in the example above. Then, the coordinates of point C can be solved directly as

$$X_C = \frac{(Y_B - Y_A) + X_A \cot\ \gamma + X_B \cot\ \alpha}{\cot\ \alpha + \cot\ \gamma}$$

$$Y_C = \frac{(X_A - X_B) + Y_A \cot\ \gamma + Y_B \cot\ \alpha}{\cot\ \alpha + \cot\ \gamma}$$

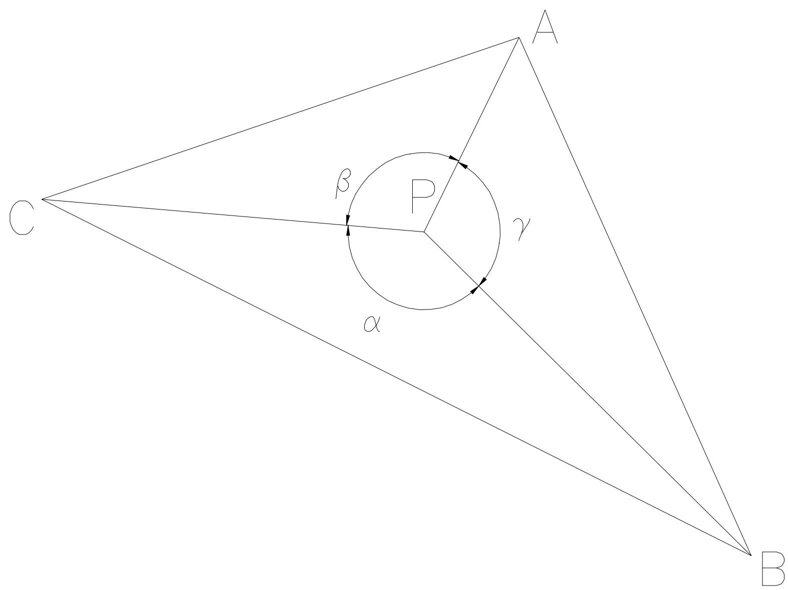
3.6.2. Resection

The three point resection problem has been conventionally defined as occupying an unknown point and measuring the angles between three, or more, stations whose coordinates are known. With the common use of total stations, it is more reasonable to expect that the situation today will also involve the measurement of distances as well as the angles. In this case, the situation is similar in approach to that of the line-line intersection problem.

The classical approach to resection is shown in Figure 3.27. The point P is occupied and the angles α, β, and γ are observed between the control points A, B, and C. There are numerous approaches to solving this problem. The easiest is called the Tienstra method.

The solution is to first compute the angles at each of the control points ($\angle_A$, $\angle_B$, and $\angle_C$). Then,

Figure 3.27
Three-Point Resection Problem.



$$\frac{1}{K_1} = \cot \angle_A - \cot \alpha$$

$$\frac{1}{K_2} = \cot \angle_B - \cot \beta$$

$$\frac{1}{K_1} = \cot \angle_C - \cot \gamma$$

The coordinates of point P are then computed from the relationships:

$$X_P = \frac{K_1 X_A + K_2 X_B + K_3 X_C}{K_1 + K_2 + K_3}$$

$$Y_P = \frac{K_1 Y_A + K_2 Y_B + K_3 Y_C}{K_1 + K_2 + K_3}$$

Geometry is critical for the solution of the three-point resection problem. If the unknown point P lies on a circle defined by the three known control points then the solution is indeterminate or not uniquely possible. In addition, there is no solution when all the points lie on a straight line or nearly straight line.