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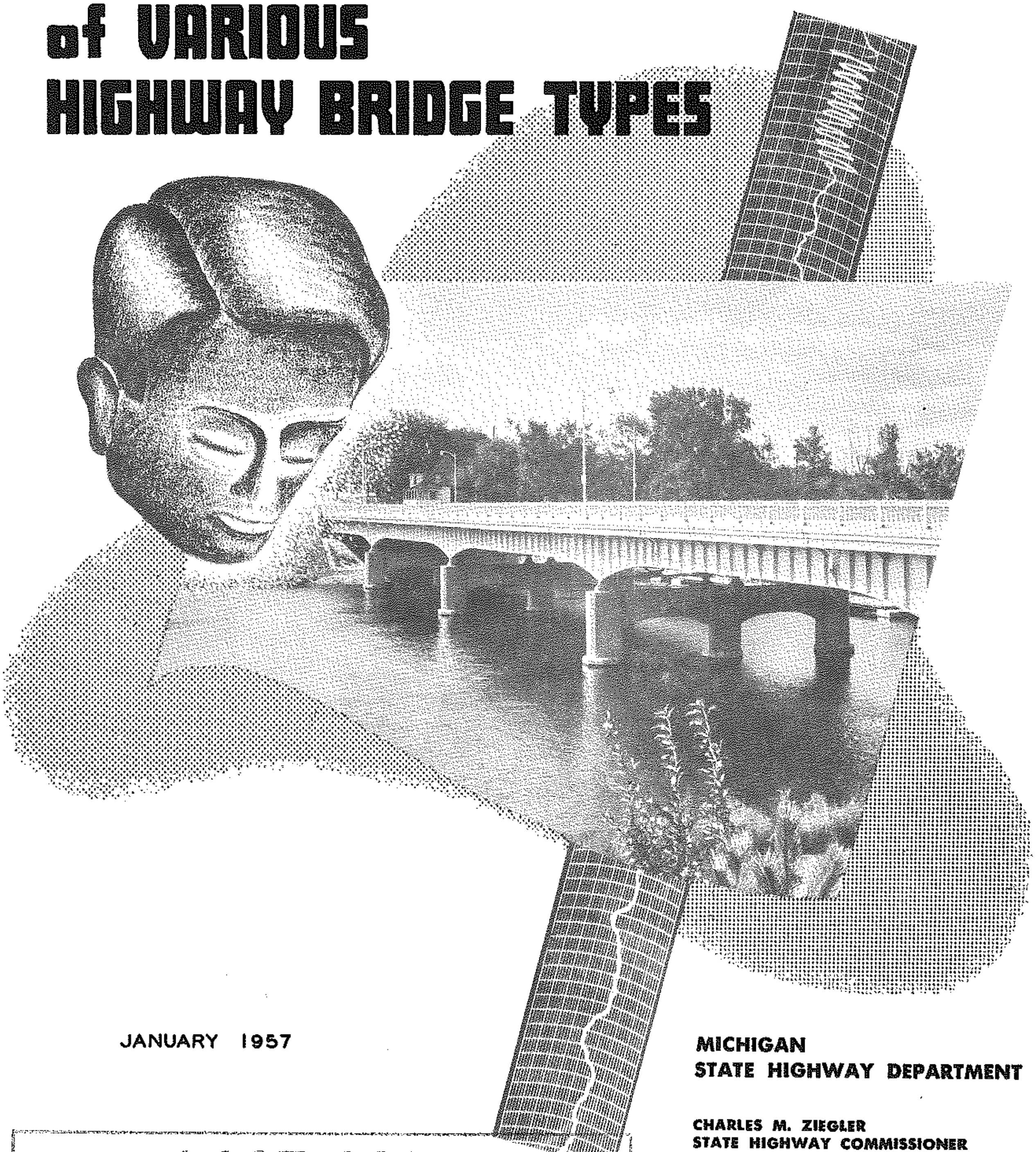
VIBRATION SUSCEPTIBILITIES of VARIOUS HIGHWAY BRIDGE TYPES



JANUARY 1957

MICHIGAN
STATE HIGHWAY DEPARTMENT

CHARLES M. ZIEGLER
STATE HIGHWAY COMMISSIONER

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VIBRATION SUSCEPTIBILITIES OF VARIOUS HIGHWAY BRIDGE TYPES

by

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Progress Report

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SYNOPSIS

This paper reports the results of deflection and vibration measurements on thirty-four spans of fifteen bridges of three types: simple-span, continuous-span, and cantilever-type. Simple-span and cantilever-type bridges included those designed both with and without composite action between the concrete deck and steel beams. The continuous-span type included both steel and reinforced concrete bridges. The same test truck with a constant load was used for testing all bridges. Similarly, the testing procedure was as uniform as possible, considering different methods of fastening deflectometers and variation in bridge site conditions.

The vibration data from these bridges indicate that the cantilever-type structure is much more susceptible to larger amplitudes of vibration than are the other types. Comparison of bridge spans designed with and without composite action indicates that the latter are much more conservatively designed, when actual performance is compared with design values. From these tests, it appears that impact, as measured by increased dynamic deflection over static deflection, is related more directly to riding quality of the roadway surface than to other factors. This report suggests methods of analysis for computing the fundamental frequency of vibration for all bridge types. These methods give reasonable agreement with the experimental values.

CONTENTS

INTRODUCTION

This research survey of the deflection and of vibratory tendencies of several types of highway bridges was made to determine the type of highway structure most readily susceptible to excessive vibration, and possibly, the range in amplitude of vibration for various types under similar loading conditions. From the outset, the study was planned as an extensive rather than an intensive one. Taking cognizance of limitations in time, equipment, and personnel, it appeared advisable to spend only a short time testing each bridge, thus testing as many bridges and bridge types as possible. With a background of vibration data on a significant number of bridges of various types, it seemed likely that improved methods for controlling or limiting vibration, might become apparent.

Previous bridge studies, conducted by the Michigan State Highway Department in 1950, dealt primarily with the lateral distribution of stress and deflection to the individual beams under design loadings for the Fennville Bridge, a six span, rolled beam, concrete deck bridge (1)*. Additional studies in 1952 and 1953 were conducted on the vibration and deflection of this bridge and of the Jackson Bypass Bridge, an eight span, plate girder bridge, consisting of five simple spans and three spans of continuous beam design (2). These studies were carried out to contribute to the research program of the ASCE Committee on Deflection Limitations of Bridges, and of the Committee on Bridges of the Highway Research Board, of which George M. Foster, Chief Deputy Commissioner, is a member. This bridge study was a project of the Research Laboratory, directed by E. A. Finney. The Laboratory is a part of the Testing and Research Division, under the direction of W. W. McLaughlin.

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*Numbers in parentheses, unless otherwise identified, refer to the Bibliography at the end of this paper.

RESEARCH OBJECTIVES

The immediate objectives of the test program were:

1. To obtain the natural frequency of vibration for several bridges of each type, including,

   (a) Simple-span bridges with concrete deck, designed with and without composite action between beams and concrete deck.

   (b) Continuous-span bridges of the rolled beam, plate girder, and reinforced concrete types.

   (c) Cantilever-type structures of rolled beam and plate girder types with concrete deck, designed with and without composite action.

2. To find the variation in amplitude of vibration for these bridge types, and their susceptibility to vibration.

3. To compare the flexibility of the various bridge types and to correlate this with susceptibility to vibration.

4. To determine effective axle load fluctuation as the test vehicle passed over the structure.

5. To compare the computed natural frequency of vibration for each type of structure, with the experimentally determined value.

TEST BRIDGES AND THEIR INSTRUMENTATION

Bridges were selected for testing on the basis of the following criteria: (1) bridges which might be flexible, built in the last ten years, (2) bridges representing as many structural types as possible, (3) bridges over ground or shallow water, rather than deep water, to simplify installation of instrumentation, (4) bridges close to Lansing, other factors being equal.

In summary, the tested bridges included:

1. Eleven simple-spans of four rolled beam bridges with concrete deck: nine spans with and two without spiral shear developers.

2. Four continuous-span structures: two of reinforced concrete, one rolled beam, and one plate girder.

3. Nine cantilever-type structures: six rolled beam with concrete deck-- five with and one without spiral shear developers, three plate girder structures-- one without floor beams and two with floor beams and stringers.

The locations of the fifteen bridges, of which thirty-four spans were tested, are shown on a map of Michigan in Figure 1. Physical limitations necessitated selecting only certain spans of these fifteen bridges for testing. Electrical lead wires longer than 150 ft. made it impossible to balance out the capacitance in the electrical bridge circuit. Therefore on symmetrical three span structures, only two spans were tested. In addition, testing of certain spans above deep water made instrumentation so difficult, that it appeared inadvisable to attempt testing these spans. Also, on multiple span structures with identical span lengths, only a few spans of each bridge were tested.

The pattern of instrumentation was kept as similar as possible for each bridge, considering site conditions at various locations. The work program was carried out so that installation of instrumentation on the bridge, testing, and subsequent removal of instrumentation, could be conducted in a single day. The bridge deflectometers for obtaining vibration and deflection data were the same ones used in the 1952-53 study (2) and are shown in Figure 2. Bridge movement could be observed visually by reading the dial gage, but a permanent record was also obtained by means of a wire resistance strain gage fastened to an aluminum cantilever beam, which was deflected by movement of the top end of the dial gage stem. Bending in the aluminum cantilever beam was accompanied by

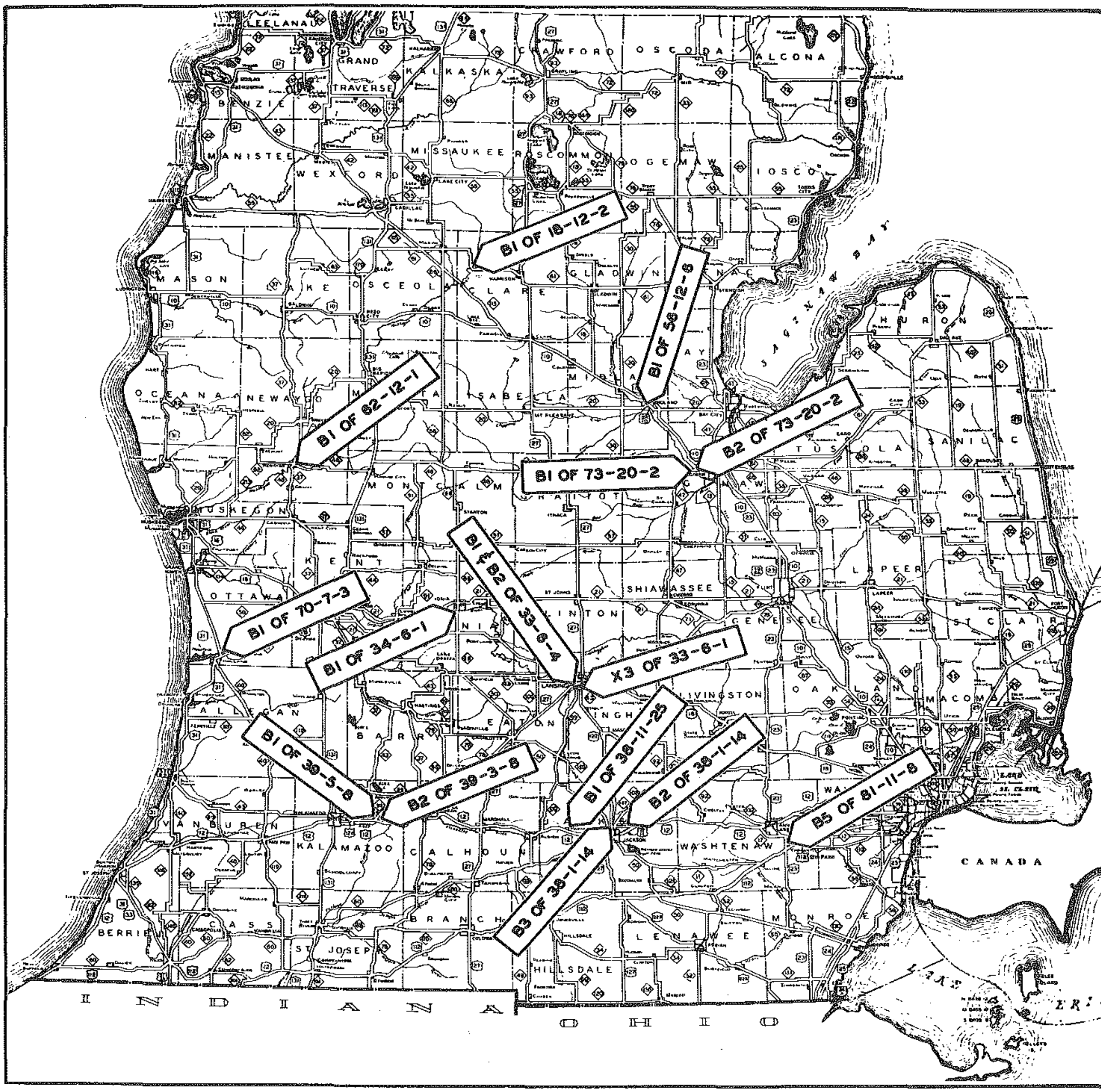


Figure 1. Location of bridges tested

change in electrical resistance of the bonded strain gage, which caused an unbalance in the electrical bridge circuit, and resulted in deflection of a light trace on a photosensitive paper strip in a Hathaway 12-channel recording oscillograph. The recording equipment is shown in Figure 3, mounted in the Instrumentation Truck. Calibration of trace deflection with bridge movement was obtained prior to each testing program.

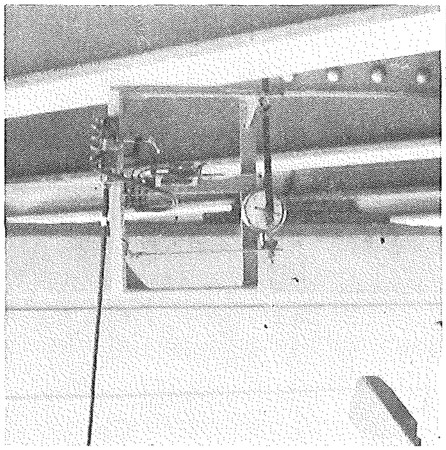
Deflectometers were mounted at mid-span for all spans tested. The lateral position of the deflectometer depended on possible methods of fastening the deflectometer to the bridge, and on the proposed path for the test vehicle. On bridges without a separate superstructure for each roadway, the test vehicle passed along the longitudinal centerline wherever possible, and the deflectometer was placed directly beneath the path of the test vehicle, and on the lower flange of rolled beam and plate girder bridges. For bridges with a separate superstructure for each roadway and a split safety median between roadways, the deflectometer was placed on the concrete median strip and the path of the test vehicle was as close as possible to this median strip.

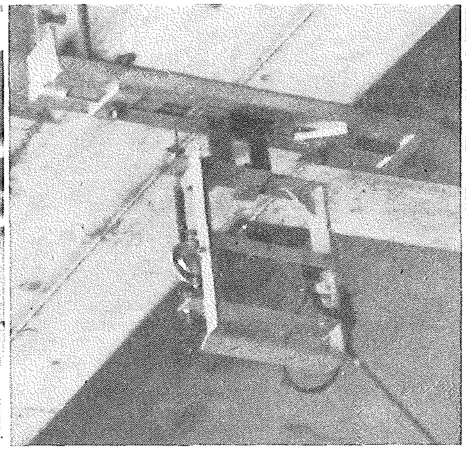
The speed of the truck and its position on the test span were recorded by pneumatic traffic cables which caused a pip on an inactive oscillograph trace as a wheel passed over the cable.

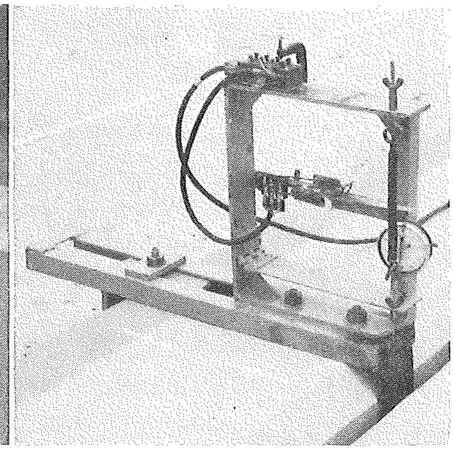
TEST TRUCK AND ITS INSTRUMENTATION

In order to obtain a comparison between bridges, the same test vehicle with identical axle loads was used for all testing (Figure 4). Prior to loading, bonded strain gages of the A-1 type were placed on the axle between the inner wheels and the springs or frame mounting on both load axles. During the loading process, the load on each pair of dual wheels and the corresponding strain reading on the axle were recorded by means of a static strain indicator. During the testing these strain gages were connected to a Brush two-channel recording oscillograph in order to note the variation in effective wheel load as the truck approached and passed over the bridge. The instrumentation panel for recording dynamic wheel load variations is shown in Figure 5.

Although the second axle of the test truck had a conventional suspension system, the third or trailer axle was not sprung, but was connected directly to the frame. A preliminary study conducted at various truck speeds showed that the variation in effective wheel load on wheels of a given axle was generally similar in pattern and amplitude. Therefore, strain gages on a given axle were wired in series, and effective axle load variation rather than effective wheel load variation was recorded for each load axle.







Deflectometer fastened to bottom flange of rolled beam bridge.

Deflectometer fastened to reinforced concrete tee-beam bridge.

Deflectometer fastened to median strip for bridges with separate superstructures for each roadway.

Figure 2. Methods of Fastening Deflectometers

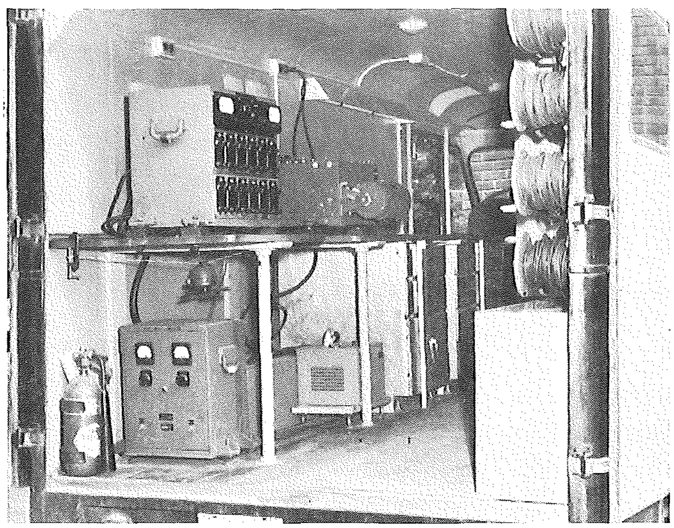


Figure 3. Hathaway 12-channel recording oscillograph in Instrumentation Truck.

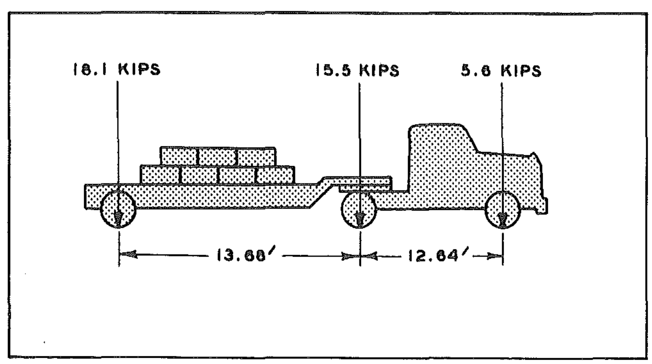


Figure 4. Test vehicle and its loading.

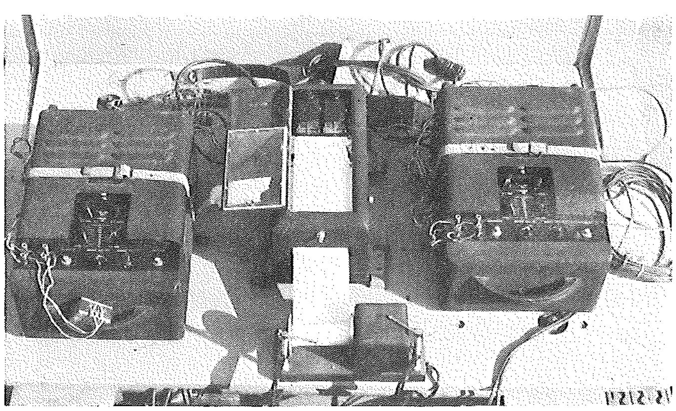


Figure 5. Instrumentation panel for determining wheel load variation on test vehicle.

The test program for each bridge was conducted in as uniform a manner as possible taking cognizance of variations in bridges, traffic, and site conditions. All runs were along a prescribed path with a variation in truck speed from creep to a maximum truck speed of 45 mph. in 5 mph. increments. Three runs were made at each truck speed, but the higher speeds could be obtained only at favorable site locations. For several bridges the maximum speed was limited to 20 or 25 mph. which no doubt reduced the magnitude of the vibration. It was noted however, that those bridges which vibrated more readily, showed a tendency toward vibration even at low speeds.

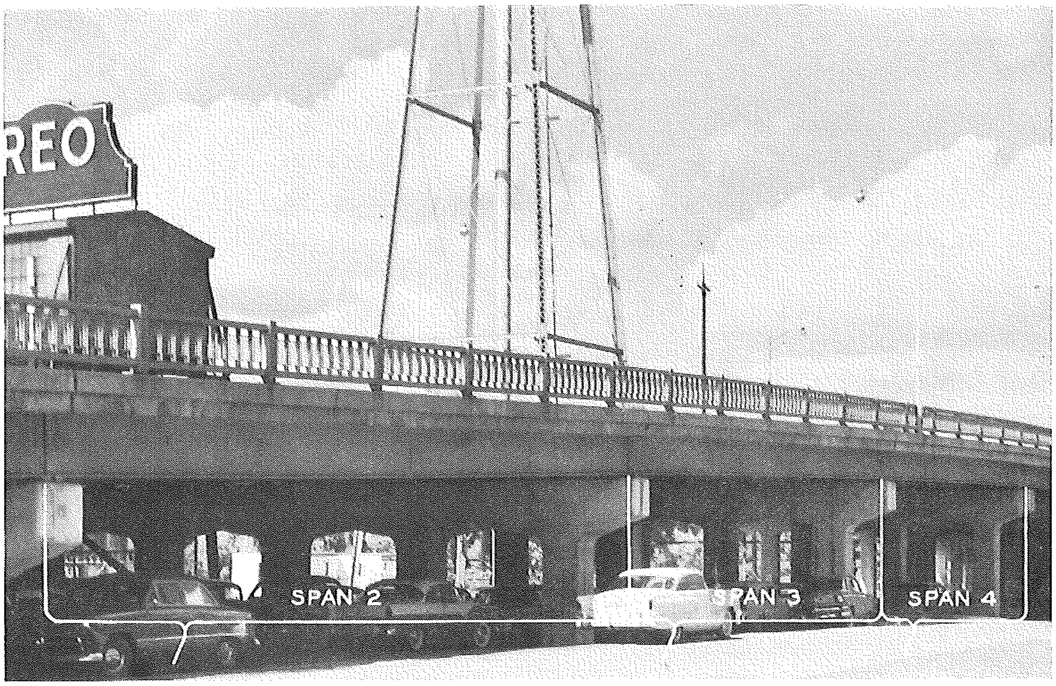
TEST RESULTS

For comparative purposes the results of tests on the thirty-four highway bridge spans will be presented according to bridge types, that is, simple-span, continuous, and cantilever types.

Simple-Span Bridges

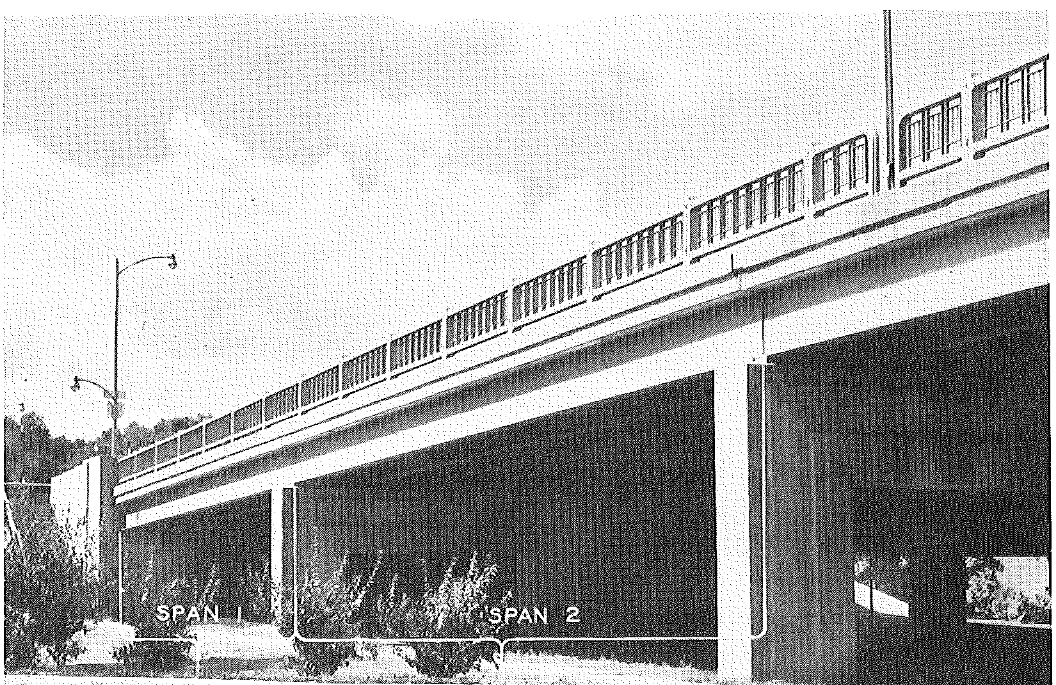
The eleven simple spans tested varied in span length from 44.8 to 64.92 ft. Photographs of these bridges are shown in Figures 6 through 10. They were all constructed with superstructures of wide flange rolled beams with concrete decks. Nine of these spans were designed for composite action between rolled beams and a concrete deck, and spiral shear developers were used to assure the composite action. The two tested spans which had been designed without the benefit of composite action were short spans, the longer being 48.65 ft. Table 1 presents a summary of vibration and deflection data for the simple spans as well as pertinent design information.

Deflection. - Deflections resulting from the test truck passing at "creep speed" over the tested spans, varied from 0.039 to 0.072 in. In order to compare the observed deflection with the theoretical values, the mid-span deflections without impact were calculated for the test truck, in accordance with the AASHO Standard Specifications for Highway Bridges, and on the basis of a ratio of modulus of elasticity of steel to concrete of 10. As shown in Table 1, the ratio of observed to theoretical deflection for the composite spans varied from 0.24 to 0.40, with an average of 0.32. The same averaged ratio for non-composite spans was 0.16. Previous tests (2) indicated comparable ratios of 0.28 and 0.14 for composite and non-composite spans respectively. These ratios verify a generally accepted view that by present specifications, non-composite spans are much more conservatively designed than are composite spans. The two



Simple-span	Simple-span	Simple-span
Span length = 60.87' c.c.	Span length = 60.87' c.c.	Span length = 60.87' c.c.
36 WF 170 lb. beams at 5' 4-3/8" c.c.	36 WF 170 lb. beams at 5' 4-3/8" c.c.	36 WF 170 lb. beams at 5' 4-3/8" c.c.
Min. concrete slab thickness = 7-1/2"	Min. concrete slab thickness = 7-1/2"	Min. concrete slab thickness = 7-1/2"
Composite design	Composite design	Composite design

Figure 6. Bridge X3 of 33-6-1. Cedar Street in Lansing.



Simple-span	Simple-span
Span length = 64.25' c.c.	Span length = 64.92' c.c.
36 WF 170 lb. beams at 5' 0" c.c.	36 WF 170 lb. beams at 5' 0" c.c.
Min. concrete slab thickness = 7"	Min. concrete slab thickness = 7"
Composite design	Composite design

Figure 7. Bridge B1 & B2 of 33-6-4. Main Street in Lansing

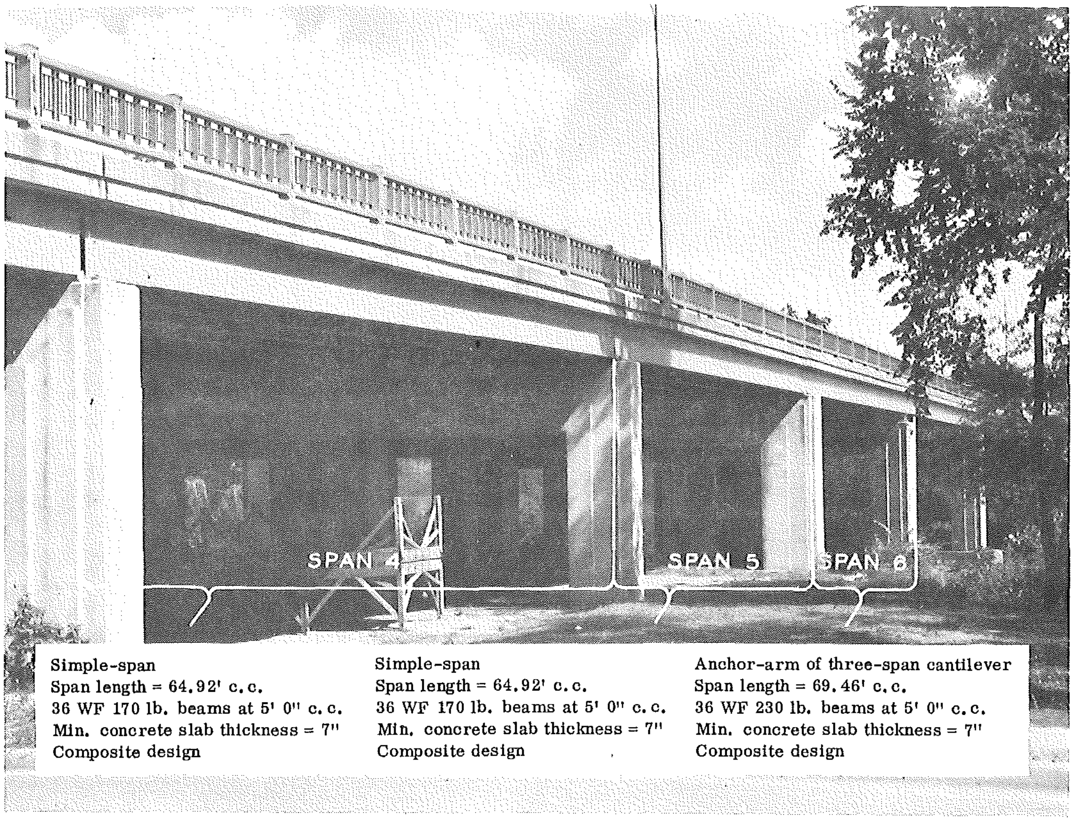
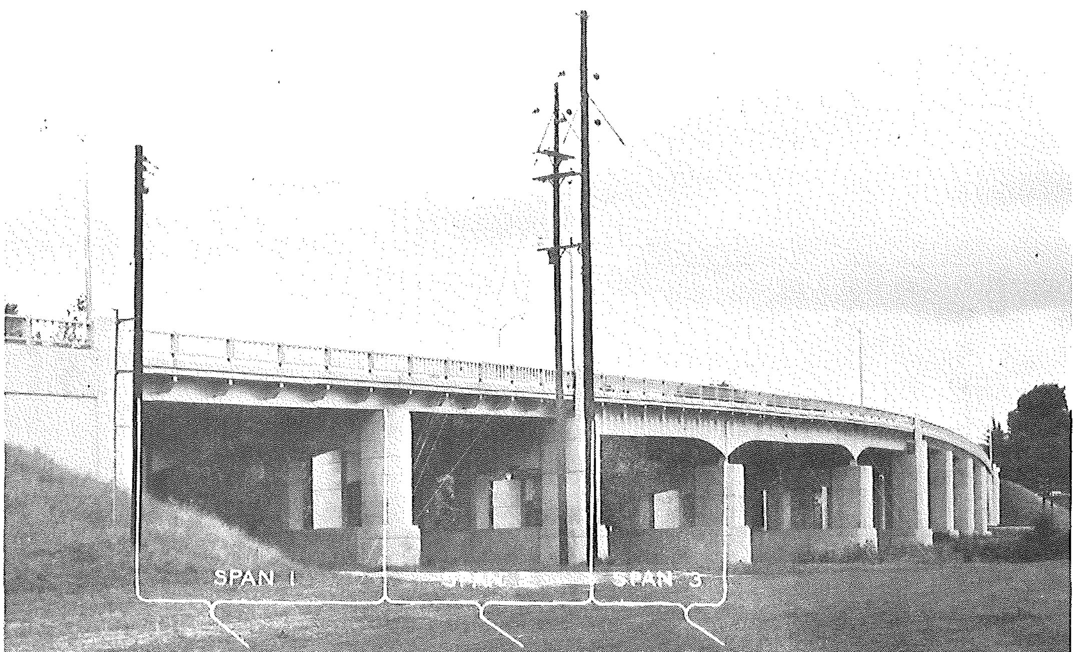


Figure 8. Bridge B1 & B2 of 33-6-4. Main Street in Lansing.



Simple-span	Simple-span	Anchor-arm of three-span cantilever
Span length = 55.30' c.c.	Span length = 55.90' c.c.	Span length = 73.90' c.c.
36 WF 150 lb. beams at 5' 6-1/2" c.c.	36 WF 150 lb. beams at 5' 6-1/2" c.c.	5' 6" min. plate-girder (2 per roadway)
Min. concrete slab thickness = 7"	Min. concrete slab thickness = 7"	Min. concrete slab thickness = 7"
Composite design	Composite design	Non-composite design

Figure 9. Bridge B1 of 56-12-6. On route M-20 in Midland.

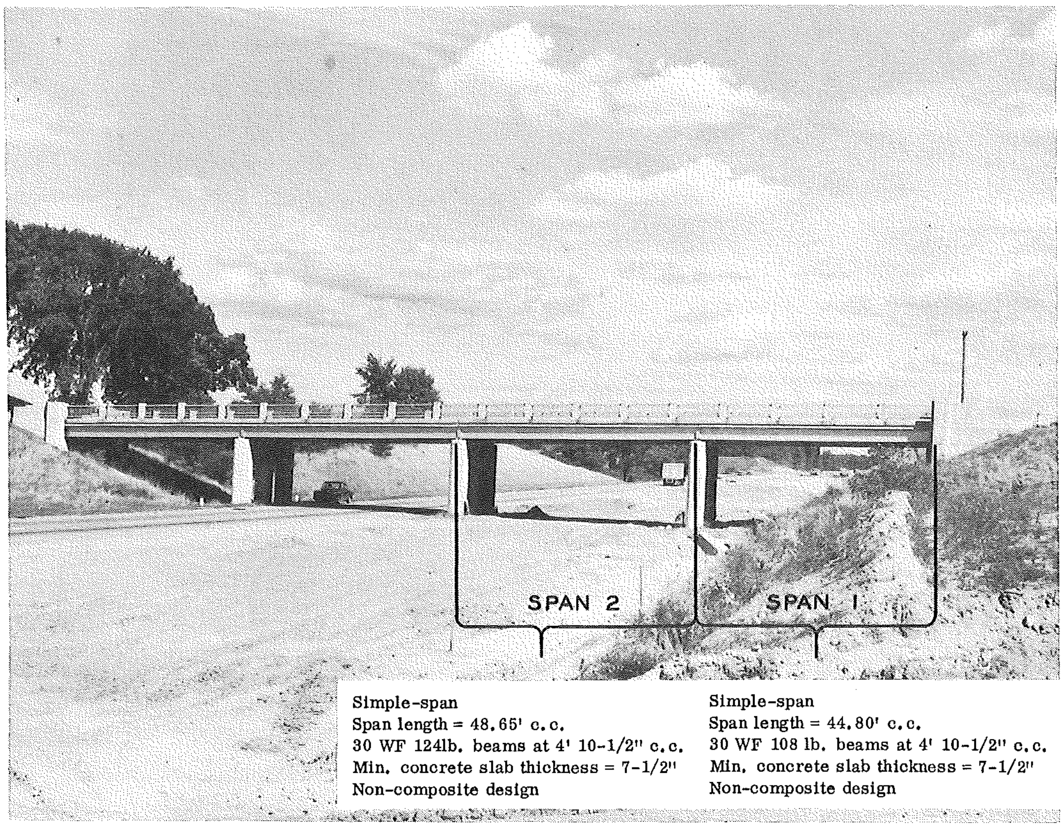


Figure 10. Bridge B2 of 39-3-8. Over route US-12 near Kalamazoo.

non-composite spans tested would have had approximately the same ratio of observed to theoretical deflection as did the composite spans, if the theoretical deflection for these spans were based on a moment of inertia considering 50 percent composite action of slab with steel beam.

Amplitude of Vibration. - The maximum amplitudes of bridge vibration for each span, for the test truck on the span, on other spans, and off the bridge, are shown in Table 1. This data indicates that seven of the eleven spans (those from bridges X3 of 33-6-1 and B1 and B2 of 33-6-4) had a tendency to vibrate at greater amplitudes than the other four spans (those from bridges B1 of 56-12-6 and B2 of 39-3-8). This is most apparent when the amplitudes of free vibration, truck on other spans, or off the bridge, are compared. The seven spans with the larger amplitudes of vibration are all over 60 ft. in span length and were designed for composite action. Of the four remaining spans, two were designed for composite action, but are only short spans of 55.3 and 55.9 ft. The two non-composite spans are also short spans, being only 44.8 and 48.6 ft. in length. There was also a marked difference in the maximum duration of free vibration for the previously-mentioned seven spans (average: 11.6 seconds) as compared to the other four spans (average: 2.1 seconds).

TABLE 1

SUMMARY OF DATA ON SIMPLE-SPAN HIGHWAY BRIDGES

	Data on Spans	Ratio of Depth to Span Length	Design L. L. Plus Impact Defl. in Inches	Deflection Due to Test Truck: Observed Deflection-In. (Creep Speed)	Deflection Due to Test Truck: Theoretical Deflection-In. (No Impact)	Deflection Due to Test Truck: Ratio: Observed / Theoretical	Max. Amplitude of Vibration in Inches: Truck On Span	Max. Amplitude of Vibration in Inches: Truck On Other Spans	Max. Amplitude of Vibration in Inches: Truck Off Bridge	Max. Duration of Vibration in Seconds After Truck was off Span	Fundamental Freq. of Vib. in c.p.s.: Observed	Fundamental Freq. of Vib. in c.p.s.: Computed Theoretical	Fundamental Freq. of Vib. in c.p.s.: % Difference Theoretical to Observed
1.	Bridge No. X3 of 33-6-1 Span 2: Length = 60.87' c.c. 36 WF 170 lb. at 5' 4-3/8" c.c. Min. concrete slab thickness = 7.5" Composite design	1/20.3	------	0.072	0.178	0.40	0.009	0.005	0.005	14.3	6.42	6.26	2.5
2.	Bridge No. X3 of 33-6-1 Span 3: Length = 60.87' c.c. 36 WF 170 lb. at 5' 4-3/8" c.c. Min. concrete slab thickness = 7.5" Composite design	1/20.3	------	0.063	0.178	0.35	0.007	0.005	0.004	13.7	6.34	6.26	1.3
3.	Bridge No. X3 of 33-6-1 Span 4: Length = 60.87' c.c. 36 WF 170 lb. at 5' 4-3/8" c.c. Min. concrete slab thickness = 7.5" Composite design	1/20.3	------	0.054	0.178	0.30	0.007	0.004	0.004	14.3	6.23	6.26	0.5
4.	Bridge No. B1 & B2 of 33-6-4 Span 1: Length = 64.25' c.c. 36 WF 170 lb. at 5' 0" c.c. Min. concrete slab thickness = 7.0" Composite design	1/21.4	approx. 0.67 or 1/1150 of span	0.070	0.211	0.33	0.009	0.005	0.001	6.5	7.77	6.34	(1)
5.	Bridge No. B1 & B2 of 33-6-4 Span 2: Length = 64.92' 36 WF 170 lb. at 5'0" c.c. Min. concrete slab thickness = 7.0" Composite design	1/21.6	0.67 or 1/1150 of span	0.066	0.211	0.31	0.013	0.004	0.002	10.0	6.35	6.24	1.7
6.	Bridge No. B1 & B2 of 33-6-4 Span 4: Length = 64.92' 36 WF 170 lb. at 5' 0" c.c. Min. concrete slab thickness = 7.0" Composite design	1/21.6	0.67 or 1/1150 of span	0.068	0.211	0.32	0.022	0.006	0.004	14.6	6.07	6.24	2.8
7.	Bridge No. B1 & B2 of 33-6-4 Span 5: Length = 64.92' 36 WF 170 lb. at 5' 0" Min. concrete slab thickness = 7.0" Composite design	1/21.6	0.67 or 1/1150 of span	0.062	0.211	0.29	0.012	0.008	0.002	8.2	(2)	6.24	----
8.	Bridge No. B1 of 56-12-6 Span 1: Length = 55.30' c.c. 36 WF 150 lb. at 5' 6-1/2" c.c. Min. concrete slab thickness = 7.0" Composite design	1/18.4	approx. 0.47 or 1/1410 of span	0.039	0.165	0.24	0.005	0.001	------	0	(2)	7.45	------
9.	Bridge No. B1 of 56-12-6 Span 2: Length = 55.90' c.c. 36 WF 150 lb. at 5' 6-1/2" c.c. Min. concrete slab thickness = 7.0" Composite design	1/18.6	0.47 or 1/1410 of span	0.053	0.171	0.31	0.010	0.001	0.001	2.4	7.25	7.31	0.8
10.	Bridge No. B2 of 39-3-8 Span 1: Length = 44.80' c.c. 30 WF 108 lb. at 4' 10-1/2" c.c. Min. concrete slab thickness = 7.5" Non-composite design	1/17.9	------	0.050	0.324	0.15	0.008	0.001	------	2.0	8.00	8.10	1.4
11.	Bridge No. B2 of 39-3-8 Span 2: Length = 48.65' c.c. 30 WF 124 lb. at 4' 10-1/2" c.c. Min. concrete slab thickness = 7.5" Non-composite design	1/19.5	------	0.057	0.360	0.16	0.005	0.001	------	4.0	7.85	7.79	0.8

(1) The percent difference was not calculated, since the computed frequency was based on simply-supported conditions, but on end spans the concrete backwall was poured around the end of the beam giving at least partial restraint. Previous studies have also shown that the end spans are much stiffer due to this partial restraint.

(2) Insufficient samples of free vibration were obtained to determine the experimental natural frequency with any accuracy.

Frequency of Vibration. - The natural frequency of vibration as observed was obtained by averaging the frequency of free vibration on runs where the vibration was uniform and sustained. The spans may again be placed in two groups with respect to the natural frequency of vibration: six in the group with lower values, and five in the higher value group. All six in the group with natural frequencies less than 6.5 cps. were also subject to higher amplitudes of vibration. In the group with the natural frequency of vibration more than 7.0 cps., were the four spans which demonstrated lower amplitudes of vibration, and one which vibrated at greater amplitudes. A partial explanation for this exception may be the unusual roughness on this bridge deck, which caused a much greater variation in effective axle loads than customary. This unusual variation in effective axle loads of the test truck may have contributed to increased amplitude of vibration for this span.

The theoretical fundamental frequency of vibration was calculated for all simple spans by a method suggested in a previous report (2). An effective superstructure cross-section was selected, composed of one or two steel beams directly beneath the path of the test truck, and of the accompanying portion of concrete deck slab above the one or two steel beams. For spans designed for composite action the entire concrete slab in the selected portion of the superstructure cross-section was considered with the steel beam in computing the effective moment of inertia. However, for non-composite spans only 50 percent of the concrete slab was considered effective. The ratio of modulus of elasticity of steel to concrete was taken as 6. The fundamental frequency of vibration was then computed on the basis of a simple beam with a uniform load.

With the exception of Span 1 of Bridge B1 & B2 of 33-6-4, the largest difference between experimental and calculated frequency of vibration was 2.8 percent, and the average error was 1.5 percent. The large difference between experimental and calculated value for Span 1 of B1 & B2 of 33-6-4 is due to the fact that the calculated value was for a simply supported beam, while this span was an end span with one end of the steel beams encased in the concrete backwall above the abutment, resulting in considerable restraint at this support. This stiffening effect was previously noted in testing a six span rolled beam bridge where all spans had the same geometric design and a nominal length of 60 ft. One of the interior spans was of composite design, but all others were non-composite. The end spans, with one end of the steel beams encased in the concrete backwall, demonstrated a higher natural frequency of vibration than all other spans, slightly higher than even the composite span.

Continuous-Span Bridges

Seven spans from four continuous-span bridges were tested. One bridge consisted of three spans and was constructed of rolled beams with concrete deck, another was a three span plate girder with concrete deck, and the remaining two were three and four span reinforced concrete tee-beam bridges. None of the steel bridges were designed for composite action. The continuous-span bridges are shown in Figures 11 through 14.

Deflection. - The observed deflections for the seven continuous-spans tested varied from 0.014 in. for the end span of a reinforced concrete bridge, to 0.072 in. for the center span of the rolled beam bridge (Table 2). The maximum deflection of 0.072 in. for the continuous-spans matches the same value for the maximum deflection for the simple-spans. The ratio of observed to theoretical deflection for the steel spans varied from 0.17 to 0.35 with an average of 0.24. The reinforced concrete tee-beam bridges, as might be expected, had much smaller deflection than the steel bridges, with an average of 45 percent less deflection for end spans and 63 percent less deflection for center spans.

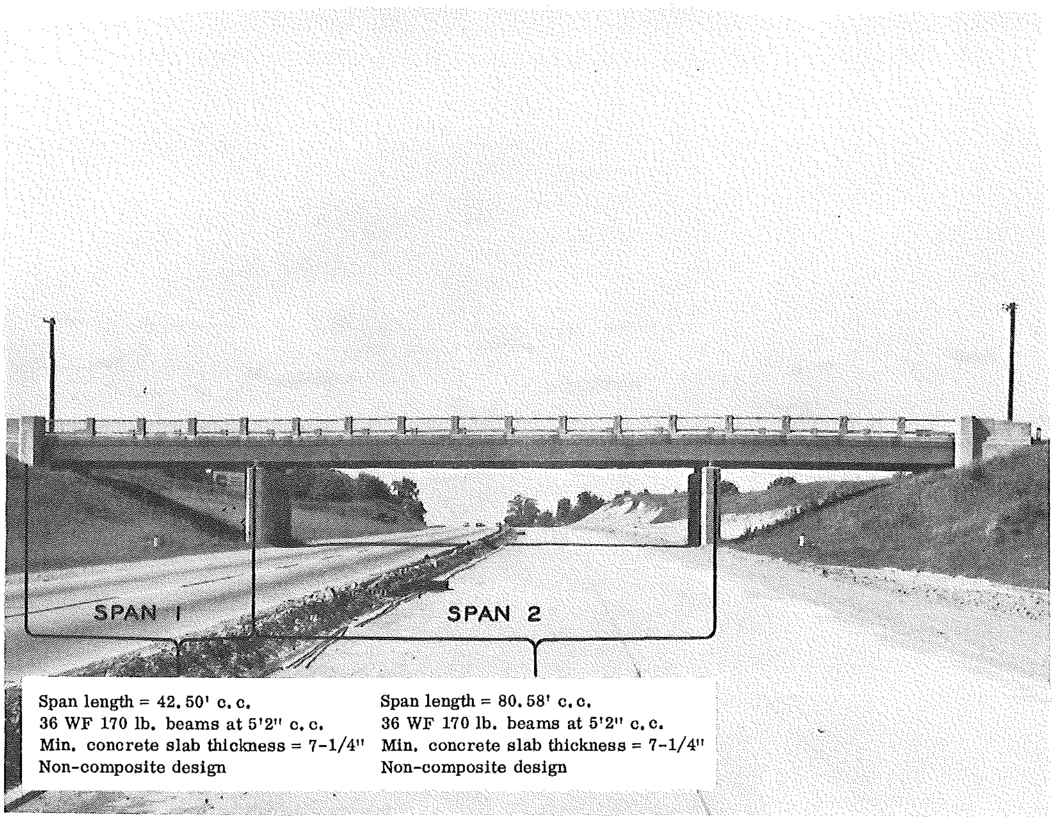


Figure 11. Bridge B2 of 38-1-14. Over US-12 east of Jackson. Three-span continuous, rolled beam bridge.

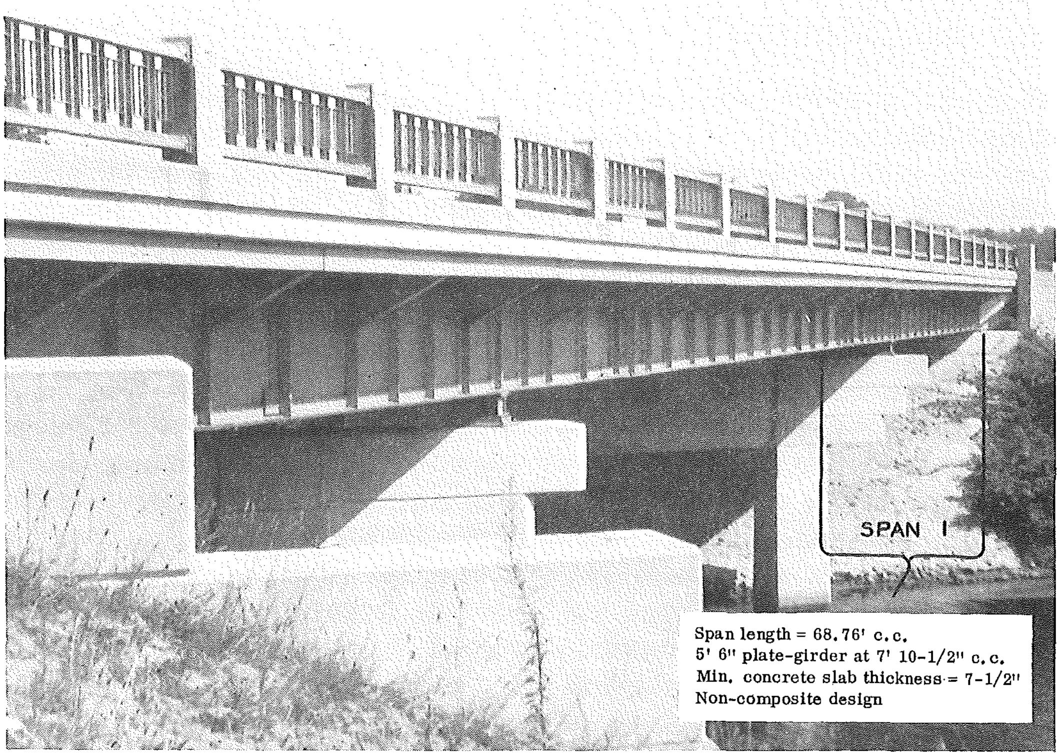


Figure 12. Bridge B1 of 70-7-3. On route US-31 north of Holland. Three-span continuous plate girder bridge.

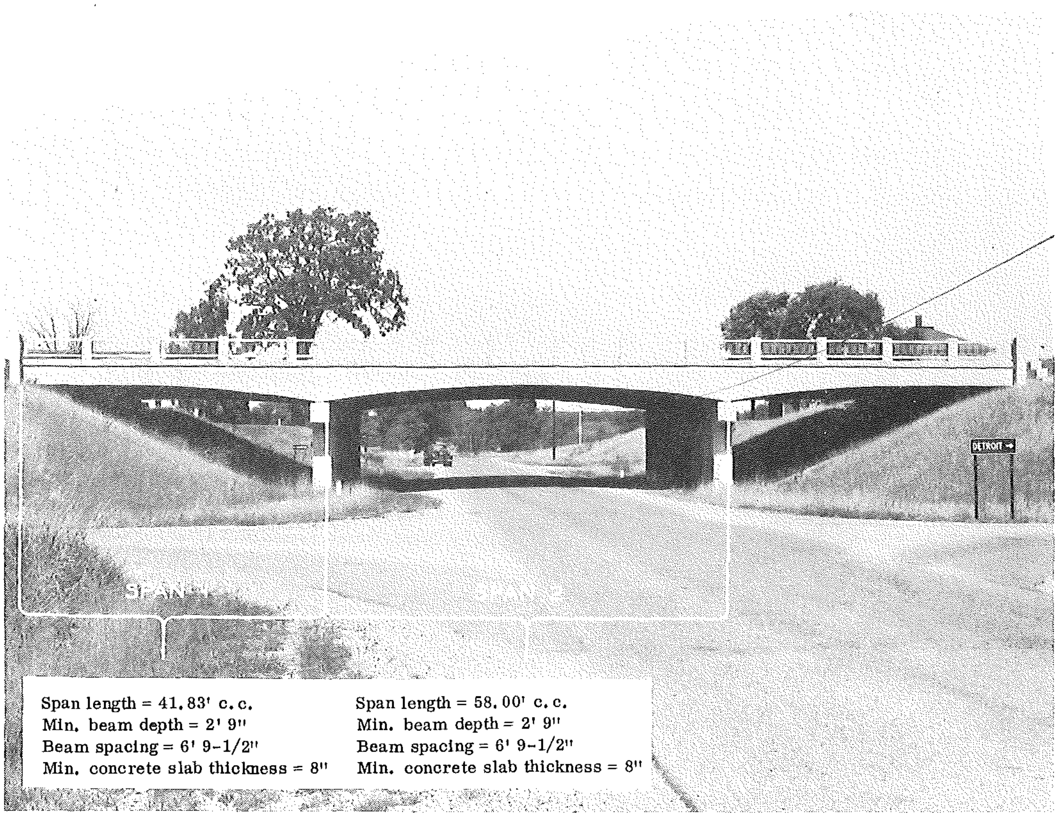


Figure 13. Bridge B1 of 38-11-25. On US-12 near Parma. Reinforced concrete tee-beam bridge.

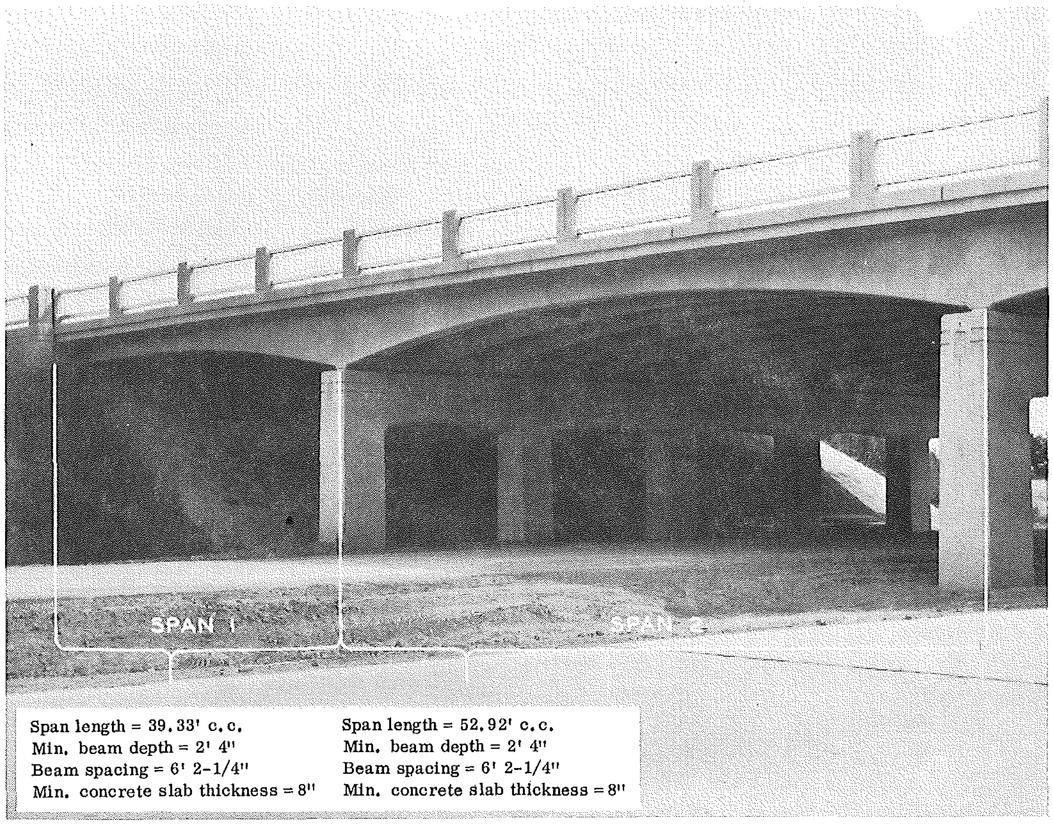


Figure 14. Bridge B5 of 81-11-8. On route US-23 south of Ann Arbor. Four-span reinforced concrete tee-beam bridge.

Amplitude of Vibration. - The reinforced concrete bridges vibrated with smaller amplitudes than the steel bridges. For the continuous-spans, the maximum amplitude of vibration for truck on span, truck on other spans, and truck off the bridge, was 0.010, 0.005, and 0.003 in. respectively. Each maximum was obtained on Span 2 of the rolled beam bridge. Unfortunately it was not possible to instrument the center span of the plate girder bridge due to site difficulties, but a comparison of results from the end spans of rolled beam and plate girder bridges indicates that the center span of the plate girder bridge would probably have vibrated with somewhat greater amplitudes than the center span of the rolled beam bridge. For the bridges tested and reported here, the simple-span bridges of composite design vibrated in general with larger amplitudes of vibration than did the non-composite continuous-span bridges.

The maximum duration of vibration after the test truck passed off the span is also given in Table 2. The averaged maximum duration of vibration for the steel spans was 12.3 seconds as compared to 7.2 seconds for the reinforced concrete bridges.

TABLE 2

SUMMARY OF DATA ON CONTINUOUS-SPAN HIGHWAY BRIDGES

Data on Spans	Ratio of Depth to Span Length[1]	Design L. L. Plus Impact Defl. in Inches	Deflection Due to Test Truck: Observed Deflection-In. (Creep Speed)	Deflection Due to Test Truck: Theoretical Deflection-In. (No Impact)	Deflection Due to Test Truck: Ratio: Observed / Theoretical	Max. Amplitude of Vibration in Inches: Truck On Span	Max. Amplitude of Vibration in Inches: Truck On Other Spans	Max. Amplitude of Vibration in Inches: Truck Off Bridge	Max. Duration of Vibration in Seconds After Truck was off Span	Fundamental Freq. of Vib. in c.p.s.: Observed	Fundamental Freq. of Vib. in c.p.s.: Computed Theoretical	Fundamental Freq. of Vib. in c.p.s.: % Difference Theoretical to Observed
1. Bridge No. B2 of 38-1-14 3-span continuous, rolled-beam bridge Span 1: Length = 42.50' c. c. 36 WF 170 lb. at 5' 2" c. c. Min. concrete slab thickness = 7.25" Non-composite design	approx. 1/10	(2)	0.027	0.077	0.35	0.003	0.002	0.001	10.4	5.23	5.28	1.0
2. Bridge No. B2 of 38-1-14 3-span continuous, rolled-beam bridge Span 2: Length = 80.58' c. c. 36 WF 170 lb. at 5' 2" Min. concrete slab thickness = 7.25" Non-composite design	approx. 1/16	1.05 or 1/922 of span	0.072	0.426	0.17	0.010	0.005	0.003	13.4	5.27	5.28	0.2
3. Bridge No. B1 of 70-7-3 3-span continuous, plate-girder bridge Span 1: Length = 68.76' c. c. 5'6" plate girder at 7'10-1/2" c. c. Min. concrete slab thickness = 7.50" Non-composite design	approx. 1/11	(2)	0.027	0.142	0.19	0.005	0.004	0.002	13.2	5.13	5.25	2.3
4. Bridge No. B1 of 38-11-25 Reinforced concrete, haunched tee-beam, continuous 3-span structure Span 1: Length = 41.83' c. c. Min. beam depth = 2'9" Beams at 6' 9-1/2" c. c. Min. concrete slab thickness = 8"	approx. 1/11	(2)	0.014	(2)	---	0.003	0.002	0.001	4.9	8.08	7.95	1.6
5. Bridge No. B1 of 38-11-25 Reinforced concrete, haunched tee-beam, continuous 3-span structure Span 2: Length = 58.00' c. c. Min. beam depth = 2'9" Beams at 6' 9-1/2" c. c. Min. concrete slab thickness = 8"	approx. 1/13	(2)	0.027	(2)	---	0.004	0.003	0.002	4.1	8.11	7.95	2.0
6. Bridge No. B5 of 81-11-8 Reinforced concrete, haunched tee-beam, continuous 4-span structure Span 1: Length = 39.33' c. c. Min. beam depth = 2'4" Beam at 6' 2-1/4" c. c. Min. concrete slab thickness = 8"	approx. 1/12	(2)	0.016	(2)	---	0.003	0.002	0.001	11.6	7.16	7.10	0.8
7. Bridge No. B5 of 81-11-8 Reinforced concrete, haunched tee-beam, continuous 4-span structure Span 2: Length = 52.92' c. c. Min. beam depth = 2'4" Beams at 6' 2-1/4" c. c. Min. concrete slab thickness = 8"	approx. 1/14	(2)	0.026	(2)	---	0.005	0.002	0.002	8.2	7.30	7.10	2.8

1 According to AASHO Specification the span length for continuous spans shall be considered as the distance between dead load points of contraflexure.

2 No data on this item was available.

Frequency of Vibration. - The range in the observed fundamental frequency of vibration for the continuous-span bridges was from 5.13 to 8.11 cps. with the steel bridges having an average of 5.21 cps. and the reinforced concrete bridges an average of 7.66 cps.

Since the continuous-span steel bridges were uniform in cross-section the simplest means of computing the natural frequency of vibration appeared to be the numerical method presented by A.S. Veletsos and N. W. Newmark(3). By this method the natural frequencies may be calculated for undamped flexural vibration of continuous beams on rigid supports and for rigid jointed plane frameworks without sidesway. However, this method as presented was restricted to span members having a uniform cross-section and mass per unit length. The additional complications of applying this method to the reinforced concrete bridges with variations of weight and moment of inertia throughout the span length, suggested that a different approach was desirable.

For the reinforced concrete bridges, the fundamental frequency of vibration was calculated by the method of Influence Coefficients.

> "For free vibrations, the system vibrating at one of its principal modes with frequency 'ω' is loaded with inertia forces '$-m_i x_i = m_i \omega^2 x_i$' of each mass, where '$x_i$' is the deflection of the mass 'm_i' at position 'i'. The equations for the deflection can hence be written as
>
> $$x_1 = a_{11}(m_1\omega^2 x_1) + a_{12}(m_2\omega^2 x_2) + a_{13}(m_3\omega^2 x_3) + \ldots$$
>
> $$x_2 = a_{21}(m_1\omega^2 x_1) + a_{22}(m_2\omega^2 x_2) + a_{23}(m_3\omega^2 x_3) + \ldots$$
>
> $$x_3 = a_{31}(m_1\omega^2 x_1) + a_{32}(m_2\omega^2 x_2) + a_{33}(m\,\omega^2 x_3) + \ldots$$
>
> .
>
> .
>
> .

. . . 'a_{ii}' being the deflection at 'i' due to a unit load at 'i', . . . and similarly 'a_{ij}' defined as the deflection at 'i' due to a unit load at 'j'" (4).

The deflection coefficients for the continuous beam may be calculated by any suitable means.

The previous equation can be modified to the following by matrix notation:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \omega^2 \begin{bmatrix} a_{11}m_1 & a_{12}m_2 & a_{13}m_3 \cdots \\ a_{21}m_1 & a_{22}m_2 & a_{23}m_3 \cdots \\ a_{31}m_1 & a_{32}m_2 & a_{33}m_3 \cdots \\ \cdot & & \\ \cdot & & \\ \cdot & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

The iteration procedure was used assuming a set of deflections x_1, x_2, x_3, etc., for the right column in the last equation and performing the proper operation. The resulting column was then normalized, by reducing one of the amplitudes to unity by dividing each term of the column by the particular amplitude. The procedure was repeated with the normalized column until the amplitudes stabilized to a definite pattern, and then the fundamental frequency was found directly.

In the computations, the inertia forces were lumped at three points for each span. The moment of inertia of the gross tee-beam cross-section was used and the modulus of elasticity of the concrete was assumed to be 5×10^6 psi.

The maximum difference between the computed fundamental frequency of vibration and the experimental value for the continuous-span bridges was 2.8 percent, while the average difference was 1.5 percent.

Cantilever-Type Bridges

Sixteen spans from nine cantilever-type bridges were tested. Six of the nine bridges were constructed with rolled beams and concrete deck, and five of these were designed for composite action between slab and beam. Anchor arm span lengths for these six bridges varied from 50 to 75 ft., while the suspended span lengths ranged from 47 to 69 ft. Two of the bridges were plate girder spans constructed with floor beams and stringers, and the remaining bridge was a combination of plate girder and rolled beam with anchor arm spans being plate girder and suspended spans rolled beam. Photographs of these bridges are shown in Figures 8, 9, and 15 through 21.

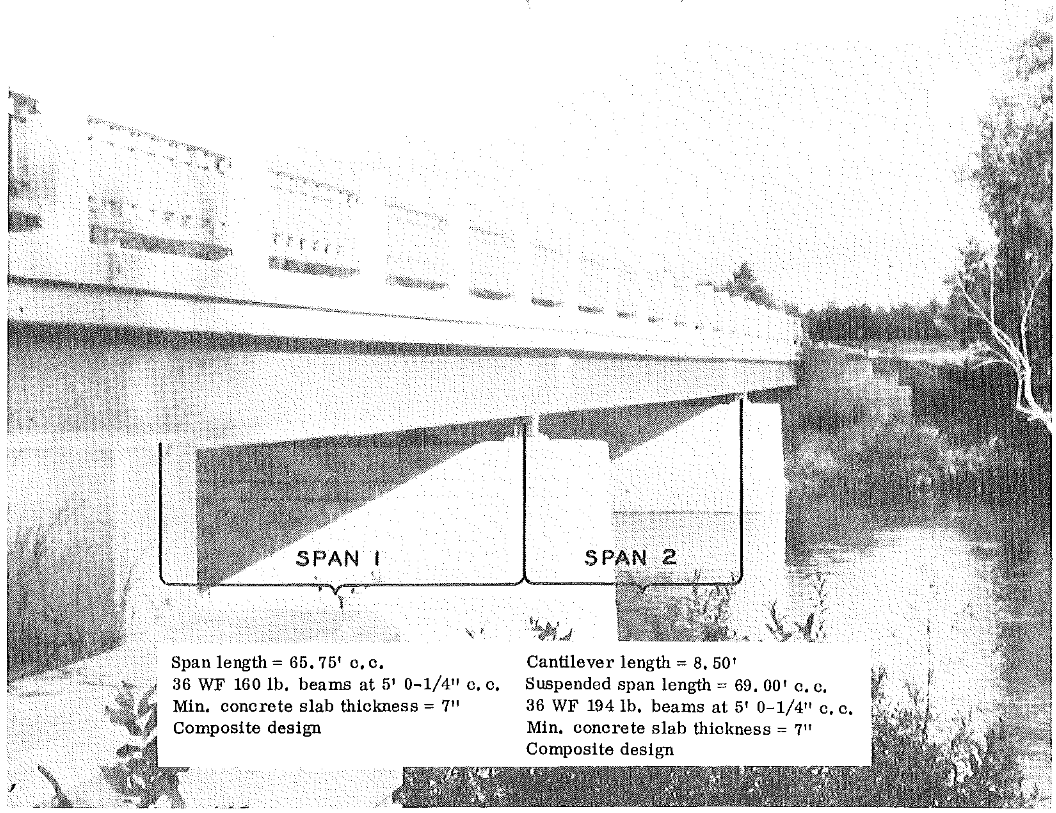


Figure 15. Bridge B1 of 18-12-2. On route M-61 near Temple. Rolled beam cantilever-type bridge.

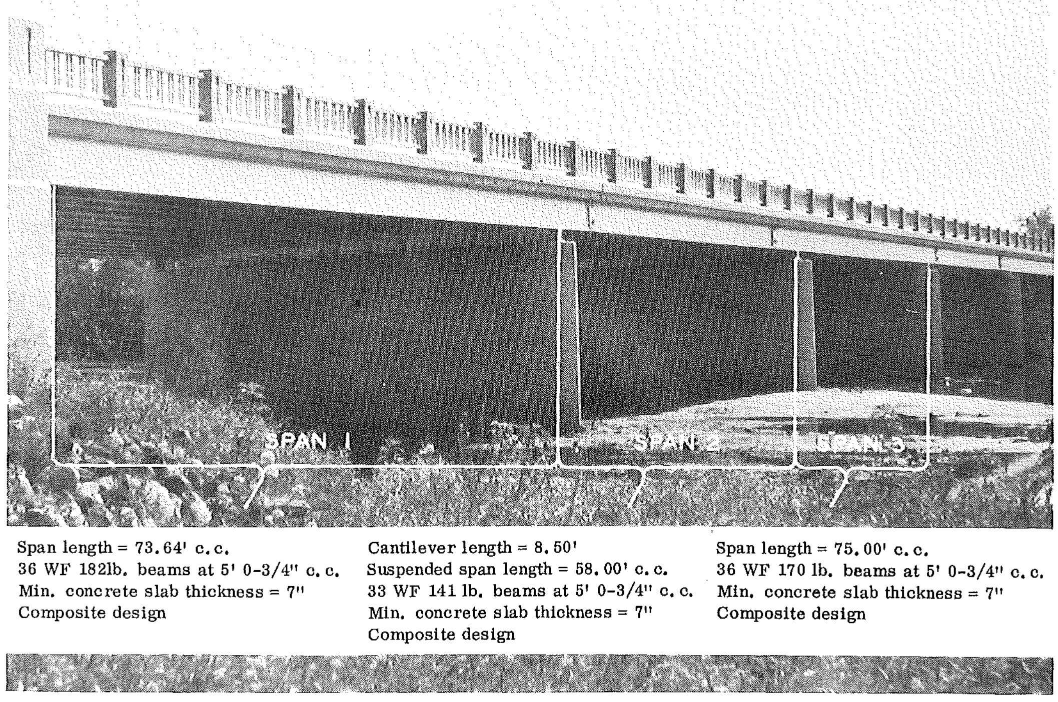


Figure 16. Bridge B1 of 73-20-2. On routes M-46 and M-47 west of Saginaw. Five-span rolled beam cantilever-type bridge.

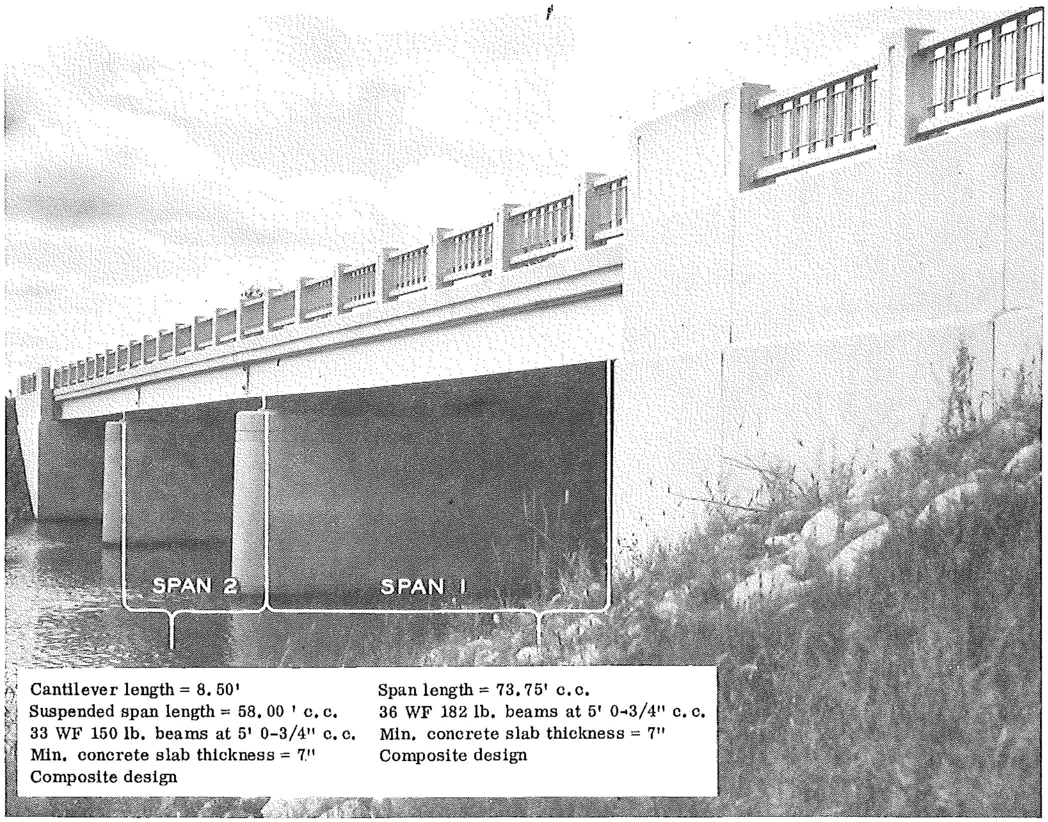


Figure 17. Bridge B2 of 73-20-2. On routes M-46 and M-47 west of Saginaw. Rolled beam cantilever-type bridge.

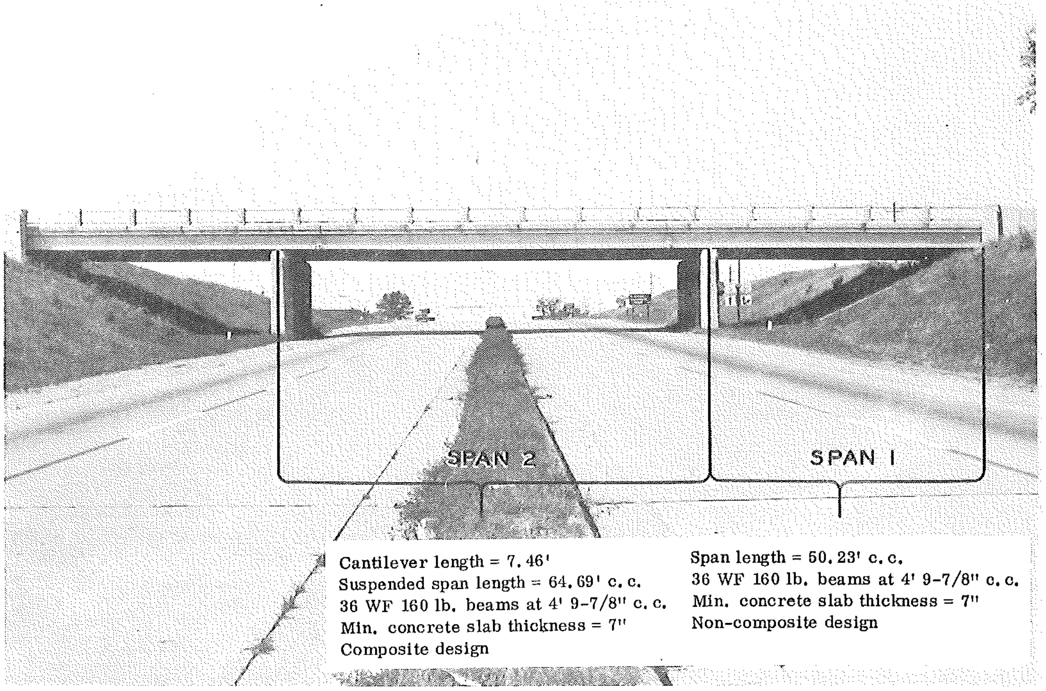


Figure 18. Bridge B3 of 38-1-14. On route US-127 over US-12 north of Jackson. Rolled beam cantilever-type bridge.

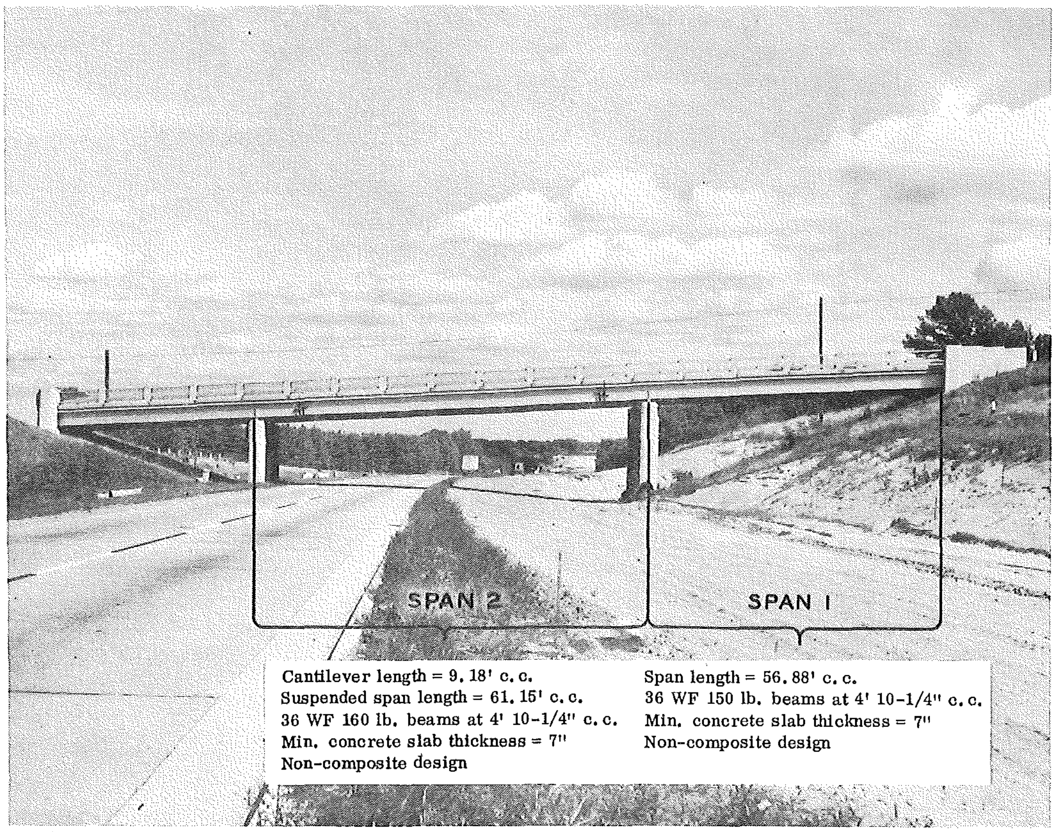


Figure 19. Bridge B1 of 39-5-8. Over route US-12 near Kalamazoo. Rolled beam cantilever-type bridge.

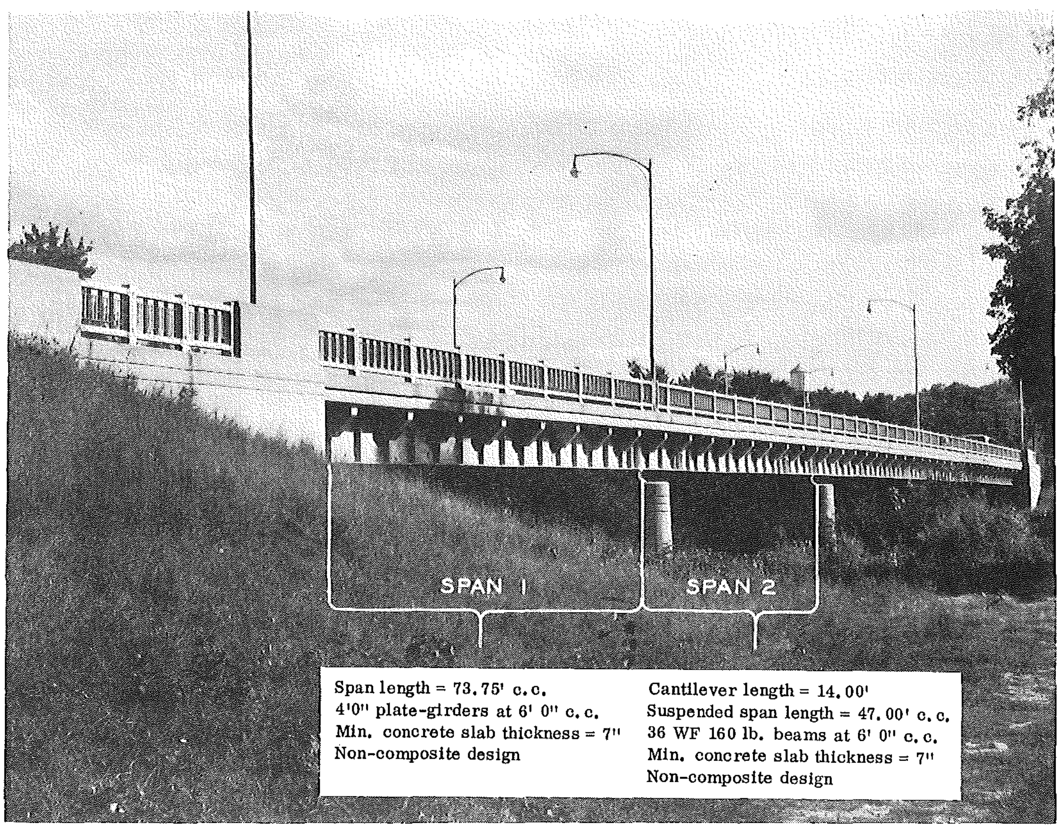


Figure 20. Bridge B1 of 34-6-1. On route M-66 south of Ionia. Five-span plate girder and rolled beam cantilever-type bridge.

Figure 21. Bridge B1 of 62-12-1. On routes M-37 and M-82 in Newaygo. Five-span plate girder cantilever-type bridge with floor beams and stringers.

<u>Deflection.</u> - The observed deflections for the sixteen spans varied from 0.049 to 0.201 in. It should be noted that this maximum of 0.201 in. is much higher than the 0.072 in. maximum which occurred on simple and continuous-span bridges. A comparison of the ratio of observed to theoretical deflection for anchor arm spans with composite design varied from 0.23 to 0.60, with an average of 0.43. For suspended spans designed for composite action this same ratio varied from 0.27 to 0.64, with an average of 0.45. In contrast, the average ratio for non-composite anchor arm spans was 0.20, and for non-composite suspended spans, 0.24.

As similarly shown for simple-spans, the non-composite spans were designed much more conservatively for deflection than the composite spans.

<u>Amplitude of Vibration.</u> - The maximum amplitudes of vibration for truck on span, truck on other spans, and truck off the bridge, were 0.037, 0.028, and 0.012 in. respectively, for the cantilever-type bridges (Table 3). Eight of the nine cantilever-type bridges appeared susceptible to larger amplitudes of vibration. These amplitudes were generally much larger than for the other types tested. The larger amplitudes of vibration

TABLE 3

SUMMARY OF DATA ON CANTILEVER-TYPE HIGHWAY BRIDGES

	Data on Spans	Ratio of Depth to Span Length	Design L. L. Plus Impact Defl. in Inches	Deflection Due to Test Truck: Observed Deflection-In. (Creep Speed)	Deflection Due to Test Truck: Theoretical Deflection-In. (No Impact)	Deflection Due to Test Truck: Ratio: Observed / Theoretical	Max. Amplitude of Vibration in Inches: Truck On Span	Max. Amplitude of Vibration in Inches: Truck On Other Spans	Max. Amplitude of Vibration in Inches: Truck Off Bridge	Max. Duration of Vibration in Seconds After Truck was off Span	Fundamental Freq. of Vib. in c.p.s.: Observed	Fundamental Freq. of Vib. in c.p.s.: Computed Theoretical	Fundamental Freq. of Vib. in c.p.s.: % Difference Theoretical to Observed
1.	Bridge No. B1 of 18-12-2 3-span, rolled-beam, cantilever structure. Span 1: Length = 65.75' c.c. 36 WF 160 lb. at 5' 0-1/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/18.6	0.675 or 1/1170 of span	0.052	0.227	0.23	0.012	0.009	0.008	19.2	4.54	4.37	3.8
2.	Bridge No. B1 of 18-12-2 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 8.50' Susp. span length = 69.00' c.c. 36 WF 194 lb. at 5' 0-1/4" c.c. Min. concrete slab thickness = 7" Composite design	1/23	(Cantilever) 0.295 or 1/346 of length. (Susp.) 0.666 or 1/250 of span.	0.070	0.259	0.27	0.017	0.017	0.012	21.2	4.53	4.37	3.5
3.	Bridge No. B1 of 73-20-2 7-span, rolled-beam, cantilever structure. Span 1: Length = 73.64' c.c. 36 WF 182 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/20.8	(1)	0.180	0.304	0.59	0.030	0.015	0.010	8.3	5.56		
4.	Bridge No. B1 of 73-20-2 7-span, rolled-beam, cantilever structure. Span 2: Cantilever = 8.50' Susp. span length = 58.00' c.c. 33 WF 141 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	1/19.3	(Cantilever) 0.344 or 1/296 of length (Susp.) 0.557 or 1/1250 of span	0.153	0.239	0.64	0.030	0.013	0.006	12.3	4.90		
5.	Bridge No. B1 of 73-20-2 7-span, rolled-beam, cantilever structure. Span 3: Length = 75.00' c.c. 36 WF 170 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/17.5	0.855 or 1/1040 of span	0.201	0.336	0.60	0.028	0.028	0.08	16.5	4.42		
6.	Bridge No. B2 of 73-20-2 3-span, rolled-beam, cantilever structure. Span 1: Length = 73.75' c.c. 36 WF 102 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/20.9	0.855 or 1/1035 of span	0.129	0.322	0.40	0.030	0.014	0.010	16.4	5.27		
7.	Bridge No. B2 of 73-20-2 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 8.50' Susp. span length = 58.00' c.c. 33 WF 150 lb. at 5' 0-3/4" c.c. Min. concrete slab thickness = 7" Composite design	1/21.1	(Cantilever) 0.344 or 1/296 of length (Susp.) 0.557 or 1/1250 of span	0.150	0.250	0.60	0.037	0.017	0.008	13.0	5.15		
8.	Bridge No. B3 of 38-1-14 3-span, rolled-beam, cantilever structure. Span 1: Length = 50.23' c.c. 36 WF 160 lb. at 4' 9-7/8" c.c. Min. concrete slab thickness = 7" Non-composite design	approx. 1/14.2	0.546 or 1/1100 of span	0.053	0.218	0.24	0.003	0.002	0.001	6.0	5.00		
9.	Bridge No. B3 of 38-1-14 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 7.46' Susp. span length = 64.69' c.c. 36 WF 160 lb. at 4' 9-7/8" c.c. Min. concrete slab thickness = 7" Composite design	1/21.6	(Cantilever) 0.250 or 1/360 of length (Susp.) 0.529 or 1/1470 of span	0.068	0.242	0.28	0.009	0.005	0.006	10.8	4.90		
10.	Bridge No. B1 & B2 of 33-6-4 3-span, rolled-beam, cantilever structure. Span 6: Length = 69.46' c.c. 36 WF 230 lb. at 5' 0" c.c. Min. concrete slab thickness = 7" Composite design	approx. 1/19.7	0.52 or 1/1600 of span	0.093	0.269	0.35	0.023	0.019	0.008	19.4	4.58		
11.	Bridge No. B1 of 39-5-8 3-span, rolled-beam, cantilever structure. Span 1: Length = 56.88' c.c. 36 WF 150 lb. at 4' 10-1/4" c.c. Min. concrete slab thickness = 7" Non-composite design	approx. 1/16.1	(1)	0.065	0.369	0.18	0.002	0.001	0.001	7.8	4.71		
12.	Bridge No. B1 of 39-5-8 3-span, rolled-beam, cantilever structure. Span 2: Cantilever = 9.18' Susp. span length = 61.15' 36 WF 160 lb. at 4' 10-1/4" c.c. Min. concrete slab thickness = 7" Non-composite design	1/20.4	(1)	0.131	0.558	0.24	0.022	0.007	0.003	14.0	4.78		
13.	Bridge No. B1 of 34-6-1 5-span, plate-girder and rolled-beam, cantilever structure. Span 1: Length = 73.75' c.c. 48" plate-girder at 6' 0" c.c. Min. concrete slab thickness = 7" Non-composite design	approx. 1/15.6	(1)	0.062	0.338	0.18	0.015	0.008	0.003	26.2	4.54		
14.	Bridge No. B1 of 34-6-1 5-span, plate-girder and rolled-beam, cantilever structure. Span 2: Cantilever length = 14.00' Susp. span length = 47.00' c.c. 36 WF 160 lb. at 6' 0" c.c. Min. concrete slab thickness = 7" Non-composite design	1/15.6	(1)	0.082	0.337	0.24	0.016	0.012	0.005	26.2	4.50		
15.	Bridge No. B1 of 62-12-1 5-span, plate-girder, cantilever structure with floor-beams and stringers. Span 1: Length = 97.25' c.c. 6' 9-1/2" min., plate-girder - 2 per roadway Min. concrete slab thickness = 7" Non-composite design	approx. 1/12.2	(1)	0.083	-----	-----	0.014	0.012	0.006	12.0	4.35		
16.	Bridge No. B1 of 56-12-6 3-span, plate-girder, cantilever structure with floor-beams and stringers. Span 3: Length = 73.90' c.c. 5' 6" min., plate-girder - 2 per roadway Min. concrete slab thickness = 7" Non-composite design	approx. 1/11.4	0.268 or 1/3300 of span	0.049	-----	-----	0.007	0.004	-----	0.7	-----		

(1) Data on a few bridges is lacking, since design computations were destroyed in a fire.

after the truck had passed off the bridge are of particular significance, for this vibration was also of longer duration. On the three span structures, the suspended spans were more flexible than the anchor arm spans and vibrated with larger amplitudes. When the maximum amplitudes of vibration for the three conditions shown in Table 3 are studied, it appears that four bridges demonstrated the most prominent vibration. These bridges are B1 of 18-12-2, B1 of 73-20-2, B2 of 73-20-2, and B1 and B2 of 33-6-4. It should be noted that all of these bridges were constructed of rolled beams and designed with composite action. The only cantilever-type bridge which did not demonstrate a tendency for larger amplitudes of vibration was a non-composite plate girder bridge with floor beams and stringers. However, it was not possible to test the suspended span on this bridge because of site difficulties, and it is expected that the suspended span would vibrate at larger amplitudes than the anchor arm span.

Frequency of Vibration. - The natural frequency of vibration for the cantilever-type bridges was lower than for the other types, and ranged from 4.35 to 5.56 cps. For all three span bridges the anchor arm and suspended spans vibrated at the same frequency during sustained vibrations. However, on Bridge B1 of 73-20-2, a seven span structure, the amplitudes of vibration were larger, but the vibration pattern was irregular and not as long in duration as might be expected. Also the spans did not appear to vibrate together in harmony, and the interaction of the different frequencies probably reduced the duration of vibration.

On the first attempt at computing the theoretical fundamental frequency of vibration for these structures, the anchor arm together with the cantilever portion of the structure was analyzed as a structural unit independent of the suspended span. This led to different computed natural frequencies for the anchor arm span and the suspended span of the same bridge, and these values were markedly different from the experimental natural frequency. In particular, the experimental frequency was much lower than the computed frequency for the suspended span, when computed as a simple beam between points of suspension.

For Bridge B1 of 18-12-2, the theoretical frequency was next computed by the method used for the reinforced concrete bridges, discussed previously in this report. The only difference in procedure was that the influence coefficients were computed on the basis of frictionless hinges at the points of suspension for the suspended span. This gave a computed fundamental frequency of 4.37 cps. as compared to an experimental frequency of 4.53 cps., a difference of 3.8 percent. As these computations are laborious and time consuming, it was not possible to complete them for the other cantilever-type structures for this report, but they will be computed as soon as possible.

One very simple method of estimating the fundamental frequency of vibration for these cantilever-type structures, was to treat the center span portion of the structure as a simple beam, with a corrected effective span length greater than the suspended span length. This rough estimate gave a maximum error of 4.4 percent and an average error of 2.7 percent when compared to the experimental frequency of vibration.

DYNAMIC AXLE LOAD VARIATION

The two load axles of the test truck were instrumented with strain gages to determine the dynamic axle load variation of the test truck as it passed over the bridge. It was determined experimentally that the natural frequency of vibration for the test truck was 3.24 cps., and the solid damping factor, approximately 0.056 lb./in. per second. Due to instrumentation difficulties the axle load variation was obtained on only twelve of the fifteen bridges tested. The maximum dynamic axle load variations for the tractor and trailer axles are shown in Table 4 for various positions of these axles, on the bridge approach, on other spans, on the test span, and off the bridge. In addition, this table gives the maximum percent impact for each span tested as obtained by using the increased dynamic deflection over static deflection as a measure of impact.

The axle load variation was seldom more than ± 4 kips, except in the case of data from two bridges. One of these bridges (B1 & B2 of 33-6-4) has had a very rough riding surface since construction. The maximum axle load variation on this bridge was ± 6.8 kips on the tractor axle, and ± 10.6 kips on the trailer axle. This departure from the static load is ± 44 and ± 58 percent, respectively. The other bridge, (B2 of 39-3-8,) had a bituminous surface for the bridge approach and a bump had formed adjacent to the north end of the bridge, causing a maximum axle load variation of ± 8.5 kips. This bridge consisted of four simple spans and the test spans were at the opposite end of the bridge. The vertical oscillation of the test truck resulting from passing over this bump, had dissipated by the time it reached the test spans.

The bridge span with the largest percent impact was also the span on which the largest axle load variation occurred (B1 and B2 of 33-6-4, Span 4). In general the percent impact for the various spans appears to be reasonably consistent with the maximum percent axle load variation recorded while the truck was on the span (Figure 22). However, there are six points which fall farther away from the general pattern. Two of these points, representing low ratios of impact to axle load variation, are from Bridge B2 of 39-3-8 which had relatively stiff simple spans. On the other

TABLE 4

SUMMARY OF DATA ON MAXIMUM IMPACT AND DYNAMIC AXLE-LOAD VARIATION FOR ALL SPANS

Bridge and Span Number		Max. Percent Impact based on:[1] (Dd − Ds) / Ds x 100	Maximum Axle-Load Variation in Kips and Percent (Test Truck at Various Positions as Noted)									
			Tractor Axle - 15.5 Kips (Sprung)					Trailer Axle - 18.1 Kips (Unsprung)				
			Bridge Approach	Other Spans	On Span	Off Bridge	Max. Percent	Bridge Approach	Other Spans	On Span	Off Bridge	Max. Percent
1. Simple-Span Bridges												
X3 of 33-6-1	Span 2	5.6	-----	±0.9	±0.8	-----	6.0	-----	±4.0	±1.5	-----	21.9
X3 of 33-6-1	Span 3	5.7	-----	±0.9	±0.8	-----	6.0	-----	±4.0	±2.6	-----	21.9
X3 of 33-6-1	Span 4	8.9	-----	±0.9	±0.9	-----	6.0	-----	±4.0	±2.7	-----	21.9
B1 & B2 of 33-6-4	Span 1	14.8	-----	±3.3	±2.5	±2.0	21.6	-----	±6.4	±4.7	±3.6	35.2
B1 & B2 of 33-6-4	Span 2	14.0	-----	±3.3	±2.5	±2.0	21.6	-----	±6.4	±5.0	±3.6	35.2
B1 & B2 of 33-6-4	Span 4	45.5	-----	±5.4	±6.8	±5.4	44.1	-----	±9.6	±10.6	±8.6	58.5
B1 & B2 of 33-6-4	Span 5	16.4	-----	±6.8	±2.0	±5.4	44.1	-----	±10.6	±4.6	±8.6	58.5
B1 of 56-12-6	Span 1	10.7			(No Data)					(No Data)		
B1 of 56-12-6	Span 2	15.7			(No Data)					(No Data)		
B2 of 39-3-8	Span 1	0.0	±1.9	±5.6	±2.7	±4.5	36.0	±2.5	±7.6	±4.1	±8.5	47.0
B2 of 39-3-8	Span 2	2.4	±1.9	±5.6	±2.8	±4.5	36.0	±2.5	±7.6	±3.5	±8.5	47.0
Average simple-span bridges		12.7	±1.9	±3.6	±2.4	±4.0		±2.5	±6.7	±4.4	±6.9	
2. Steel Continuous-Span Bridges												
B2 of 38-1-14	Span 1	8.3	±2.5	±2.5	±2.0	-----	16.3	±2.3	±2.4	±2.2	-----	13.6
B2 of 38-1-14	Span 2	10.2	±2.5	±2.5	±1.5	-----	16.3	±2.3	±2.4	±1.4	-----	13.6
B1 of 70-7-3	Span 1	13.0	±2.5	±2.3	±2.4	±2.0	16.3	±4.9	±5.7	±4.0	±8.8	31.7
Average steel continuous-span bridges.		10.8	±2.5	±2.4	±2.0	±2.0		±3.2	±3.5	±2.5	±3.8	
3. Concrete Continuous-Span Bridges												
B1 of 38-11-25	Span 1	8.3	±2.8	±2.6	±2.6	±3.6	23.0	±2.7	±2.6	±2.8	±3.2	17.7
B1 of 38-11-25	Span 2	14.3	±2.8	±2.6	±2.6	±3.6	23.0	±2.7	±2.8	±2.6	±3.2	17.7
B5 of 81-11-8	Span 1	9.7	±1.4	±2.2	±1.4	±1.4	14.0	±2.4	±2.1	±2.1	±2.7	15.0
B5 of 81-11-8	Span 2	12.0	±1.4	±2.2	±2.0	±1.4	14.0	±2.4	±2.1	±2.1	±2.7	15.0
Average concrete continuous-span bridges.		11.1	±2.1	±2.4	±2.2	±2.5		±2.6	±2.4	±2.4	±3.0	
4. Cantilever-Type Bridges												
B1 of 18-12-2	Span 1	23.9	±3.0	±2.9	±2.6	±3.2	20.4	±4.2	±4.1	±3.7	±3.8	23.2
B1 of 18-12-2	Span 2	23.8	±3.0	±2.9	±2.3	±3.2	20.4	±4.2	±4.1	±3.4	±3.8	23.2
B1 of 73-20-2	Span 1	11.5	±2.1	±2.2	±1.5	-----	14.1	±2.2	±2.4	±1.8	-----	13.6
B1 of 73-20-2	Span 2	8.5	±2.1	±2.2	±1.7	-----	14.1	±2.2	±2.4	±2.4	-----	13.6
B1 of 73-20-2	Span 3	3.5	±2.1	±2.2	±1.6	-----	14.1	±2.2	±2.4	±1.7	-----	13.6
B2 of 73-20-2	Span 1	18.3	±1.7	±2.5	±2.5	±1.8	16.0	±3.2	±2.5	±3.6	±3.6	19.8
B2 of 73-20-2	Span 2	15.8	±1.7	±2.5	±2.5	±1.8	16.0	±3.2	±3.6	±2.4	±3.6	19.8
B3 of 38-1-14	Span 1	8.9			(No Data)					(No Data)		
B3 of 38-1-14	Span 2	7.1			(No Data)					(No Data)		
B1 & B2 of 33-6-4	Span 6	28.2	-----	±6.8	±4.4	±5.4	44.1	-----	±10.6	±7.8	±8.6	58.5
B1 of 39-5-8	Span 1	24.1	±2.7	±1.9	±4.0	±3.8	25.6	±2.6	±1.6	±3.5	±3.6	19.8
B1 of 39-5-8	Span 2	22.1	±2.7	±4.0	±1.9	±3.8	25.6	±2.6	±3.5	±1.6	±3.6	19.8
B1 of 34-6-1	Span 1	27.1			(No Data)					(No Data)		
B1 of 34-6-1	Span 2	16.1			(No Data)					(No Data)		
B1 of 62-12-1	Span 1	11.8	±2.6	±1.5	±2.4	±2.2	15.3	±2.6	±1.7	±2.3	±2.2	14.6
B1 of 56-12-6	Span 3	12.5			(No Data)					(No Data)		
Average cantilever-type bridges		16.4	±2.4	±2.9	±2.5	±3.2		±2.9	±3.5	±3.1	±4.2	

[1] Dd = Dynamic deflection
Ds = Static deflection

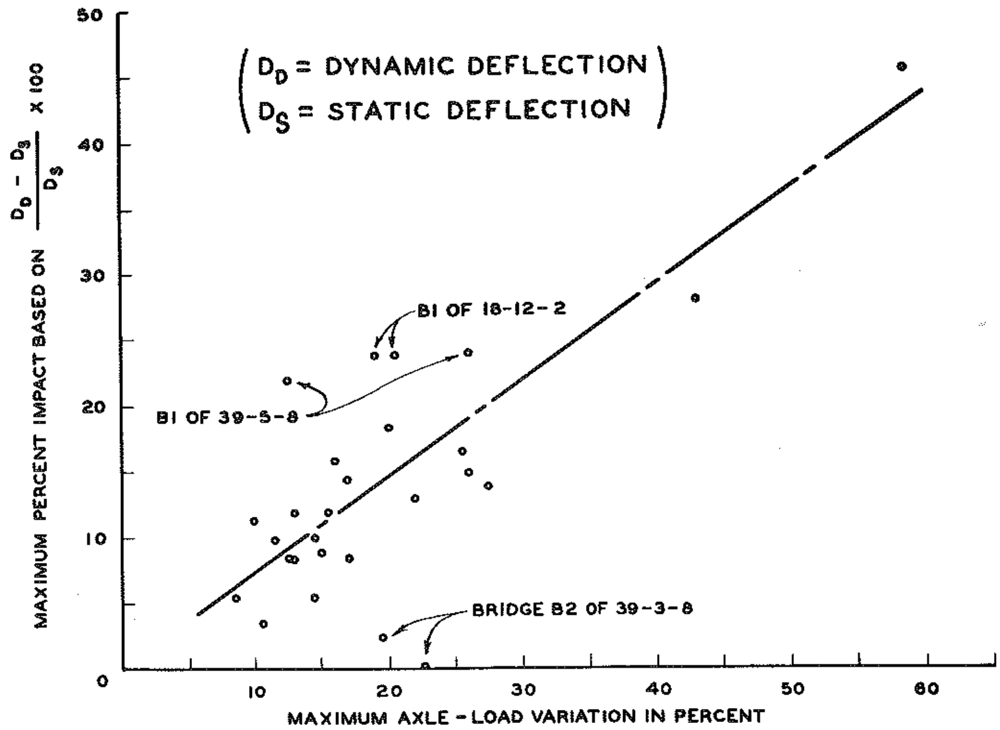


Figure 22. Maximum impact compared with maximum axle-load variation for all types of highway spans.

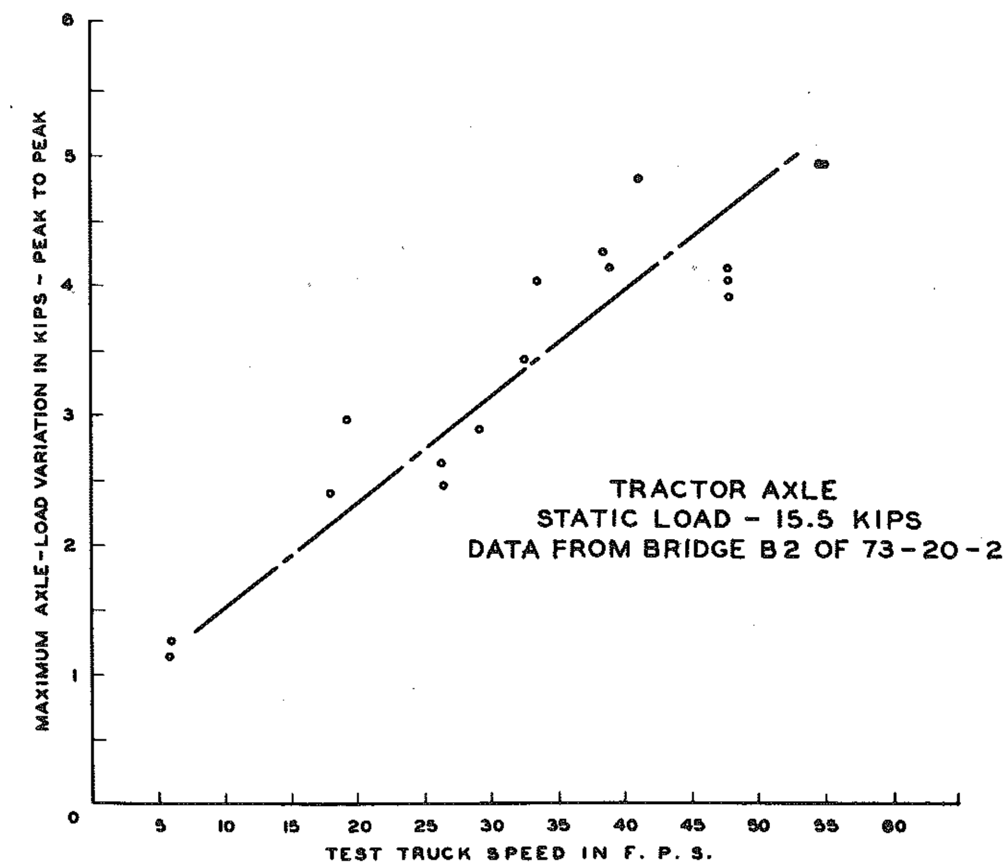


Figure 23. Maximum axle-load variation compared with test truck speed.

hand, the four points representing high ratios of impact to axle load variation were from two of the more flexible cantilever-type bridges.

The axle load variation increased in proportion to test truck speed. In Figure 23 the data from the tractor axle on Bridge B2 of 73-20-2 was used to illustrate this correlation.

An incidental, but interesting observation is shown in Figure 24, where three test truck runs were made on Bridge B2 of 73-20-2, at approximately the same speed. Both spans of this bridge responded in almost identical pattern and amplitude for all three of these test runs. For two of the three test runs the dynamic axle load variation was also recorded and these two traces are also similar to each other, although not as uniform in pattern as the bridge oscillations.

COMPARISON OF BRIDGE TYPES

In comparing the vibration behavior of various bridges, the same grouping as before, of simple-span, continuous-span, and cantilever-type will be employed. However a differentiation will be made between composite and non-composite structures, and in the case of continuous-span bridges, a distinction between those constructed of steel and of reinforced concrete. No distinction will be made between rolled beam and plate girder bridges of the various types.

Figures 25 through 29 show the subject data averaged for each bridge grouping. Figure 25 compares these bridge types on the basis of observed deflection and it illustrates the fact that the cantilever-type with composite design had the largest average deflection, followed by the same type without composite design. The reinforced concrete bridges, as might be expected, deflected least. In Figure 26 the bridge types are compared with respect to the ratio of observed to theoretical deflection. This graph shows that the non-composite spans had a 50 percent lower ratio, compared to the composite spans, for both simple-span and cantilever-type bridges. Composite and non-composite spans of the cantilever-type bridges had higher ratios than the corresponding simple-span bridges.

A comparison of the amplitudes of vibration in Figure 27 illustrates the much greater susceptibility of the cantilever-type bridges, especially those designed for composite action, to larger amplitudes of vibration. It should be noted that the cantilever-type spans tested were not especially long. The maximum anchor arm span length was 75 ft. and the longest suspended span length was 69 ft. In general, the cantilever-type bridges

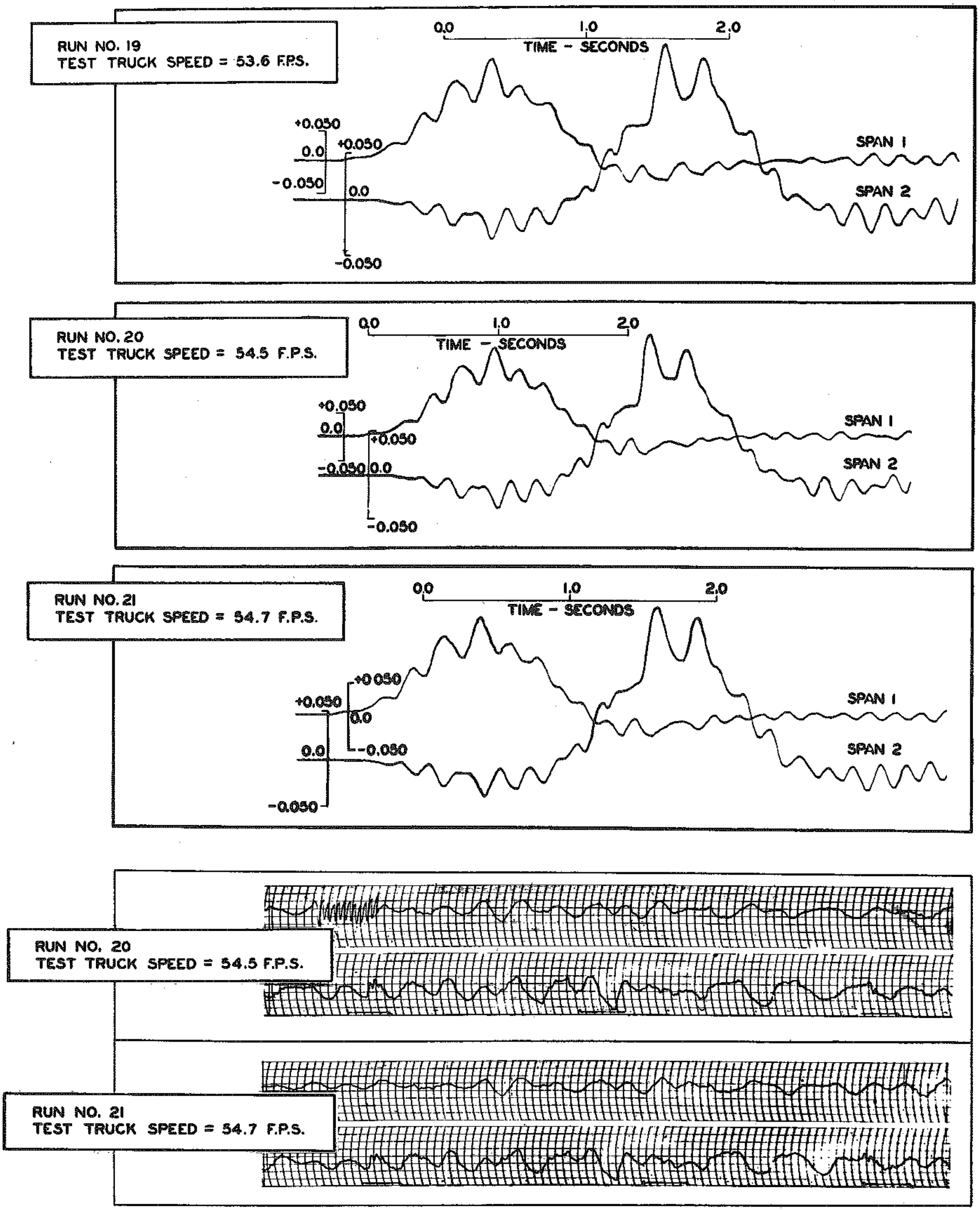


Figure 24. Bridge oscillation and axle-load variation for three runs of test truck on Bridge B2 of 73-20-2 at approximately the same speed.

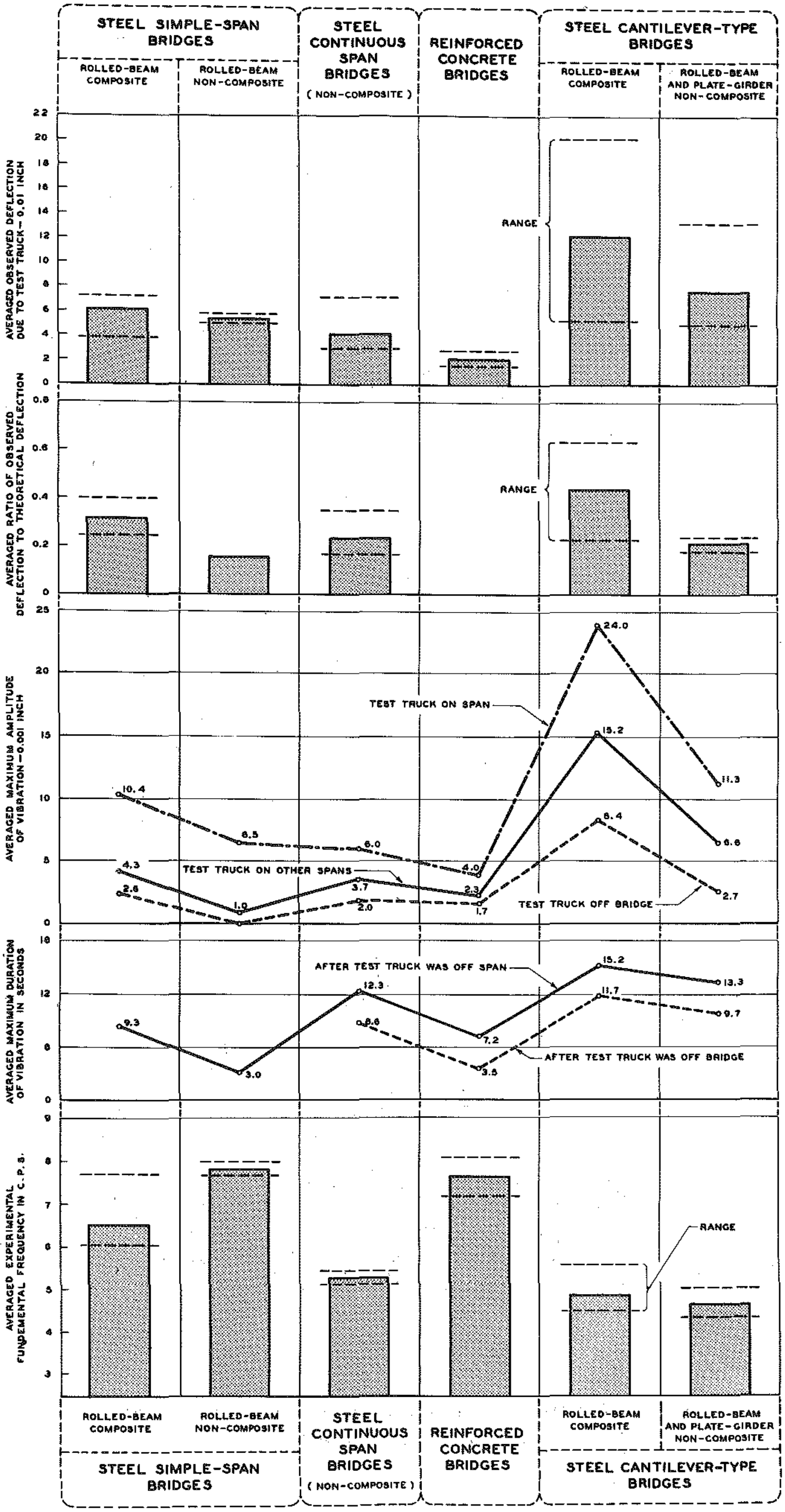


Figure 25. Observed deflection for various types of highway bridges.

Figure 26. Ratio of observed deflection to theoretical deflection for various types of highway bridges.

Figure 27. Maximum observed amplitude of vibration averaged for various types of highway bridges.

Figure 28. Maximum duration of vibration after truck was off span or bridge - averaged for various types of highway bridges.

Figure 29. Experimental fundamental frequency - averaged for various types of highway bridges.

designed without composite action had longer span lengths, but these did not vibrate with as large an amplitude of vibration.

The maximum duration of vibration, averaged for each bridge group, is shown in Figure 28. Again, the cantilever-type bridges lead the others, but steel continuous-span bridges follow rather closely non-composite cantilever-type bridges. A possible explanation for the longer duration of vibration for the steel continuous-span bridges may lie in the fact that the average solid damping factor for this type is the same as the average for the cantilever-type bridge. The solid damping factors shown in Table 5 were computed from examples of sustained free vibration. The average damping factor for the simple span bridges was not sufficiently higher than the other types to be conclusive, but the reinforced concrete bridges had an average damping factor approximately twice as large as that of the other types.

Figure 29 gives the experimental natural frequency, averaged for each bridge type. The cantilever-type bridges had the lowest natural frequency, while the steel continuous-span bridges had the second lowest. A comparison of the amplitude of free vibration with fundamental frequency is shown for all spans in Figure 30. Although the relationship between amplitude and fundamental frequency is not well defined, it does appear that the amplitude of free vibration has a tendency to increase as the fundamental frequency of the span is reduced. This figure also shows that the cantilever type bridges are grouped in the low frequency and high amplitude part of this graph.

Another way of evaluating the various types of bridges is by comparing the oscillograph traces of the most prominent vibrations for each type of structure. This has been done in Figure 31. In this evaluation, it is again clear that the greatest amplitude and duration of vibration is obtained on the cantilever-type structures. Figure 32 shows the oscillograph traces of two other cantilever-type bridges in order to emphasize the fact that all but one of the bridges of this type were readily susceptible to sustained vibration.

On one particular cantilever-type bridge, B1 of 39-5-8, the flexibility of the suspended span was noted during the testing. In order to demonstrate this flexibility one man starting jumping at the center of this span producing perceptible vibration. Figure 33 shows the oscillograph trace resulting from three men weighing a total of less than 500 lb. jumping in phase and at a frequency close to the natural frequency of the bridge. This resulted in an amplitude of vibration of 0.010 in.

TABLE 5

SUMMARY OF DATA ON DAMPED FREE VIBRATION FOR ALL SPANS

Bridge and Span Number		Logarithmic Decrement	Solid Damping Factor
1. Simple-Span Bridges			
X3 of 33-6-1	Span 2	0.065	0.021
" " "	" 3	0.050	0.016
" " "	" 4	0.096	0.030
B1 & B2 of 33-6-4	Span 1	-----	-----
" " "	" 2	-----	-----
" " "	" 4	0.069	0.022
" " "	" 5	0.091	0.028
B1 of 56-12-6	Span 1	-----	-----
" " "	" 2	-----	-----
B2 of 39-3-8	Span 1	-----	-----
" " "	" 2	0.076	0.024
Average simple-span bridges		0.074	0.024
2. Steel Continuous-Span Bridges			
B2 of 38-1-14	Span 1	-----	-----
" " "	" 2	0.044	0.014
B1 of 70-7-3	Span 1	0.079	0.025
Average steel continuous-span bridges		0.062	0.020
3. Concrete Continuous-Span Bridges			
B1 of 38-11-25	Span 1	-----	-----
" " "	" 2	0.115	0.036
B5 of 81-11-8	Span 1	0.122	0.039
" " "	" 2	0.142	0.043
Average concrete continuous-span bridges		0.126	0.039
4. Cantilever-Span Bridges			
(A) Anchor-Arm Spans			
B1 of 18-12-2	Span 1	0.038	0.012
B1 of 73-20-2	Span 1	0.084	0.027
" " "	" 3	0.071	0.023
B2 of 73-20-2	Span 1	0.073	0.023
B3 of 38-1-14	Span 1	-----	-----
B1 & B2 of 33-6-4	Span 6	0.040	0.013
B1 of 39-5-8	Span 1	-----	-----
B1 of 34-6-1	Span 1	0.057	0.018
B1 of 62-12-1	Span 1	0.075	0.024
B1 of 56-12-6	Span 3	-----	-----
Average anchor-arm spans		0.063	0.020
(B) Suspended Spans			
B1 of 18-12-2	Span 2	0.044	0.014
B1 of 73-20-2	Span 2	0.074	0.024
B2 of 73-20-2	Span 2	0.057	0.018
B3 of 38-1-14	Span 2	0.075	0.024
B1 of 39-5-8	Span 2	0.070	0.022
B1 of 34-6-1	Span 2	0.044	0.014
Average suspended spans		0.061	0.019
Average cantilever-span bridges		0.062	0.020

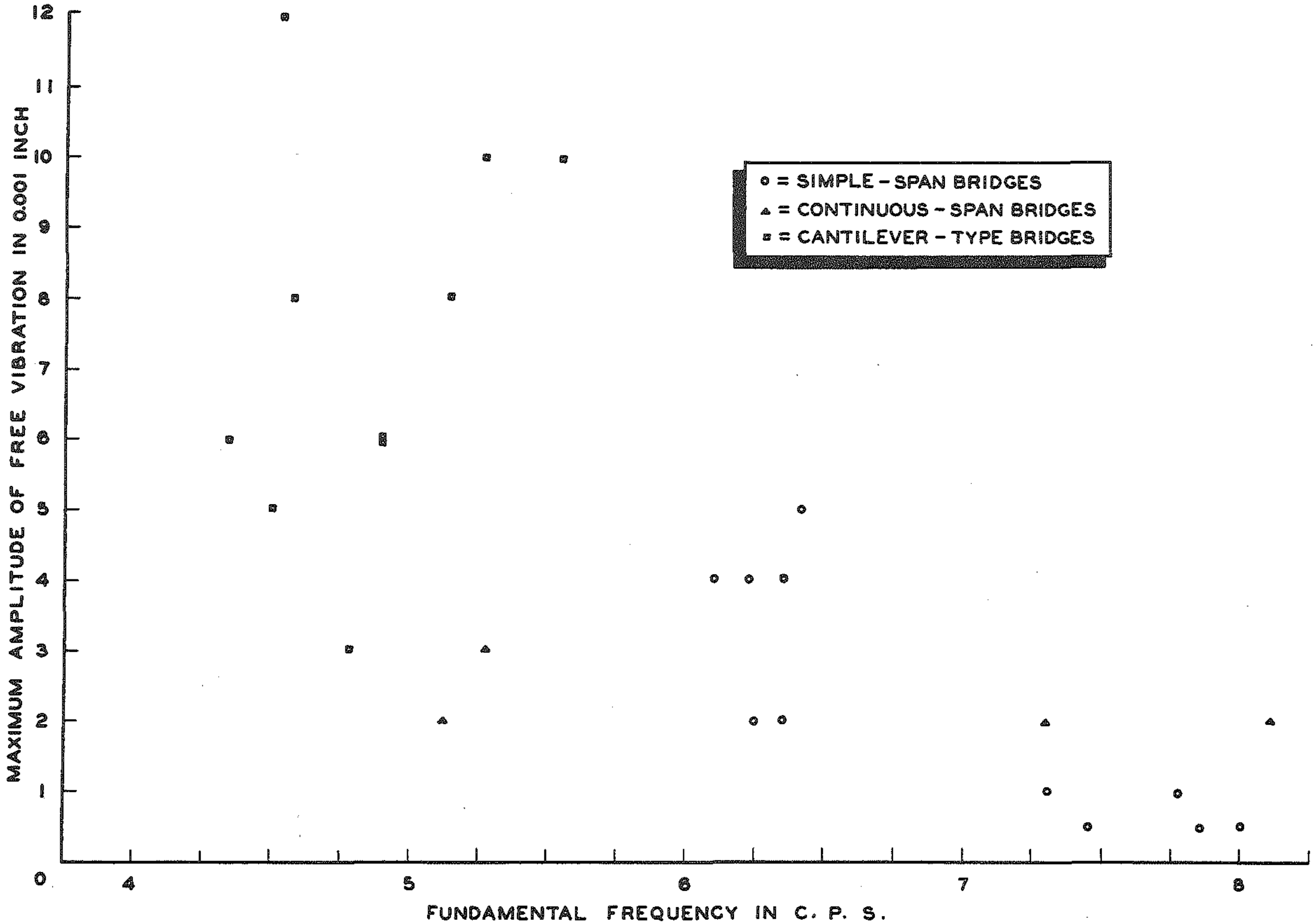


Figure 30. Maximum amplitude of free vibration compared with fundamental frequency for all bridge types.

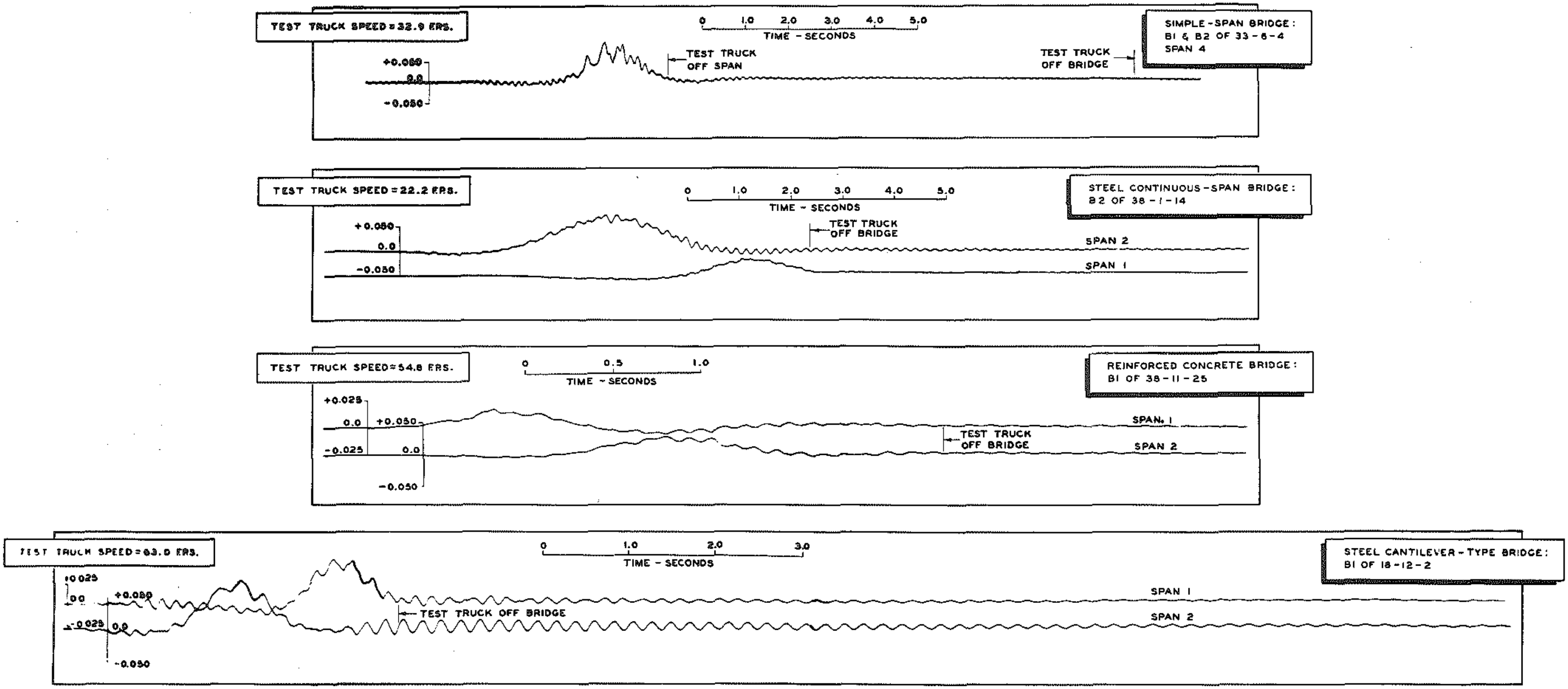


Figure 31. Comparison of oscillograph traces of most pronounced vibration for each bridge type.

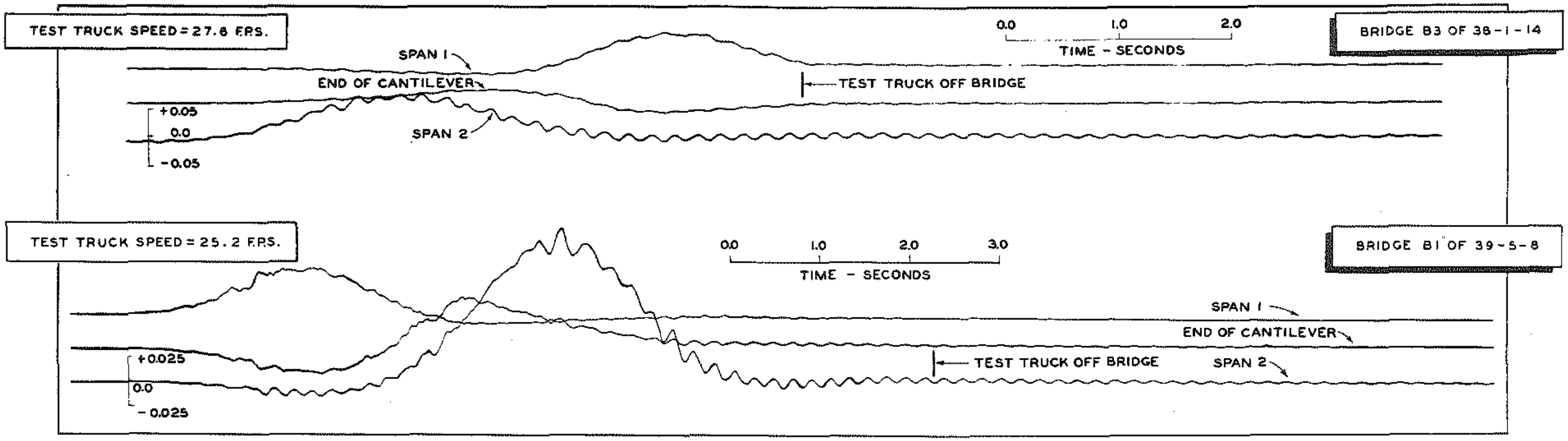


Figure 32. Oscillograph traces showing pronounced vibration of other cantilever-type bridges.

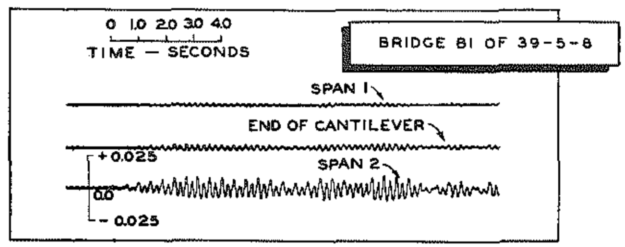


Figure 33. Oscillograph trace showing effect of three men jumping at center of suspended span of Bridge B1 of 39-5-8

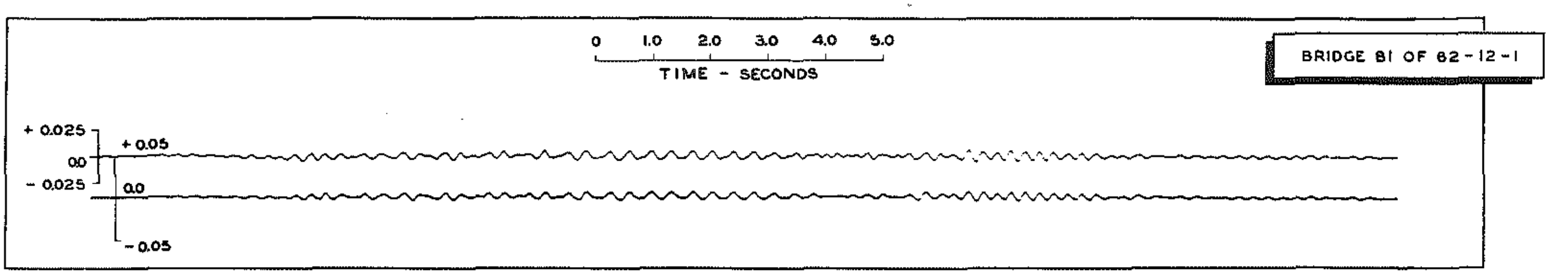


Figure 34. Oscillograph trace showing effect of truck passing over opposite roadway on a bridge with separate superstructure for each roadway.

In previous bridge tests it had been noted that in the case of bridges built with common piers and abutments, but with separate superstructures for each roadway, a noticeable amount of vibration occurred on one superstructure when a truck passed over the opposite one. Since this was particularly noticeable on Bridge B1 of 62-12-1, an oscillograph trace was obtained showing vibration caused by the test truck on the opposite roadway superstructure. This is shown in Figure 34, where the maximum amplitude of vibration under this condition was 0.005 in. It appears that the only way this vibration can be transmitted from one superstructure to the other is through the common piers and abutments. Since this effect had been noted previous to the bridge tests reported here, traffic was stopped temporarily on both roadways during the test program, so that recorded vibrations would be caused solely by passage of the test truck.

PSYCHOLOGICAL REACTION TO VIBRATION

These bridges were not analyzed for susceptibility to vibration on the basis of any fear that such vibration might lead to harmful structural effects. The primary concern in limiting vibration rises from the possibility that vibration may have discomforting psychological effects on pedestrians or motorists. In current and previous testing, the existing magnitudes of bridge vibration apparently would not appreciably affect a motorist in a vehicle travelling across the bridge. However, to a pedestrian walking across the bridge or to a motorist seated in a stalled vehicle on the bridge, the magnitude of the bridge vibration experienced in these tests might have a discomforting effect.

Individual sensitivity to such vibration varies and therefore it is difficult to set an exact limit as to what amplitude of vibration for a given frequency is perceptible, unpleasant, or intolerable. Janeway (5) has recommended certain safe limits for amplitude of vibration at various frequencies of vibration (Figure 35). These limits were based on data from subjects standing, or sitting on a hard seat. From one to six cycles per second, the recommended amplitude limits are based on the equation $af^3 = 2$, where 'a' is the amplitude, and 'f' is the frequency of vibration. From six to twenty cycles per second, the recommended amplitude limits are based on the equation $af^2 = 1/3$.

On this same Figure the various bridge vibration amplitudes and frequencies have been plotted in order to compare them with the Janeway limits. The amplitude of vibration is shown with the test truck on the span, and off the bridge. For amplitudes of vibration with the test truck on the span, seven cantilever-type spans and seven simple-spans had points falling above the limits. However, the amplitude of vibration with the span

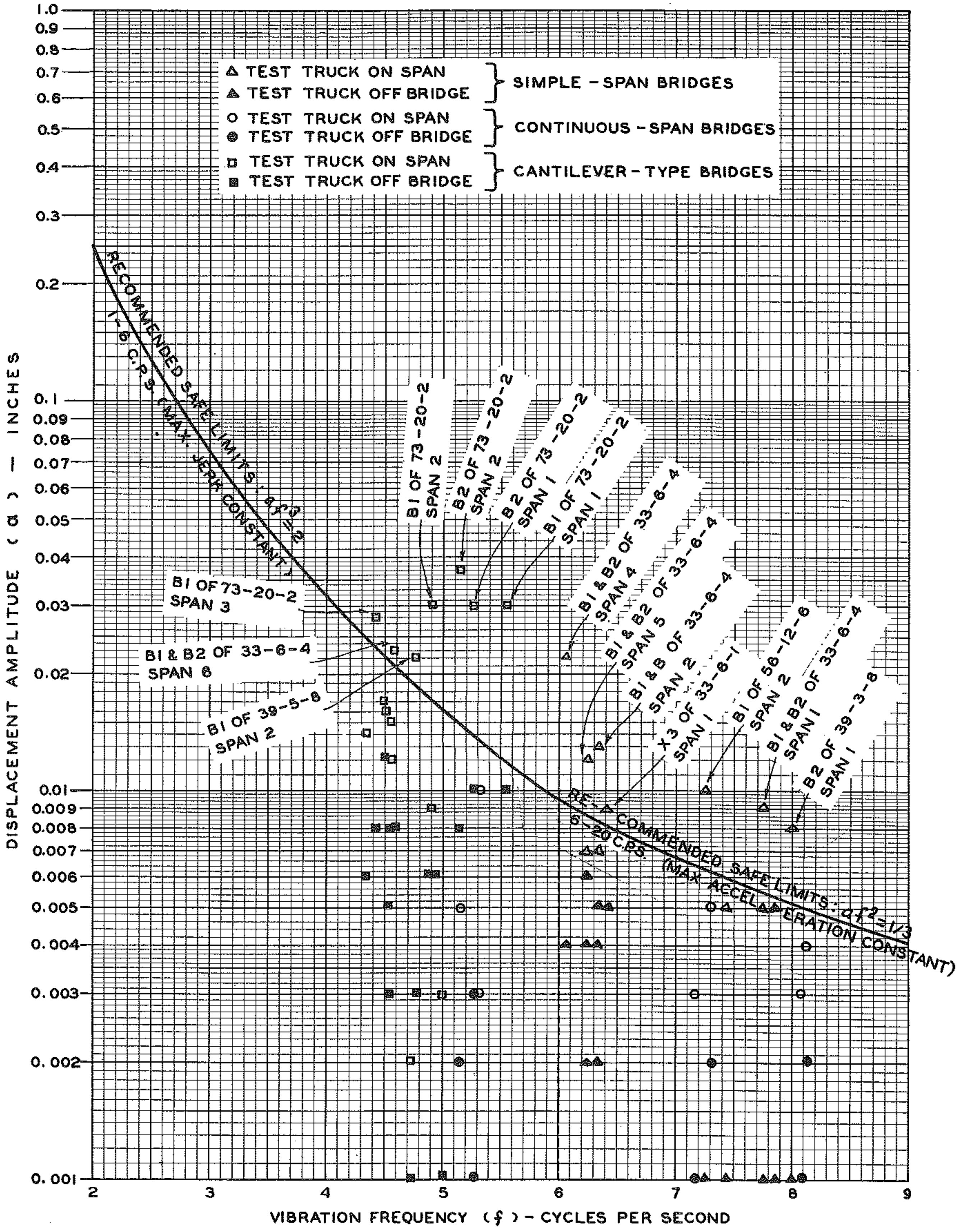


Figure 35. Observed amplitude and frequency of bridge vibration compared with recommended safe limits by Janeway[5].

loaded was never more than one or two cycles at the magnitude shown; therefore, it is felt that this vibration would not cause discomfort to someone on the bridge. However, several of the bridges tested (B1 of 73-20-2, Span 1; B2 of 73-20-2, Span 1; and B1 and B2 of 33-6-4, Span 4) had amplitudes of vibration with the test truck off the bridge which closely approached the limit line. This free vibration was sustained for a considerable number of cycles on some bridges. Further, data from other tests (2) indicates that in certain instances, normal truck traffic would produce larger amplitudes of vibration than those produced by the test truck.

Personal reaction of personnel engaged in these tests appears somewhat counter to the comparison of bridge vibration with recommended safe limits shown in Figure 35. The vibration of the simple-span and steel continuous-span bridges was perceptible, but was not sufficiently extensive to become discomforting. However, greater amplitude of vibration, although at a lower frequency, was somewhat discomforting on several of the cantilever-type bridges (B1 of 62-12-1, B1 of 73-20-2, B2 of 73-20-2 and Span 6 of B1 & B2 of 33-6-4). For the first three bridges, the instrumentation truck was parked on the roadway superstructure opposite from the one being tested. One person was seated on a hard stool in the instrumentation truck during the test program which lasted several hours. He was subjected to the bridge oscillation from the test truck on the other roadway, plus that due to the passage of normal truck traffic on the same roadway, which occurred between test truck runs. Due to this bridge oscillation he felt mild discomfort leading to a headache.

CONCLUSIONS

Assumptions and Limitations

The theoretical computations were based on the following assumptions:

For deflection calculations --

1. The modulus of elasticity of concrete was assumed to be 3×10^6 psi.

2. The effective moment of inertia of the spans designed for composite action and non-composite action was as recommended by the AASHO Specifications. The lateral distribution of load to stringers was also based on this specification.

For natural frequency of vibration calculations --

3. The modulus of elasticity of concrete was assumed to be 5×10^6 psi.

4. The effective moment of inertia for spans designed for composite action was based on considering 100 percent of the concrete deck above the beam or beams in question as effective.

5. The effective moment of inertia for spans designed for non-composite action was based on considering 50 percent of the concrete deck above the beam or beams in question as effective.

6. For reinforced concrete bridges the moment of inertia was based on the full gross tee-beam cross section.

The following findings are based solely on the thirty-four spans of fifteen bridges which were tested in this program. Cognizance should be taken that selection of bridges, slight differences in testing procedure, and slight variations in roadway roughness, could have had some influence on results.

General Findings

The following findings appear sufficiently conclusive to warrant careful consideration:

1. On the basis of deflection the cantilever-type bridges were definitely the most flexible bridge type tested.

2. The cantilever-type bridges were much more susceptible to large amplitudes and longer duration of vibration than the other types tested.

3. The average ratio of observed to theoretical deflection was 50 percent less for non-composite spans as compared to spans designed with composite action. This was true for both simple-span and cantilever-type bridges.

The following points are indicated from the present data, but more extensive testing or refinement in instrumentation may modify some of these concepts:

4. As a general rule, the bridges with the lower fundamental frequency of vibration were more susceptible to larger amplitudes of vibration.

5. Personal reaction bordering on discomfort was experienced only from vibration of the cantilever-type bridges.

6. In comparing bridges of a given type, designed with and without composite action, those designed with composite action were more susceptible to larger amplitudes of vibration.

7. The maximum percent impact as measured by the increase in dynamic deflection over the static deflection appeared to be related reasonably well to the maximum dynamic axle load variation of the test truck while on the span.

8. The magnitude of the dynamic axle load variation of the test truck increased with test truck speed.

9. It appears possible to compute the fundamental frequency of any of the bridge types tested, with sufficient accuracy, by using the methods and assumptions suggested in this report.

Discussion of Findings Pertinent to Bridge Design

The results of these bridge tests as discussed previously, indicate that there is an inequality in present design methods between bridges designed with and without, composite action. This was shown by the 100 percent higher ratio of actual deflection to theoretical deflection for the spans designed for composite action compared to those designed without it.

These tests also indicate that the present deflection limitations, when applied in design to simple span, continuous and cantilever-type bridges, do not appear to result in equitable stiffness for the various bridge types. If modifications in the deflection limitations are proposed, or if other means of controlling susceptibility to larger amplitudes of vibration are contemplated, then a thorough study of the cantilever-type highway bridge is imperative.

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