

**METHODOLOGY OF HOMOGENEOUS AND
NON-HOMOGENEOUS MARKOV CHAINS FOR
MODELLING BRIDGE ELEMENT DETERIORATION**

Final Report

to

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1.1 Background and Organisation of Report

Bridge management is an important activity of transportation agencies in the US and in many other countries. A critical aspect of bridge management is to reliably predict the deterioration of bridge structures, so that appropriate or optimal actions can be selected to reduce or minimize the deterioration rate and maximize the effect of spending for replacement or maintenance, repair, and rehabilitation (MR&R). In the US, Pontis is the most popular bridge management system used among the state transportation agencies. Its deterioration model uses the Markov Chain, with a statistical regression to estimate the required transition probabilities. This is the core part of deterioration prediction in Pontis.

This report focuses on the Markov Chain model used in Pontis, which is vital to the understanding and implementation of the Pontis software. Emphasis has been made on the limitations of Pontis methodology, and establishing a new method for transition probability estimation. This is because predicting deterioration is the basis for decision-making with respect to MR&R. The next portion of this chapter presents a literature review on the subject of estimating transition probability matrix using Markov Chain for modeling deterioration in civil engineering facilities, such as bridges, pavements, and waste water systems.

This report consists of seven additional chapters. Chapter 2 will present the results of a survey regarding the application of bridge management in the US and Canadian

transportation agencies. In Chapter 3, the basic concept of Markov Chain will be introduced in a simplified way with an emphasis on application. Simple examples are included to facilitate easy understanding. That chapter covers both homogeneous and non-homogeneous Markov chains for background information and comparison. Pontis uses a homogenous Markov chain model.

Chapter 4 presents a simple arithmetic algorithm for quickly estimating the transition probabilities, which will aid in understanding the concept of “transition”. However, this approach is not appropriate or proposed for routine application for predicting deterioration, since it lacks a capability to statistically cover possible random variation in condition data. This is to be seen in the examples included in this report. Chapter 5 discusses the approach used in Pontis for estimating the transition probability matrix. Chapter 6 highlights the issues associated with the Pontis approach for updating the transition probabilities, especially for Michigan. Chapter 7 presents the proposed method for transition probability estimation, based on the concept of non-homogenous Markov Chain. Here the proposed method will be compared with the existing Pontis approach, and the arithmetic method. Chapter 8 offers a summary for this research effort and the conclusions reached.

1.2 Literature Review:

The Markov Chain has been the most commonly used methodology to predict the deterioration of bridge structures and elements. The core of Markov Chain is the transition probability matrix because it is the basis for deterioration prediction. Thus, realistically estimating this matrix is critical. To understand the state of the art in the

estimation of the transition probability matrix in Markov Chain a literature review was performed particularly for transportation application facilities such as bridges and pavements. As a result three relevant papers are summarized below. There were also other papers found to be helpful, which are included in the reference list of this report.

1.2.1 Review

(1) José J. Ortiz-García; Seósamh B. Costello, and Martin S. Snaith (2006), “Derivation of Transition Probability Matrices for Pavement Deterioration Modeling,” *Journal of Transportation Engineering*, Vol. 132, No. 2, Feb 1, 2006

In this paper pavement deterioration modeling in general is divided into two broad groups, the deterministic- and the probabilistic-based approaches. Mathematical models are used to predict the deterioration (or the future condition state) as a fixed value in the deterministic approach, and as a probability for a particular condition in the probabilistic approach. It was pointed out in the paper that out of these two broadly classified models, the probabilistic modeling using Markov prediction is more frequently used.

This research effort attempted to compare methods for estimating the transition probabilities. Three candidate methods were tested on six sets of artificial data specifically synthesized for this purpose.

The first method directly uses historical condition data of the system for selecting the optimal transition probabilities available. The second utilizes a regression curve obtained from the original data as a criterion for estimation, and the third assumes that the distributions of condition are available to assist in the process. More details are to be

presented and discussed below about these different methods for estimating the transition probabilities.

The six artificial data sets were generated using the same basic concepts for condition rating. The system's condition is defined on a scale ranging from 0 to 100, with 0 for intact condition and 100 for complete disintegration. The score range is further divided into 10 intervals each having a width of 10 and defined as a condition state. The midpoints of these intervals are taken as the names of the condition states, namely 95, 85, 75, 65, 55, 45, 35, 25, 15, and 5. So data c_{jt} is the condition State j at time t . Each data set is to simulate annually collected conditions for 30 sites of a network over a 20-year period. Data Set 1 represents an S-shaped deterioration curve typical of the trend associated with either cracking or raveling progression in pavements. Data Set 2 represents a deterioration curve where the rate of progression starts slowly but increases with age. Data Sets 3 to 5 represent deterioration curves where the rate of progression starts fast but decreases with age, with each of the data sets representing a different rate of deterioration. Finally, Data Set 6 represents a completely random rate of progression.

Method A – This method is to find the transition probabilities by minimizing the sum of the squared differences between each of the data points and the average condition calculated from the distributions of condition. The objective of the estimation is to minimize

$$\text{Objective Function } Z = \sum_t \sum_j [c_{jt} - \bar{y}(t)]^2 \quad (1.1)$$

where c_{jt} is the condition data for State j at time t as defined earlier, and $\bar{y}(t)$ is the condition at time t weighted by the distribution vector using the estimated transition probabilities

$$\bar{y}(t) = \mathbf{a}(t) \cdot \mathbf{c} \quad (1.2)$$

where $\mathbf{c} = (95, 85, 75, 65, 55, 45, 35, 25, 15, 5)$ is the vector indicating the midpoints of the condition intervals between 90 and 100, 80 and 90, 70 and 80, 60 and 70, 50 and 60, 40 and 50, 30 and 40, 20 and 30, 10 and 20, 0 and 10, respectively. In other words, these midpoints can be viewed as the nominal values for the 10 condition level in the Markov Chain. $\mathbf{a}(t)$ is the probability distribution at time t .

$$\mathbf{a}(t) = \{a_1(t), a_2(t), a_3(t), \dots, a_{10}(t)\}^T \quad (1.3)$$

where $a_i(t)$ for $i = 1, 2, \dots, 10$ are the probabilities for the respective states at time t , $\bar{y}(t) = 95a_1(t) + 85a_2(t) + 75a_3(t) + 65a_4(t) + 55a_5(t) + 45a_6(t) + 35a_7(t) + 25a_8(t) + 15a_9(t) + 5a_{10}(t)$. Vector $\mathbf{a}(t)$ depends on the estimated transition probability matrix \mathbf{P} , which is selected to minimize the objective function Z in Equation (1.1)

Method B - In this method, a regression is performed first using the collected data c_{jt} . This results in $y(t)$ as a relation between the condition (defined as 95, 85, 75, 65, ..., and 5 as above) and time t . The objective function Z for this case is defined to be minimized as follows:

$$\text{Objective Function} \quad Z = \sum_t \left[y(t) - \bar{y}(t) \right]^2 \quad (1.4)$$

The objective of this method is to minimize this function as the squared distance between the regression curve $y(t)$ and the transition-probability-matrix-fitted curve $\bar{y}(t)$.

Method C - In this method, the raw data are presented in the form of distributions. The objective function Z is calculated as

$$\text{Objective Function } Z = \sum_t \sum_i [a_i(t) - a_i'(t)]^2 \quad (1.5)$$

where $a_i(t)$ has been defined in Equation (1.3), and $a_i'(t)$ for $i = 1, 2, 3, 4, \dots, 10$ are probabilities for condition i at time t obtained using raw data c_{jt} .

For all the test data sets, Method C yielded distributions closer to the “observed” distributions (i.e., distributions based on the synthesized inspection data) than Methods A and B. The distributions determined from Method C were also comparable, in many cases almost identical, to the “observed” ones. It is thus concluded that Method C is most appropriate for estimating the transition probability matrix.

Note that Method C directly used the observed distributions $a_i'(t)$ for $i=1, 2, 3, \dots, 10$ in the optimization requirement “Z” defined in Equation (1.5), and Methods B and C does not. Therefore, the above conclusion is not surprising. Furthermore using $a_i'(t)$ as a criterion for estimating the transition probabilities is a realistic and thus reasonable approach, because it is important to determine the transition probabilities to be able to reliably predict future conditions of the system.

(2) G. Morcous (2006), “Performance Prediction of Bridge Deck Systems Using Markov Chains,” *Journal of Performance of Constructed Facilities*, Vol. 20, No. 2, May 1, 2006.

In this work it is stated that the stochastic Markov-Chain models are used in current bridge management systems for performance prediction because of their ability to capture the time dependence in predicting bridge deterioration. The required life-cycle cost assessment of bridges is based on these predictions. It is a decision making process

based on the total cost for the bridge over its lifetime depending on the need for maintenance or demolition. The various costs involved in bridge management are construction cost, maintenance cost, demolition cost, and the user cost (indirect costs caused by detour, accidents, etc).

Bridge management systems such as Pontis and BRIDGIT adopt the Markov-Chain model for performance prediction of components, systems, and networks. The criterion used in this research for transition probability estimation is to minimize

$$\begin{aligned} \text{Objective Function } Z &= \sum_t |C(t) - E(t)| \\ \text{subject to } 0 &\leq p_{ij} \leq 1 & i, j = 1, 2, \dots, n & \quad (1.6) \\ \sum_j p_{ij} &= 1 & i = 1, 2, \dots, n & \end{aligned}$$

where $C(t)$ is the system condition rating at time t based on regression. This function describes a statistical relation between the condition and time t , obtained by regression analysis using data from inspection of the bridges. $E(t)$ is the expected rating at time t based on the Markov Chain using the estimated transition probabilities. This method appears to be similar to Method B in (Ortiz-Garcia et.al 2006) discussed earlier.

(3) Hyeon-Shik Baik; Hyung Seok (David) Jeong; and Dulcy M. Abraham (2006) “Estimating Transition Probabilities in Markov Chain-Based Deterioration Models for Management of Wastewater Systems,” *Journal of Water Resources Planning and Management*, Vol. 132, No. 1, January 1, 2006.

The so called ordered probit model was used in this work to estimate the transition probabilities for a Markov Chain based deterioration model for wastewater

systems. It uses a non-homogeneous Markov Chain for this purpose. The condition assessment data set used to evaluate the developed method was obtained from the City of San Diego. It was concluded that the ordered probit model approach seemed to provide a theoretically and statistically more robust model as compared to the nonlinear optimization-based approach for the estimation of transition probabilities. In the nonlinear optimization based approach, the transition probabilities are estimated by minimizing the following objective function,

$$\begin{aligned} \text{Objective Function } Z &= \sum_t \sum_n |Y(t) - E(n, \mathbf{P})| \\ \text{subject to } 0 &\leq p_{ij} \leq 1 \quad i, j = 1, 2, \dots, 5 \end{aligned} \quad (1.7)$$

$$\sum_j p_{ij} = 1 \quad i = 1, 2, \dots, 5$$

where t is the system's age, n is the number of transitions and \mathbf{P} is the transition probability matrix. $Y(t)$ is the average condition rating at t based on a regression analysis:

$$Y(t) = e^{-0.949 + 0.044t} \quad (1.8)$$

using the condition data. $E(n, \mathbf{P})$ is the predicted condition rating after n transitions based on the Markov Chain model using the estimated transition probabilities in \mathbf{P} . This approach appears to be also similar to that used in Equation (1.6).

It was also pointed out that, for developing accurate models using the ordered probit model, it is necessary to have panel data that span over multiple time periods. In order to predict more accurate and detailed deterioration patterns of wastewater systems,

factors such as the depth of the installation, the soil condition, the groundwater level, and the frequency of sewage overflows should be collected and evaluated.

A major drawback that has been discussed in this paper is that, in the current inspection practices, the information discussed here is not readily available for wastewater systems. It is emphasized that a standardized condition rating system is required to generate a more robust deterioration model and to evaluate the deterioration processes of wastewater systems among different municipalities. By employing a standardized condition rating system, current management practices and future investment planning can be evaluated. Another problem is that, currently each municipality uses a different rating system for its wastewater systems. These different condition-rating systems prevent comparison of the effects of maintenance and information sharing regarding condition assessment among municipalities.

Based on this discussion, the ordered probit model does not appear to be suitable for current practice of bridge management, due to its higher requirement for a large amount of data to allow modeling non-homogenous stochastic processes.

1.2.2 Summary

The literature review shows that Markov Chain is a popular and plausible tool to model system or element deterioration and improvement. For bridge management, it appears to be reasonable as well. In addition, the computation effort for using Markov Chain is also affordable for bridge management, considering several to tens of thousand bridges involved in a typical state.

In applying the Markov Chain model, the critical step is to estimate the transition probability matrix based on observation data. Several approaches have been proposed for this purpose. It seems to be agreeable that the ability to reliably predict deterioration comparable with observed deterioration is required. On the other hand, it should be emphasized that this requirement can only be used for the time period in which observation or inspection data have been collected. Predictions to the future beyond this time period cannot be evaluated until more data become available.

In this chapter the experiences of US and Canadian transportation agencies with bridge management system is reviewed based on their response to a questionnaire. The questionnaire developed in this project included two parts. The first one had a set of 13 questions categorized under “General Questions”. The second one was framed as “Additional Information and Comments”. The complete questionnaire is included in the appendix to this report. A total of thirty one agencies responded to the survey. The details of these two categories and the agency responses are presented in this chapter.

In the “General Questions” groups, there were a total of 13 questions aimed to understand various aspects of bridge management system application. These questions are further categorized below for analysis. The first sub-category of questions is to gather general information on the type of bridge management system (BMS) and on the bridge condition data available. There are 3 questions in this sub-category as follows:

1. Which BMS does your agency use?
2. Approximately how many years of bridge condition data (inspection and/or asset management data) does your agency have in your database?
3. What bridge condition data are used within your BMS?

Table 2.1 exhibits the responses to these questions. It shows Pontis as the most popular system currently being used. Also more agencies (16) have data between 4 to 10 years, compared with 11 agencies with more than 10 years of data. For those agencies

that use Pontis, all of them use both the NBI and the CoRe formats except Florida that uses CoRe only. Note that six US agencies using Pontis are also using data other than NBI and CoRe.

Table 2.1: Bridge Management Systems in and Condition Data Available

States	1	2	3
	Which BMS is used ?	No. of years of bridge condition data	Type of condition data
Alaska	Pontis	>10	NBI / CoRe
Alberta	In-House system	> 10	Other
Arizona	In-House system	4 to 10	NBI / CoRe
Arkansas	Pontis	1 to 3	NBI / CoRe
Colorado	Pontis	4 to 10	NBI / CoRe / Other
Delaware	Pontis	4 to 10	NBI / CoRe / Other
District of Columbia		4 to 10	NBI / CoRe
Florida	Pontis	4 to 10	CoRe
Georgia	Pontis	0 to 1	NBI / CoRe
Hawaii	Pontis	4 to 10	NBI / CoRe
Illinois	Pontis	> 10	NBI / CoRe
Iowa	In-House system (Code Sheets)	1 to 3	NBI
Kansas	Pontis	> 10	NBI / CoRe
Maine	Pontis	>10	NBI / CoRe / Other
Maryland	In-House system	4 to 10	NBI / CoRe / Other
Minnesota	Pontis	4 to 10	NBI / CoRe
Mississippi	Pontis	4 to 10	NBI / CoRe

Montana	Pontis	> 10	NBI / CoRe
Nevada	Pontis	> 10	NBI / CoRe
New Mexico	Pontis	4 to 10	NBI / CoRe
New York	In-House system (Bridge Needs Assessment Model)	> 10	Other
Ohio	In-House system	>10	Other
Ontario	In-House system	4 to 10	Other (element condition state)
Puerto Rico	Pontis	4 to 10	NBI / CoRe
South Carolina	Pontis	>10	NBI / CoRe
Tennessee	Pontis	> 10	NBI / CoRe
Utah	Pontis	1 to 3	NBI / CoRe / Other
Vermont	Pontis	4 to 10	NBI / CoRe
Virginia	Pontis	4 to 10	NBI / CoRe
Washington	Pontis	4 to 10	NBI / Other
Wyoming	Pontis	4 to 10	NBI / CoRe / Other

The second sub-category of questions was to know the practice in agency-specific application. Such application involves element definition, development of maintenance, rehabilitation, and repair (MR&R) policies, and cost estimation. These are questions were used to gather relevant information:

4. If your agency is a Pontis user, have you made modifications to the AASHTO CoRe elements, and/or have you added additional elements?

5. Has your agency developed bridge preservation policies, for maintenance, rehabilitation, and repair (MR&R)?
6. What cost data do you use to determine cost parameters for projects in your BMS?

Table 2.2: Agency Specific Application or Modification

States	4	5	6
	Modifications to CoRe Elements (If Pontis user)	MRR policy developed?	Type of Cost Data
Alaska	Yes	No	Past Bid / Bridge Maintenance Crew
Alberta		No	Past Bid
Arizona		No	
Arkansas	Yes	No	Past Bid
Colorado	Yes	No	Past Bid
Delaware	Yes	Yes	Past Bid
District of Columbia		No	Past Bid
Florida	Yes	No	Past Bid
Georgia	Yes	No	Past Bid
Hawaii	No	No	Past Bid
Illinois	Yes	Yes	Past Bid
Iowa		No	Past Bid
Kansas	Yes	No	Past Bid
Maine	Yes	Yes	Past Bid
Maryland	Not a Pontis user	No	
Minnesota	Yes	Yes	Past Bid
Mississippi	Yes	No	

Montana	Yes	Yes	Past Bid
Nevada	Yes	No	Default data in Pontis
New Mexico	No	Yes	Past Bid
New York		No	Past Bid
Ohio			Past Bid
Ontario		Yes	Past Bid
Puerto Rico	No	No	Past Bid
South Carolina	Yes	Yes	MR&R cost - SCDOT, Others – contract
Tennessee	Yes	No	Past Bid
Utah	Yes	Yes	Past bid compared with annual cost and average MRR costs
Vermont	No	No	Past Bid
Virginia	Yes	Yes	Past Bid
Washington	Yes	Yes	Past Bid
Wyoming	Yes	No	Past Bid

Table 2.2 shows that 19 out of the 23 Pontis states have modified the CoRe definitions and 4 have not. On the other hand, only about a half (11) of them have develop their own MR&R policies. A vast majority (26) of the 31 agencies are using bid prices for cost estimation, only one is using the Pontis default cost values.

The next sub-category questions focused on the transition probabilities in the Markov Chain modeling. There were three questions on how the transition probabilities are obtained or estimated, and the satisfaction associated with it. The three questions in this sub-category are:

7. Does your agency use deterioration rates based on transition probabilities?
8. If your agency is a Pontis user, are you satisfied with the resulting transition probabilities or deterioration rates (Do you think they model the situation realistically)?
9. How does your agency determine the transition probabilities or deterioration rates for a bridge element?

Table 2.3 Generation and Application of Condition Transition Probabilities

States	7	8	9
	Deterioration rates based on Transition Probabilities?	Satisfied with Transition Probabilities? (If Pontis User)	How to determine Transition Probability or Deterioration rate?
Alaska	Yes	Need to evaluate	Historic and Expert Elicitation
Alberta	No		Historic and Expert Elicitation
Arizona	No		
Arkansas	Yes	Yes	Expert Elicitation
Colorado	No	Not implemented	Not implemented
Delaware	Yes	Yes	Expert Elicitation
District of Columbia	Yes		Historic and Expert Elicitation
Florida	Yes	Yes	Expert Elicitation
Georgia		No	Historic and Expert Elicitation
Hawaii	Yes	Under development	Expert Elicitation
Illinois	Yes	Yes	Historic and Expert Elicitation
Iowa	No		
Kansas	Yes	Yes	Historic and Expert Elicitation
Maine	No	Yes / Partially	Historic

Maryland	No	No	
Minnesota	Yes	Partially	Expert Elicitation
Mississippi	No		
Montana	Yes	Yes	Historic and Expert Elicitation
Nevada	As in Pontis	Not yet started using	
New Mexico	No	Haven't Tried	Historic and Expert Elicitation
New York	No		Historic Data
Ohio			Historic only
Ontario	Yes		Expert Elicitation
Puerto Rico	No		
South Carolina	Yes	Yes / Partially	Historic and Expert Elicitation
Tennessee	No	Yes	Historic and Expert Elicitation
Utah	Yes	No	Expert Elicitation
Vermont	Yes	Partially	Historic and Expert Elicitation
Virginia	Yes	Partially	Expert Elicitation
Washington			Historic and Expert Elicitation
Wyoming	Yes	Yes	Historic and Expert Elicitation

Sixteen agencies here use deterioration rates based on the transition probabilities, and 11 said not using. This seems to indicate that the condition transition probabilities are considered important to the agencies. Out of the 23 Pontis states, 8 reported satisfaction with the transition probabilities produced in Pontis, 5 said partially satisfied, 3 said not satisfied, and 5 yet to evaluate (or not responding to the question). This distribution of response indicates a reasonable success with Pontis but also some room for improvement as well. For estimating the transition probabilities, 13 out of the 23

Pontis states use historic data and elicitation, 7 use elicitation only, and the rest either did not respond or have not reached this stage of implementation. This situation of a large number of agencies using elicitation is perhaps because the available historic data still do not meet the need for reliable estimation of the transition probabilities.

The last sub-category of questions in the general questions section included miscellaneous subjects related to application and satisfaction as follows:

10. Has your agency compared your BMS with your traditional approach for bridge management decision making?
11. Do you think your agency's BMS fully meets your need for bridge management?
12. Please describe how your agency determines the discount rate for project cost projection to the future.
13. How does your agency perform rulemaking and project prioritization within the BMS?

Table 2.4 shows the responses received to these questions. Fourteen agencies out of the 31 that responded reported experience with comparison of the BMS and traditional approaches, 15 said not, and the rest did not respond and most likely did not have such experience. We believe such an experience is important for the calibration of the BMS. As to the question whether the BMS meets the agency's needs, 17 out of 31 said no in addition to one that did not respond to this particular question. For comparison, 12 confirmed that the BMS does meet their needs. This large number of negative response to the question deserves attention. One possible

explanation to this situation is that, as mentioned earlier, available data are not adequate to model the behavior of the bridges and thus to help decision making.

For determining the discount rate, only very few agencies spend an effort to estimate the rate, many agencies use the Pontis default, and a large number (17) of the agencies do not use it, yet to determine it, or did not respond. To the last question in this group of general questions, most (20) agencies either did not respond or reported that rulemaking and project prioritization are not done currently. Others reported some traditional ways of practice, including “Collaboration between main office and field staff”, “BMS, maintenance engineers and bridge review team”, “Rules based on bridge administrative manual”, “NBI ratings”, “Cost benefit analysis & bridge condition index”, “Bridge Condition ratio along with engineering judgment”, etc.

For the second group of questions for additional information and comments, including whether the agencies are familiar with other relevant work and have additional comments, a few agencies responded positively. We then followed up by phone calls or e-mails to clarify and/or locate the specific information to acquire written documentations for the specific experience. In addition, 25 out of the 31 agencies that responded answered positively to the question whether they would like to have the results of this survey. It shows a strong interest in this research work.

Table 2.4: Changes Made for Prioritization

States	10	11	12	13
	BMS vs Traditional Approach Compared	Does your BMS meet all your needs?	Discount rate methodology	How rule making and project prioritization is performed?
Alaska	No	no	Default used	Not done
Alberta	Yes	No	4% based on Historic Data	Not done
Arizona		Yes		
Arkansas	No	Yes		
Colorado	Yes	Yes	Not implemented	Not done
Delaware	Yes			Rules are written so as to be compatible with Pontis
District of Columbia	No	No	Not used	
Florida	No	No	Work program Office	
Georgia	No	Yes	Not Determined	Not Determined
Hawaii	Yes		As in Pontis	Under implementation
Illinois	Yes	Yes	Limited by design	Top down approach
Iowa	No	No	National Inflation Index	Collaboration between main office and field staff
Kansas	Yes	Yes	As in Pontis	As in Pontis
Maine	Yes	No	Default used - Its thought that Pontis is not sensitive to this value	BMS, maintenance engineers and bridge review team
Maryland	No	Yes		
Minnesota	Yes	No	As in Pontis	Not done
Mississippi	No	No		
Montana	Yes	No	Bonding rate	Rules based on bridge administrative manual
Nevada	No	No		
New Mexico	No	Yes	Guessing	NBI ratings
New York	No	No		No
Ohio	No	No		
Ontario	No	Yes	Provincial govt.'s finance ministry	Cost benefit analysis & bridge condition index
Puerto Rico		No		

South Carolina	Yes	Yes	Pontis MR&R and Own inflation rate	Bridge Condition ratio along with engineering judgment
Tennessee	Yes	No	Yet to set up	Not done
Utah	Yes	No	Inflation Rate (4%)	Concept Report (Scope, schedule and budget is defined)
Vermont	Yes	Yes	Work is being done	Work is being done
Virginia	Yes	No	As in Pontis	In the development stage
Washington	No	Yes		
Wyoming	No	No	As in Pontis	Not done

This chapter presents the basic framework of Markov Chain as a modeling tool for bridge management. It summarizes the concept and defines the symbols used, with simple examples for illustration.

3.1 Markov Chain as A Stochastic Process

The Markov Chain is a stochastic process as a mathematical model for a system or an element that has random outcomes. These outcomes are viewed as a function of independent variables such as a temporal or spatial factor. For example, the condition of a bridge element is modeled in Pontis as a stochastic process with time t as the independent variable, because the future condition cannot be predicted with certainty. A stochastic process can be formally defined as follows.

Consider a series of discrete time points $\{t_k\}$ for $k = 1, 2, \dots$ and let ξ_{t_k} be a random variable as the condition of a bridge element, which describes its state at time t_k . The family of random variables $\{\xi_{t_k}\}$ $k = 1, 2, \dots$ is then said to form a stochastic process. The total number of considered states, in general, may be finite or infinite. For application in bridge management, a finite number of states is used. For example in Pontis, up to 5 states are used to describe the condition of a bridge element, with 1 for the best and 5 for the worst condition.

A Markov Chain is a stochastic process for which a future state depends only on the immediately preceding state, not any further previous states. This Markovian property for $\{\xi_{t_k}\}$ can be mathematically expressed as

$$P\{\xi_{t_n} = x_n \mid \xi_{t_{n-1}} = x_{n-1}, \dots, \xi_{t_0} = x_0\} = P\{\xi_{t_n} = x_n \mid \xi_{t_{n-1}} = x_{n-1}\} \quad (3.1)$$

Here symbol $|$ means “the given condition”, and P means the probability and $x_0, x_1, x_2, \dots, x_{n-1}$, and x_n are states of $\xi_{t_0}, \xi_{t_1}, \xi_{t_2}, \dots, \xi_{t_{n-1}}$, and ξ_{t_n} . Equation (3.1) reads as “the probability of ξ_{t_n} being equal to x_n given that $\xi_{t_{n-1}}, \dots$, and ξ_{t_0} are equal to x_{n-1}, \dots , and x_0 is equal to the probability of ξ_{t_n} being equal to x_n only if $\xi_{t_{n-1}}$ is equal to x_{n-1} ”. Namely, the conditions at time $t_{n-2}, t_{n-3}, \dots, t_0$ do not affect the condition at t_n , only the condition at t_{n-1} does.

3.2 Transition Probability in Markov Chain

The probability P defined in Equation (3.1) is called the transition probability which can be written in short as follows

$$P\{\xi_{t_n} = x_n \mid \xi_{t_{n-1}} = x_{n-1}\} = p_{x_{n-1} x_n} \quad (3.2)$$

This is the conditional probability of the system or element being in state x_n at t_n , given that it was in state x_{n-1} at t_{n-1} . This probability is also referred to as the one-step transition probability, since it describes the transition of the condition between times t_{n-1} and t_n , over one time step or one time interval.

For example, $p_{34} = 30\%$ for a bridge element means that the probability that this element will be in State 4 at t_n , if it was in State 3 at t_{n-1} , is 30 percent. Here t_n can be, for example, Year 1997, and t_{n-1} Year 1996. This also indicates that the prediction based on the Markov Chain is probabilistic, or with uncertainty taken into account. Note also that a 30% transition probability over one year from State 3 to State 4 in reality means a very high deterioration rate, because State 4 is considered a poor condition and State 3 significantly more acceptable.

Similarly an m -step transition probability is thus defined as

$$P_{x_n \quad x_{n+m}} = P \left\{ \xi_{t_{n+m}} = x_{n+m} \mid \xi_{t_n} = x_n \right\} \quad (3.3)$$

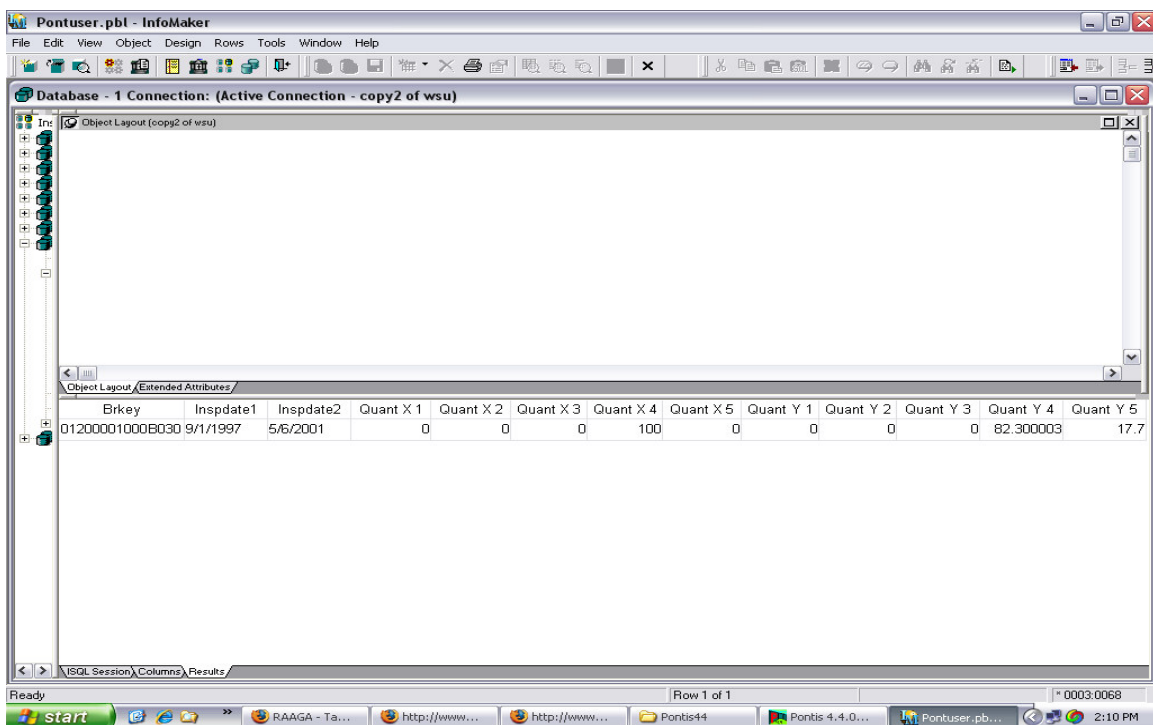
Here $(n + m) - n = m$ steps indicating the time difference between t_{n+m} and t_n . Each step here can be defined as a day, a month, a year, 2 years, 10 years, etc., depending on the system and its states of interest. For bridge management, Pontis uses a year as a typical time step. Namely the transition probability matrices for bridge elements are implicitly for 1-year periods.

In Pontis, a total of 5 condition states are used to describe the condition of bridge elements. Initially at time t , the system may be in any one of these states. Pontis uses $X = \{x_1, x_2, \dots, x_5\}^T$ to express the probability distribution for an element at the “before” time or at the last inspection. Superscript “ T ” here means transpose (to a column vector). For example, using the MODT Pontis data for Element 107 of bridge 01200001000B030 in Environment 1 and inspected in 1997, this distribution is expressed as $X = \{x_1, x_2, x_3, x_4, x_5\}^T = \{0, 0, 0, 100, 0\}^T$. It indicates that Element 107 has 0% in

State 1, State 2, State 3, 100% in State 4, and 0% in State 5. This means that Element 107 of this bridge is entirely in State 4.

Pontis also uses $Y = \{y_1, y_2, \dots, y_5\}^T$ to express the probability distribution for the system or the element at the “after” time or at the later inspection. For the same Element 107 above in Environment 1 but inspected in 2001, this distribution is $Y = \{y_1, y_2, y_3, y_4, y_5\}^T = \{0, 0, 0, 82.3, 17.7\}^T$. It indicates that Element 107 still has 0% in State 1, State 2, State 3, but 82.3% in State 4, and 17.7% in State 5. Here t_n is 2001 as the later inspection time, and t_{n-4} is 1997 as the previous inspection time. Namely over 4 years, part (17.7%) of the bridge element has deteriorated from State 4 to State 5, which is the worst state in the Pontis condition rating system.

Figure 3.1: Example Infomaker Screen For Probability Distribution For Element 107 In Environment 1



In Figure 3.1 this example is shown in InfoMaker. The first column “Brkey” shows the bridge identification number. “Inspdate 1” indicates the “before” inspection date, which is at time t_{n-4} (1997). “Quantity X 1” to “Quantity X 5” are probabilities in the distribution $\{x_1, x_2, x_3, x_4, x_5\}$. “Inspdate 2” indicates the “after” inspection date, which is at time t_n (2001). “Quantity Y 1” to “Quantity Y 5” are probabilities in the distributions $\{y_1, y_2, y_3, y_4, y_5\}$.

If the stochastic process is assumed Markovian, then according to Equation (3.2)

$$p_{ij} = P \left\{ \xi_{t_n} = j \mid \xi_{t_{n-4}} = i \right\} \quad i, j = 1, 2, 3, 4, 5 \quad (3.4)$$

are the 4-year transition probabilities for the system to change from State i at t_{n-4} (1997) to State j at t_n (2001). These transition probabilities can be more conveniently arranged in the matrix form \mathbf{P} as follows

		<i>To</i>	<i>State</i>	<i>at</i>	<i>year</i>	2001	
		1	2	3	4	5	
$\mathbf{P} =$	<i>From</i> 1	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	(3.5)
	<i>State</i> 2	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	
	<i>at</i> 3	p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	
	<i>year</i> 4	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	
	1997 5	p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	

In general, the size of this transition probability matrix depends on the total number of possible outcomes considered. For the cases of Pontis the possible outcomes are the 5 condition states, thus the size of the matrix is 5 x 5.

Some of the transition probabilities for Element 107 can be estimated as shown in the following matrix

$$\mathbf{P} = \begin{array}{cc|ccccc} & & \textit{To} & \textit{State} & \textit{at} & \textit{year} & 2001 \\ & & 1 & 2 & 3 & 4 & 5 \\ \hline \textit{From} & 1 & p_{11} & p_{12} & p_{13} & p_{14} & p_{15} \\ \textit{State} & 2 & p_{21} & p_{22} & p_{23} & p_{24} & p_{25} \\ \textit{at} & 3 & p_{31} & p_{32} & p_{33} & p_{34} & p_{35} \\ \textit{year} & 4 & 0 & 0 & 0 & p_{44} = \frac{82.3}{100} & p_{45} = \frac{17.7}{100} \\ 1997 & 5 & p_{51} & p_{52} & p_{53} & p_{54} & p_{55} \end{array} \quad (3.6)$$

Many transition probabilities in Equation (3.6) could not be estimated except p_{41} , p_{42} , p_{43} , p_{44} , and p_{45} , and thus no value has been given for them. It is because no data are provided for States 1, 2, 3, and 5. Based on the data given for Element 107 of Bridge 01200001000B030, p_{44} is estimated as 82.3 / 100 because out of the 100% of Element 107, 82.3% remained in State 4 (or transferred from State 4 to State 4). Similarly, $p_{45} = 17.7/100$ because 17.7% out of the 100% of Element 107 deteriorated from State 4 to State 5 (or transferred from State 4 to State 5). $p_{41} = p_{42} = p_{43} = 0$ because over the 4-year period, no repair or rehabilitation work was done, not possible to cause the condition state to become better.

The above simple example also shows how a bridge element may deteriorate and such deterioration may be modeled using a Markov Chain with estimated transition probabilities over a time interval. In general, such deterioration may vary with time. When this time-dependent variation is not significant, we use the so-called homogenous Markov Chain to approximately model the situation, which is to be discussed next.

3.3 Homogeneous Markov Chain

A Markov Chain is called homogeneous if its transition probabilities p_{ij} defined in Equations (3.1) to (3.3) are constant or independent of time. That means, for example, for the same length of 2 years as a time step, the deterioration follows the same pattern no matter when it started (i.e., no matter in which year the transition started):

$$P\{\xi_{t_k} = j | \xi_{t_{k-2}} = i\} = P\{\xi_{t_n} = j | \xi_{t_{n-2}} = i\} \quad (3.7)$$

for $n \neq k$. Note that Pontis uses (or assumes) homogenous Markov Chains for all bridge elements. Namely, it uses all inspection data to estimate one transition probability matrix for one element in one environment, no matter when the inspections were done as long as the same amount of time or approximately same amount of time has elapsed between two inspections. Then this matrix is used in predicting or projecting future condition in the probability sense for the same environment. This assumption for homogenous deterioration is actually questionable because the environment condition for an element does vary with time, especially when a long time period is concerned, such as the typical life span of bridge.

3.4 Non-Homogeneous Markov Chain

By the name, the non-homogeneous Mark Chain model does not assume a homogeneous behavior of the stochastic process. In other words, the transition probability matrix \mathbf{P} is not a constant but a function of time. Time here can be the absolute calendar time, age, or both. Age can be viewed as a relative measure of time, independent from the absolute time. For bridge management application, we consider the age of the bridge element in this report. Including the absolute time represents a further

more general treatment of the subject. An example appropriate for such a treatment is a scenario of special climate in a certain year that significantly alters the mechanism of bridge element deterioration, such as a very warm winter that accelerates steel corrosion. Since bridge elements have relatively long lives in tens to hundreds of years, an individual year of such abnormal condition may still have limited or little influence on deterioration and can be “averaged” out in modeling. Therefore, we include only age as the factor to account for non-homogeneity.

3.5 Properties of Transition Probabilities

It also should be noted that p_{ij} defined in Equations (3.1) to (3.3) must satisfy the following conditions,

$$\sum_{j=1, 2, \dots, 5} p_{ij} = 1 \quad \text{for all } i, \quad (3.8)$$

$$p_{ij} \geq 0 \quad \text{for all } i \text{ and } j$$

Equation (3.8) means that 1) each row of the transition probability matrix adds to 1 and 2) all probabilities are non-negative. They are valid because p_{ij} is a non-negative probability of transition from condition State i to State j , and from a State i the condition can only become 1, 2, 3, 4, or 5. Thus the total probability (i.e., the sum) of all these possibilities has to be 1. For example, Equation (3.6) shows the transition probabilities for Element 107 from State 4 in Row 4, for a time period of 4 years. The sum of these p_{ij} in that row is 1.0, because the total probability for Element 107 to transfer from State 4 to all these states is 1.0.

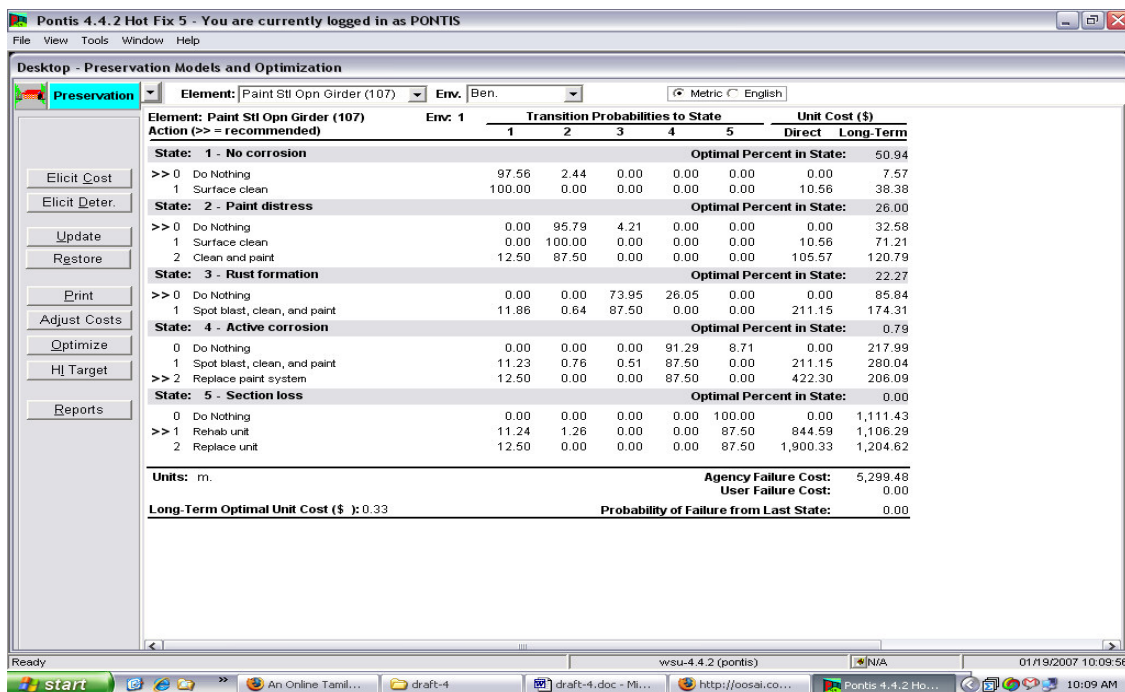
3.6 “Do-Nothing” Transition Probabilities

Note that the matrix in Equation (3.5) in Pontis appears as a matrix for “Do-nothing” mixed with other transition probability matrices for other MR&R options. One example is shown in Figure 3.2. The five rows of p_{ij} for one-year marked as “Do-nothing” are shown as:

$$P = \begin{bmatrix} 97.56 & 2.44 & 0.00 & 0.00 & 0.00 \\ 0.00 & 95.79 & 4.21 & 0.00 & 0.00 \\ 0.00 & 0.00 & 73.95 & 26.05 & 0.00 \\ 0.00 & 0.00 & 0.00 & 91.29 & 8.71 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \quad (3.9)$$

By comparison, one can see that this matrix being the transition probabilities for “Do-nothing” for one-year is taken out from the other transition probabilities for other different MR&R options. It should be noted that for searching for the optimal MR&R strategy, an elicitation is needed for the last transition probability (p_{55} here for this example) (AASHTO Pontis Manual).

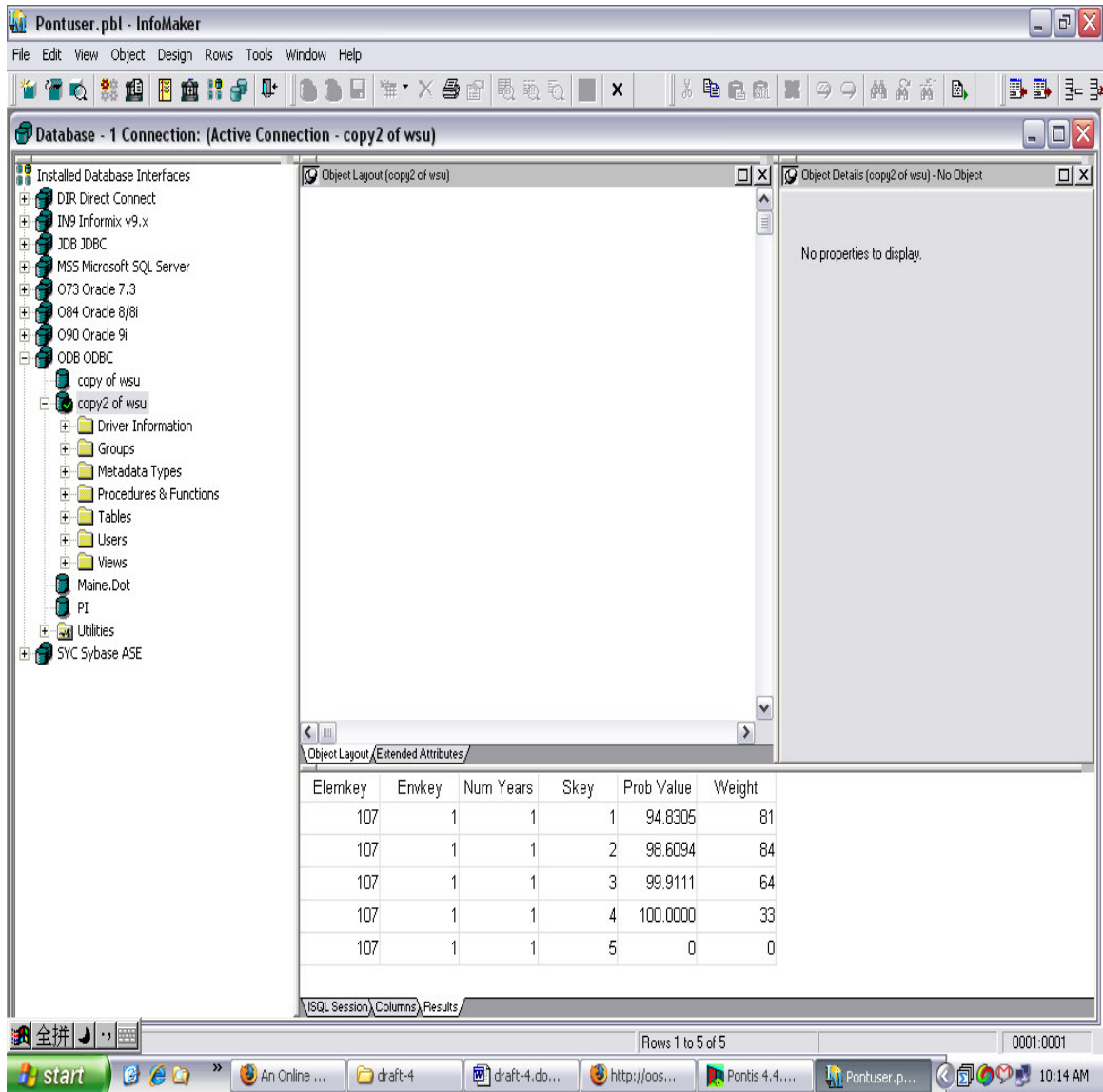
Figure 3.2: An Example Pontis Screen of Transition Probabilities



Further note that in InfoMaker, the transition probabilities in Equation (3.5) is shown as a vector $\{p_{11}, p_{22}, p_{33}, p_{44}, p_{55}\}^T$ for convenience. An example screen of this is shown in Figure 3.3. As seen the probabilities in the vector are 94.8305, 98.6094, 99.1111, 100.00, 0. Note that Pontis has a 1.0 for p_{55} because an element already in State 5 will never change its state if no maintenance work is done to it. Actually, other p_{ij} 's do not need to be shown, as seen in Figure 3.3 because the transition probability matrix is set to have $p_{13} = p_{14} = p_{15} = p_{21} = p_{24} = p_{25} = p_{31} = p_{32} = p_{35} = p_{41} = p_{42} = p_{43} = p_{51} = p_{52} = p_{53} = p_{54} = 0$. This means that 1) transition can only occur between two consecutive states (or no transition skipping a state can take place) and 2) improvement in condition state is impossible under the “do-nothing” assumption. The second assertion is based on an assumption of deterioration if nothing such as repair or rehab is done. Thus the controlling or independent items in the matrix are those on the diagonal (i.e., p_{11} , p_{22} , p_{33} , p_{44} , and p_{55}), and p_{12} , p_{23} , p_{34} , and p_{45} can be obtained using p_{11} , p_{22} , p_{33} , and p_{44} according to Equation (3.8), namely

$$\begin{aligned}
 p_{12} &= 1 - p_{11} \\
 p_{23} &= 1 - p_{22} \\
 p_{34} &= 1 - p_{33} \\
 p_{45} &= 1 - p_{44}
 \end{aligned}
 \tag{3.10}$$

Figure 3.3: An Example InfoMaker Screen of Transition Probabilities in A Vector



As seen above, the transition probability matrix is an important component in the Markov Chain model. Therefore, the criticality of reliably estimating the transition probabilities cannot be over-emphasized. This chapter presents a simple algorithm for quickly estimating the transition probabilities. It should be emphasized that this estimation approach suffers from lack of a statistical basis. On the other hand, its simple structure offers a quick estimation and understanding of the transition nature.

This simple method uses the observed condition change data over a period of time and thereby estimates the transition probabilities. The estimation is done by creating a transition probability matrix that can produce exactly the observed condition changes.

To illustrate the arithmetic method, let us consider an example for Element 12 (Concrete Deck – Bare in Environment 3, in square meters) for all MDOT bridges with an inspection interval of 2 years. The condition state distributions for these bridges are given in Table 4.1.

Table 4.1 Condition State Distributions for Element 12 in Environment 3 for MDOT

$\{ X^{(0)} \}$	$x_1^{(0)}$	$x_2^{(0)}$	$x_3^{(0)}$	$x_4^{(0)}$	$x_5^{(0)}$
in square meters	24896	34104.	15800	10300	5200
$\{ Y^{(1)} \}$	$y_1^{(1)}$	$y_2^{(1)}$	$y_3^{(1)}$	$y_4^{(1)}$	$y_5^{(1)}$
in square meters	17399	34801	17200	13200	7700

This table indicates that for these bridges at the beginning of the 2-year period, 24896 sq.m of the bare concrete deck was in State 1, 34104 in State 2, 15800 in State 3, 10300 in State 4, and 5200 in State 5. Two years later, the same concrete bare decks now have 17399 in State 1, 34801 in State 2, 17200 in State 3, 13200 in State 4 and 7700 in State 5. Note that $\{ X^{(0)} \}$ and $\{ Y^{(1)} \}$ here are given in physical quantity unit (square meters), note that this can also be represented by percentage. These two ways of presentation actually are equivalent with a difference of multiplicative factor. All the physical quantities can be transferred to percentage or probability by dividing each of the 5 components of the two vectors in Table 4.1 by the total quantity. Since the total quantity $x_1^{(0)} + x_2^{(0)} + x_3^{(0)} + x_4^{(0)} + x_5^{(0)} = y_1^{(1)} + y_2^{(1)} + y_3^{(1)} + y_4^{(1)} + y_5^{(1)}$ is 90300 sq.m, dividing $\{ X^{(0)} \}$ and $\{ Y^{(1)} \}$ in Table 4.1 by 90300 will give the vectors in percentage or probability.

By comparison of $\{ X^{(0)} \}$ and $\{ Y^{(1)} \}$ in Table 4.1, it is seen that 17399 sq.m of the concrete decks remained in State 1 after 2 years of service. In other words, 7497 sq.m (= 24896 - 17399) of the 24896 sq.m deteriorated to State 2. Out of the 34104 sq.m of bare decks that were in State 2, 27304 sq.m [= 34801 - (24896 - 17399)] stayed in State 2 and 6800 sq.m (= 34104 - 27304) became State 3. Out of 15800 sq.m, in State 3, 10400 sq.m [= 17200 - (34104 - 27304)] stayed in State 3 and 5400 sq.m (= 15800 - 10400) moved to State 4. Out of 10300 sq.m, in State 4, 7800 sq.m [= 13200 - (15800 - 10400)] stayed in State 4 and 2500 sq.m (= 10300 - 7800) moved to State 5. Finally, 5200 sq.m [= 7700 - (10300 - 7800)] out of the 5200 sq.m, which was in State 5, stayed in State 5 with 2500 sq.m (= 7700 - 5200) coming from State 4. This analysis is also documented in the last row of Table 4.2. For convenience of review, Table 4.1 is duplicated in Table 4.2.

Table 4.2 An Example of Estimating Transition Probabilities

$X^{(0)}$	$x_1^{(0)}$	$x_2^{(0)}$	$x_3^{(0)}$	$x_4^{(0)}$	$x_5^{(0)}$
In square meters	24896	34104	15800	10300	5200
$Y^{(1)}$	$y_1^{(1)}$	$y_2^{(1)}$	$y_3^{(1)}$	$y_4^{(1)}$	$y_5^{(1)}$
In square meters	17399	34801	17200	13200	7700
Quantity that stayed in same state	17399	$34801 - (24896 - 17399) = 27304$	$17200 - (34104 - 27304) = 10400$	$13200 - (15800 - 10400) = 7800$	$7700 - (10300 - 7800) = 5200$

Accordingly, the transition probability $p_{ij} = P \left\{ \xi_{t_n} = j \mid \xi_{t_{n-2}} = i \right\}$ for this element (within the MDOT bridges) to change from State i at t_{n-2} to State j at t_n can be estimated as follows using the results of Table 4.2

$$\mathbf{P} = \begin{array}{c|ccccc} & \text{State} & \text{at} & \text{2 year} & & \\ & & & & & \\ \hline & & 1 & 2 & 3 & 4 & 5 \\ \hline \text{State} & 1 & p_{11} = \frac{17399}{24896} & p_{12} = \frac{24896-17399}{24896} & p_{13} = 0 & p_{14} = 0 & p_{15} = 0 \\ \text{at} & 2 & p_{21} = 0 & p_{22} = \frac{27304}{34104} & p_{23} = \frac{34104-27304}{34104} & p_{24} = 0 & p_{25} = 0 \\ \text{0 year} & 3 & p_{31} = 0 & p_{32} = 0 & p_{33} = \frac{10400}{15800} & p_{34} = \frac{15800-10400}{15800} & p_{35} = 0 \\ & 4 & p_{41} = 0 & p_{42} = 0 & p_{43} = 0 & p_{44} = \frac{7800}{10300} & p_{45} = \frac{10300-7800}{10300} \\ & 5 & p_{51} = 0 & p_{52} = 0 & p_{53} = 0 & p_{54} = 0 & p_{55} = \frac{5200}{5200} \end{array} \tag{4.1a}$$

and thus

$$\mathbf{P} = \begin{array}{c|ccccc} & \text{State} & \text{at} & \text{2 year} & & \\ & & & & & \\ \hline & & 1 & 2 & 3 & 4 & 5 \\ \hline \text{State} & 1 & p_{11} = 0.69887 & p_{12} = 0.30113 & p_{13} = 0 & p_{14} = 0 & p_{15} = 0 \\ \text{at} & 2 & p_{21} = 0 & p_{22} = 0.80061 & p_{23} = 0.19939 & p_{24} = 0 & p_{25} = 0 \\ \text{0 year} & 3 & p_{31} = 0 & p_{32} = 0 & p_{33} = 0.65822 & p_{34} = 0.34178 & p_{35} = 0 \\ & 4 & p_{41} = 0 & p_{42} = 0 & p_{43} = 0 & p_{44} = 0.75727 & p_{45} = 0.24273 \\ & 5 & p_{51} = 0 & p_{52} = 0 & p_{53} = 0 & p_{54} = 0 & p_{55} = 1 \end{array} \tag{4.1b}$$

Note that p_{11} is the probability for this element to remain in State 1 after 2 years of service, estimated as $\frac{17399}{24896} = 0.69887$. p_{12} is the probability for it to deteriorate to State 2 from State 1 after 2 years or $1 - 0.69887 = 0.30113$, because no transition from State 1 to States 3, 4, and 5 is observed. Furthermore p_{22} is the probability for this element to remain in State 2, estimated as $\frac{27304}{34104} = 0.80061$ with 27304 found in Table 4.2 as the difference of 34801 sq.m in State 2 after 2 years and 7497 sq.m from State 1. Similarly $p_{23} = 1 - p_{22} = 1 - 0.80061 = 0.19939$. The rest of the matrix in Equations (4.1) is estimated according to the same concept and results in Table 4.2. It is also seen that all rows of the matrix in Equation (4.1b) add to 1 satisfying Equation (3.8).

As noted earlier that the matrix in Equation (4.1) is shown as a vector (0.69887, 0.80061, 0.65822, 0.75727, 1.000) instead of a matrix, because all other terms are zero except p_{12} , p_{23} , p_{34} , and p_{45} that are equal $1 - p_{11}$, $1 - p_{22}$, $1 - p_{33}$, and $1 - p_{44}$. Thus the only independent terms are the diagonal terms, which can be conveniently expressed in a vector, without losing generality.

Thus this arithmetic method is useful in understanding the concept of transition, particularly when used for small data sets. When used for a larger data set, it however loses reliability due to lack of a statistical basis. This fact will be highlighted in Chapter 7 where the arithmetic method is compared with both the Pontis and the proposed non-homogeneous Markov Chain approaches to estimating transition probabilities.

In this chapter the methodology used in Pontis is presented for estimating the transition probabilities. In Chapter 6, issues related to the Pontis approach are summarized based on the discussion here.

Pontis updates the transition probabilities using two sources. One is expert elicitation and the other historical inspection data. The expert elicitation is simply input by the user, which can be based on experience without use of inspection data at all. Note that at this early stage of Pontis application most of experience perhaps has to be derived from inspection data. In this study the focus is on how to use historical inspection data to estimate or update the transition probabilities (for “do-nothing”) to model the reality for Michigan.

To determine the transition probability matrix for a bridge element in an environment in the jurisdiction of an agency, two phases of calculations are used in Pontis. The first one is to estimate such matrices using inspection data according to their inspection intervals. The second one is to combine these matrices into one. The need for the first phase is due to the reality that not all bridges are inspected with a constant time interval. Section 5.1 presents the Pontis approach for the first phase, and Section 5.2 deals with the second phase.

5.1 Estimation of Transition Probabilities for One Time Step Using Inspection Data

Estimating the transition probabilities in Pontis for modeling deterioration for one time step is proceeded as follows: (1) Identifying pairs of the “before” (at t_{n-1}) and the “after” (at t_n) condition data. (2) Using the identified paired data to compute or estimate the transition probability matrix by regression. Note that the time step here can be one year, two years, three years, etc. The first step of identifying data pairs is to prepare relevant data for the second step of computation based estimation. It includes assembling pairs of condition inspection data over time for the specific element and making sure of consistent time intervals between inspections.

For each observation pair of inspection data, vector h_j is used to record the pair (Pontis technical manual 4.4):

$$h_j = \{ x_1^j, x_2^j, x_3^j, x_4^j, x_5^j; y_1^j, y_2^j, y_3^j, y_4^j, y_5^j \} \quad (5.1)$$

where x_k^j is the bridge element in condition state k that has been observed in the earlier (“before”) observation of pair j , and y_k^j is the element quantity observed in the k^{th} condition state in the later (“after”) observation for the same bridge. Hence x_k^j and y_k^j ($k = 1, 2, 3, 4, 5$) form the pair.

For example, consider a Michigan bridge *01200001000B030*, which has Element 107 (Open Steel Beam Painted) in the condition states as tabulated below for Year 1997 and Year 2001

Table 5.1 Observed Pair of Condition Ratings for Bridge 01200001000B030's

Open Steel Beam Painted

Bridge	Inspection Date	Percent for State				
		1	2	3	4	5
01200001000B030	9/1/1997	0	0	0	100	0
01200001000B030	5/6/2001	0	0	0	82.3	17.7

For this data set, the observation pair h_j is then formed as follows

$$h_j = \{ x_1^j = 0, x_2^j = 0, x_3^j = 0, x_4^j = 100, x_5^j = 0; y_1^j = 0, y_2^j = 0, y_3^j = 0, y_4^j = 82.3, y_5^j = 17.7 \} \quad (5.2)$$

or

$$h_j = \{ 0, 0, 0, 100, 0; 0, 0, 0, 82.3, 17.7 \}$$

This pair vector means that Element 107 for bridge *01200001000B030* had 100% of the element in State 4 at Year 1997 and 4 years later 82.3% and 17.7% in States 4 and 5, respectively. This also indicates that 17.7% of this element has deteriorated from State 4 to State 5 over the 2-year time period. Note that Equation (5.2) for h_j uses percentage, which can be converted to the physical quantity by simply multiplying h_j with the total quantity. Further note that when those quantities for the same element from different bridges are summed, all h_j vectors, $j = 1, 2, \dots$ need to be in physical quantity, not percentage. They are then used for estimating the transition probabilities.

For a bridge network and perhaps also a specific environment, there could be M such observation pairs for a specific element. This results in the following vectors X and Y

$$\begin{aligned}
X &= (x_1, x_2, x_3, x_4, x_5) \\
&= \left(\sum_{j=1}^M x_1^j, \sum_{j=1}^M x_2^j, \sum_{j=1}^M x_3^j, \sum_{j=1}^M x_4^j, \sum_{j=1}^M x_5^j \right) \quad (5.3)
\end{aligned}$$

$$\begin{aligned}
Y &= (y_1, y_2, y_3, y_4, y_5) \\
&= \left(\sum_{j=1}^M y_1^j, \sum_{j=1}^M y_2^j, \sum_{j=1}^M y_3^j, \sum_{j=1}^M y_4^j, \sum_{j=1}^M y_5^j \right) \quad (5.4)
\end{aligned}$$

Note that after the summation these two vectors can be divided by the total quantity

$$\sum_{j=1}^M x_1^j + \sum_{j=1}^M x_2^j + \sum_{j=1}^M x_3^j + \sum_{j=1}^M x_4^j + \sum_{j=1}^M x_5^j = \sum_{j=1}^M y_1^j + \sum_{j=1}^M y_2^j + \sum_{j=1}^M y_3^j + \sum_{j=1}^M y_4^j + \sum_{j=1}^M y_5^j \quad \text{to express}$$

them in percentage or probability. Pontis then uses these vectors to estimate the transition probabilities through a regression procedure as follows.

Based on the total probability theorem, transition probabilities p_{ki} ($i = 1, 2, 3, 4, 5$) need to satisfy the following equation according to the total probability theorem.

$$y_i = p_{1i}x_1 + p_{2i}x_2 + p_{3i}x_3 + p_{4i}x_4 + p_{5i}x_5 \quad (i = 1, 2, 3, 4, 5) \quad (5.5)$$

Note that there are five such equations in Pontis for $i = 1, 2, 3, 4, 5$ to include all 25 transition probabilities in the matrix defined in Equation (5.3).

Due to random behavior of deterioration and possible variation in inspection data y_i and x_i ($i = 1, 2, 3, 4, 5$), Equation (5.5) cannot be satisfied exactly. In estimating the transition probabilities p_{ij} , Pontis uses the concept of regression, although there can be other approaches to finding them. Namely, Pontis finds such p_{ij} ($i, j = 1, 2, 3, 4, 5$) values that minimize the differences between the two sides of Equation (5.5). This is the difference between the predicted and the observed conditions. This difference is defined as the sum of the squared residuals as follows

$$\Delta_i^2 = \sum_{j=1}^M (y_i^j - p_{1i}x_1^j - p_{2i}x_2^j - p_{3i}x_3^j - p_{4i}x_4^j - p_{5i}x_5^j)^2 \quad i = 1, 2, 3, 4, 5 \quad (5.6)$$

To minimize Δ_i^2 differentiating this quantity with respect to p_{1i} , p_{2i} , p_{3i} , p_{4i} , and p_{5i} , and then equating the partial derivatives to zero, we obtain the following five linear equations as a set.

$$\begin{aligned} \sum_{j=1}^M x_1^j y_i &= p_{1i} \sum_{j=1}^M (x_1^j)^2 + p_{2i} \sum_{j=1}^M x_1^j x_2^j + p_{3i} \sum_{j=1}^M x_1^j x_3^j + p_{4i} \sum_{j=1}^M x_1^j x_4^j + p_{5i} \sum_{j=1}^M x_1^j x_5^j \\ \sum_{j=1}^M x_2^j y_i &= p_{1i} \sum_{j=1}^M x_2^j x_1^j + p_{2i} \sum_{j=1}^M (x_2^j)^2 + p_{3i} \sum_{j=1}^M x_2^j x_3^j + p_{4i} \sum_{j=1}^M x_2^j x_4^j + p_{5i} \sum_{j=1}^M x_2^j x_5^j \\ \sum_{j=1}^M x_3^j y_i &= p_{1i} \sum_{j=1}^M x_3^j x_1^j + p_{2i} \sum_{j=1}^M x_3^j x_2^j + p_{3i} \sum_{j=1}^M (x_3^j)^2 + p_{4i} \sum_{j=1}^M x_3^j x_4^j + p_{5i} \sum_{j=1}^M x_3^j x_5^j \\ \sum_{j=1}^M x_4^j y_i &= p_{1i} \sum_{j=1}^M x_4^j x_1^j + p_{2i} \sum_{j=1}^M x_4^j x_2^j + p_{3i} \sum_{j=1}^M x_4^j x_3^j + p_{4i} \sum_{j=1}^M (x_4^j)^2 + p_{5i} \sum_{j=1}^M x_4^j x_5^j \\ \sum_{j=1}^M x_5^j y_i &= p_{1i} \sum_{j=1}^M x_5^j x_1^j + p_{2i} \sum_{j=1}^M x_5^j x_2^j + p_{3i} \sum_{j=1}^M x_5^j x_3^j + p_{4i} \sum_{j=1}^M x_5^j x_4^j + p_{5i} \sum_{j=1}^M (x_5^j)^2 \end{aligned} \quad (5.7)$$

for $i = 1, 2, 3, 4, 5$

All these linear equations can be expressed in the matrix form as:

$$[XX] = \begin{pmatrix} \sum_{j=1}^M (x_1^j)^2 & \sum_{j=1}^M x_1^j x_2^j & \sum_{j=1}^M x_1^j x_3^j & \sum_{j=1}^M x_1^j x_4^j & \sum_{j=1}^M x_1^j x_5^j \\ \sum_{j=1}^M x_2^j x_1^j & \sum_{j=1}^M (x_2^j)^2 & \sum_{j=1}^M x_2^j x_3^j & \sum_{j=1}^M x_2^j x_4^j & \sum_{j=1}^M x_2^j x_5^j \\ \sum_{j=1}^M x_3^j x_1^j & \sum_{j=1}^M x_3^j x_2^j & \sum_{j=1}^M (x_3^j)^2 & \sum_{j=1}^M x_3^j x_4^j & \sum_{j=1}^M x_3^j x_5^j \\ \sum_{j=1}^M x_4^j x_1^j & \sum_{j=1}^M x_4^j x_2^j & \sum_{j=1}^M x_4^j x_3^j & \sum_{j=1}^M (x_4^j)^2 & \sum_{j=1}^M x_4^j x_5^j \\ \sum_{j=1}^M x_5^j x_1^j & \sum_{j=1}^M x_5^j x_2^j & \sum_{j=1}^M x_5^j x_3^j & \sum_{j=1}^M x_5^j x_4^j & \sum_{j=1}^M (x_5^j)^2 \end{pmatrix} \quad (5.8)$$

$$[XY]_i = \begin{pmatrix} \sum_{j=1}^M x_1^j y_i^j \\ \sum_{j=1}^M x_2^j y_i^j \\ \sum_{j=1}^M x_3^j y_i^j \\ \sum_{j=1}^M x_4^j y_i^j \\ \sum_{j=1}^M x_5^j y_i^j \end{pmatrix} \quad i = 1, 2, 3, 4, 5 \quad (5.9)$$

$$a_i = \begin{pmatrix} p_{1i} \\ p_{2i} \\ p_{3i} \\ p_{4i} \\ p_{5i} \end{pmatrix} \quad i = 1, 2, 3, 4, 5 \quad (5.10)$$

We can write the solution for the regression Equation (5.7) as

$$a_i = [XX]^{-1} [XY]_i \quad i = 1, 2, 3, 4, 5 \quad (5.11)$$

where superscript “-1” means inverse of matrix.

For the solution to exist the matrix $[XX]$ must be nonsingular and thus invertible.

Note that vectors $[XY]_i$ ($i = 1, 2, \dots, 5$) can be assembled to one matrix $[XY]$ as follows:

$$[XY] = \left[[XY]_1, [XY]_2, [XY]_3, [XY]_4, [XY]_5 \right] \quad (5.12)$$

Then the transition probability matrix \mathbf{P} for one-time step can be written as follows

according to Equation (5.11)

$$[a_1, a_2, a_3, a_4, a_5] = \mathbf{P}_e = [XX]^{-1} [XY] \quad (5.13)$$

This is the calculation in Pontis for estimating \mathbf{P} over a time step or a time interval associated with the observation pair X and Y . The estimated matrix is now denoted as \mathbf{P}_e , with a subscript “e” for “estimated”.

Figure 5.1: Example InfoMaker Screen Showing [XY] Matrix for Element 12 in Environment 3 for MDOT Bridges with 2-year Inspection Interval

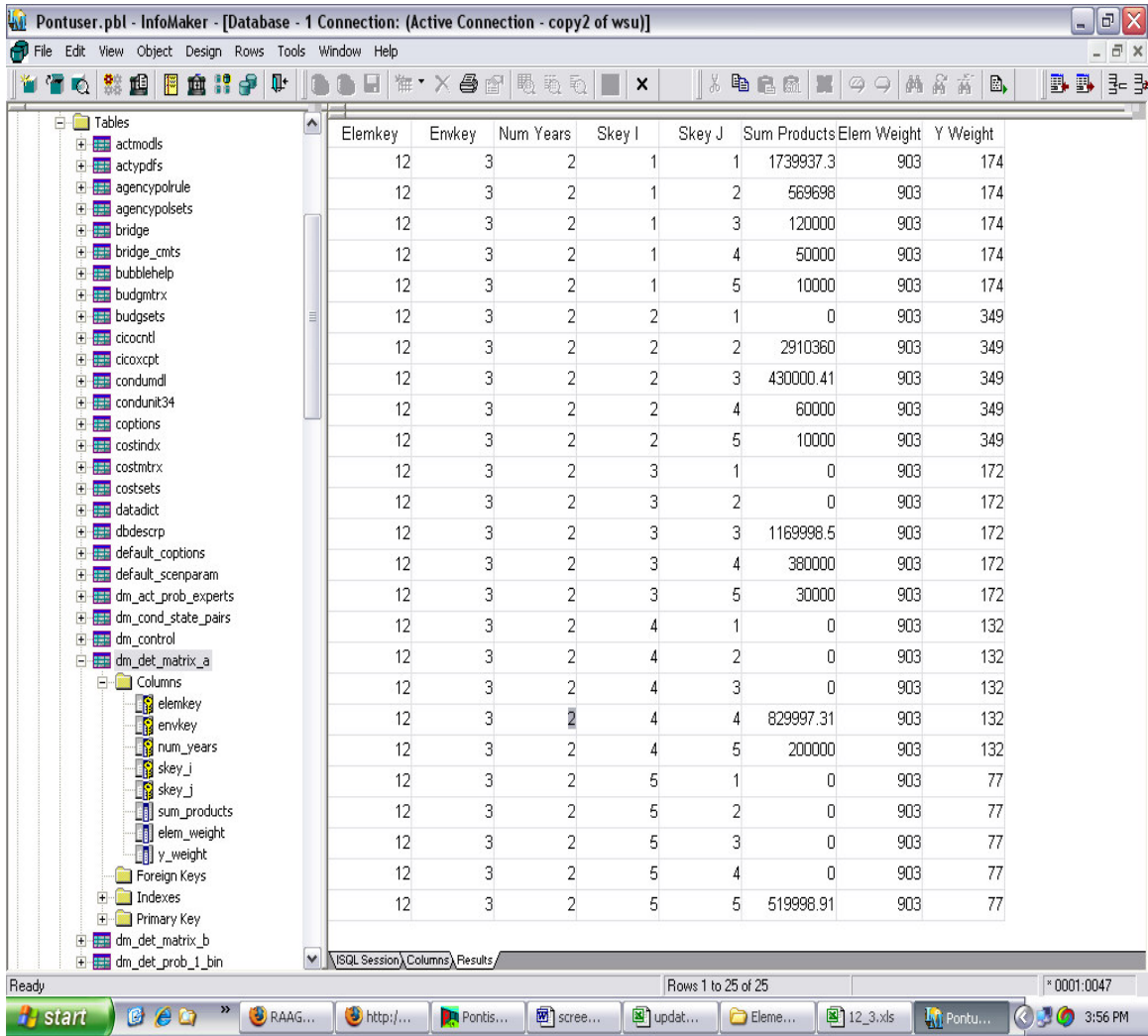


Table 5.2: [XY] Matrix for Element 12 in Environment 3 for MDOT Bridges with 2-year Inspection Interval

Key j \ Key i	1	2	3	4	5
1	1739937.3	569698	120000	50000	10000
2	0	2910360	430000.41	60000	10000
3	0	0	1169998.5	380000	30000
4	0	0	0	829997.31	200000
5	0	0	0	0	519998.91

As an example, Figure 5.1 shows the $[XY]$ matrix for Element 12 in Environment 3 for MDOT bridges with an inspection interval of 2 years. The first column “Elemkey” identifies the element number, which is 12 for this case. The second column “Envkey” indicates the environment in which this element is in, and it is 3 for this case. The third column “Num Years” presents the number of years between two inspections, in this case 2 years is the inspection interval selected. The fourth column “Skey I” refers to the row for condition State i , in the $[XY]$ matrix and the fifth column “Skey J” the column for condition State j ; these values of $[XY]$ are assembled in Table 5.2 in matrix form for reference. Namely in Table 5.2 and Figure 5.1, we have

$$\begin{aligned}
 [XY]_1 &= \begin{bmatrix} 1739937.3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; & [XY]_2 &= \begin{bmatrix} 569698 \\ 2910360 \\ 0 \\ 0 \\ 0 \end{bmatrix}; & [XY]_3 &= \begin{bmatrix} 120000 \\ 430000.41 \\ 1169998.5 \\ 0 \\ 0 \end{bmatrix}; \\
 [XY]_4 &= \begin{bmatrix} 50000 \\ 60000 \\ 380000 \\ 829997.31 \\ 0 \end{bmatrix}; & [XY]_5 &= \begin{bmatrix} 10000 \\ 10000 \\ 30000 \\ 200000 \\ 519998.91 \end{bmatrix} & & (5.14)
 \end{aligned}$$

The fifth column “Sum Products” gives these components of the entire matrix $[XY]$ in a vector format.

Figure 5.2: InfoMaker Screen Showing the Values of [XX] matrix for Element 12 in Environment 3 for MDOT Bridges with 2-Year Inspection Interval.

Elemkey	Envkey	Num Years	Skey I	Skey J	Sum Products	X Weight
12	3	2	1	1	2489282.3	249
12	3	2	1	2	354.06744	249
12	3	2	1	3	0	249
12	3	2	1	4	0	249
12	3	2	1	5	0	249
12	3	2	2	1	354.06744	343
12	3	2	2	2	3410010	343
12	3	2	2	3	0	343
12	3	2	2	4	0	343
12	3	2	2	5	0	343
12	3	2	3	1	0	158
12	3	2	3	2	0	158
12	3	2	3	3	1580000	158
12	3	2	3	4	0	158
12	3	2	3	5	0	158
12	3	2	4	1	0	103
12	3	2	4	2	0	103
12	3	2	4	3	0	103
12	3	2	4	4	1030000	103
12	3	2	4	5	0	103
12	3	2	5	1	0	52
12	3	2	5	2	0	52
12	3	2	5	3	0	52
12	3	2	5	4	0	52
12	3	2	5	5	520000	52

Table 5.3: [XX] Matrix for Element 12 in Environment 3 for MDOT Bridges with 2-Year Inspection Interval

Key j \ Key i	1	2	3	4	5
1	2489282.3	354.06744	0	0	0
2	354.06744	3410010	0	0	0
3	0	0	1580000	0	0
4	0	0	0	1030000	0
5	0	0	0	0	520000

In Figure 5.2, the matrix $[XX]$ defined in Equation (5.8) is shown for the same element on Infomaker screen. The format for $[XX]$ is the same as for $[XY]$. Table 5.3 shows the $[XX]$ matrix in the matrix form for reference.

According to Equation (5.11) or (5.13), $[XX]$ needs to be inverted to find the estimated transition probability matrix \mathbf{P} . This inverse is shown in Table 5.4 for the same example of element.

Table 5.4: Inverse of $[XX]$ Matrix for Element 12 in Environment 3 for MDOT
Bridges with 2-year Inspection Interval
($[XX]$ matrix shown in Table 5.2)

Key j \ Key i	1	2	3	4	5
1	0.0000004	0.0000000	0.0000000	0.0000000	0.0000000
2	0.0000000	0.0000003	0.0000000	0.0000000	0.0000000
3	0.0000000	0.0000000	0.0000006	0.0000000	0.0000000
4	0.0000000	0.0000000	0.0000000	0.0000010	0.0000000
5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000019

It is seen that this $[XX]$ matrix is inverted. However, this may not be always the case for other possible data. For example for the same element in the same environment for MDOT bridges but with an inspection interval 3 years, the $[XX]$ matrix is non-invertible. This $[XX]$ matrix is shown in Figure 5.3 from InfoMaker and its matrix form is shown in Table 5.5 for reference. Apparently, when inadequate data are available, the $[XX]$ matrix becomes not invertible. Lack of data often occurs to State 5 since usually not many bridges or elements are kept at this worst state for a long period of time. Non-invertible $[XX]$ matrix may also occur when the data are not consistent, for example, for a worst state to become better when no work was done.

Figure 5.3: Example InfoMaker Screen Showing [XX] Matrix for Element 12 in Environment 3 for MDOT Bridges with 3-Year Inspection Interval

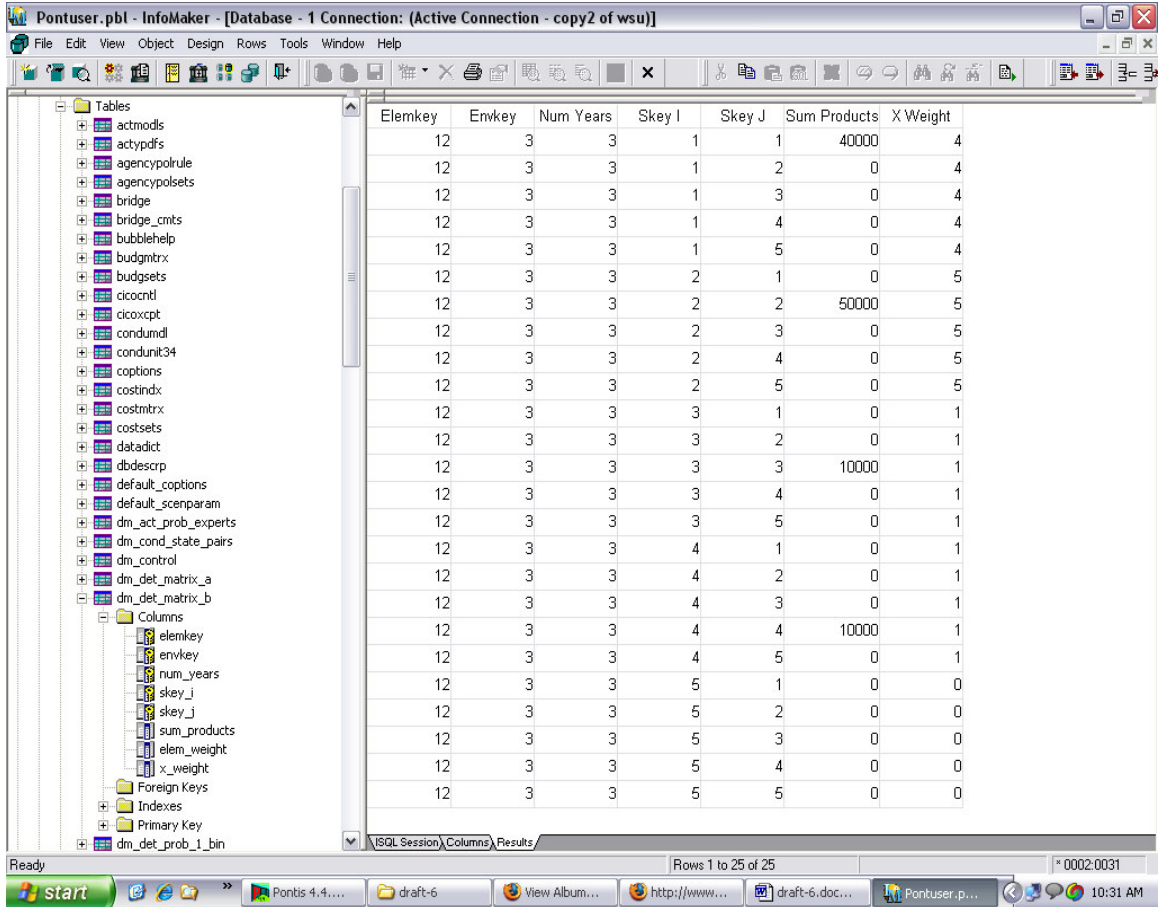


Table 5.5: [XX] Matrix for Element 12 in Environment 3 for MDOT Bridges with 3-Year Inspection Interval

Skey j \ Skey i	1	2	3	4	5
1	40000	0	0	0	0
2	0	50000	0	0	0
3	0	0	10000	0	0
4	0	0	0	10000	0
5	0	0	0	0	0

For the case where $[XX]$ is invertible, i.e., Element 12 in Environment 3 for 2-year inspection interval, Equation (5.13) gives the following result:

$$\begin{aligned}
 \mathbf{P}_e &= [XX]^{-1}[XY] = \\
 &= \begin{bmatrix} 2489282.3 & 354.06744 & 0.000000 & 0.000000 & 0.000000 \\ 354.06744 & 3410010 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1580000.0 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 1030000.0 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 520000.0 \end{bmatrix}^{-1} \begin{bmatrix} 1739937.3 & 56968 & 12000 & 5000 & 1000 \\ 0.000000 & 2910360 & 430000.41 & 6000 & 1000 \\ 0.000000 & 0.000000 & 1169998.5 & 38000 & 3000 \\ 0.000000 & 0.000000 & 0.000000 & 829997.31 & 20000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 519998.91 \end{bmatrix} \\
 &= \begin{bmatrix} 0.0000004 & 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.0000003 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.0000006 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.0000010 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 0.0000019 \end{bmatrix} \begin{bmatrix} 1739937.3 & 56968 & 12000 & 5000 & 1000 \\ 0.000000 & 2910360 & 430000.41 & 6000 & 1000 \\ 0.000000 & 0.000000 & 1169998.5 & 38000 & 3000 \\ 0.000000 & 0.000000 & 0.000000 & 829997.31 & 20000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 & 519998.91 \end{bmatrix} \\
 &= \begin{bmatrix} 0.70 & 0.23 & 0.05 & 0.02 & 0.0 \\ 0.00 & 0.85 & 0.13 & 0.02 & 0.00 \\ 0.00 & 0.00 & 0.74 & 0.24 & 0.02 \\ 0.00 & 0.00 & 0.00 & 0.81 & 0.19 \\ 0.00 & 0.00 & 0.00 & 0.00 & 1.00 \end{bmatrix} \tag{5.15}
 \end{aligned}$$

This estimated transition probability matrix is for time steps with a length of 2 years.

It is seen in Equation (5.15) that p_{13} , p_{14} , p_{15} , p_{24} , p_{25} , p_{35} , p_{21} , p_{31} , p_{32} , p_{41} , p_{42} , p_{43} , p_{51} , p_{52} , p_{53} , and p_{54} , are not necessarily zero. For example $p_{13} = 0.05$, $p_{14} = 0.02$, $p_{24} = 0.02$, $p_{35} = 0.02$. This is because the regression process does not require all these terms to be zero. In addition each row also may not add to 1, again because it is not required in the regression approach used in Pontis.

Nevertheless, after the calculation shown in Equation (5.15), Pontis takes only the diagonal terms for the transition probability matrix, sets a zero to p_{13} , p_{14} , p_{15} , p_{24} , p_{25} , p_{35} ,

$p_{21}, p_{31}, p_{32}, p_{41}, p_{42}, p_{43}, p_{51}, p_{52}, p_{53}, p_{54}$, compute $1 - p_{11}$ as p_{12} , $1 - p_{22}$ as p_{23} , $1 - p_{33}$ as p_{34} , $1 - p_{44}$ as p_{45} , and sets 1 to p_{55} . Now the rows are forced to add to 1.0.

5.2 Combination of Estimated Transition Probability Matrices for Different Time-Steps

In reality, not all “before” and “after” inspections are done with an exactly same constant time difference or interval. For example, bridge inspections may be performed with several months apart to several years apart, although 2 years apart is the norm in the US. Inspection data obtained with different time intervals should not be mixed in one estimation calculation as formulated in Equation (5.13). For example, three one-year transition probability matrices multiplied with each other gives a three-year matrix, which should not be mixed with one-year matrices.

Instead, the data need to be grouped according to the length of inspection interval. For each group with the same inspection interval, Equation (5.13) can be computed, which will result in \mathbf{P} for that particular inspection interval or time step. In order to combine these transition probability matrices estimated using data with different time intervals, Pontis does offer a function to do just that, which is presented below.

Based on the homogeneous Markov Chain concept, the transition probability matrix for n -step (over n time intervals) is defined as the product of n one-step (one-time interval) transition probability matrices:

$$\mathbf{P}^{n^T} = \underbrace{\mathbf{P}^T \mathbf{P}^T \dots \mathbf{P}^T}_{n \text{ matrices multiplied}} \quad (5.16)$$

According to this concept, Pontis determines a transition probability matrix \mathbf{P} for one year as one-step by combining equivalent one-step (one-year) transition matrices. Each of

the equivalent one-step matrices is obtained from an n -step (n -year) matrix. This weighted combination is done one row at a time because the weight for each row of each matrix can be different.

This process can be described as follows

$$[\mathbf{P}]_{\text{row}_i} = [\mathbf{P}_e]_{\text{row}_i} w_{1_i} + [\mathbf{P}_e^2]_{\text{row}_i} w_{2_i} + [\mathbf{P}_e^3]_{\text{row}_i} w_{3_i} + \dots + [\mathbf{P}_e^{10}]_{\text{row}_i} w_{10_i} \quad (5.17)$$

$$(i = 1, 2, 3, 4, 5)$$

where $\mathbf{P}_e, \mathbf{P}_e^2, \mathbf{P}_e^3, \dots$, and \mathbf{P}_e^{10} are the transition probability matrices estimated using inspection data respectively with 1-year, 2-year, 3-year,, and 10-year time intervals. Note that in real data, two inspection dates are never exactly n years apart ($n=1,2,3,\dots$). Thus the real intervals are rounded in the Pontis calculation. In Equation (5.17) w_{1_i}, w_{2_i}, \dots , and w_{10_i} are weights for these 10 matrices and row i respectively.

They should satisfy

$$w_{1_i} + w_{2_i} + w_{3_i} + \dots + w_{10_i} = 1 \quad (5.18)$$

Each of the transition probability matrices in Equation (5.17) for different time intervals can be expressed as follows with their transition probabilities identified:

$$\mathbf{P}_e = \left\{ \begin{array}{ccccc} p_{11} & 1-p_{11} & 0 & 0 & 0 \\ 0 & p_{22} & 1-p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & 1-p_{33} & 0 \\ 0 & 0 & 0 & p_{44} & 1-p_{44} \\ 0 & 0 & 0 & 0 & p_{55} \end{array} \right\} \quad (5.19)$$

$$\mathbf{P}_e^2 = \left\{ \begin{array}{ccccc} \sqrt{p_{11}^2} & 1 - \sqrt{p_{11}^2} & 0 & 0 & 0 \\ 0 & \sqrt{p_{22}^2} & 1 - \sqrt{p_{22}^2} & 0 & 0 \\ 0 & 0 & \sqrt{p_{33}^2} & 1 - \sqrt{p_{33}^2} & 0 \\ 0 & 0 & 0 & \sqrt{p_{44}^2} & 1 - \sqrt{p_{44}^2} \\ 0 & 0 & 0 & 0 & \sqrt{p_{55}^2} \end{array} \right\} \quad (5.20)$$

$$\mathbf{P}_e^3 = \left\{ \begin{array}{ccccc} \sqrt[3]{p_{11}^3} & 1 - \sqrt[3]{p_{11}^3} & 0 & 0 & 0 \\ 0 & \sqrt[3]{p_{22}^3} & 1 - \sqrt[3]{p_{22}^3} & 0 & 0 \\ 0 & 0 & \sqrt[3]{p_{33}^3} & 1 - \sqrt[3]{p_{33}^3} & 0 \\ 0 & 0 & 0 & \sqrt[3]{p_{44}^3} & 1 - \sqrt[3]{p_{44}^3} \\ 0 & 0 & 0 & 0 & \sqrt[3]{p_{55}^3} \end{array} \right\} \quad (5.21)$$

⋮
⋮

$$\mathbf{P}_e^n = \left\{ \begin{array}{ccccc} \sqrt[n]{p_{11}^n} & 1 - \sqrt[n]{p_{11}^n} & 0 & 0 & 0 \\ 0 & \sqrt[n]{p_{22}^n} & 1 - \sqrt[n]{p_{22}^n} & 0 & 0 \\ 0 & 0 & \sqrt[n]{p_{33}^n} & 1 - \sqrt[n]{p_{33}^n} & 0 \\ 0 & 0 & 0 & \sqrt[n]{p_{44}^n} & 1 - \sqrt[n]{p_{44}^n} \\ 0 & 0 & 0 & 0 & \sqrt[n]{p_{55}^n} \end{array} \right\} \quad (5.22)$$

where p_{11} , p_{22} , p_{33} , p_{44} and p_{55} are diagonal terms of the transition probability matrix \mathbf{P}_e estimated using inspection data spanning over one year. The regression procedure described in Section 5.1 (Equation 5.13) is used to find these probabilities. The probabilities p_{11}^2 , p_{22}^2 , p_{33}^2 , p_{44}^2 and p_{55}^2 are obtained using data over 2 years for the “before” and “after” inspections. The exact same procedure in Section 5.1 (Equation 5.13) is supposed to be used to find these terms. Please notice that the superscript “2”

here indicates the time interval of 2 years, and it is not an exponent. Similarly $p_{11}^n, p_{22}^n, p_{33}^n, p_{44}^n$, and p_{55}^n are the same except using data over n years.

It is seen in Equations (5.19) to (5.22) that the diagonal terms of the transition probability matrix for n years are taken n^{th} root for $n = 2, 3, \dots, 10$ to be combined with the one-year matrix as defined in Equation (5.17). Further, all other probabilities in the matrices are set to zero except the ones next to the diagonal terms to the immediate right, as discussed earlier. This is done based on two assumptions: 1) the condition will not improve without repair or rehabilitation; 2) deterioration will not take place in the form of skipping a condition state (i.e., from State 1 to 3, from State 2 to 4, or from State 3 to 5). The second assumption may be true for short time periods such as one or two years, but questionable for longer periods such as 8, 9, and 10 years. Practically, however, this is not a serious concern at this point, because perhaps no bridge was inspected that many years apart. Nevertheless, transition probabilities over 3 or 4 years may very well be non-zero skipping a state. Equation (5.24) shows an example of a 6.0% transition probability for States 1 to 3, 6.8% from State 2 to 4, and 8.3% from States 3 to 5, for Element 12 in Environment 3 of Michigan. Ignoring these (i.e., setting them to zero) apparently will result in a lower deterioration rate.

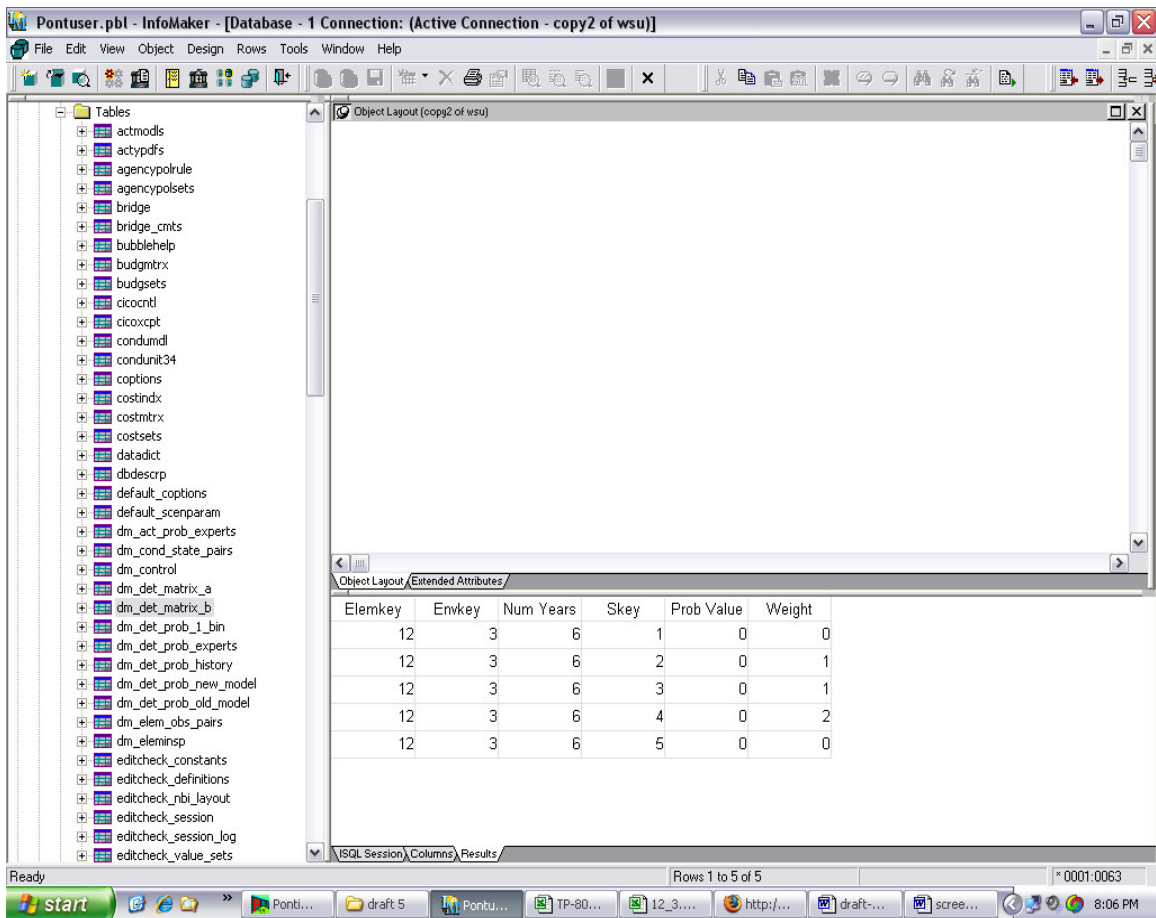
The weights in Equation (5.17) are set in Pontis as follows, depending on the number of data pairs used to estimate $\mathbf{P}_e, \mathbf{P}_e^2, \mathbf{P}_e^3, \dots, \mathbf{P}_e^{10}$ for each row, respectively

$$w_{ji} = \frac{N_{ji}}{\sum_k N_{ki}} \quad i= 1, 2, 3, \dots, 5; \quad j= 1, 2, 3, \dots, 10 \quad (5.23)$$

where N_{ji} is the number of data pairs with j -years apart and transition (deterioration) starting from State i , used to estimate probability matrix \mathbf{P}_e^j . For example, Figure 5.4

shows an example of N_{6i} for \mathbf{P}_e^6 and $i = 1, 2, 3, 4, 5$. The first column “Elemkey” indicates Element 12, “Envkey” for Environment 3, “Num Years” for inspection interval 6 years, “Skey” for row $i = 1, 2, 3, 4, 5$, and “Weight” for N_{6i} in Equation (5.23). In other words, the more data pairs are used for a transition probability matrix, the heavier the resulting matrix will be weighted when combined with other matrices for different years. Also the more data pairs there are for a row in a matrix, that row will be weighted more when combined with the same row in other matrices.

Figure 5.4: Example Infomaker Screen for Combination Weights



Note that the combined resulting transition matrix is to be applied to the same element from all the bridges in the same environment and for all the future years, based on the homogenous Markov Chain assumption discussed in Chapter 3. Since that matrix is for one-step equal to one year, for n -step (n -year) transition, n one-year matrices will be multiplied to obtain the n -year transition probability matrix, according to Equation (5.16). Then it is multiplied by the corresponding initial distribution to find the predicted condition distributions in the future.

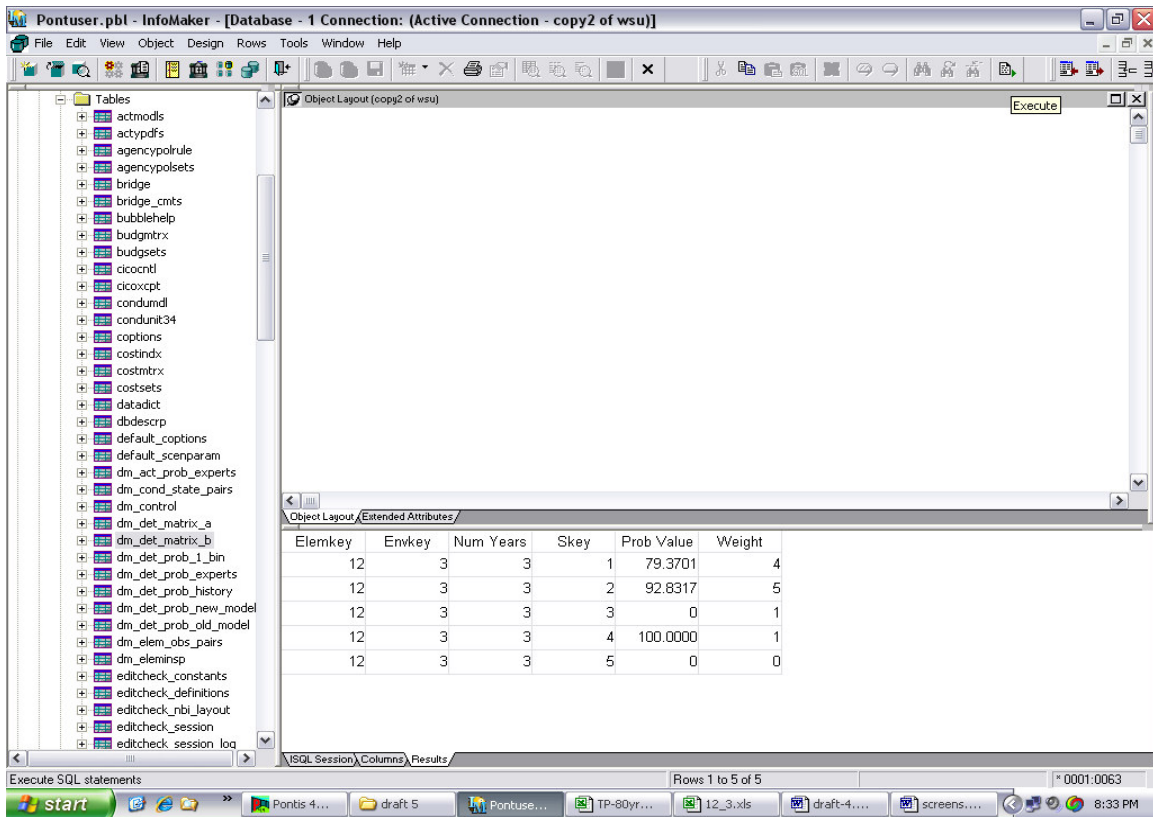
This chapter discusses several issues related to Pontis in estimating the transition probability matrix for “do-nothing” action to be used in bridge element condition prediction. As discussed earlier this transition probability matrix is critical because the predicted long-term deterioration can be significantly affected.

6.1 Non-invertible [XX] Matrix:

Equation (5.13) shows that the [XX] matrix needs to be inverted to obtain the estimated transition probability matrix \mathbf{P}_e . In other words, if [XX] is not invertible, \mathbf{P}_e cannot be found. When this occurs it appears that Pontis sets certain values in the [XX] matrix to make it invertible so that the calculation can proceed. For the example of Element 12 in Environment 3 discussed in Chapter 5 for 3-year inspection interval the [XX] matrix is not invertible. This [XX] matrix was shown in Table 5.5.

Though the [XX] matrix is not invertible Pontis still outputs transition probabilities. Figure 6.1 shows these transition probability values from InfoMaker. The first column “Elemkey” identifies the element number, which is 12 for this case. The second column “Envkey” indicates the environment in which this element is in, and it is 3. The third column “Num Years” presents the number of years between two inspections, in this case 3 years is the inspection interval selected. The fourth column “Skey” refers to the row of condition States (i.e., first row for State 1, second for State 2,, fifth for State 5). The fifth column “Prob Value” gives the transition probability corresponding to the respective “Skey”. The last column “Weight” is respective N_{3i} $i = 1, 2, 3, 4, 5$.

Figure 6.1: InfoMaker Screen Showing Transition Probabilities for Element 12 in Environment 3 for MDOT Bridges with 3-Year Inspection Interval ([XX] Matrix Non-invertible but Transition Probabilities Obtained)



It appears that a 1 is added to p_{55} to obtain the inverse of the $[XX]$ matrix and then the calculation proceeds. This needs to be further investigated to fully understand the procedure used for making the matrix invertible. Further the implications of this procedure to the future distribution prediction also need to be fully understood.

6.2 Negative Transition Probabilities

Another issue with the Pontis approach is that negative transition probabilities may be found in the regression procedure formulated in Equation (5.13). According to

the theory of probability, no probability should be smaller than 0 or negative. However negative transition probabilities may result from Equation (5.13) as shown in the following example.

Consider Element 107 in Environment 1 for MDOT bridges with an inspection interval of 2 years. The InfoMaker screen showing the $[XY]$ matrix for this case is given in Figure 6.2 and the same assembled in the matrix form is shown in Table 6.2.

Figure 6.2: InfoMaker Screen of $[XY]$ Matrix for Element 107 in Environment 1 for MDOT Bridges with 2-Year Inspection Interval

Elemkey	Envkey	Num Years	Skey I	Skey J	Sum Products
107	1	2	1	1	3117124.8
107	1	2	1	2	644948.13
107	1	2	1	3	116792.44
107	1	2	1	4	15121.051
107	1	2	1	5	0
107	1	2	2	1	262242.28
107	1	2	2	2	1589668.5
107	1	2	2	3	289768.38
107	1	2	2	4	43271.488
107	1	2	2	5	1792.6422
107	1	2	3	1	44386.844
107	1	2	3	2	212238.38
107	1	2	3	3	468168.56
107	1	2	3	4	51607.371
107	1	2	3	5	2168.1489
107	1	2	4	1	5422.813
107	1	2	4	2	25704.6
107	1	2	4	3	38155.164
107	1	2	4	4	56548.727
107	1	2	4	5	748.5238
107	1	2	5	1	0
107	1	2	5	2	1421.5549
107	1	2	5	3	1212.417
107	1	2	5	4	731.744
107	1	2	5	5	493.615

Table 6.2: [XY] Matrix for Element 107 in Environment 1 for MDOT Bridges with 2-Year Inspection Interval

Skey j \ Skey i	1	2	3	4	5
1	3117124.8	644948.13	116792.44	15121.051	0
2	262242.28	1589868.5	289768.38	43271.488	1792.6422
3	44386.844	212238.38	468168.56	51607.371	2168.1489
4	5422.813	25704.6	38155.164	56548.727	748.5258
5	0	1421.549	1212.417	731.744	493.615

The [XX] matrix for this example is given in Figure 6.3 and assembled in the matrix form in Table 6.3.

Figure 6.3: InfoMaker Screen of [XX] Matrix for Element 107 in Environment 1 for MDOT Bridges with 2-year Inspection Interval

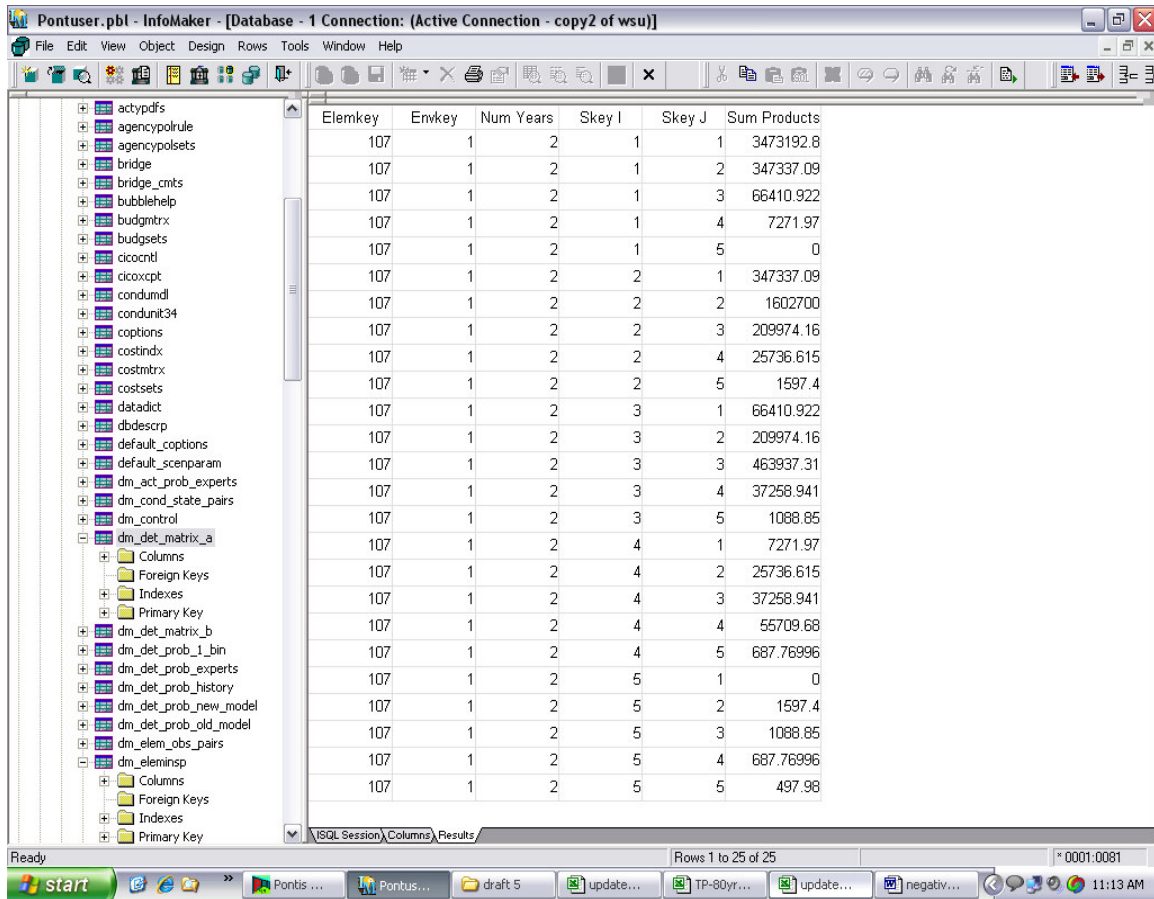


Table 6.3: [XX] Matrix for Element 107 in Environment 1 for MDOT Bridges with 2-Year Inspection Interval

Skey j \ Skey i	1	2	3	4	5
1	3473192.8	347337.09	66410.922	7271.97	0
2	347337.09	1602700	209974.16	25736.615	1597.4
3	66410.922	209974.16	463937.31	37258.941	1088.85
4	7271.97	25736.615	37258.941	55709.68	687.76996
5	0	1597.400	1088.85	687.76996	497.98

For this case the [XX] matrix is invertible and the inverse result is given in Table 6.4 computed using Excel. Transition probabilities for this case is calculated using Equation (5.13) and the results are presented in Table 6.5.

Table 6.4: Inverse Matrix for [XX] Matrix for Element 107 in Environment 1 for MDOT Bridges with 2-Year Inspection Interval

Skey j \ Skey i	1	2	3	4	5
1	0.0000003	-0.0000001	0.0000000	0.0000000	0.0000002
2	-0.0000001	0.0000007	-0.0000003	-0.0000001	-0.0000014
3	0.0000000	-0.0000003	0.0000024	-0.0000014	-0.0000023
4	0.0000000	-0.0000001	-0.0000014	0.0000192	-0.0000231
5	0.0000002	-0.0000014	-0.0000023	-0.0000231	0.0020497

Table 6.5: Transition Probability Matrix for Element 107 in Environment 1 for MDOT Bridges with 2-year Inspection Interval

Skey j \ Skey i	1	2	3	4	5
1	0.90	0.09	0.01	0.00	0.00
2	-0.03	0.97	0.05	0.01	0.00
3	-0.02	0.01	0.98	0.03	0.00
4	0.01	0.00	0.00	0.99	0.00
5	0.13	-0.28	0.12	0.01	0.99

Pontis then takes the diagonal terms 0.90, 0.97, 0.98, 0.99, and 0.99 as p_{11} , p_{22} , p_{33} , p_{44} and p_{55} . Further it calculates $P_{12} = 1 - p_{11}$, $p_{23} = 1 - p_{22}$, $p_{34} = 1 - p_{33}$, $p_{45} = 1 - p_{44}$ and ignores the negative probabilities.

Although it appears that the transition probabilities shown in Pontis are not negative, the negative probabilities shown in Table 6.5 actually have affected the probabilities on the diagonal, because they were obtained from the same process defined in Equation (5.13).

6.3 Possible Non-zero p_{13} , p_{14} , p_{15} , p_{24} , p_{25} , p_{35} , p_{21} , p_{31} , p_{32} , p_{41} , p_{42} , p_{43} , p_{51} , p_{52} , p_{53} , and/or p_{54} values

In Table 6.5, $p_{13} = 0.01$ and $p_{24} = 0.01$. They show an example of non-zero probabilities for transitions skipping a state level, namely from State 1 to State 3 and State 2 to State 4. Recall that we have seen another such case earlier. Apparently, this situation violates one of the assumptions used in Pontis that such skipping transition is impossible. As a matter of fact, data showing such skipping are excluded in Pontis in the stage of valid pair identification i.e., when X and Y are identified in Equation (5.3) and (5.4). However, the regression procedure defined in Equation (5.13) does not eliminate such possibility of having transition probabilities become non-zero.

Furthermore, Table 6.5 shows $p_{32} = 0.01$, $p_{51} = 0.13$, $p_{53} = 0.12$, $p_{54} = 0.01$, which also violates another assumption in Pontis that the probabilities of transition from a worse state to a better states are zero indicating no MR&R action is taken. Again these non-zero values affect the probabilities on the diagonal.

Note also that theoretically the diagonal terms in the estimated transition probability matrix \mathbf{P}_e can become negative, and other off diagonal terms can become non-zero or even negative. It is because the regression process defined in Equation (5.13) does not prevent these possibilities. This, situation may also cause the diagonal terms (p_{11} , p_{22} , p_{33} , p_{44} , p_{55}) taken in Pontis for future prediction to become unreliable. Thus, this is a significant issue related to using the Pontis approach for future condition prediction and the related optimization for MR&R action.

7.1 Optimization Formulation for Estimating Transition Probabilities

The Pontis approach to estimating the homogenous transition probability matrix focuses on the error or difference between the predicted probabilistic conditions and the inspection based conditions (or measured conditions). After formulating this sum of errors for all the inspection data used for estimation, a formal minimization procedure is applied to find the appropriate transition probabilities such that the error sum is reduced to the minimum. This process produces the minimizing transition probability matrix. Again this matrix is constant for all ages of the element.

A similar approach is adopted here for estimating the non-homogeneous transition probability matrices. Namely the error or the difference between the predicted and measured bridge element conditions is minimized. Since the element conditions are associated with a probability distribution, the mean of the distribution is used for this optimization. Namely, estimating the transition probability matrices is accomplished here by minimizing the sum of the differences between the inspection based conditions and the predicted conditions. The prediction process no longer assumes a constant transition probability matrix as for the homogeneous Markov Chain. Instead, the following formulation is developed in this project for estimating or identifying the age dependent (*i.e.*, non-homogeneous) transition probability matrices for each bridge element:

$$\text{Minimize } \sum_{i=1, \dots, N} |Y_i - \text{Predicted } [Y_i, P(A_i)]|^2 \quad (7.1)$$

$$\text{Subject to } \sum_{k=1, 2, \dots, 5} p_{jk}(A_i) = 1 \quad \text{for all } j$$

$$p_{jk}(A_i) \geq 0 \quad \text{for all } k \text{ and } j$$

where N = total number of condition transitions used (*i.e.*, the number of data pairs used); Y_i = condition state vector right after the i th transition (for i th pair of transition data); Predicted $[Y_i, P(A_i)]$ = Predicted condition state vector for the same element involved in the i th transition, using the transition probability matrix $P(A_i)$ depending on the element's age A_i . The symbol $|x|$ means the magnitude or modulus of vector x . The transition probabilities p are the elements of the matrix P . The conditions for them to meet in Equation (7.1) are there to satisfy Equation (3.8) for consistency. These conditions are met in Pontis, rather, after the transition probability matrix is found, by simply setting p values equal to 0 if obtained negative, or the diagonal terms 1.0 if obtained to be larger than 1.0.

Note also that the Markov Chain model used here is more general than the Pontis' homogeneous model, for its non-homogeneity. Therefore, the transition probability matrices $P(A)$ are shown as functions of age A . It means that $P(A_i)$ can be different according to age A_i of the element involved in transition i . For application to bridge management focused herein, we consider only the effect of age, not the absolute time. This can be seen more easily in a simple application example in the next section.

7.2 A Simple Illustrative Example

Consider Element 215 (reinforced concrete abutment) in Environment 1 for the MDOT bridges with an inspection interval of 2 years. So $t_n - t_{n-1}$ is 2 years for this example. A sample set of 3 pairs of condition transition data for this element is used in this example for illustration, as displayed in Table 7.1. For this element, the total number of condition states is 4 (i.e., $S=4$).

Table 7.1 Sample Data for Element 215 in Environment 1 for MDOT

Bridge Key	age-x	X_1	X_2	X_3	X_4	age-y	Y_1	Y_2	Y_3	Y_4
16116021000B010	3	1.00	0.00	0.00	0.00	5	0.93	0.07	0.00	0.00
15115051000B010	36	0.94	0.06	0.00	0.00	38	0.89	0.11	0.00	0.00
05105011000B010	42	0.95	0.05	0.00	0.00	44	0.84	0.16	0.00	0.00

In this table, column “Bridge Key” is the identification for each bridge. Column “age-x” records the age of the element at time t_{n-1} for this case (Year 1995), and similarly “age-y” the age at time t_n (Year 1997). Column X_1 shows the percentage of element 215 in Condition State 1 in the bridge at time t_{n-1} (Year 1995). Similarly, X_2 , X_3 , and X_4 are the percentages of the element in Condition States 2, 3, and 4 respectively at time t_{n-1} , and Y_1 , Y_2 , Y_3 , and Y_4 at time t_n (Year 1997).

For easy illustration without loss of generality, it is assumed that for the age range of 0 to 20 years, the transition probability matrices for every two years are constant and designated as U_{0-20} . The subscript 0-20 indicates the applicable age range. For the age range of 21 to 40 years, another constant transition probability matrix V_{21-40} is used to

model the transition (deterioration) for every 2 years in the same format. For the age range of 41 and older, a third transition probability matrix W_{41-} is used to model transition (deterioration) for every two years also in the same format but without specifying the ending age. Since the sample data set includes bridges only up to 42 years of age, no further transition probability matrices could be used. Otherwise more matrices would be considered and used for modeling. The indefinite upper age limit for W_{41-} means that for ages older than 41 years, this transition probability matrix is to be used for prediction. Accordingly, Equation (7.1) is specifically formulated as follows for $N=3$:

$$\begin{aligned}
 & \text{Minimize } | \{0.93, 0.07, 0.00, 0.00\} - \{1.00, 0.00, 0.00, 0.00\} U_{0-20} |^2 \\
 & \quad + | \{0.89, 0.11, 0.00, 0.00\} - \{0.94, 0.06, 0.00, 0.00\} V_{21-40} |^2 \\
 & \quad + | \{0.84, 0.16, 0.00, 0.00\} - \{0.95, 0.05, 0.00, 0.00\} W_{41-} |^2
 \end{aligned} \tag{7.2}$$

$$\text{Subject to } \sum_{k=1,2,\dots,4} u_{jk,0-20} = 1 \quad \text{for all } j, \quad \text{and} \quad u_{jk,0-20} \geq 0 \quad \text{for all } k \text{ and } j$$

$$\sum_{k=1,2,\dots,4} v_{jk,21-40} = 1 \quad \text{for all } j, \quad \text{and} \quad v_{jk,21-40} \geq 0 \quad \text{for all } k \text{ and } j$$

$$\sum_{k=1,2,\dots,4} w_{jk,41-} = 1 \quad \text{for all } j, \quad \text{and} \quad w_{jk,41-} \geq 0 \quad \text{for all } k \text{ and } j$$

The lower case non-bolded letters u , v , and w are the transition probabilities in the matrices U_{0-20} , V_{21-40} , and W_{41-} , with the age applicability ranges also indicated. The first term in the magnitude signs $| \cdot |$ is associated with the first bridge “16116021000B010”, where the first vector $\{0.93, 0.07, 0.00, 0.00\}$ is the probability distribution of the

element condition for this bridge after the transition at time t_n (Year 1997). The second half in the magnitude signs (*i.e.*, the terms after the minus sign) is a product of a vector and the transition probability matrix. The vector in the big brackets is the probability distribution of the element condition of that bridge at time t_{n-1} (Year 1995). This vector multiplied with the transition probability matrix U_{0-20} gives the predicted probability distribution of the element condition state at time t_n (Year 1997). The following two magnitudes squared have the same physical meaning and structure as the first one but for the other two bridges. Then the transition probability matrices U_{0-20} , V_{21-40} , and W_{41-} are to be found to have the sum of the differences or errors minimized. In this example, the following probabilities are pre-set for impossible transitions and consistency, respectively.

$$\begin{aligned}
 u_{jk,0-20} &= 0 & \text{if } j > k; & & u_{jk,0-20} &= I - u_{jj,0-20} & \text{if } k-j=I; & & 0 & \text{otherwise} \\
 v_{jk,21-40} &= 0 & \text{if } j > k; & & v_{jk,21-40} &= I - v_{jj,21-40} & \text{if } k-j=I; & & 0 & \text{otherwise} \\
 w_{jk,41-} &= 0 & \text{if } j > k; & & w_{jk,41-} &= I - w_{jj,41-} & \text{if } k-j=I; & & 0 & \text{otherwise} \\
 u_{44,0-20} &= v_{44,21-40} &= w_{44,41-} &= I & & & & & & (7.3)
 \end{aligned}$$

This makes only the first three diagonal terms in each of the three transition probability matrices unknown to be found in the minimization process: $u_{11,0-20}$; $u_{22,0-20}$; $u_{33,0-20}$; $v_{11,21-40}$; $v_{22,21-40}$; $v_{33,21-40}$; $w_{11,41-}$; $w_{22,41-}$; $w_{33,41-}$.

As noted earlier, Equation (7.2) is a simple example of Equation (7.1) in that the transition probability matrices U_{0-20} , V_{21-40} , and W_{41-} are constants respectively for age ranges 0 to 20 years, 21 to 40 years, and 41 years and older. In other words, the transition rates or deterioration rates are assumed constant within each of the specified

age ranges, but variable for the entire service life of the element. In general, the transition probabilities over each of the 20-year or longer periods may not be very constant. The model in Equation (7.1), nevertheless, allows more general cases of one transition probability matrix for each time unit of interest. On the other hand, the number of unknown transition probabilities to be found will accordingly increase and the required computation effort. However, for certain bridge elements with low deterioration rates (*i.e.*, those with long service lives), assuming a constant transition probability matrix for a number of years is not unreasonable. It is because the rate (or probability) of transition from a better state to a worse state (or the rate of deterioration) for this situation is relatively low, and thus variation of this rate from year to year is not significant. Therefore a constant transition probability matrix for an age range may be realistic and practical.

It may be also interesting to point out that the Pontis approach based on homogeneous Markov Chain actually can be viewed as a special case of non-homogeneous Markov Chain, with only one constant transition probability matrix for all time periods over the entire life span of the element.

7.3 An Application Example

In this application example, the same Element 215 is used. The inspection data of the element are taken from the MDOT Pontis database. It is well known that high quality of input data can never be over-emphasized. We thus have examined the data cleaning procedure used in Pontis and found that the process is not consistent enough because it misses invalid pairs. We have thus developed a more rigorous data cleaning procedure,

which has been used here to screen out invalid data pairs that were not identified by the Pontis screening process. A typical example is presented below for illustration:

Table 7.2: Condition State Distribution for Element 215 in MDOT Bridge
27127022000B030 in Environment 1

$X^{(0)}$	$x_1^{(0)}$	$x_2^{(0)}$	$x_3^{(0)}$	$x_4^{(0)}$
In %	77.778	14.815	7.407	0
$Y^{(1)}$	$y_1^{(1)}$	$y_2^{(1)}$	$y_3^{(1)}$	$y_4^{(1)}$
In %	0	96.296	3.704	0

In this table, as indicated earlier in this report, vectors $X^{(0)}$ and $Y^{(1)}$ are the condition state distribution in the previous year and the current years respectively. Note that the sum of $x_1^{(0)}$ and $x_2^{(0)}$ for condition states 1 and 2 should be greater than or equal to the sum of $y_1^{(1)}$ and $y_2^{(1)}$, because the latter two quantities can only come from the former two for the do-nothing situation. However, it is not true as seen in Table 7.2, as the sum of $x_1^{(0)}$ and $x_2^{(0)}$ is 92.593, which is less than the sum of $y_1^{(1)}$ and $y_2^{(1)}$, 96.296, in Table 7.2. There is a number of possible causes for the sum of $y_1^{(1)}$ and $y_2^{(1)}$ to be larger than that of $x_1^{(0)}$ and $x_2^{(0)}$. 1) Simple error of input such as typo. 2) Some quantity from $x_3^{(0)}$ has moved to $y_2^{(1)}$ for improvement in condition, which violates the “Do-Nothing” assumption. Our screening algorithm targets at such observed inconsistencies.

In this first step of development work, to avoid the issue of collectively using data with different inspection intervals and weighting them differently, only the pairs with 2-year inspection interval for Element 215 are used here in this example. So $t_n - t_{n-1} = 2$ years is used with approximation of rounding. Equation (7.1), as specified in Equation

(7.3), is applied, except that more data pairs are included, using all available MDOT data up to Year 2000.

It should be noted that Equation (7.1) for estimating non-homogeneous transition probability matrices can easily accommodate data obtained with different inspection intervals, such as 2, 3, or any other number of years. In Pontis, in contrast, data from unevenly intervalled inspections are grouped according to the inspection interval, because mixing them would cause unacceptable approximation. Namely, data with 1-year intervals (with rounding) are put in one group, 2-year intervals (also with rounding) in another group, and so on. Each group is used separately for estimating the transition probability matrix for that group. Then the resulting matrices for different groups are averaged with weights proportional to the number of data pairs used in each group. This approach also requires critical examination, because longer intervalled inspection data usually are more reliable than those shorter intervalled. For example, a data pair intervalled by one year is usually not as good in quality as another pair separated for longer time. It is because for shorter intervals, the condition change is very little and the inspection results may not reflect such fine changes. In the Pontis approach, however, this higher quality data pair intervalled longer is treated the same way as another pair intervalled more closely.

In applying Equation (7.1) to this data set, three transition probability matrices are used to model the element's deterioration: U_{0-20} , V_{21-40} , and W_{41-} , as done in the small problem shown previously. They respectively cover age ranges of 0 to 20 years, 21 to 40 years, and 41 years and beyond.

Using inspection data up to Year 2000, Table 7.3 shows the resulting transition matrices U_{0-20} , V_{21-40} , and W_{41-} for Environment 1 in Michigan, compared with the constant transition probability matrix obtained using the Pontis approach. According to Equation (7.3) only the 9 probabilities on the diagonal are the unknowns. Table 7.3 displays these 9 terms, along with the last terms in the probability matrices ($u_{44}=v_{44}=w_{44}=1$). Since this last term is not treated as an unknown for the non-homogeneous Markov Chain approach, it is shown lightened.

It is seen in Table 7.3 that the Pontis transition probabilities are mostly between the maximum and minimum values of those in the matrices U_{0-20} , V_{21-40} , and W_{41-} . This actually shows the essence of the homogeneous Markov Chain application here: modeling a non-homogeneous Markov Chain with compromise. Nevertheless, constrained by the homogeneity assumption, it would not be able to realistically model the non-homogeneous stochastic process. For projecting to a far future, the difference between the two approaches can be significant.

For other three environments, similar comparisons are observed, showing the Pontis obtained transition matrices to be a compromise to fit the data. This is seen more clear in Table 7.4 when the non-homogeneous Markov Chain approach used 6 transition probability matrices U , V , W , X , Y , and Z (instead of three tried first) to respectively cover age ranges of 0-10, 11-20, 21-30, 31-40, 41-50, and 51 years and older.

Table 7.3 Comparison of Transition Probability Matrices between the Proposed Method Using 3 Matrices, Pontis Approach, and Arithmetic Method for Element 215 in Environment 1

	p_{11}	p_{22}	p_{33}	p_{44}
U_{0-20}	0.977	0.986	1.000	1.000
V_{21-40}	0.968	0.986	0.941	1.000
W_{41-}	0.960	0.982	1.000	1.000
Pontis	0.963	0.985	0.990	1.000
Arithmetic	0.962	0.963	1.000	1.000

Table 7.4 Comparison of Transition Probability Matrices between the Proposed Method Using 6 Matrices, Pontis Approach, and Arithmetic Method for Element 215 in Environment 1

	p_{11}	p_{22}	p_{33}	p_{44}
U_{0-10}	0.972	0.989	0.995	1.000
V_{11-20}	0.989	0.980	0.990	1.000
W_{21-30}	0.973	0.959	0.990	1.000
X_{31-40}	0.966	0.992	0.978	1.000
Y_{41-50}	0.949	1.000	0.990	1.000
Z_{51-}	0.967	0.976	0.999	1.000
Pontis	0.963	0.985	0.990	1.000
Arithmetic	0.962	0.963	1.000	1.000

To evaluate the proposed non-homogeneous Markov Chain approach, the resulting transition probabilities are used to predict the element's immediate future

distribution vector at the network level. Then this predicted distribution is compared with the measured distribution vector using the inspection data. A relative error is then calculated to quantitatively evaluate the effectiveness of the prediction. It is important to note that this future distribution vector based on inspection results was not used in the process of estimation for the transition probabilities. Essentially, this evaluation simulates a practical application of using the latest inspection data to predict the future bridge condition at the network level. Tables 7.5 and 7.6 display the results for this evaluation between the proposed approach and the Pontis approach, using the Element 215 data of MDOT. Largely as expected, the proposed approach has produced smaller errors, mainly due to the higher modeling resolution using several transition probability matrices for the entire life span of the element. This is also seen more clearly when comparing the results of Table 7.5 to those in Table 7.6. It is seen that using 6 transition probability matrices (each covering about 10 years except the last matrix) has performed generally better than using 3 matrices (each covering about 20 years except the last matrix).

Note also that when Year 2002 data are used for evaluation, inspection data up to Year 2000 are used for estimating the transition probability matrices. When Year 2004 data are used for evaluation, inspection data up to Year 2002 are used for the estimation. Again this is to simulate a case of realistic practice of bridge management. These cases are indicated in Tables 7.5 and 7.6.

For comparison, the arithmetic method presented earlier is also applied to this example, and the results are included in both Tables 7.5 and 7.6. It is seen that that method did not perform as well as the Pontis and the proposed approaches. It is

understood that the main reason for this performance is its lack of statistical foundation of the arithmetic method. That method simply “fits” the data into the assumed structure of the transition probability matrix. Therefore, it cannot be expected to predict the future behavior of the element’s deterioration.

Table 7.5 Comparison of Errors (in %) for Proposed Method Using 3 Matrices, Pontis Approach, and Arithmetic Method – Element 215

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	2.11	3.05	Env3	Proposed Approach	5.67	1.01
	Pontis Approach	2.89	3.88		Pontis Approach	7.11	2.03
	Arithmetic Method	2.72	4.07		Arithmetic Method	7.54	2.19
Env2	Proposed Approach	3.12	3.95	Env4	Proposed Approach	1.95	6.10
	Pontis Approach	3.83	4.48		Pontis Approach	2.82	6.97
	Arithmetic Method	3.80	4.27		Arithmetic Method	3.7	7.44

Table 7.6 Comparison of Errors (in %) for Proposed Method Using 6 Matrices, Pontis Approach, and Arithmetic Method – Element 215

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	2.04	3.00	Env3	Proposed Approach	6.81	0.94
	Pontis Approach	2.89	3.88		Pontis Approach	7.11	2.03
	Arithmetic Method	2.72	4.07		Arithmetic Method	7.54	2.19
Env2	Proposed Approach	2.89	2.99	Env4	Proposed Approach	1.50	5.81
	Pontis Approach	3.83	4.48		Pontis Approach	2.82	6.97
	Arithmetic Method	3.80	4.27		Arithmetic Method	3.7	7.44

It is worth mentioning that, again, more transition probability matrices can be used in the proposed non-homogeneous Markov Chain model to improve modeling when warranted. Of course, this will increase the requirement for computation effort.

It is also important to note that the Pontis approach for estimating the transition probability matrix may cause a probability value to become negative and the sum of a row not to add to 1, especially when the number of valid data pairs is small. This problem is completely avoided or resolved in formulating the optimization problem of Equation (7.1), shown in Equation (7.3) for this example as the constraints for minimization. The condition of Equation (3.8) (i.e., non negative probabilities summed to 1.0 for each row) can be enforced as in Equation (7.3). The optimization process as formulated in Equation (7.1) then will not produce those violating values as solutions. Note that the Pontis approach, though, is different for enforcing these constraints. It rather solves a least square fitting problem by minimizing the squared error sum without avoiding these violating values. When such values do result, they are simply deleted and then replaced by artificially determined values. Therefore, the Pontis approach is not expected to produce reliable results every time when applied. This is seen in the results in Tables 7.5 and 7.6. It is also seen in the results for more elements presented next.

7.4 More Application Examples and Discussions

More elements are used in this section for application of the proposed non-homogeneous Markov Chain modeling and prediction. They include Elements 104, 106, and 210. The results will have both the three and six matrix cases as done above. Tables 7.7 and 7.8 show the results for Element 104 - Precast Prestressed Box Beams.

Table 7.7 Comparison of Errors (in %) for Proposed Method Using 3 Matrices, Pontis Approach, and Arithmetic Method – Element 104

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	0.06	0.01	Env3	Proposed Approach	0.69	0.35
	Pontis Approach	0.06	0.35		Pontis Approach	1.85	1.04
	Arithmetic Method	0.06	0.35		Arithmetic Method	1.88	1.04
Env2	Proposed Approach	0.26	0.07	Env4	Proposed Approach	0.48	0.34
	Pontis Approach	0.78	0.03		Pontis Approach	0.44	0.36
	Arithmetic Method	1.08	0.04		Arithmetic Method	0.44	0.44

It is shown in Table 7.7 that the first set of results for Environment 1 using year 2002 data for evaluation involves the same error for all three approaches used. The reason for this is that there is no deterioration for all the data points (data pairs) used for estimating the transition probabilities. To be exact, there are 38 data points for this environment, and all of them have 100% of the element in State 1 before and after inspection. Thus the estimations of the transition probabilities or their interpretations using these different methods actually lead to the same result. In other words, the data show no deterioration, and these different methods have consistent interpretation for the underlying non-deteriorating mechanism. The 0.06 % error in Table 7.7 is simply due to the inconsistency of the future (Year 2002) data with those used in estimating the transition probabilities.

Table 7.8 displays the same comparison but using 6 different transition matrices in the proposed method based on a non-homogeneous Markov Chain. In general, these results show improvement from those in Table 7.7 with reduced errors for the proposed method.

Table 7.8 Comparison of Errors (in %) for Proposed Method Using 6 Matrices, Pontis Approaches, and Arithmetic Method - Element 104

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	0.06	0.02	Env3	Proposed Approach	0.27	0.01
	Pontis Approach	0.06	0.35		Pontis Approach	1.85	1.04
	Arithmetic Method	0.06	0.35		Arithmetic Method	1.88	1.04
Env2	Proposed Approach	0.27	0.10	Env4	Proposed Approach	0.46	0.17
	Pontis Approach	0.78	0.03		Pontis Approach	0.44	0.36
	Arithmetic Method	1.08	0.04		Arithmetic Method	0.44	0.44

Table 7.9 Comparison of Errors (in %) for Proposed Method Using 3 Matrices, Pontis Approach, and Arithmetic Method – Element 210

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	6.73	0.74	Env3	Proposed Approach	2.62	0.64
	Pontis Approach	6.72	8.54		Pontis Approach	5.24	2.04
	Arithmetic Method	6.72	7.22		Arithmetic Method	4.81	1.35
Env2	Proposed Approach	3.89	1.50	Env4	Proposed Approach	6.0	4.60
	Pontis Approach	3.14	3.27		Pontis Approach	7.43	7.56
	Arithmetic Method	3.44	3.53		Arithmetic Method	7.46	9.50

Table 7.9 shows the results for another Element 210 - Reinforced Concrete Pier Wall. It highlights, the vulnerability of the arithmetic method with consistently larger errors than the other two methods for all four cases of environment. As mentioned earlier, this vulnerability is due to lack of a statistical basis for the arithmetic method.

Focusing on the proposed method, it is seen that except for Environments 1 and 2 using year 2002 data for evaluation, the proposed method is shown performing more reliably than the Pontis approach. Even in these cases, the errors for the proposed method are not significantly larger (6.73% versus 6.72% and 3.89% versus 3.14%).

Table 7.10 includes the results using 6 transition probability matrix for the proposed method, for further taking advantage of the non-homogeneous Markov Chain model. It is seen that, by comparison of Tables 7.8 and 7.9, using more matrices helps in reducing error for more cases. This also highlights the advantage of the proposed method in its flexibility for different data sets or its ability to treat a variety of situations.

Table 7.10 Comparison of Errors (in %) for Proposed Method Using 6 Matrices, Pontis Approach, and Arithmetic Method – Element 210

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	6.56	1.20	Env3	Proposed Approach	2.98	0.80
	Pontis Approach	6.72	8.54		Pontis Approach	5.24	2.04
	Arithmetic Method	6.72	7.22		Arithmetic Method	4.81	1.35
Env2	Proposed Approach	3.26	1.22	Env4	Proposed Approach	7.93	4.43
	Pontis Approach	3.14	3.27		Pontis Approach	7.43	7.56
	Arithmetic Method	3.44	3.53		Arithmetic Method	7.46	9.50

In the final part of this section, the results for Element 106 – Steel Girder/Beams Not Painted are presented. Both cases of using 3 and 6 matrices for the non-homogeneous method are included.

Table 7.11 Comparison of Errors (in %) for Proposed Method Using 3 Matrices, Pontis Approaches, and Arithmetic Method – Element 106

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	ND	ND	Env3	Proposed Approach	1.23	11.60
	Pontis Approach	ND	ND		Pontis Approach	16.24	12.78
	Arithmetic Method	ND	ND		Arithmetic Method	14.29	11.69
Env2	Proposed Approach	4.59	5.56	Env4	Proposed Approach	40.65	26.12
	Pontis Approach	7.59	13.04		Pontis Approach	45.09	30.30
	Arithmetic Method	12.14	13.82		Arithmetic Method	40.49	29.36

ND= No data available

Table 7.12 Comparison of Errors (in %) for Proposed Non-homogeneous Markov Chain Using 6 Matrices, Pontis Approaches, and Arithmetic Method – Element 106

		Using 2002 data for evaluation	Using 2004 data for evaluation			Using 2002 data for evaluation	Using 2004 data for evaluation
Env1	Proposed Approach	ND	ND	Env3	Proposed Approach	1.23	11.60
	Pontis Approach	ND	ND		Pontis Approach	16.24	12.78
	Arithmetic Method	ND	ND		Arithmetic Method	14.29	11.69
Env2	Proposed Approach	4.59	5.56	Env4	Proposed Approach	41.45	27.16
	Pontis Approach	7.59	13.04		Pontis Approach	45.09	30.30
	Arithmetic Method	12.14	13.82		Arithmetic Method	40.49	29.36

ND - No data available

It is indicated in Tables 7.11 and 7.12 that the first set of results for Evaluation 1 of year 2002 is not given due to lack of data. For the remaining cases, it is seen that the proposed method performed better than the Pontis approach, showing smaller errors for prediction. These examples show that the proposed method is more reliable, compared with the other two.

This research effort has gathered information on the current state of art and practice of bridge management system in the US. Most state agencies use Pontis, although the level of experience varies. The most experienced states have collected more than 10 years of condition inspection data. The level of satisfaction with Pontis also varies among the states. A critical issue is the estimation of the transition probability matrices, which describe or model the deterioration of bridge elements.

The Pontis bridge management system has been reviewed in this study to present the technical background and identify areas for improvement. In summary, they include 1) possible negative transition probabilities although set to zero when found through the regression estimation process; 2) possible larger than 1 transition probabilities, set to 1 when found; 3) inadequate consistency-screening of raw data for more reliable estimation results; and 4) assumed homogeneity of Markov Chain.

A non-homogeneous Markov Chain model has been developed in this study and proposed in this report, for improving modeling element deterioration. The homogeneous Markov Chain model used in Pontis is a special case of this new model and approach. Application examples show that this new method can better predict bridge element deterioration trends.

References

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- [2] José J. Ortiz-García; Seósamh B. Costello, and Martin S. Snaith (2006), “*Derivation of Transition Probability Matrices for Pavement Deterioration Modeling,*” Journal of Transportation Engineering, Vol. 132, No. 2, Feb 1, 2006.
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- [6] AASHTO Pontis Technical Manual, Release 4.4

Appendix

Questionnaire – State use of Bridge Management Systems

Dear Colleagues:

Wayne State University is assisting the Michigan Department of Transportation (MDOT) calibrate tools and input data for their Bridge Management System (BMS). MDOT uses National Bridge Inspection (NBI) data, AASHTO Commonly Recognized (CoRe) Elements, and Michigan specific elements in their BMS. MDOT is a user of the AASHTOWARE Pontis software. MDOT has been working on implementation of BMS for several years and is facing a number of tasks and issues in the process. We would like to learn your experience in these areas to benchmark. We would be very much grateful if you could kindly complete the following questionnaire and return it by e-mail, fax, or US mail to:

Dr. Gongkang Fu, PE, Professor and Director,
Center for Advanced Bridge Engineering
Department of Civil and Environmental Engineering
Wayne State University; voice: 313-577-3842;
Fax: 313-577-3881; gfu@eng.wayne.edu

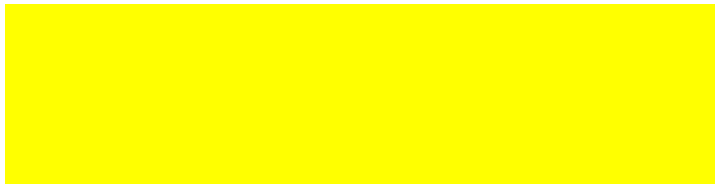
If you have questions about this survey you can also contact:

Dave Juntunen
Engineer of Bridge Operations
Michigan Department of Transportation
Phone (517) 322-5688
E-mail: juntunend@michigan.gov

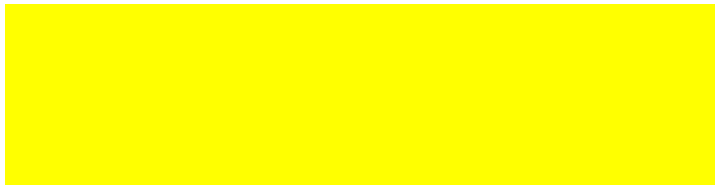
MDOT will gladly share the results of this questionnaire upon request.

Please return this questionnaire by **May 31, 2006.**

Name:
Title:
Organization:
Phone:
E-mail:



Name:
Title:
Organization:
Phone:
E-mail:



In case you are unable to answer some of these questions, you may leave them unanswered, but please return this questionnaire with the above section filled. Thank you!

I. General Questions

I-1. Which BMS does your agency use? Pontis , BRIDGIT , an in-house system (give name) _____, other system (give name) _____.

I-2. Approximately how many years of bridge condition data (inspection and/or asset management data) does your agency have in your database?
0 to 1 , 1 to 3 , 4 to 10 , more than 10 .

I-3. What bridge condition data are used within your BMS? NBI , CoRe (Pontis) , Other , (specify):

I-4. If your agency is a Pontis user, have you made modifications to the AASHTO CoRe elements, and/or have you added additional elements? Yes No . If yes, please list modified or additional elements, and mark those that have proven to be useful (If you have more changes or additions that can fit here, please indicate in the last text box):
- _____
- _____
- _____
- _____
- _____
- _____

I-5. Has you agency developed bridge preservation policies, for maintenance, rehabilitation, and repair (MRR)? Yes . No . If yes, please provide a copy of the policies and/or describe how the policies were developed:

I-6. What cost data do you use to determine cost parameters for projects in your BMS? Past bid prices for your agency . Other (please specify and discuss any issues you may be having).

I-7. Do you use deterioration rates based on transition probabilities? Yes No . If yes, how?

[Redacted]

I-8. If you are a Pontis user, are you satisfied with the resulting transition probabilities or deterioration rates (Do you think they model the situation realistically)? Yes . No . Partially . If not yes, why?

[Redacted]

I-9. How do you determine the transition probabilities or deterioration rates for a bridge element? Use historic data only Use expert elicitation only . Use historic data and expert elicitation. Other (please specify)

[Redacted]

I-10. Has your agency compared your BMS with your traditional approach for bridge management decision making? Yes No . If yes, please describe your comparison results.

[Redacted]

I-11. Do you think your agency's BMS fully meets your need for bridge management? Yes No If no, what enhancements would you like to have?

[Redacted]

I-12. Please describe how your agency determines the discount rate for project cost projection to the future.

[Redacted]

I-13. How do you perform rulemaking and project prioritization within your BMS?

[Redacted]

II. Additional Information and Comments

II-1. If you are aware of any effort spent towards improving BMS, please kindly provide contact information below to allow us to have access to the information. Add more sheets if needed.

Subject:
Name:
Organization:
Phone:
e-mail:



Subject:
Name:
Organization:
Phone:
E-mail:



II-2. If you have any further comments/questions relevant to this questionnaire or this synthesis topic, please add them here.



II-3. Would you like to receive a copy of the questionnaire results? Yes , No

***You have completed this survey. Remember to save your work (We recommend that you add your state name to the file name)
Please return the survey **May 31, 2006.**
Thank you very much!***