MICHIGAN
STATE HIGHWAY DEPARTMENT

THE DESIGN

of

CONCRETE PAVEMENTS

for

POST WAR CONSTRUCTION

Research Laboratory
Testing and Research Division
March 15, 1945

FOREWORD

The following text is a reproduction of part one of an original copy of Research Laboratory Report 68a titled "The Design of Concrete Pavements for Post War Construction" approved by the Department in April, 1945, for use in relation to the post war construction of concrete pavements in Michigan.

The original report also includes material on joint spacing and joint design covered in parts 2 and 3 respectively. These parts were not included here because it was believed that their contents were not pertinent to the matter of determining slab thickness.

This material is intended to supplement the narrative text prefacing the Department's reply to the Bureau of Public Roads, Inquiry No. 7 pertaining to the "Incremental Design Standards and Cost Factors for Road Construction" designed to implement the studies required by Section 210 of the Highway Revenue Act of 1956.

INTRODUCTION

The concrete pavement design presented in this report has been made with due consideration to all known requirements for a modern highway and is based on principles established by theory, research and experience. The recommendations contained herein have been prepared for consideration in connection with Michigan's post-war highway construction program.

The design as developed in the report recognizes four distinct classes of highways based on traffic conditions and embodies other features in pavement construction in keeping with the modern trend. The design features are briefly summarized as follows:

1. Highway Classes: I Expressway, II Heavy Primary, III Light Primary and IV Secondary.
2. The pavement will be of uniform thickness and properly reinforced.
3. Slab thickness according to class is respectively 9-10, 9-10, 7-9 and 6-8 inches.
4. Slab length between breaks in reinforcement is 100 feet. No intermediate joints.
5. Weight of reinforcement per 100 square feet for each thickness will be approximately [illegible], [illegible], 185 and [illegible] pounds respectively.
6. Expansion joints will not be installed during summer construction except at structures, intersections and at other locations specified. During fall construction, commencing September 15th, expansion joints will be placed at intervals of not less than 400 feet or not greater than that of one days pour as directed by the Engineer. #1
7. Wood boards preformed one inch wide will be used for expansion joint filler.
8. Contraction joints will be installed at 100 foot intervals.
9. Contraction joints will be formed by a groove 1/2 inch wide and 2 inches deep in the surface, and in addition, a metal parting strip 1 inch high will be installed at the bottom of the joint.
10. A galvanized metal shield will be required at the bottom and edges of joints to prevent infiltration of foreign matter. It will be fabricated in conjunction with the dowel bar assembly.

11. Dowel bars, 1-1/4" x 15" spaced at 12 inch centers will be required at all transverse joints for the prevention of faulting.

12. All joints will be sealed with an asphaltic-latex joint sealing compound.

The selection of this design was influenced by the facts that: (1) every joint is potentially a source of structural weakness contributing to the ultimate deterioration of the pavement accompanied by increasing maintenance costs, (2) it is possible to design a concrete pavement slab up to 200 feet and possibly more in length of sufficient strength to withstand combined warping, load and friction stresses for any predictable traffic condition and subgrade support satisfying certain assumptions; (3) the service performance of 100 foot slabs constructed ten or more years ago on suitable subgrades has been very satisfactory. It would appear then to be good engineering practice to reduce the number of joints to an absolute minimum consistent with sound design practices and economic considerations.

The report has been presented in three parts. Part I explains in detail the method of determining cross-section thickness by stress analysis taking into account the principle of fatigue. Part II presents a discussion of several important factors which influence transverse joint spacing substantiated by theoretical considerations and the results from observational studies, including slab movement measurements on the Michigan Test Road. Part III covers the design of transverse joints including provision for slab movement, the exclusion of foreign matter and the design and spacing of load transfer devices. Detailed conclusions and recommendations are presented at the end.

For convenience to the reader, it has been thought advisable to preface the main body of the report by a summary of assumptions on which the design is based.

ASSUMPTIONS CONSIDERED IN THE DESIGN

1. The load and warping stresses are evaluated by the H. M. Westergaard theory with all its assumptions. The corner stresses are evaluated in the case of upward warping by the Public Roads Administration experimental formula and the corner temperature stresses by the formula proposed by Bradbury (See Text).

2. Slab thickness computations have been based on the assumption that the combined load, warping and friction stresses do not exceed values sufficient to produce cracks due to fatigue before 20 years for the traffic under consideration. This 20 year period, referred to as "crack expectancy", is believed reasonable and one possible to obtain under modern construction methods.

3. Westergaard's analysis considers loads applied in the interior, or at the edge, or corner of the slab. With the advent of wider pavements the possibility of all wheel loads reaching the extreme edge or corner positions is reduced. Thus, consideration has been given to both 100 percent and 50 percent wheel load applications at the critical points.

4. The subgrade is assumed to be uniform throughout, well drained, stable and to have a definite numerical value for subgrade reaction. The subgrade reaction "p" at a point is proportionate to the deflection at that point and to a coefficient "k" which is the same for all points of the slab and which is called the subgrade modulus.

5. Preliminary subgrade studies by the Department indicate that the modulus of subgrade reaction for subbases of sandy or granular nature are in the order of 100 to 300 p.c.i. Since granular subbases are utilized in Michigan, the extreme values of 100 and 300 p.c.i. have been assumed for subgrade modulus "k" in computing slab thicknesses.

6. Laboratory and field studies by the Department and other highway organizations indicate that the subgrade friction coefficient varies between approximately 1.0 for light granular soils to 2.0 for heavier clay soils. A friction coefficient of 1.0 was used with "k" equal to 100 and a value of 2.0 with "k" equal to 300 for the determination of slab thicknesses.

7. The friction forces are assumed to produce only uniformly distribued direct tension or compression stresses without any bending of the slab.

8. The subgrade friction stresses for slabs of equal length and thickness are considered identical for both types of construction.

9. All slabs are considered to be unrestrained at the ends. Therefore, forces which may be caused by dowel bar friction, temperature or other sources are not considered in the analysis.

10. All joints are assumed to act as <u>ideal hinges</u> without the transmission of any bending moments and without permitting infiltration or affecting riding qualities.

11. Contraction joints containing load transfer devices are the only type of plane of weakness joints considered.

12. Expansion joints are planes of total slab separation in the pavement located at breaks in reinforcement. They usually contain a premolded filler material suitable for preventing infiltration and capable of absorbing compression forces. Slip dowels and other devices are generally included to preventing faulting of the slab ends.

13. For the type of pavement construction under consideration sufficient reinforcement is provided so that the width of transverse cracks, if they should appear, should remain very small in order to prevent the infiltration of foreign matter and to provide a certain amount of edge support. The whole friction force is assumed to be taken up by the longitudinal bars of the reinforcement without exceeding the yield point of the steel.

14. Under present construction methods dowels or other devices are considered necessary at expansion and contraction joints to prevent faulting only. Their ability to reduce edge stresses by load transfer is not considered in the design analysis, except in special cases when 50% load transfer features are provided. Consequently, an impact factor of 1.5 is utilized in computing slab thickness at transverse edges and at corners. At all other points the impact factor is considered to be 1.00.

15. Cracks occurring in reinforced pavement are assumed to permit 50% load transfer.

16. In the computation of warping stresses, the length of slab is considered to be the distance between any two consecutive joints.

17. The average interpolated slab thickness between the ones for (k = 100, F = 1.00) and (k = 300, F = 2.00) represents the thickness for k = 200, f = 1.50 for the same traffic and same crack expectancy.

18. Certain physical properties of concrete are assumed to have the following values in the analysis:

Modulus of Rupture	750 p.s.i.
Modulus of Elasticity	$5x10^6$ p.s.i.
Ultimate Compression	5,000 p.s.i.

Poisson's Ratio is 0.15 for loads and zero for temperature or warping stresses.

The Temperature Expansion Coefficient is 5.5×10^{-6}.

19. Four classes of traffic are considered defining the load forces.

20. The radius of load distribution is defined by Bradbury (See Text).

21. Temperature differentials of 3h in summer, 2h in winter and (h) at night are considered where "h" equals slab thickness.

PART ONE

DETERMINATION OF SLAB THICKNESS

The thickness requirements for the types of concrete pavements under consideration have been determined by stress analysis, since methods are available for calculating the stresses induced in a concrete pavement slab by external forces and temperature, and since the stress-resisting properties of the concrete material are well established.

In connection with the method of stress analysis, the principle of fatigue has been utilized as a means for analytically predicting the age at which cracks may be expected to occur in concrete pavements of different thicknesses, because it is possible to ascertain, with a reasonable degree of certainty, the particular traffic characteristics to which the pavement is to be subjected and the frequency of combined load, warping and friction stresses.

Special consideration has been given in the analysis to such factors as: wheel load frequencies based on actual traffic survey data, the latest recommendations of the Public Roads Administration in regard to stress analysis, the use of definite design values obtained from field and laboratory studies conducted by the Department and the introduction of an advanced method for determining the number of stress repetitions to cause failure.

The cross-section thicknesses for several types of pavement construction have been determined on the basis of combined load, warping and subgrade friction stresses employing the assumptions and data outlined early in the text.

In determining slab cross-section thickness, consideration is given to continuous slab lengths of 10, 20, 30, 40, 60, 100, 140 and 200 feet.

Traffic In Relation To Wheel Load Frequencies And Highway Classes

The daily frequency of wheel loads in excess of 4,000 pounds for the different classes of highways can be predicted with a reasonable degree of accuracy from existing traffic survey data. For the purpose of the design analysis, traffic data were obtained from the Planning and Traffic Division for the years 1936, 1942 and 1943. These data are presented in Tables 1, 2 and 3. Figure 1 is a graphical presentation of the data in Table 1. This graph has been used as a basis for estimating the future wheel load frequency for each class of load in terms of percentage of total daily axle loads occurring on certain routes representative of a particular class of highway.

The four highway classes recognized by the Design Division are:

I.	Express Way	- Divided Lane
II.	Heavy Primary	- 2 Lane
III.	Light Primary	- 2 Lane
IV.	Secondary	- 2 Lane

The classification of these types of highways by traffic characteristics is made in Table 4.

Determination of Critical Wheel Load Frequency. Tables 4 and 5 also contain the computed critical wheel load frequency per day and per year respectively. The daily wheel load frequency is determined by multiplying the percentage for each wheel class by total daily axle loads. The total critical wheel loads per year in Table 5 were obtained by multiplying the values in Table 4 by 365.

Load Stress Calculations

The magnitude of the stress developed in a pavement slab by a definite wheel load will be determined by considering: (1) The vertical component of the load, (2) The position of load with respect to edge of slab, (3) The area over which the load is distributed on the pavement surface, (4) Certain physical characteristics of the pavement material, (5) Certain physical characteristics of the subgrade, (6) Time duration of the load, and (7) Impact of load.

Formulas I, II, III and IV below are used for the calculation of stress values. These are derived from Westergaard's analysis (1)*.

Case I, for maximum unit stress at interior of slab (S_i)

$$S_i = \frac{1.1\,(1+u)}{h^2}\,P\left[\log_{10}\left(\frac{h}{b}\right) + 1/3\,\log_{10}\left(\frac{E}{K}\right) - 0.089 - 13.64\left(\frac{t}{L}\right)^2 Z\right], \text{ or}$$

$$= \frac{1.265\,P}{h^2}\left[\log_{10}\left(\frac{t}{b}\right) + 0.15831\right] \text{ - - - - - - - - - - - - - - I}$$

Case II, for maximum unit stress at edge of slab (S_e)

$$S_e = \frac{2.117 \times (1 + 0.54u)P}{h^2}\left[\log_{10}\left(\frac{h}{b}\right) + 1/3\,\log_{10}\left(\frac{E}{K}\right) - 0.2666\right], \text{ or}$$

$$= \frac{2.288\,P}{h^2}\left[\log_{10}\left(\frac{t}{b}\right) + 0.0897667\right] \text{ - - - - - - - - - - - - - - II}$$

Case III, for maximum unit stress at Corner (S_c)

$$S'_c = \frac{3P}{h^2}\left[1 - \frac{a\sqrt{2}}{t}^{\,0.6}\right] \text{ for downward warping - - - - - - - III}$$

$$S''_c = \frac{3P}{h^2}\left[1 - \frac{a\sqrt{2}}{t}^{\,1.2}\right] \text{ for upward warping - - - - - - - IV}$$

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*(1) Westergaard, H. M.: "Computation of Stresses in Concrete Roads", Proceedings of Highway Research Board, 1925. Part I.

GRAPHIC PRESENTATION OF WHEEL LOAD FREQUENCY BASED ON 1936 TRAFFIC COUNT

FROM REPORT TO PUBLIC ROADS ADMINISTRATION
BY PLANNING AND TRAFFIC DIVISION 1938

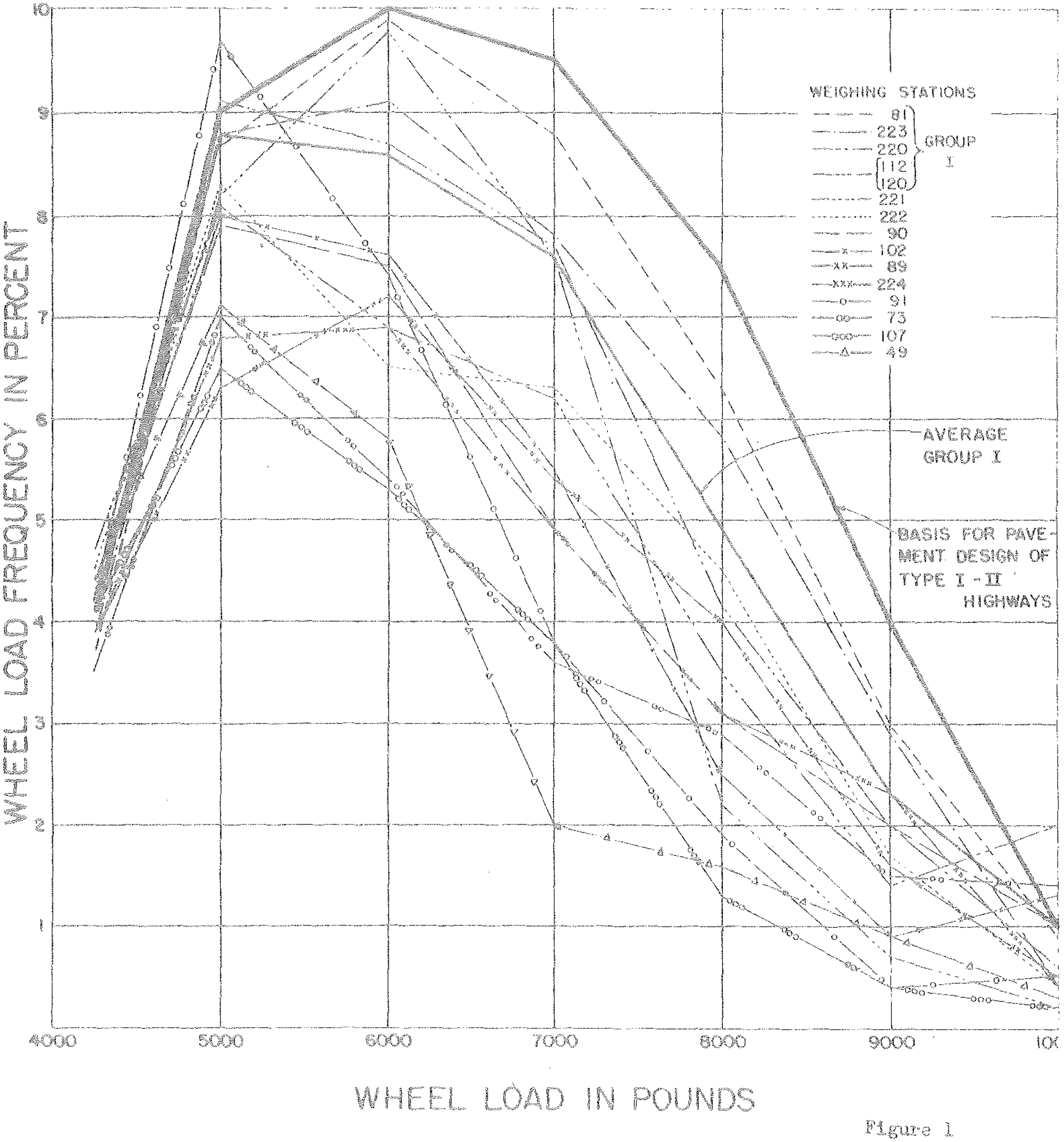

Figure 1

CONCRETE PAVEMENT DESIGN STUDY
TABLE NO. I
DETERMINATION OF WHEEL LOAD FREQUENCY
BASED ON 1936 TRAFFIC COUNT

Weight Station No.	Route No.	Time	Direction of Traffic	Total Vehicles Weighed	Total Axles Weighed	Ratio Axles to Vehicles	WHEEL LOADS															
							Under 4000	%	4000-4400 4250	%	4500-5400 5000	%	5500-6400 6000	%	6500-7400 7000	%	7500-8400 8000	%	8500-9400 9000	%	Over 9400 10,000	%	% Wheel Load
81	US 24		1	2,987	7,638	2.56	4,487	58.7	285	3.7	667	8.7	754	9.9	670	8.8	477	6.3	229	3.0	69	0.9	41.3
223	US 24	Aug.-Sept.	2	7,131	18,235	2.56	11,174	61.3	716	3.9	1,608	8.8	1,657	9.1	1,427	7.8	1,059	5.8	523	2.9	71	0.4	38.7
220	US 16	Feb.-Mar.	2	1,958	4,978	2.54	3,167	63.6	204	4.1	453	9.1	434	8.7	383	7.7	207	4.2	102	2.0	28	0.6	36.4
221	US 12	Apr.-May	2	3,943	9,926	2.51	6,633	66.9	438	4.4	809	8.2	971	9.8	757	7.6	216	2.2	76	0.7	16	0.2	33.1
112 & 120	US 12		1	1,384	3,587	2.59	2,407	67.2	169	4.7	291	8.1	247	6.9	222	6.2	127	3.5	52	1.4	72	2.0	32.8
222	US 112	June-July	2	4,343	10,696	2.46	7,248	67.8	483	4.5	887	8.3	701	6.5	673	6.3	480	4.5	179	1.7	45	0.4	32.2
90	US 24		1	1,127	2,928	2.60	2,023	69.2	130	4.4	231	7.9	221	7.5	144	4.9	91	3.1	60	2.0	28	1.0	30.8
108	US 12		1	1,407	3,192	2.27	2,234	70.0	135	4.2	254	8.0	241	7.6	176	5.5	81	2.5	29	0.9	42	1.3	30.0
69	US 10		1	1,570	3,938	2.51	2,797	71.1	153	3.9	247	6.3	285	7.2	214	5.4	158	4.0	63	1.6	21	0.5	28.9
224	US 10	Sept.-Oct.	2	2,787	6,532	2.34	4,693	71.8	251	3.8	444	6.8	450	6.9	317	4.9	201	3.1	150	2.3	26	0.4	28.2
91	M 78		1	912	2,160	2.37	1,557	72.1	91	4.2	209	9.7	161	7.4	83	3.8	39	1.9	9	0.4	11	0.5	27.9
73	US 16		1	1,244	2,987	2.40	2,228	74.7	103	3.5	212	7.0	160	5.4	107	3.6	90	2.9	46	1.5	41	1.4	25.3
49	US 131		1	856	1,963	2.29	1,527	77.9	87	4.4	140	7.1	113	5.8	40	2.0	32	1.6	18	0.9	6	0.3	22.1
107	US 127		1	1,469	3,197	2.18	2,505	78.4	130	4.1	207	6.5	170	5.3	121	3.8	45	1.3	12	0.4	7	0.2	21.6
				AVERAGE		2.44		69.3		4.3		7.9		7.4		5.6		3.4		1.6		0.7	
				ESTIMATE		2.50	HIGHEST VALUES			4.7		9.7		9.9		8.8		6.3		3.0		2.0	
Estimated Wheel Load Frequency on Basis of Traffic Study										4.0		8.0		10.0		9.5		7.5		4.0		1.0	

CONCRETE PAVEMENT DESIGN STUDY
TABLE II
SUMMARY OF LOADOMETER SURVEY
BY PLANNING AND TRAFFIC DIVISION
1942

Weight Station No.	Route No.	Time	Direction of Traffic	Total Vehicles Weighed	Total Axles Weighed	Ratio Axles to Vehicles	WHEEL LOADS															
							Under 4000	%	4000-4400 4850	%	4500-5400 5000	%	5500-6400 6000	%	6500-7400 7000	%	7500-8400 8000	%	8500-9400 9000	%	Over 9400 10,000	%	% Wheel Load
106	US 16	Aug. 13		117	296	2.53	187	63.2	11	3.7	9	3.0	15	5.1	22	7.4	24	8.1	20	6.8	8	2.7	36.8
109	US 131	Aug. 3		163	416	2.55	266	63.9	18	4.3	35	8.4	21	5.1	30	7.2	29	7.0	13	3.1	4	1.0	36.1
115	US 112 US 131	July 30		110	306	2.78	200	65.3	10	3.3	18	5.9	27	8.8	25	8.2	21	6.9	4	1.3	1	0.3	34.7
39	US 10 M 15	Aug. 7		183	457	2.49	318	69.6	16	3.5	25	5.5	38	8.3	28	6.1	24	5.3	6	1.3	2	0.4	30.4
31	US 24 US 25	Aug. 12		195	507	2.60	369	72.8	20	3.9	20	3.9	18	3.6	24	4.7	26	5.1	18	3.6	12	2.4	27.2
112	US 12	July 31		240	592	2.46	450	76.1	16	2.7	28	4.7	30	5.1	31	5.2	31	5.2	6	1.0	0	0	23.9
82	US 25 M 97	Aug. 11		216	479	2.22	366	76.2	15	3.1	30	6.3	18	3.8	18	3.8	19	4.0	7	1.5	6	1.3	23.8
52	US 27	Aug. 6		130	316	2.43	242	76.6	14	4.4	15	4.7	10	3.2	22	7.0	9	2.8	4	1.3	0	0	23.4
27	US 131	Aug. 5		117	290	2.48	227	78.3	8	2.8	9	3.1	10	3.4	13	4.5	15	5.2	7	2.4	1	0.3	21.7
40	US 31	Aug. 4		121	275	2.27	226	82.2	8	2.9	15	5.5	11	4.0	7	2.5	7	2.5	1	0.4	0	0	17.8
				AVERAGE		2.48		72.5		3.5		5.1		5.0		5.7		5.2		2.3		0.8	

CONCRETE PAVEMENT DESIGN STUDY
TABLE III
SUMMARY OF LOADOMETER SURVEY
BY PLANNING AND TRAFFIC DIVISION
1943

Weight Station No.	Route No.	Time	Total Vehicles Weighed	Total Axles Weighed	Ratio Axles to Vehicles	WHEEL LOADS																
						Under 4000	%	4000-4400 4250	%	4500-5400 5000	%	5500-6400 6000	%	6500-7400 7000	%	7500-8400 8000	%	8500-9400 9000	%	Over 9400 10,000	%	% Wheel Load
118	US 112 US 131	July 29	70	197	2.81	103	52.4	6	3.0	25	12.7	27	13.7	18	9.1	14	7.1	4	2.0	0	0.0	47.6
81	US 24 US 25	Aug. 13	201	562	2.79	314	56.0	23	4.1	30	5.3	44	7.8	71	12.6	44	7.8	23	4.1	13	2.3	44.0
112	US 12	July 30	112	315	2.81	179	56.8	15	4.8	26	8.3	25	7.9	38	12.0	22	7.0	6	1.9	4	1.3	43.2
89	US 10 M 15	Aug. 9	104	264	2.54	165	62.4	11	4.2	14	5.3	14	5.3	32	12.1	20	7.6	6	2.3	2	0.8	37.6
109	US 131	Aug. 2	86	225	2.62	148	65.8	10	4.4	22	9.8	11	4.9	12	5.3	13	5.8	6	2.7	3	1.3	34.2
82	US 25 M 97	Aug. 10	130	323	2.48	214	66.3	12	3.7	26	8.0	19	5.9	13	4.0	22	6.8	11	3.4	6	1.9	33.7
108	US 16	Aug. 12	156	406	2.60	269	66.3	20	4.9	22	5.4	30	7.4	25	6.2	19	4.7	13	3.2	8	2.0	33.7
27	US 131	Aug. 5	59	146	2.48	104	71.2	6	4.1	7	4.8	7	4.8	5	3.4	10	6.9	5	3.4	2	1.4	28.8
52	US 27	Aug. 6	64	164	2.56	120	73.2	6	3.7	11	6.7	9	5.5	5	3.0	8	4.9	3	1.8	2	1.2	26.8
40	US 31	Aug. 3	60	136	2.27	104	76.5	4	2.9	10	7.4	7	5.1	5	3.7	3	2.2	3	2.2	0	0.0	23.5
			AVERAGE		2.60		64.7		4.0		7.4		6.8		7.1		6.1		2.7		1.2	

CONCRETE PAVEMENT DESIGN STUDY
TABLE NO. IV
COMPUTATION OF CRITICAL WHEEL LOAD FREQUENCY, PER DAY

Class	Highway Type	Total Daily Traffic Capacity per Lane	Commercial Traffic Percent	Commercial Traffic Per Lane	Estimated Com. Traffic per Lane	Ratio of Axles to Vehicles	Total Daily Axle Loads	Daily Wheel Load Frequency for Each Class						
								4000-4400 4800 4 %	4500-5400 5000 9%	5500-6400 6000 10%	6500-7400 7000 9.5%	7500-8400 8000 7.5%	8500-9400 9000 4%	Over 9400 10,000 1%
I	Express Way Divided Lane	12,000	15	1800	2000	2.6	5200	208	468	520	494	390	208	52
II	Heavy Primary 2 Lane	12,000	8	960	1000	2.6	2600	104	234	260	247	195	104	26
III	Light Primary 2 Lane	12,000	2	240	250	2.6	650	26	59	65	62	49	26	6.5
IV	Secondary 2 Lane	12,000	1	120	125	2.6	325	13	29	33	31	24	13	3

TABLE NO. V
COMPUTATION OF CRITICAL WHEEL LOAD FREQUENCY, PER YEAR

Class	Highway Type	Critical Wheel Load Per Year						
		4800	5000	6000	7000	8000	9000	10,000
I	Express Way - Divided Lane	75,920	170,820	189,800	180,310	142,350	75,920	18,980
II	Heavy Primary - 2 Lane	37,960	85,410	94,900	90,155	71,175	37,960	9,490
III	Light Primary - 2 Lane	9,490	21,353	23,725	22,539	17,794	9,490	2,373
IV	Secondary - 2 Lane	4,745	10,677	11,863	11,270	8,897	4,745	1,187

where:

P = applied load, in pounds

h = thickness of slab, in inches

a = radius of wheel load distribution in inches
The values are given in Table VI

t = radius of relative stiffness, in inches ("t" substituted for "l" in Westergaard's equation) See Formula V

b = radius of resisting section, in inches, and is computed by formula VI

E = Modulus of elasticity of concrete in pounds per square inch

u = Poisson's ratio for concrete

$Z = \frac{t}{L} = 0.2$

<u>Radius of Wheel Load Distribution</u>: Load distribution radii for various wheel loads according to Bradbury (2)* are given below in Table VI.

TABLE VI

Load-Distribution Radius, "a", in inches

Wheel Load	Location of wheel on slab			
P	Corner	Interior	Transverse Edge	Longitudinal Edge
4000	4.6	4.6	5.5	6.7
5000	5.0	5.2	6.1	7.4
6000	5.3	5.7	6.6	8.0
7000	5.6	6.1	7.0	8.6
8000	5.8	6.5	7.4	9.1
9000	6.0	6.9	7.8	9.6
10000	6.3	7.3	8.1	10.1

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*(2) Bradbury, R. D.: "Reinforced Concrete Pavements". Wire Reinforcement Institute, 1938.

The Radius of Relative Stiffness:. In regard to the radius of relative stiffness Bradbury states that, "a concrete pavement slab functions essentially as a flat plate resting upon a continuous but yielding support. Any tendency for the slab to deflect downward when a load is applied on the pavement surface is restrained to a certain extent by an upward induced reaction exerted by the subgrade. The degree of resistance to slab deflection thus offered by the subgrade is dependent upon the stiffness or pressure-deformation properties of the subgrade material. But the tendency of the slab itself to deflect is dependent upon its properties of flexural stiffness. Thus, the net resultant deflection of the slab, which is also the deformation of the subgrade, which, in turn, is a direct measure of the magnitude of reacting subgrade pressure, becomes a function of the relative stiffness of the slab to that of the subgrade. According to Westergaard, this factor termed the 'radius of relative stiffness' - takes the form of a linear dimension and may be expressed by the formula:"

$$t = \sqrt[4]{\frac{Eh^3}{12\,(1-u^2)K}} \qquad - - - - - - - - V$$

in which:

t = radius of relative stiffness, in inches

E = modulus of elasticity of the concrete, in pounds per square inch

h = slab thickness, in inches

k = subgrade modulus in pounds per cubic inch

u = Poisson's ratio for the concrete

Radius of Resisting Section: The slab thickness obviously has the effect of distributing the bending moment over some effective section of resistance. Also, the radius of load distribution is a factor in determining the extent of moment distribution. According to Westergaard, the equivalent radius of the resisting section may be approximated in terms of the radius of load distribution and slab thickness by the following formulae.

$$b = \sqrt{1.6\,a^2 + h^2} - 6.75h \text{ when "a" is less than } 1.724h \text{ - - - - VI}$$

$$b = \text{"a" when "a" is greater than } 1.724h$$

in which

b = equivalent radius of resisting section, in inches

a = radius of wheel load distribution, in inches

h = slab thickness, in inches

Determination of Subgrade Modulus. In Westergaard's original theory it was assumed that the reactions of the subgrade are vertical only and are proportional to the deflections of the slab. The subgrade reaction per unit area at a given point is the product of the deflection at that point and a coefficient of subgrade stiffness "k" which was termed the modulus of subgrade reaction. This modulus is expressed in pounds per square inch per inch of deflection, or pounds per cubic inch.

To make practical use of the analysis one must be able to assign a value to the modulus of subgrade reaction for the particular soil structure with which he is concerned. There are three recognized field procedures by which the load sustaining ability of the subgrade can be measured,

1. Load-displacement tests on rigid plates, in which the loads are applied at the center of rigid plates of relatively small size. The subgrade modulus is obtained from the load-penetration relationship. The method for computing "k" is shown in Figure 2, part 1.

2. Load-deflection tests on flexible circular plates. The load is applied at the center of flexible circular plates of relatively large dimensions. The subgrade modulus "k" is computed by two methods designated as the volumetric displacement method and the deflection method.

Method A. In the volumetric method, the shape of the deflected plate must be determined precisely and its vertical displacement measured in order to estimate accurately the volumetric displacement of the soil that is caused by the test load on the plate. The modulus of subgrade reaction is computed by dividing the load in pounds by the volume of the displaced soil in cubic inches.

Method B. The deflection method of computing "k" offers a much more precise treatment of the problem than any of the methods so far advanced, since it is based upon a rigorous theory for finite circular plates of uniform thickness, symmetrically loaded and supported. The theory of these plates was developed by Ferdinand Schleicher (3). The subgrade modulus can be readily ascertained (from a deflection measured) at any point under the plate from specially prepared graphs. The theory lends itself to the determination of deflections at positions not immediately under the load, a determination which is valuable as a check and means of detecting any distortions of the plate during the test caused by temperature and moisture conditions. See Figure 2, part 2.

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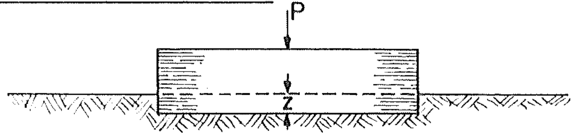
(3) Schleicher, Ferdinand: Kreisplatten auf Elasticher Unterlage. (Circular Plates on Elastic Subgrades) 1926.

1. RIGID CIRCULAR PLATE



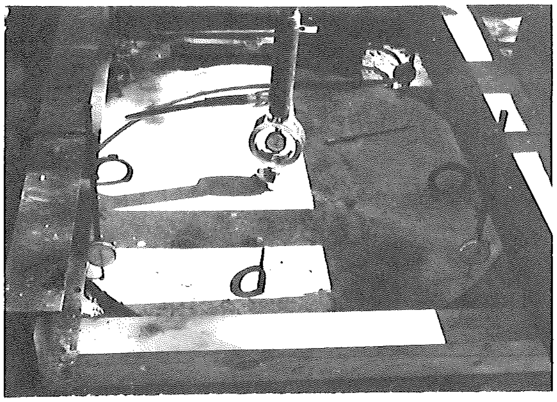
$$k = \frac{P}{A \cdot z}$$

k = subgrade modulus

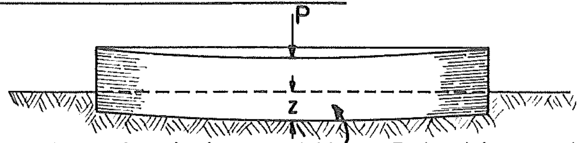
P = load in pounds

A = area in square inches

z = deflection in inches



2. FLEXIBLE CIRCULAR PLATE



Method A: $k = \frac{P}{V}$

$V = A \cdot z_{av.}$

P = load in pounds

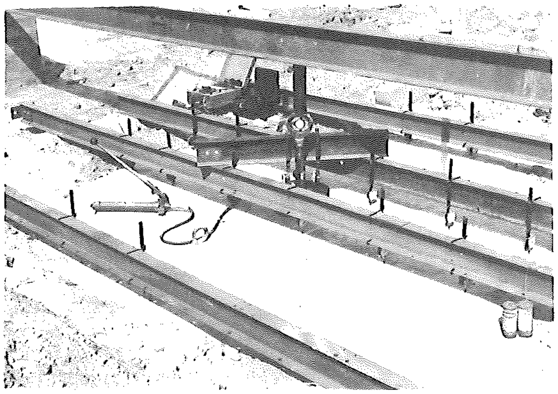
V = volumetric displacement of soil in cu. in.

z = deflection in inches

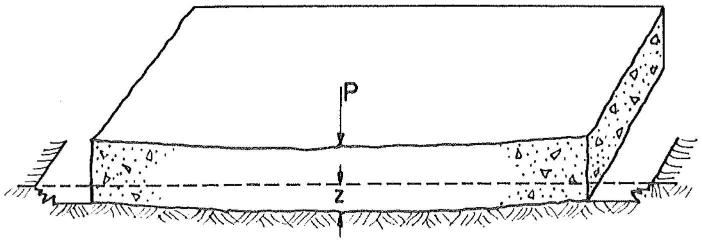
k = subgrade modulus

A = area of plate in sq. inches

Method B: "k" can be determined from prepared graphs based on relationships between load deflection and physical properties of the plate.



3. FULL SIZE PAVEMENT SLABS



For Interior: $k = \frac{3(1-u^2)P^2}{16Eh^3z^2}$

For Free Edge: $k = \frac{2(1-u^2)(1+0.4u)^2P^2}{Eh^3z^2}$

k = subgrade modulus, p.c.i.

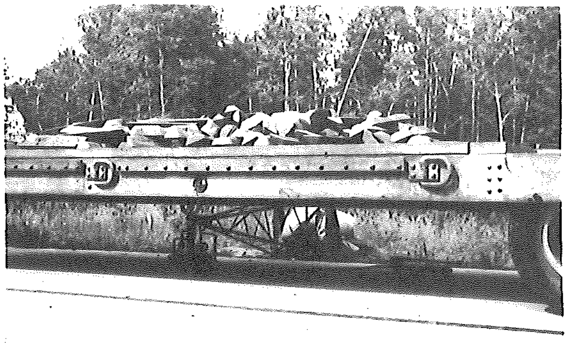
u = Poissons ratio

E = modulus of elasticity, p.s.i.

P = load in pounds

z = deflection in inches

h = slab thickness in inches



3. Load-deflection tests on full size pavement slabs. This procedure consists of applying test loads at the free edge or interior point of a pavement slab of uniform thickness and of normal size. If the elastic modulus of the concrete in the slab is known, it is possible to estimate the value of the subgrade modulus from the slab deflection under the applied load by means of deflection formulas given by Westergaard (4). See Figure 2, part 3.

All of the three procedures for determining the subgrade modulus have been tried by the Department on various occasions. It has been found that the procedure employing the flexible circular plate, with the subgrade modulus computed by the deflection method (Method 2B), gave much more consistent and reproducible results than did any of the other procedures.

Recent field studies by this method on sandy and sandy clay subgrade materials gave values for subgrade modulus of approximately 200 and 300 p.c.i., respectively. Since it is not anticipated that the subgrade modulus under pavement slabs will exceed 300 p.c.i. under normal construction conditions employing sand subbase material, extreme values for "k" of 100 and 300 p.c.i. have been assumed in the calculations for pavement thickness.

Physical Properties of Concrete. The physical characteristics of concrete involved in Method 2B and in Westergaard's equations for stress determinations are: modulus of elasticity and Poisson's ratio.

The modulus of elasticity of concrete produced under modern design requirements and Michigan specifications is approximately 5,000,000 to 6,000,000 pounds per square inch. A value of 5,000,000 was used in computing unit stresses.

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(4) Westergaard, H. M.: "Computation of Stresses in Concrete Roads." Proceedings of Highway Research Board, 1925. Part I.

Poisson's ratio for concrete was considered to be 0.15 for stress due to load, and zero for warping stresses.

Calculation of Wheel Load Stresses. The maximum unit tensile stress in the concrete has been calculated by equations I, II and III for wheel loads ranging from 4,000 to 10,000 pounds when they occur at the four critical points such as (1) at the interior, (2) at the free longitudinal edge, (3) at a transverse joint edge and (4) at a corner. In the calculation of unit stresses, slab thicknesses from 6 to 12 inches in one-inch increments were considered. Two extreme conditions of subgrade support were assumed, with values for subgrade modulus "k" of 100 and 300 pounds per cubic inch.

The computed stresses for the conditions described above are presented in Table VII. They are also presented graphically in Figures 3,4,5.

Calculation of Temperature - Warping Stresses.

Warping stresses were computed for temperature differentials in the slab of "3h" in spring and summer, "2h" in fall and winter and "h" at night, where "h" equals slab thickness, according to Westergaard's analysis (5)* and assuming Poisson's ratio equal to zero. The equations for computing warping stresses are given as follows:

For interior, longitudinal and transverse joint edges.

$$Swx = S_o \cdot C_x \text{ and } Swy = S_o \cdot C_y \text{ - - - - - - - - - - - - - - - - - } VII$$

$$S_o = \frac{E.e.d.}{2}, \quad C'_x = \frac{12\ L}{t\sqrt{8}}, \quad C'_y = \frac{12\ W}{t\sqrt{8}}$$

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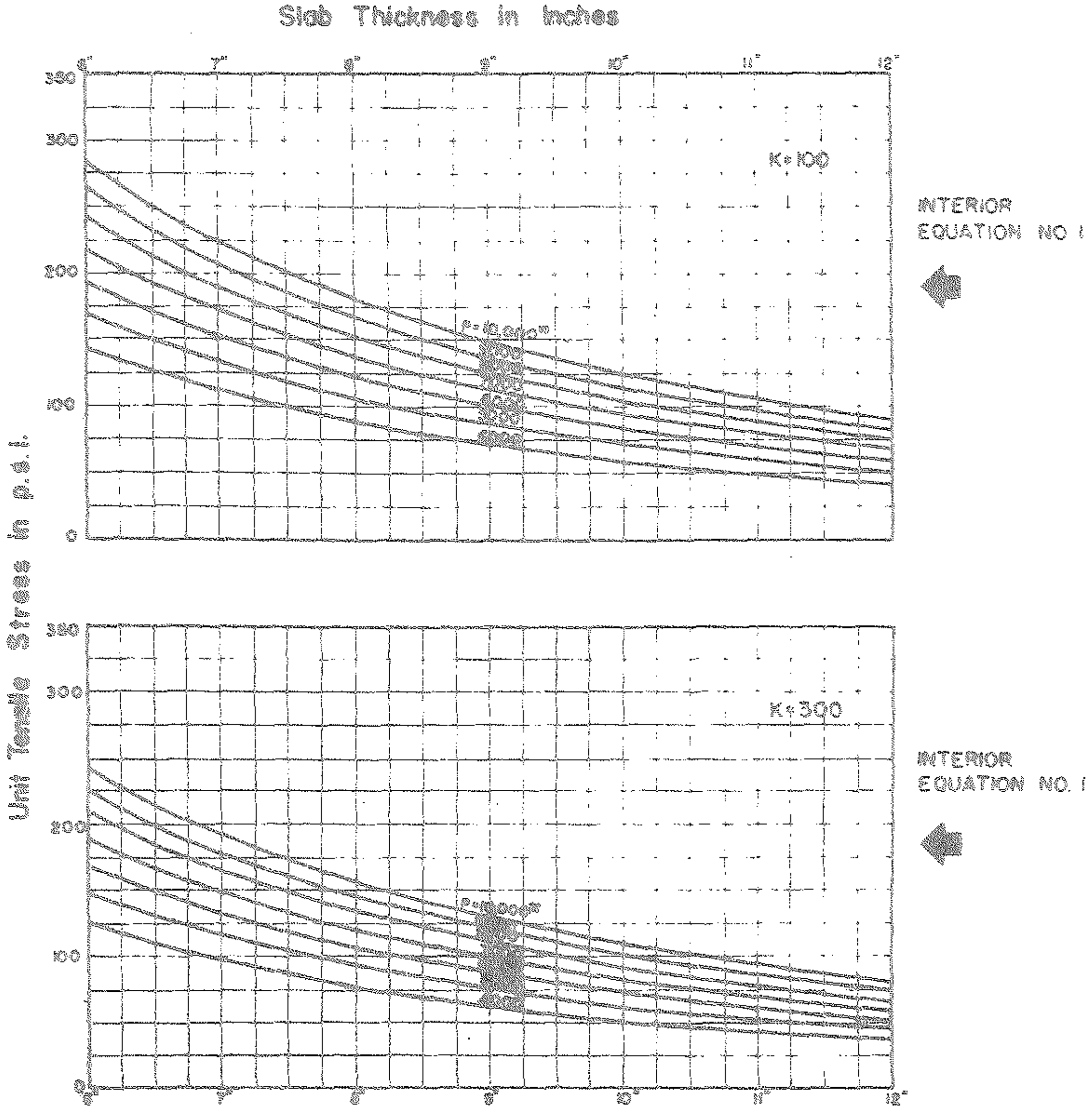
*(5) Westergaard, H. M.:"Analysis of Stresses in Concrete Pavements Due to Variations of Temperature." Proceedings of Highway Research Board, 1926.

TABLE NO. VII

SUMMARY OF WHEEL LOAD STRESSES DETERMINED BY WESTERGAARD'S EQUATIONS

Wheel Load P	K = 100 - Slab thickness in inches							K = 300 - Slab thickness in inches						
	6"	7"	8"	9"	10"	11"	12"	6"	7"	8"	9"	10"	11"	12"
Wheel Load Stresses at Interior of Slab by Equation No. I														
4000	143	109	87	70	57	48	40	125	96	77	62	51	42	36
5000	169	131	104	84	70	59	50	148	115	92	75	62	52	45
6000	194	151	121	98	81	68	58	169	133	106	87	72	61	52
7000	219	172	137	111	92	78	67	190	149	120	98	82	69	60
8000	242	189	152	124	103	87	75	209	165	134	110	91	78	66
9000	264	206	167	136	114	96	83	226	178	145	120	100	86	73
10000	283	223	180	148	124	105	90	241	192	156	129	109	93	80
Wheel Load Stresses at Longitudinal Edge of Slab by Equation No. II														
4000	198	156	126	103	86	73	63	167	133	108	90	76	64	55
5000	233	184	149	124	104	88	76	195	156	128	107	90	77	66
6000	265	211	172	142	120	102	88	219	177	146	122	104	89	77
7000	293	234	191	159	135	116	99	239	195	162	136	116	99	86
8000	320	258	211	177	150	128	111	259	213	177	150	128	110	96
9000	346	279	230	192	164	140	122	277	229	193	161	139	121	105
10000	369	298	247	207	177	152	132	292	243	204	173	151	129	113
Wheel Load Stresses at Transverse Edge of Slab by Equation No. II														
4000	222	172	137	112	93	77	66	190	149	120	98	82	69	59
5000	262	205	165	134	112	94	80	223	177	143	117	99	84	71
6000	301	236	190	156	130	109	94	254	202	164	136	114	97	83
7000	337	266	215	176	148	125	108	284	227	185	153	130	110	95
8000	373	294	238	198	166	140	120	311	250	204	170	144	123	106
9000	404	322	260	216	182	154	132	335	272	222	185	158	135	117
10000	437	347	285	235	199	168	145	360	291	242	202	171	147	127
Wheel Load Stresses at Corner of Slab by Equation No. III														
4000	202	155	123	100	83	70	59	179	139	111	91	76	64	55
5000	245	188	149	122	101	85	73	214	167	135	110	92	78	67
6000	287	221	176	143	119	101	86	249	195	157	129	108	92	79
7000	325	252	201	164	137	116	99	280	221	179	147	123	105	90
8000	366	284	226	185	154	130	112	313	247	200	165	139	118	102
9000	405	315	252	205	171	145	125	344	273	221	183	154	131	113
10000	439	341	274	224	187	158	136	368	294	239	199	167	143	123
Wheel Load Stresses at Corner of Slab by Equation No. IV														
4000	282	212	166	132	108	90	76	262	199	157	126	104	87	74
5000	346	261	203	163	134	112	95	318	243	192	155	128	107	91
6000	409	309	241	194	159	133	113	373	287	226	183	151	127	108
7000	469	356	278	224	184	154	131	426	328	260	211	174	146	125
8000	531	403	316	254	210	175	149	479	369	294	239	197	166	142
9000	591	450	353	284	234	196	167	530	411	326	266	220	185	158
10000	646	493	388	313	258	216	183	574	447	356	291	241	203	174

WHEEL LOAD STRESSES



WHEEL LOAD STRESSES

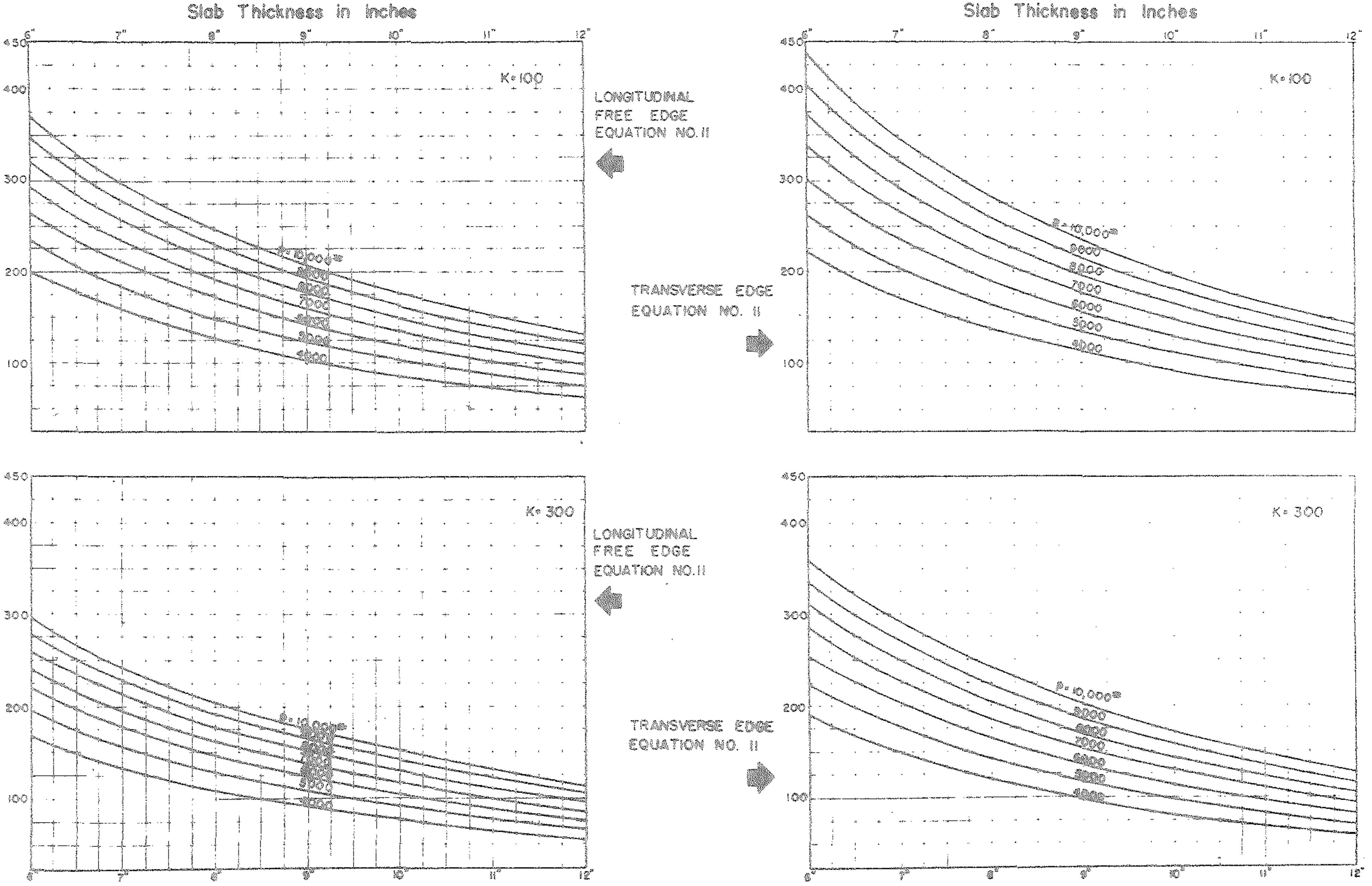


Figure 4

WHEEL LOAD STRESSES

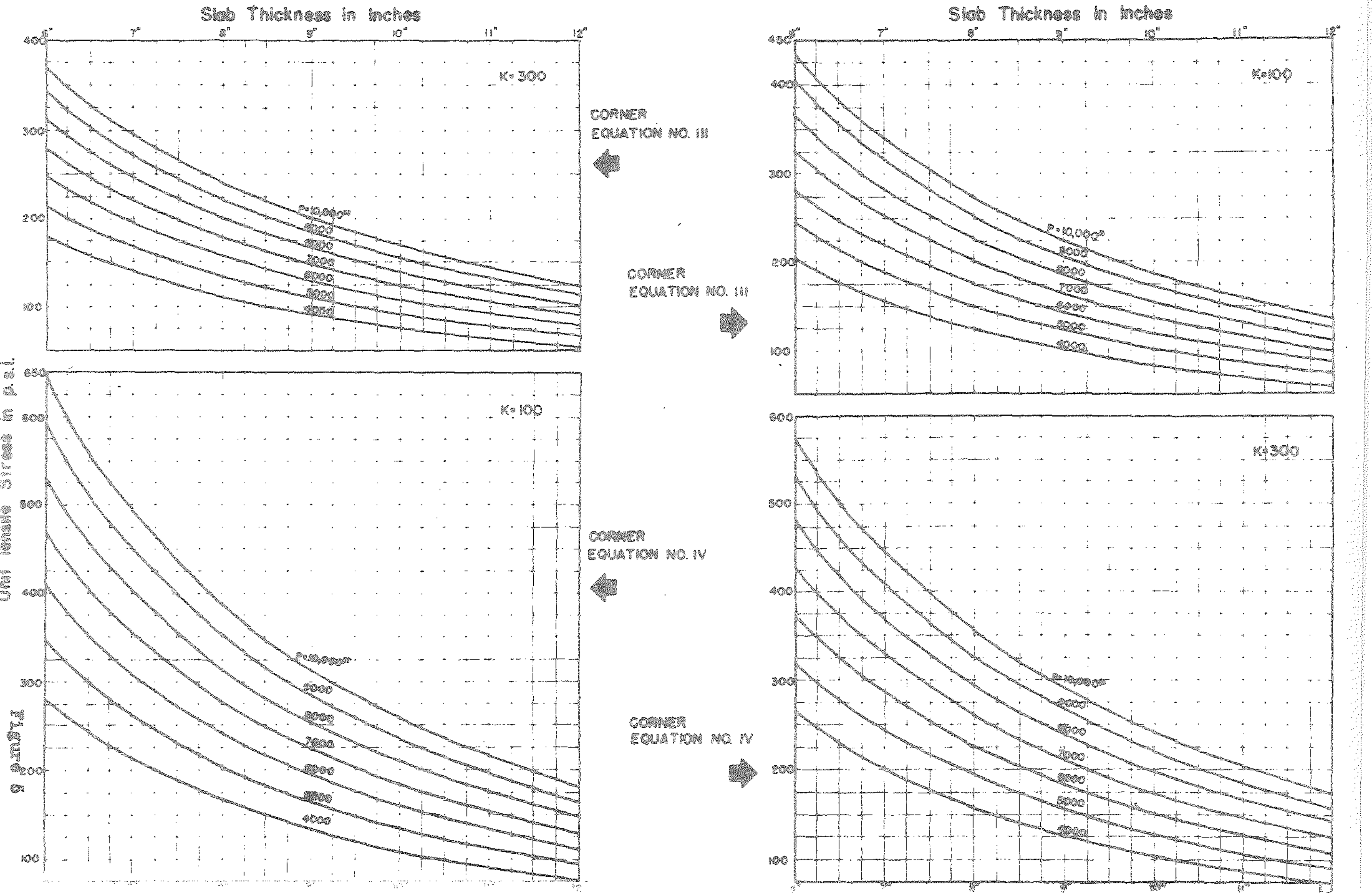


Figure 5

Near corner of slab, according to Bradbury (6)*

$$Sw_c = \frac{E.e.d.}{3(1-u)} \sqrt{\frac{a}{t}} \quad \text{which reduces to}$$

$$Sw_c = 9.167.d \sqrt{\frac{a}{t}} \quad \text{- - - - - - - - - - - - - - - - - - VIII}$$

where:

Swx and Swy = Warping stress in pounds per square inch in x (transverse) and y (longitudinal) direction respectively

Sw_c = Warping stress near corner of slab

Cx and Cy = Coefficients defined by graph in figure 6 in relationship with C'_x and C'_y

E = Modulus of elasticity of concrete in p.s.i.

e = Thermal coefficient of concrete

d = Temperature differential between top and bottom of slab

t = Radius of relative stiffness in inches (Equation V)

L = Length of slab between joints in feet

W = Width of slab in feet

For slab lengths greater than (12t) the warping stresses at a distance of $\pi t \sqrt{2}$ from slab end was considered to be 1.043 . S_c, in accordance with Bradbury (6)*. A complete summary of warping stresses for critical points is presented in Table VIII and IX.

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(6)* Bradbury, R. D.: Reinforced Concrete Pavements. Wire Reinforcement Institute, 1938.

Subgrade Friction Stresses

The subgrade friction stresses in the concrete at the middle of the slabs "L" feet long were determined by the following equation:

$$S_f = \frac{f.W.L.}{288} \qquad \text{IX}$$

where:

S_f = Friction stress in pounds per square inch

f = Coefficient of subgrade friction

W = Unit weight of concrete assumed at 150 pounds per cubic feet

L = Length of slab in feet

Laboratory and field studies by the Department and other Highway Organizations indicate that the subgrade friction coefficient varies between approximately 1.0 for light granular soils to 2.0 for heavier clay soils. A friction coefficient of 1.0 was used with "k" equal to 100 and a value of 2.0 with "k" equal to 300 for the determination of slab thickness. A value of 1.5 was assumed for "k" = 200. The calculated friction stresses are presented in table X

TABLE X

SUMMARY OF SUBGRADE FRICTION STRESSES

Length of Slab	Stress p.s.i. k = 100 f = 1.0	Stress p.s.i. k = 300 f = 2.0
10	5.2	10.4
15	7.8	15.6
20	10.4	20.8
30	15.6	31.2
40	20.8	41.6
60	31.2	62.4
100	52.0	104.0
140	72.8	145.6
200	104.0	208.0

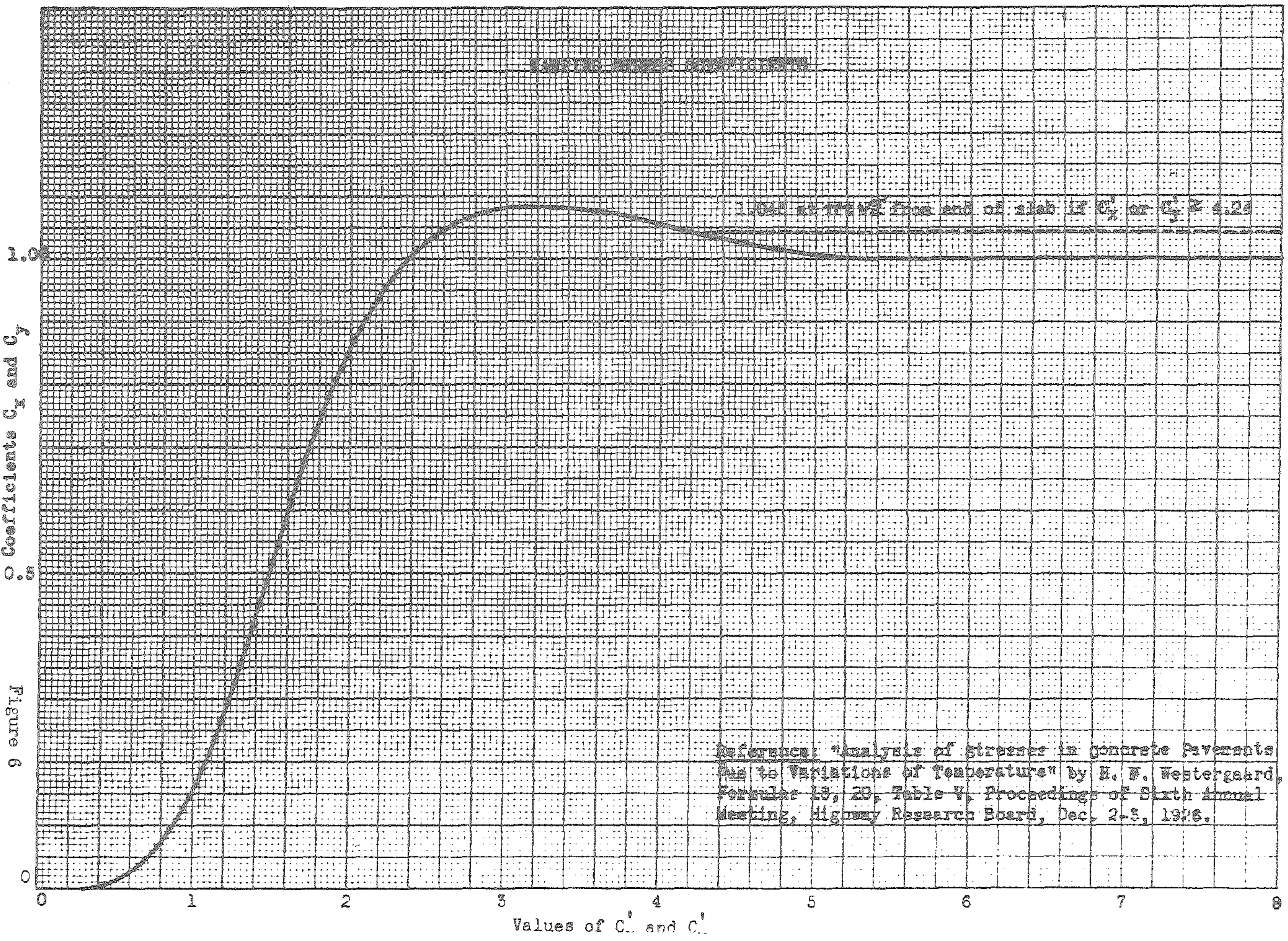


Figure 6

Reference: "Analysis of Stresses in Concrete Pavements Due to Variations of Temperature" by H. M. Westergaard, Formulas 19, 20, Table V, Proceedings of Sixth Annual Meeting, Highway Research Board, Dec. 2-3, 1926.

TABLE NO. VIII

SUMMARY OF WARPING STRESSES

AT INTERIOR AND LONGITUDINAL EDGE OF SLAB BY EQUATION VII

Slab Thickness In Inches	AT A POINT MIDWAY BETWEEN ENDS LENGTH OF SLAB IN FEET 10		11		15		20		30		40		60		or $\frac{16\ell}{12}$		At a distance of $\frac{\ell\sqrt{2}}{12}$ feet from end of slab	
	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300
6	109	184	139	211	220	263	265	267	258	248	257	248	248	248	248	248	258	258
7	87	173	113	208	221	297	295	315	311	294	300	289	289	289	289	289	302	302
8	69	159	96	195	214	311	314	350	358	348	343	330	330	330	330	330	344	344
9	60	141	81	178	197	319	353	394	403	391	390	372	371	371	371	371	388	388
10	48	124	68	163	181	320	386	425	443	444	442	426	413	413	413	413	430	430
11	41	109	59	146	163	313	329	443	476	493	491	458	455	454	454	454	473	473
12	35	94	50	130	147	300	307	457	517	538	539	500	500	495	495	495	516	516

TABLE IX

WARPING STRESSES AT CORNER OF SLAB BY EQUATION VIII

Slab Thickness In Inches	LOAD ON SLAB IN POUNDS 4000		5000		6000		7000		8000		9000		10000	
	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300	k = 100	k = 300
6	21	24	22	25	23	26	23	27	24	27	24	28	25	29
7	23	27	24	28	25	29	26	30	26	30	27	31	27	32
8	25	29	27	30	27	31	28	32	29	33	29	33	30	34
9	27	31	28	33	29	34	30	35	31	35	32	36	32	37
10	29	34	30	35	31	36	32	37	33	38	33	38	34	40
11	31	36	32	37	33	38	34	39	35	40	35	41	36	42
12	33	38	34	39	35	41	36	42	37	42	37	43	38	44

ℓ = radius of relative stiffness, See Equation No. V, page 12.

Determination of Combined Unit Stresses

Unit stresses including those produced by wheel loads, warping and friction forces have been combined for each of the critical load positions designated as the interior, longitudinal free edge, transverse joint edge, and corner. In the case of the transverse edge and corner positions, combined stresses have also been computed considering 50% load transfer without impact and no load transfer with 1.50 impact.

Special combined stress summaries were prepared for the interior and longitudinal free edge of continuous slab construction at a distance of $\pi t \sqrt{2}$ from the free ends when the slab lengths in feet were longer than 12t, where "t" equals radius at relative stiffness (See Equation V).

The combined stresses were calculated separately for warping and friction stresses and for warping, friction and load stresses as follows:

1. Warping and friction stresses at interior and longitudinal free edge

$Ssd = S_{w3} - S_f$ for summer days
$Ssn = -S_{w1} - S_f$ for summer nights
$Swd = S_{w2} + S_f$ for winter days
$Swn = -S_{w1} + S_f$ for winter nights

2. Warping stresses for transverse edge

$Ssd = S_{w3}$ for summer days
$Ssn = S_{w1}$ for summer nights
$Swd = S_{w2}$ for winter days
$Swn = S_{w1}$ for winter nights

3. Warping and friction stresses at corner

$Ssd = S_{w3} - S_f$ for summer days
$Ssn = S_{w1} - S_f$ for summer nights
$Swd = -S_{w2} + S_f$ for winter days
$Swn = S_{w1} + S_f$ for winter nights

4. Combined warping, friction and load stresses

$S_c = Ssd + S_L$ for summer days

$S_c = Ssn + S_L$ for summer nights

$S_c = Swd + S_L$ for winter days

$S_c = Swn + S_L$ for winter nights

where S represents unit stress and subscripts denote:

s, Summer

n, Night

d, Day

w, Warping

f, Friction

L, Load

c, combined unit stress

1-2-3,=1h-2h-3h or temperature differential where h = slab thickness

Free body diagrams illustrating the action of the combined unit stresses for various load positions and time of day and year are presented in figure 7.

Method of Combining Wheel-Load Applications With Warping. It is obvious that critical combinations of load and warping stresses at any given section of slab can occur only for a few hours per day during certain seasons of the year. Therefore, in order to arrive at a logical estimate as to the number of wheel load applications which can be combined with temperature-warping, the following assumptions have been made:

1. Approximately 75 percent of commercial traffic occurs between 6 A. M. and 6 P. M. (considered daytime traffic). This figure is derived from Table XI, which is compiled from traffic studies by the Planning and Traffic Division.
2. Critical warping takes place during a 5-hour period in the daytime and during the same period of time in the night. This is established by pavement temperature studies by the Public Roads Administration and verified by observations on the Michigan Test Road.

3. The critical period of stress application may be reduced one-half because of gradual variation of temperature differential in the slab for the 5-hour period.

4. In Michigan, 25 percent of days in the year are bright according to U. S. Weather Bureau records at East Lansing. At night, 100 percent of wheel loads are considered critical because of the considerable temperature changes during that period which will influence warping of the slab more than temperature fluctuations occurring on dull days.

5. The numerically greatest value of these daily maxima is apt to occur either during the spring and summer months, or fall and winter months. Each period is considered 1/2 year.

On the basis of the above assumptions, the percent of total critical wheel loads is computed as shown below, where each factor is taken in order of presentation above.

<u>Day Time</u>

For spring and summer: $0.75 \times \frac{5}{12} \times 1/2 \times 0.25 \times 1/2 \times 100 = 2.0\%$

For fall and winter : $0.75 \times \frac{5}{12} \times 1/2 \times 0.25 \times 1/2 \times 100 = 2.0\%$

<u>Night Time</u>

For spring and summer: $0.25 \times \frac{5}{12} \times 1/2 \times 1 \times 1/2 \times 100 = 2.6\%$

For fall and winter : $0.25 \times \frac{5}{12} \times 1/2 \times 1 \times 1/2 \times 100 = 2.6\%$

On the basis of the above assumptions and calculations, a value of 4% has been assumed for the purpose of computing the number of wheel load applications per year combined with warping. The total critical wheel load applications per year considered in the computation of slab thickness are given in Table XII.

TABLE XI

HOURLY DISTRIBUTION OF ANNUAL AVERAGE DAILY TRUCK TRAFFIC FROM A COMPOSITE OF 3 TRAFFIC COUNT STATIONS

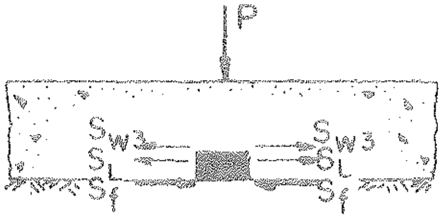
Based on 1 Week from Each Month of the Year 1936

By Planning and Traffic Division
Michigan State Highway Department

	SINGLE TRUCKS		TRAILER COMBINATIONS		TOTAL TRUCKS & TRAILER COMBINATIONS	
HOUR	VOLUME	PERCENT	VOLUME	PERCENT	VOLUME	PERCENT
12 PM - 1 AM	16.79	1.36	53.85	5.22	70.64	3.12
1 AM - 2 AM	14.75	1.20	57.06	5.53	71.81	3.17
2 AM - 3 AM	15.85	1.28	52.72	5.11	68.57	3.03
3 AM - 4 AM	17.66	1.43	51.78	5.01	69.44	3.06
4 AM - 5 AM	18.99	1.54	49.59	4.80	68.58	3.03
5 AM - 6 AM	26.16	2.12	50.07	4.85	76.23	3.36
6 AM - 7 AM	38.32	3.11	47.18	4.57	85.50	3.77
7 AM - 8 AM	51.66	4.19	36.27	3.51	87.93	3.88
8 AM - 9 AM	69.45	5.63	32.18	3.12	101.63	4.48
9 AM - 10 AM	84.97	6.89	36.31	3.52	121.28	5.35
10 AM - 11 AM	86.73	7.03	37.69	3.65	124.42	5.49
11 AM - 12 M	86.66	7.02	38.68	3.75	125.34	5.53
12 M - 1 PM	78.87	6.39	40.37	3.91	119.24	5.26
1 PM - 2 PM	82.88	6.72	41.08	3.98	123.96	5.47
2 PM - 3 PM	84.99	6.89	41.70	4.04	126.69	5.59
3 PM - 4 PM	83.39	6.76	41.81	4.05	125.20	5.53
4 PM - 5 PM	83.95	6.81	43.17	4.18	127.12	5.61
5 PM - 6 PM	78.91	6.40	39.10	3.79	118.01	5.21
6 PM - 7 PM	57.81	4.69	37.46	3.63	95.27	4.20
7 PM - 8 PM	47.12	3.82	37.31	3.61	84.43	3.73
8 PM - 9 PM	36.11	2.93	39.29	3.80	75.40	3.33
9 PM - 10 PM	27.74	2.25	38.60	3.74	66.34	2.93
10 PM - 11 PM	23.13	1.87	42.59	4.12	65.72	2.90
11 PM - 12 PM	20.64	1.67	46.59	4.51	67.23	2.97
24-Hour Total	1233.53	100.00	1032.45	100.00	2265.98	100.00
6 AM - 6 PM Total		73.84		46.07		61.17

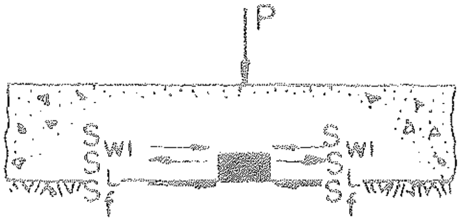
COMBINED STRESSES FOR INTERIOR & LONGITUDINAL FREE EDGE

SUMMER



DAYTIME

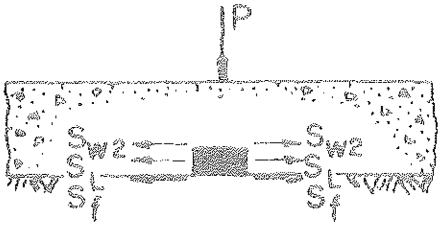
$S_C = S_L + S_{sd} = S_L + S_{w3} - S_f$



NIGHT TIME

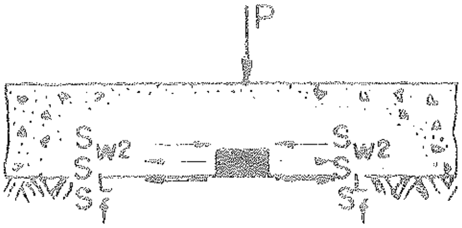
$S_C = S_L + S_{sn} = S_L - S_{w1} - S_f$

WINTER



DAYTIME

$S_C = S_L + S_{wd} = S_L + S_{w2} + S_f$

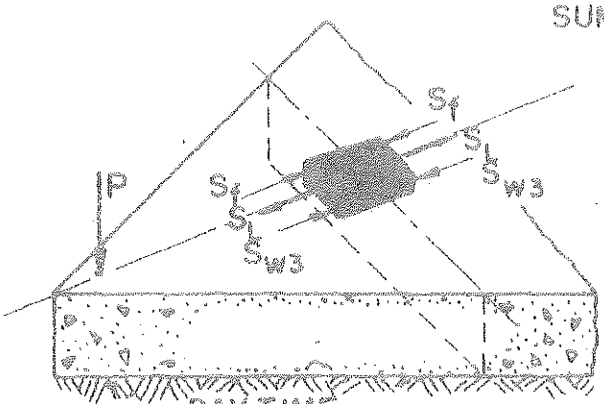


NIGHT TIME

$S_C = S_L + S_{wn} = S_L - S_{w1} + S_f$

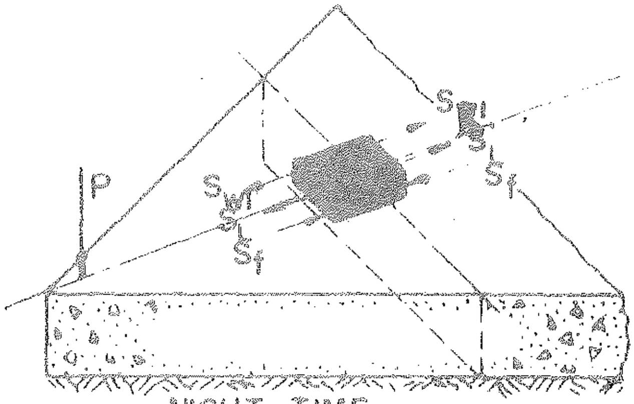
COMBINED STRESSES AT CORNER

SUMMER



DAYTIME

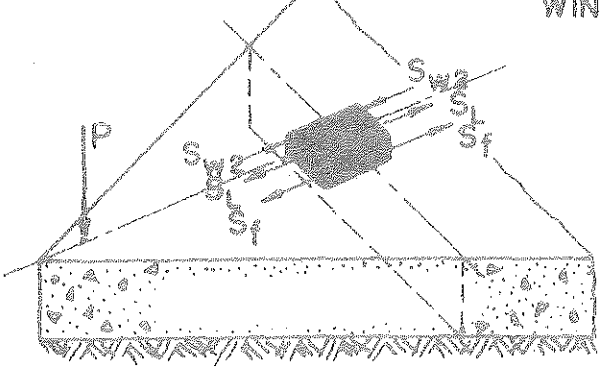
$S_C = S_L + S_{sd} = S_L + S_{w3} - S_f$



NIGHT TIME

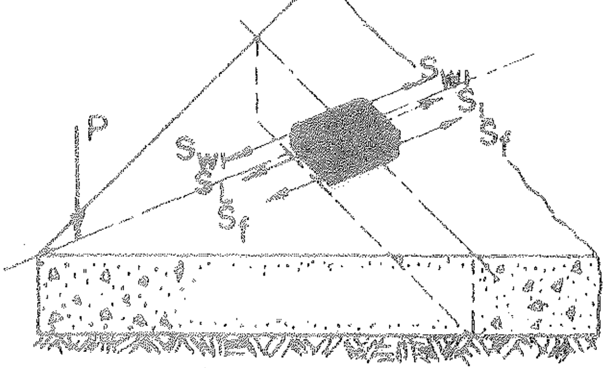
$S_C = S_L + S_{sn} = S_L + S_{w1} - S_f$

WINTER



DAYTIME

$S_C = S_L + S_{wd} = S_L - S_{w2} + S_f$



NIGHT TIME

$S_C = S_L + S_{wn} = S_L + S_{w1} + S_f$

Figure 7

TABLE XII

NUMBER OF CRITICAL WHEEL LOAD APPLICATIONS PER YEAR

	HIGHWAY CLASS I		HIGHWAY CLASS II	
Wheel Loads	Total Critical Wheel Loads	Number Combined with warping 4% Spring & Summer or 4% Fall & Winter	Total Critical Wheel Loads	Number Combined with warping 4% Spring & Summer or 4% Fall & Winter
4000	75,920	3,037	37,960	1.518
5000	170,820	6,833	85,410	3,416
6000	189,800	7,592	94,900	3,796
7000	180,310	7,212	90,155	3,606
8000	142,350	5,694	71,175	2,847
9000	75,920	3,037	37,960	1,518
10000	18,980	759	9,490	380
	HIGHWAY CLASS III		HIGHWAY CLASS IV	
4000	9,490	380	4,745	190
5000	21,353	854	10,677	427
6000	23,725	949	11,863	475
7000	22,539	902	11,270	451
8000	17,794	712	8,897	356
9000	9,490	380	4,745	190
10000	2,373	95	1,187	48

DETERMINATION OF SLAB THICKNESS

The determination of slab thickness is based upon the crack expectancy of the pavement slab. The "crack expectancy" of concrete pavements is defined as the number of years elapsing between construction of the pavement slab and the time at which cracks are expected to occur. It being assumed that the cracks when they occur are caused by fatigue in the concrete due to repetitions of unit stresses of magnitudes exceeding 50 percent of the ultimate rupture strength of the concrete. Laboratory studies on the fatigue of concrete indicate that unit stresses of magnitudes less than 50 percent of the ultimate rupture strength of the concrete are not harmful to the structural integrity of the concrete structure.

Stress Cycles. When computing crack expectancy for any part of a pavement slab it is necessary to associate maximum stress possibilities with stress cycles and the principle of fatigue. A stress cycle is considered as one complete change in unit stress from minimum to maximum. The number of stress cycles per year which are caused by the frequency of load applications and temperature changes during that period are determined in the following manner:

1. Stress cycles due to load and temperature; cycles varying from:

S_{sd} to $(S_{sd} + S_L)$
S_{wd} to $(S_{wd} + S_L)$
S_{sn} to $(S_{sn} + S_L)$
S_{wn} to $(S_{wn} + S_L)$

with load frequencies as given in Table V.

2. Stress cycles due to warping only:

S_{sd} to S_{sn}
S_{wd} to S_{wn}

Since 1/2 year is considered as spring and summer or fall and winter and 25 percent of the days in the year are bright, the number of complete warping cycles per year are computed as follows:

$$1/2 \times 0.25 \times 365 = 46 \text{ cycles}$$

$$1/2 \times 0.25 \times 365 = 46 \text{ cycles}$$

3. Stress cycles where friction force varies from:

$+S_f$ to $-S_f$ with an annual frequency of unity

4. Any other possible stress cycles were assumed not to affect the crack expectancy.

<u>Fatigue of Concrete</u>. The number of stress applications to cause failure of the concrete was determined from the specially prepared graph presented in Figure 8. This tentative fatigue graph has been prepared on the basis of concrete fatigue data determined by the Illinois Division of Highways (7)*, which has been interpreted graphically by Bradbury. The graph presented by Bradbury has been modified and extended voluntarily by the authors to take into consideration the relationship of fatigue due to stress repetitions when the loads are applied to concrete slabs already under varying magnitudes of stress. This action is based upon the work of German investigators who have made similar studies with metals (8)*. In Bradbury's interpretation of the fatigue principle he assumes that the concrete in the pavement has zero stress at time of load application.

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*(7) Illinois Division of Highways, Engineering Report No. 34-1, Fatigue Curves according to R. D. Bradbury. Reinforced Concrete Pavements-1938.

*(8) Report No. 42, October 21, 1933. Verein Deutscher Ingenieure (VDI)

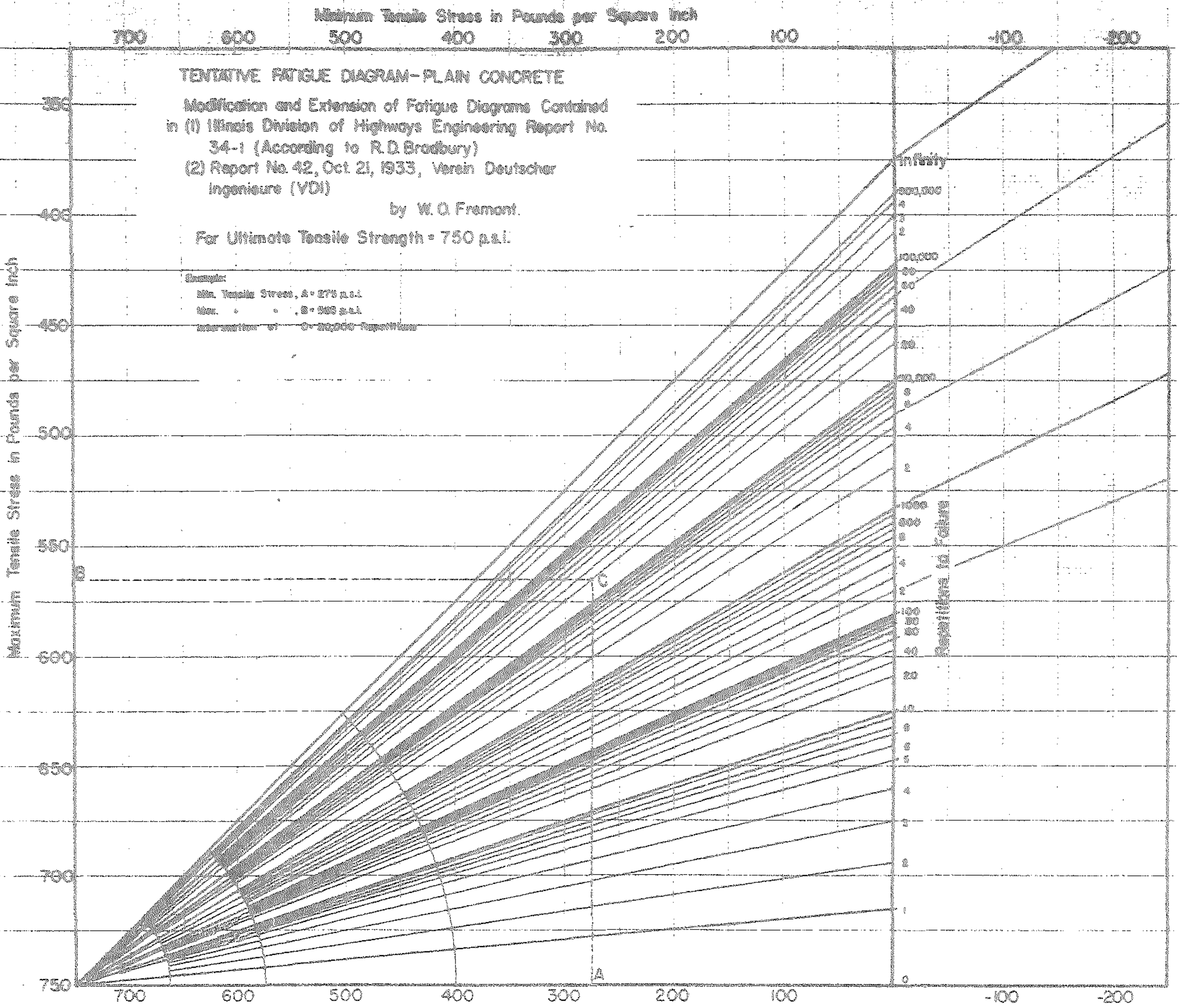
Calculation of Crack Expectancy. The design factors which affect crack expectancy in pavement slabs are: thickness, slab length, subgrade modulus, uniformity of cross section, and traffic volume. The crack expectancy for each case is calculated separately in the following manner:

With the subgrade assumed constant, slab length and thickness fixed, the warping stresses and load stresses are computed for critical points in the slab. These stress values are then combined and tabulated. The maximum and minimum stresses for each wheel load class are selected for the spring and summer seasons, and also for the fall and winter. Repetitions of cycles to failure are read from Figure 8. The repetitions of cycles to failure are designated "R". For each wheel load class the annual load frequency is selected from Table XII. This value is termed "r". The ratio $\frac{r}{R}$ is then computed. A similar computation is made for warping stresses only, again using Figure 8 for R, but using $r = 46$ (see stress cycles). The contribution of stress due to friction is negligible, since the annual frequency is unity. The ratios $\frac{r}{R}$ are summed, and the reciprocal of this value is the crack expectancy in years for the particular condition considered. The procedure employed in calculating crack expectancy is shown in the following example.

EXAMPLE OF CRACK EXPECTANCY CALCULATIONS

Assume for example a point at the interior of a slab, having a length of 20 feet, a thickness of 6", $k = 100$, and $f = 1.0$. The crack expectancy computation is as follows:

From previous calculations the following warping and warping plus friction stresses were obtained:



TENTATIVE FATIGUE DIAGRAM–PLAIN CONCRETE

Modification and Extension of Fatigue Diagrams Contained
in (1) Illinois Division of Highways Engineering Report No.
34-1 (According to R.D. Bradbury)
(2) Report No. 42, Oct. 21, 1933, Verein Deutscher
Ingenieure (VDI)

by W. O. Fremont.

For Ultimate Tensile Strength = 750 p.s.i.

Example:
Min. Tensile Stress, A = 275 p.s.i.
Max. " " , B = 565 p.s.i.
Intersection at C = 20,000 Repetitions

Figure 8

(A) Warping Stresses from Equation VII

When $d = h$ then $S_{w1} = 88$ p.s.i.
$d = 2h$ then $S_{w2} = 176$ p.s.i.
$d = 3h$ then $S_{w3} = 265$ p.s.i.

(B) Friction Stresses from Equation IX

When $L = 20$ feet then $S_f = 11$ p.s.i.

(C) Combined Warping and Friction Stresses From Figure 4

$S_{sd} = S_{w3} - S_f = 254$ p.s.i.
$S_{sm} = -S_{w1} - S_f = -99$ p.s.i.
$S_{wd} = S_{w2} + S_f = 187$ p.s.i.
$S_{wm} = -S_{w1} + S_f = -77$ p.s.i.

The warping and friction stresses are independent of the load "P" and are combined with load stress S_L as follows:

TABLE XIII

COMBINED LOAD AND WARPING STRESSES

Load P	Load Stresses S_L (from table VII)	Combined Stresses			
		$S_{wd} + S_L$	$S_{wm} + S_L$	$S_{sd} + S_L$	$S_{sm} + S_L$
4000	143	330	66	397	44
5000	169	356	92	423	70
6000	194	381	117	448	95
7000	219	406	142	473	120
8000	242	429	165	496	143
9000	264	451	187	518	165
10000	283	470	206	537	184

To determine crack expectancy, repetitions to failure "R" must be determined from the graph of Figure 8. This necessitates selection of maximum and minimum tensile stresses. These are tabulated as follows:

TABLE XIV

DETERMINATION OF CRACK EXPECTANCY

Load P	Minimum Tensile Stress, from (C)	Maximum Tensile Stress, from Table XIII	R (from fig. 8)	r (from table XII)	$\frac{r}{R}$
	S_{sd}	$S_{sd} + S_L$			
		(Spring and Summer)			
4000	254	397	∞	3037	0
5000	254	423	∞	6833	0
6000	254	448	∞	7592	0
7000	254	473	∞	7212	0
8000	254	496	∞	5694	0
9000	254	518	300,000	3037	0.0101
10000	254	537	70,000	759	0.0108
		(Fall and Winter)			
	S_{wd}	$S_{wd} + S_L$			
4000	187	330	∞	3037	0
5000	187	356	∞	6833	0
6000	187	381	∞	7592	0
7000	187	406	∞	7212	0
8000	187	429	∞	5694	0
9000	187	451	∞	3037	0
10000	187	470	∞	759	0
		(Warping Only)			
	S_{sn}	S_{sd}			
-	-99	254	∞	46	0
	S_{wn}	S_{wd}			
-	-77	187	∞	46	0
	S_f	S_f			
-	-11	+11	∞	1	0
				Total	0.0209

$$\text{Crack Expectancy} = \frac{1}{\frac{r}{R}} = \frac{1}{0.0209} = 48.0 \text{ years}$$

Determination of Slab Thickness For A Desired Pavement Crack Expectancy. The crack expectancy calculation is made for each type of pavement to be considered. These include thickness from 6" to 12", slab lengths from 10 ft. to 200 ft. and various joint spacing. The results are plotted on coordinate paper with pavement thickness against crack expectancy. Curves are drawn joining points of the same slab length. A sample of such a family of curves is shown in Figure 9.

To determine the pavement thickness necessary for a given crack expectancy, it is only necessary to choose a definite time period, on the graph draw a horizontal line through that point and note the abscissae of the points of intersection of this line with the crack expectancy curves. As an illustration see Figure 9. The required time interval is 20 years, and the dashed line at this level intersects the curves as follows:

TABLE XV

PAVEMENT THICKNESS DETERMINATION

Curve No.	Slab Length In Feet	Location	Pavement Thickness In Inches
1.	100	Longitudinal Edge	6.5
2.	140	Longitudinal Edge	7.5
3	Any length	Corner	9.3

A design diagram may now be drawn by graphing these data, using slab lengths as abscissae and pavement thicknesses as ordinates. Similar diagrams may be made for each Highway class and for definite design conditions

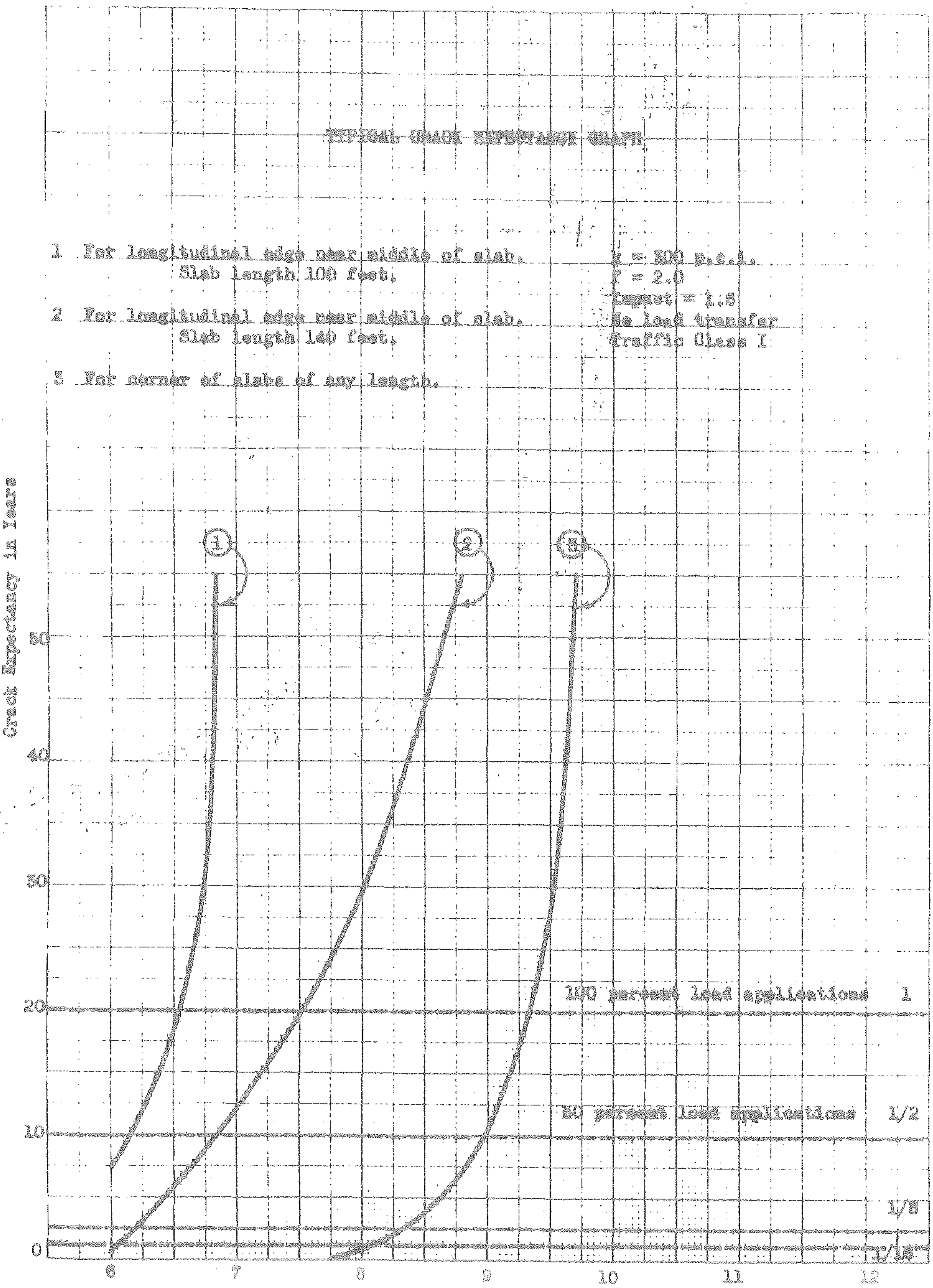
Preparation of Design Diagrams, Uniform Cross Section.

In preparing the design curves for uniform pavements, as shown in Figure 10, the slab thicknesses were determined first for Class I highways. The slab thicknesses for pavements in the other three classes were determined from the design data established from Class I in the following manner:

The load frequencies for traffic Classes II, III, and IV are definite fractions of the frequency for Class I, (See Tables IV and V) and the ratios of these frequencies to that of Class I are respectively 1/2, 1/8, and 1/16. Hence the pavement life period, or crack expectancy, would be expected to be 2, 8, and 16 times that of the expectancy for Class I if the slab thickness remained unchanged.

To determine the pavement thicknesses for a given crack expectancy, a horizontal line is drawn at the given crack expectancy value in a graph similar to that of Figure 9. The abscissae of the points of intersection of this line and each curve gives the thicknesses required for each condition specified for Class I traffic. Since Class II traffic has half the frequency, a horizontal line at one-half the value for Class I traffic will intersect the curves at points whose abscissae give thicknesses required for Class II traffic. Correspondingly, lines at 1/8 and 1/16 the values of Class I crack expectancy yield thicknesses required for Classes III and IV (See Figure 9).

Design curves have also been prepared for two cases of highway loadings, (1) when 100 percent of all wheel loads act at the free edge and corner and (2) when only 50 percent of all wheel loads reach the same positions. For the case of 50 percent wheel load applications the slab


TYPICAL CRACK EXPECTANCY GRAPH
1 For longitudinal edge near middle of slab. Slab length 100 feet.
2 For longitudinal edge near middle of slab. Slab length 140 feet.
3 For corner of slabs of any length.
k = 200 p.c.i.
F = 2.0
Impact = 1.5
No load transfer
Traffic Class I
①
②
③
Crack Expectancy in Years
50
40
30
20
10
0
100 percent load applications 1
50 percent load applications 1/2
1/8
1/16
6
7
8
9
10
11
12
Slab Thickness "h" in Inches

Fiture 9

DESIGN DIAGRAMS FOR CONTINUOUS UNIFORM REINFORCED CONCRETE PAVEMENT

FOR HIGHWAY CLASSES I-II-III-IV

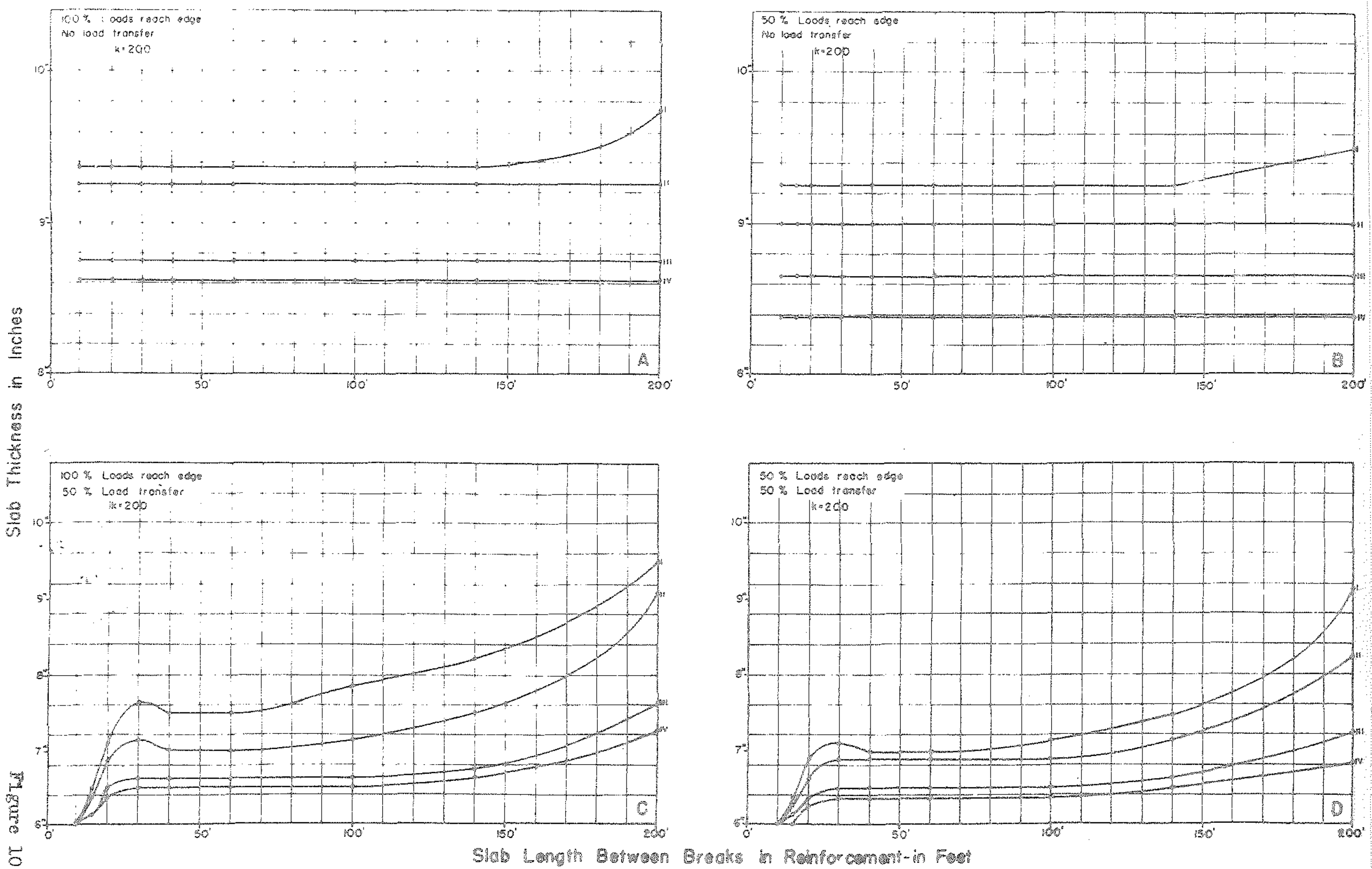


Figure 10

thicknesses for the various design requirements are obtained from the crack expectancy graphs prepared for Class I highways in the manner described above for different highway classes but taking in this case a 10 year period for crack expectancy (See Figure 9).

In constructing the design curves for uniform pavements, given in Figure 10, largest slab thickness was selected for the subgrade modulus and design requirements for "k" = 100, with f = 1.00 and "k" = 300 with f = 2.00. The average of the above found values were used for expressing the slab thickness versus slab lengths for continuous and hinged slabs when "k" = 200 with f = 1.50.

Twelve Foot Lanes or 24 Foot Pavement Widths.

As previously mentioned, all design calculations have been based upon 11 foot lanes or 22 foot pavements. However, some 24 foot pavements have been constructed and others are contemplated. In that connection, supplementary tables and graphs were prepared showing the effect on slab thickness of increasing "W" (width) by one foot.

The data from the study indicate that critical stresses in pavements of uniform thickness occur at the corners, and since corner stresses are not affected by changes in slab width it is obvious that the increase in width does not affect the slab thickness at least for a change in lane width from 11 to 12 feet. Therefore, the design data previously presented for the 22 foot pavement will suffice for 24 foot pavement widths.

With the increase of slab width the steel reinforcement requirements per 100 square feet do not change, but the cross-sectional area of the longitudinal steel must be increased in the ratio of $\frac{24}{22}$. Graphs showing the reinforcement requirements for 22 and 24 foot pavements will be found in Part II under a separate discussion of steel reinforcement in relation to joint spacing.

PART II

JOINT SPACING

The question of joint spacing in concrete pavements has concerned highway engineers for a great many years. At the present time the subject still seems to be quite controversial although there is available sufficient information on slab behavior to afford a logical solution of this problem.

From all viewpoints, an ideal concrete pavement surface would be one consisting of a continuous ribbon of concrete of uniform construction with no breaks in its continuity. Unfortunately such a concrete pavement cannot be achieved under present day practice. Because of its inherent weakness in tension, concrete is highly susceptible to cracking under tensile stresses particularly those induced in pavement slabs by volume changes due to temperature fluctuation. The nearest approach to obtaining an ideal pavement free of transverse cracks is best accomplished by dividing the pavement into sections or slabs, each slab as long as possible consistent with practical design requirements and within economic limitations.

To satisfy these conditions, joints are normally provided in concrete pavements to reduce to safe values the stresses caused by expansion, contraction and warping of the concrete, and by subgrade friction forces. Thus in accordance with the adopted design procedure, presented in Part I, the spacing of joints is limited, although not necessarily fixed, by permissible maximum stress intensities which should provide reasonable freedom from cracks in a period of 20 years. Joints in this catagory are generally known as expansion and plane of weakness, the latter type being commonly designated contraction or dummy joints depending upon the manner in which they are constructed.