Final Report No. 2

THE MICHIGAN INTERCITY PASSENGER DEMAND MODEL

Prepared for:

INTERAGENCY TRANSPORTATION COUNCIL
STATE OF MICHIGAN

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Final Report No. 2 *June 1971*

THE MICHIGAN INTERCITY PASSENGER DEMAND MODEL

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CONTRACT 1-1970

SRI Project MSH-8476

PREFACE

This report, the second in a series of transportation planning reports prepared by SRI for the Michigan Interagency Transportation Council (ITC), documents the specification, calibration, and testing of a demand model developed to predict intercity travel by mode within and around the State of Michigan. Examples of the application of the model in transportation planning studies may be found in Reports 3 and 4 of the series. The work described in this report was conducted jointly by SRI and ITC staff. At SRI, model development work was performed by John Billheimer under the supervision of Dan G. Haney. William Kasip of ITC provided most of the input data required for model calibration and testing.

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I INTRODUCTION

This report documents the specification, calibration, and performance of a mathematical model developed to predict intercity travel by mode, within and around the State of Michigan. The performance of different existing demand models is studied, and a model is formulated that uses income data and the cost, time, and frequency of modal service to predict the travel volumes linking a wide range of city sizes. Model parameters are estimated using a constrained search calibration technique. Model performance is documented, and the sensitivity of this performance to changes in input data and calibration parameters is discussed.

II MODEL SPECIFICATION

Range of Performance

The State of Michigan covers an area of 58,216 square miles. Its populated areas range in size and type from isolated rural hamlets to the heavily industrialized Detroit area. An intercity passenger demand model designed for use in the state must be capable of predicting traffic by mode between cities of widely varying population densities separated by distances ranging from 50 to 600 miles. To assist in the formulation and calibration of a model having this capability, a set of 15 sample origin-destination pairs was selected to reflect the range of population-distance combinations existing in and around Michigan. These sample pairs are listed in Table 1. The designations "small," "medium," and "large" displayed in Table 1 fall into the following population categories:

Size Designation	Population Range
Small	20,000 to 50,000
Medium	200,000 to 1,000,000
Large	2,000,000 to 5,000,000

For the purpose of estimating intercity passenger traffic, the population of an origin or destination was taken to be the population of the entire Standard Metropolitan Statistical Area (SMSA). In the case of areas too small to receive an SMSA designation, the county was taken as the basic population unit.

Performance of Existing Models

As a first step toward selecting a model for use in predicting Michigan intercity traffic, the 15 sample origin-destination pairs were used to test several existing single-mode and multimode demand models. A list of the models tested appears at the top of the page following Table 1.

Table 1

SAMPLE ORIGIN-DESTINATION PAIRS

City Size	Distance: Short (50-100 Miles)	Distance: Medium (150-250 Miles)	Distance: Long (400-600 Miles)
Small to small	(1) ALP to SSM	(2) SSM to HOU	
Small to medium	(3) ALP to FLI	(4) SSM to FLI	
Small to large	(5) CAR to DET	(6) SSM to DET	(7) HOU to DET
Medium to medium	(8) FLI to KAL	(9) FLI to COL	(10) FLI to DES
Medium to large	(11) FLI to DET	(12) FLI to CHI	(13) FLI to PHL
Large to large		(14) DET to CHI	(15) DET to PHL

Notes:

(x) = Pair number

City abbreviations: ALP = Alpena

SSM = Sault Sainte Marie

HOU = Houghton

FLI = Flint

CAR = Caro

DET = Detroit

KAL = Kalamazoo

COL = Columbus, Ohio

DES = Des Moines

CHI = Chicago

PHL = Philadelphia

Model No.	Developer	Calibration Area	Mode Split Capability	References
1	Unknown	Detroit	Single mode	1
2	Stanford Research Institute	California	Multimode	2
3	Wilbur Smith	Illinois	Single mode	3
4	Arthur D. Little	Michigan	Single mode	4
5	Department of Transportation (Office of High Speed Ground Transportation)	Northeast Corridor	Multimode	5

Details regarding the structure of the above model may be found in the indicated references. All of these models represent traffic between zones as a function of trip generation characteristics and some measure of interzonal impedance. As such, all of the models are descendants of the gravity model. In fact, the first two models, calibrated for Detoit and California, respectively, follow the format of the basic gravity model, in which the demand for interzonal travel, D, is expressed as follows:

$$D = \frac{K\ (\text{origin population} \times \text{destination population})^{\alpha}}{(\text{distance})^{\beta}}$$

where K, α, and β are calibration coefficients.

The third model varies this basic format only slightly by expressing the impedance measure as an exponential function of distance. In the fourth model, developed and calibrated by Arthur D. Little to predict highway traffic in the State of Michigan, the trip generation characteristic depends both on the population of the origin and destination zones and on the population surrounding these zones. The impedance function in this model is segmented so that it is relatively flat for low interzonal travel times, increases rapidly for moderate travel times, and flattens out again for travel times exceeding five hours.

In the fifth model, the number of families in each zone whose annual income exceeds $10,000 is used as a trip generation characteristic, while the impedance function is a composite of the time, cost, and frequency of service experienced on each interzonal mode of travel.

Table 2 shows the results of using each of the demand models to predict traffic between the 15 sample city pairs. This table also contains actual measurements of Michigan intercity automobile traffic, as compiled by the Department of State Highways in studies conducted between 1964 and 1968. A comparison of predicted and observed values shows that existing models generally are unable to cope with the range of city sizes and distances to be found in Michigan. As might be expected, Model 4, developed and calibrated specifically for use in the state, does the best job of reproducing actual traffic measurements, although it appears to overstate traffic between cities separated by short distances. Unfortunately, this model is limited to the prediction of automobile traffic. Furthermore, it is somewhat unwieldy in that it is necessary to define an entire zone structure, complete with surrounding zones, and generate trips from every zone merely to compute the traffic between a single city pair.

The performance of the remaining models hardly can be termed promising. The two classic gravity models, Models 1 and 2, perform poorly when uprooted from their place of calibration and applied to the range of city-pairs existing in Michigan. Model 3, which woefully understates traffic, might profit slightly from a change of coefficients. The DOT model, calibrated on the large cities of the Northeast Corridor, performs poorly in estimating traffic between the smaller Michigan cities.

The discrepancies between predicted and actual values depicted in Table 2 highlight the difficulty of predicting intercity passenger demand over a wide range of distances and city sizes. This difficulty is multiplied by the problem of designating passenger mode preferences. No gravity model exists that can be pulled off the shelf and used with confidence to predict travel patterns in any arbitrary area. In this sense, the term gravity model, implying as it does an immutable law, is a misnomer. Isaac Newton himself might have had second thoughts about the validity of his gravity model had it been necessary to reformulate it for different masses and recalibrate it for different points on the earth's surface.

Model Selection

The ability to reproduce observed travel data with a reasonable degree of fidelity over the range of city sizes and separations encountered in Michigan was but one of the criteria considered in selecting

Table 2

COMPARISON OF DEMAND MODEL PREDICTIONS
One-Way Person Trips by Auto Per Day
(Using 1960 Population Data)

City Pair	Model Results: Unknown	SRI	W. Smith	A. D. Little	Department of Transportation	Actual 1964-1968
ALP-SSM	1	2	1	22	0	19
SSM-HOU	0	1	0	7	0	11
ALP-FLI	4	17	2	29	1	27
SSM-FLI	1	6	0	14	1	51
CAR-DET	280	1,837	138	910	51	660
SSM-DET	8	46	0	62	4	274
HOU-DET	3	15	0	60	1	62
FLI-KAL	56	327	31	78	29	58
FLI-COL	37	231	1	n.a.	33	16
FLI-DES	2	8	0	n.a.	2	3
FLI-DET	7,021	59,378	1,302	24,859	2,877	14,600
FLI-CHI	262	2,032	1	127*	148	77
FLI-PHL	22	151	0	n.a.	14	5
DET-CHI	2,635	25,732	2	597*	1,391	775
DET-PHL	279	2,461	0	n.a.	161	74

* Estimated from Illinois-Wisconsin travel.

an intercity demand model. In addition to this important consideration, it was desired that the model have the following attributes:

- Simplicity
- Sound theoretical structure
- Ability to reflect the intermodal consequences of system changes.

Since each of the tested models failed to reproduce modal preferences over the range of sample city sizes, the selection process centered on these additional attributes. Once a model having these attributes was identified, an attempt was made to extend its range of applicability to include the city sizes of interest in Michigan.

A review of existing intercity demand models led to the selection of the basic model developed by McLynn,[5] modified by the National Bureau of Standards,[6] and summarized in the Northeast Corridor Transportation Project Report.[7] The variables considered by the model in determining the traffic by a mode m between origin-destination pair (i,j) are defined as follows:

t_m = total (i → j) travel time for the m-th mode (hours)

c_m = total (i → j) out-of-pocket <u>per</u> <u>capita</u> cost (dollars)

f_m = frequency of (i → j) service (trips per day)

F = number of families with annual incomes exceeding \$10,000 (families × 10^{-5}) in the SMSA or county of the origin or destination city

Using these variables, the modified demand model may be defined by the following relationships:

$$w_m = \begin{cases} a_m t_m^{\alpha(1)} c_m^{\alpha(2)} [1 - \exp(-Kf_m)]^{\alpha(3)} & (m \neq \text{auto}) \\ t_m^{\alpha(4)} (c_m/1.7)^{\alpha(5)} & (m = \text{auto}) \end{cases} \qquad (1)$$

$$W = \sum_m w_m \qquad (2)$$

$$D = \begin{cases} \beta(0)(F_iF_j)^{\beta(1)}W^{\beta(2)} & F_iF_j > G \\ \beta'(0)(F_iF_j)^{\beta'(1)}W^{\beta(2)} & F_iF_j \le G \end{cases} \tag{3}$$

$$D_m = Dw_m/W \tag{4}$$

The terms w_m and W may be regarded as modal conductance and total $(i \rightarrow j)$ conductance, respectively. D_m and D are daily one-directional modal $(i \rightarrow j)$ demand and total $(i \rightarrow j)$ demand, respectively (measured in persons).

In an attempt to adapt the model to the wide range of city sizes of interest in Michigan transportation studies, the demand model was segmented as indicated in Equation (3). Thus the demand for travel between origin-destination pairs whose population product F_iF_j was below a specified value, G, received different treatment than city-pairs having larger population products. The task of calibrating the above model is described in the next section.

III MODEL CALIBRATION

Data Development

In calibrating the demand model, numerical values were assigned to each of the model's parameters and the effect of each assigned value on the model's ability to reproduce actual travel data was observed. The basic demand data used in this calibration process consisted of observed one-way travel volumes by air, rail, bus, and auto between 20 origin-destination pairs for the base year 1967. The 20 origin-destination pairs were comprised of the 15 sample city pairs listed in Table 1 and the following five additional pairs:

Detroit - Cleveland
Detroit - Pittsburgh
Detroit - Milwaukee
Flint - Cleveland
Flint - Milwaukee

These five origin-destination pairs were added to broaden the data base and to place additional emphasis on travel between larger cities.

Estimates of base year airline travel between the sample origins and destinations were taken from published surveys conducted by the Civil Aeronautics Board.[8] Assessments of automobile travel were obtained from different screenline interviews conducted by the Michigan Department of State Highways. Intercity bus and rail traffic was estimated from a knowledge of passenger embarkations and debarkations, and the number of buses and trains scheduled between the sample city pairs. The resulting estimates of observed one-way traffic between the 20 city-pairs are shown in Table 3. These estimates of the true state of intercity travel were used as a basis for calibrating the intercity passenger demand model.

The cost, time, and frequency of common carrier service between each pair of cities were obtained from published schedules, and access times and costs were computed for each city. In calculating automobile costs and times, operating costs of $.04 per mile were assumed and average speeds of 60, 30, and 15 miles per hour were associated with freeways, arterials, and local streets. An average automobile occupancy of 1.7 persons per vehicle was assumed.

Table 3

"OBSERVED" ONE-WAY PERSON TRIPS PER DAY

City Pair	Air	Rail	Bus	Auto	Total
ALP-SSM	0	0	1	19	20
SSM-HOU	0	0	0	11	11
ALP-FLI	0	0	2	27	29
SSM-FLI	0	0	5	51	56
CAR-DET	0	0	20	660	680
SSM-DET	5	0	10	274	289
HOU-DET	7	0	0	62*	69
FLI-KAL	0	3	25	58	86
FLI-COL	0	2	2	16†	18
FLI-DES	1	0	1	3‡	5
FLI-DET	9	30	250	4618§	4907
FLI-CHI	31	5	20	77‡	143
FLI-PHL	4	0	2	5	11
DET-CHI	631	80	150	775	1636
DET-PHL	251	5	20	74	350
DET-CLE	137	3	25	572	737
DET-PIT	139	2	10	103	254
DET-MIL	134	2	10	41	187
FLI-CLE	9	1	4	22	36
FLI-MIL	3	0	1	4	8

* Estimated from Iron Mountain-Detroit data.
† Estimated from Flint-Ohio data.
‡ Estimated from Flint-Illinois and Wisconsin data.
§ Estimated from Flint-Wayne and Flint-Macomb data. (Flint-Oakland data were eliminated because of the biasing effect of local trips between the conterminous counties.)

Census data from 1960 were extrapolated to 1967 in estimating the number of families in each origin or destination zone having a real income exceeding $10,000 per year. Underlying this extrapolation process was the assumption, made after consideration of a number of alternatives, that the ratio of families in this income bracket to total population would remain constant over the years. This assumption made it possible to use existing population forecasts as a basis for estimating the required input data.

Calibration Technique

References 5 and 6 outline a technique for using least-squares regression analysis to calibrate the basic McLynn demand model. This technique begins with the empirical determination of certain parameters and proceeds through a series of two log-linear regression analyses. Attempts to apply this technique to calibrate the model formulated in Equations (1) through (4) using Michigan demand data proved unsuccessful. Part of the explanation for this lack of success may be traced to the failure of the log-linear regression format to deal adequately with the range of city sizes under consideration. This is not surprising, in view of the fact that the two-stage calibration technique developed in References 5 and 6 achieved only limited success in representing travel between the larger Northeast Corridor cities.

In lieu of regression analysis, the Michigan Intercity Passenger Demand Model was calibrated by means of a constrained search technique. Through a combination of past experience and a knowledge of the model's structure, upper and lower bounds were set on acceptable values of each model parameter. A limited search was undertaken within these constraints for the combination of parameters that minimized a series of error functions describing model performance. The parameter bounds and error functions used in this constrained search calibration process are described below.

Parameter Bounds

The following logical bounds were imposed on the model parameters in advance of the calibration process.

$$0 \leq \beta'(0) \leq \beta(0)$$

$$0 \leq \beta'(1) \leq \beta(1) \leq 1.1$$

$$0 \leq \beta(2) \leq 1$$

$$-5 \le \alpha(j) \le 0 \qquad j = 1,2,4,5$$

$$\alpha(3) = .3247$$

$$K = 0.12$$

$$0 \le a_m \le 5$$

The model's consistency of behavior was assured by imposing a positive or a negative constraint on each parameter. In addition, the positively constrained parameters $\beta'(1)$, $\beta(1)$, and $\beta(2)$ each had logical upper bounds. Experience with gravity models has shown that the exponent $\beta(1)$ associated with the population product rarely exceeds 1.1. Were this exponent to be higher, population increases would have a disproportionate effect on predicted travel demand. Further, the exponent $\beta'(1)$ associated with small city pairs cannot exceed the large-city exponent $\beta(1)$. This relationship is indicated by empirical data relating intercity travel to population product for the sample city pairs.

Consideration of the conductance exponent $\beta(2)$ shows that the value of this exponent cannot exceed unity. Otherwise, a decrease in the time or cost of travel by one mode could cause corresponding increases in travel over competing modes. This can be shown by considering that, for small changes in time or cost, demand changes may be expressed as a function of the partial derivative of demand with respect to the changing variable. If the cost, c_m, of travel by mode m between two cities were to be changed, the effect on a competing mode n can be represented as follows:

$$\Delta D_n = \frac{\partial D_n}{\partial c_m} \Delta c_m$$

$$\Delta D_n = \frac{\alpha(2)}{c_m} [\beta(2) - 1] D_n \frac{W_m}{W} \Delta c_m \tag{5}$$

where ΔD_n represents a small change in demand D_n for a competing mode n

Δc_m represents a small change in cost c_m.

Thus the intermodal effects predicted by the proposed demand model will remain consistent only so long as $\beta(2) \le 1$.

Model consistency also demands that the modal conductance exponents $\alpha(1)$, $\alpha(2)$, $\alpha(4)$, and $\alpha(5)$, associated with time and cost, be negative. If these exponents are allowed to become too large, however, small changes in time or cost will have a disproportionate effect on demand. If $\alpha(2) \leq -5$, for example, sensitivity analysis shows that a 10 percent decrease in the cost of mode m could cause more than a 50 percent increase in the demand for that mode. Accordingly, a lower limit of -5 was placed on exponents $\alpha(1)$, $\alpha(2)$, $\alpha(3)$, and $\alpha(4)$ to forestall such unlikely results.

In calibrating his basic demand model, McLynn empirically set K = 0.12.[5] This value was used in the Michigan model, as was the McLynn-calibrated value $\alpha(3) = 0.3247$. An upper limit of 5 was placed on the common carrier conductance multiplier a_m, because it was felt that larger values of a_m would create unrealistic imbalances between common carrier traffic and automobile traffic.

Error Functions

As the parameter values were varied within the established bounds, certain error functions were computed and monitored to determine the overall effect of each parameter on the demand model's ability to reproduce observed travel data. These error functions are described below.

RMS Error. The model rms error, E^m_{rms}, was computed by taking the square root of the sum of the squares of the differences between calculated and observed modal travel values. That is,

$$E^m_{rms} = \sqrt{\sum_{j-1}^{20} (D_{mj} - D'_{mj})^2} \qquad (6)$$

where D_{mj} = calculated travel by mode m for case j

D'_{mj} = observed travel by mode m for case j.

Absolute Error. The absolute error, E^m_{abs}, was computed by taking the sum of the absolute values of the difference betwwen calculated and observed modal travel values. That is,

$$E^m_{abs} = \sum_{j=1}^{20} \left| D_{mj} - D'_{mj} \right| \quad .$$

<u>Gap Error</u>. The gap error, E^{m}_{gap}, was computed by counting the number of calculated travel values that fell outside a predetermined range surrounding the observed value.

$$E^{m}_{gap} = \sum_{j-1}^{20} \mu_{mj}$$

where

$$\mu_{mj} = \begin{cases} 1, & \text{if} \quad D_{mj} \leq (1 - \delta)D'_{mj} \\ 0, & \text{if} \quad (1 - \delta)D'_{mj} \leq D_{mj} \leq (1 + \delta)D'_{mj} \\ 1, & \text{if} \quad D_{mj} \geq (1 + \delta)D'_{mj} \end{cases} \qquad (8)$$

In the calibration process, E^{m}_{gap} was recorded for three separate settings of the gap value δ: $\delta = 0.1$, $\delta = 0.25$, and $\delta = 0.5$. These three settings recorded the number of calculated demand values that failed to fall within 10 percent, 25 percent, and 50 percent of the observed demand.

<u>Calibration Procedure</u>

In calibrating the segmented demand model, attention was first directed toward the determination of the parameters $\beta(0)$ and $\beta(1)$, which were associated with larger city pairs. Once these parameters were fixed, the search for $\beta'(0)$ and $\beta'(1)$ was undertaken. In the case of larger city pairs, the constrained search calibration procedure followed the steps outlined below.

Step (1) Set $\beta(0) = a_m = 1$

Step (2) Select values for $\beta(1)$ and $\beta(2)$

Step (3) Select values for $\alpha(1)$, $\alpha(2)$, $\alpha(4)$, and $\alpha(5)$

Step (4) Compute D_{mj} for each city pair

Step (5) Compute error functions

Step (6) Adjust a_m to approximate mode split proportions

Step (7) Adjust $\beta(0)$ to minimize error functions

Step (8) Return to Step (3) and try another combination of $\alpha(i)$. Repeat until no further improvement in the error functions appears possible for the combination of $\beta(1)$ and $\beta(2)$ selected in Step (2). Then try another combination of $\beta(1)$ and $\beta(2)$.

In the actual calibration process, $\beta(1)$ and $\beta(2)$ are varied in increments of 0.1 until a combination was found that appeared to fit the observed travel data associated with larger city pairs. At this point, $\beta(0)$, $\beta(1)$, and $\beta(2)$ were fixed, and a search was undertaken for appropriate values of $\beta'(0)$ and $\beta'(1)$. The results of this calibration process are described below.

Calibration Results

The above calibration process resulted in the identification of the following parameter values:

$$a_m = \begin{cases} 1.5, & m = \text{air} \\ 0.75, & m = \text{bus, rail} \end{cases}$$

$$\alpha(1) = \alpha(2) = -1.5$$

$$\alpha(3) = 0.3247, \qquad K = 0.12$$

$$\alpha(4) = \alpha(5) = -1.8$$

$$\beta(0) = 25{,}000, \qquad \beta'(0) = 2{,}500$$

$$\beta(1) = 1.0, \qquad \beta'(1) = 0.1$$

$$\beta(2) = 0.9$$

$$G = 0.075$$

Table 4 contains a comparison of the demand calculated through the use of the above parameters and the observed travel between each of the 20 sample city pairs. The error functions associated with this comparison are shown in Table 5.

Table 4

COMPARISON OF CALCULATED AND OBSERVED VALUES

	Air		Rail		Bus		Auto		Total	
Case	Calculated	Observed	Calculated	Observed	Calculated	Observed	Calculated	Observed	Calculated	Observed
ALP-SSM	0	0	0	0	0	1	14	19	14	20
SSM-HOU	0	0	0	0	0	0	3	11	3	11
ALP-FLI	0	0	0	0	2	2	13	27	15	29
SSM-FLI	0	0	0	0	1	5	3	51	4	56
CAR-DET	3	0	0	0	4	20	70	660	76	680
SSM-DET	1	5	0	0	1	10	3	274	4	289
HOU-DET	1	7	0	0	0	0	1	62	1	69
FLI-KAL	0	0	3	3	14	25	60	58	77	86
FLI-COL	4	0	1	2	1	2	13	16	19	20
FLI-DES	1	1	0	0	0	1	0	3	2	5
FLI-DET	51	9	55	30	269	250	4,096	4,618	4,470	4,907
FLI-CHI	29	31	4	5	9	20	59	77	101	133
FLI-PHL	14	4	0	0	1	2	2	5	18	11
DET-CHI	660	631	97	80	149	150	802	775	1,708	1,636
DET-PHL	155	251	5	5	19	20	40	74	220	350
DET-CLE	332	137	41	3	94	25	650	572	1,117	737
DET-PIT	188	139	34	2	62	10	127	103	411	254
DET-MIL	115	134	16	2	28	10	42	41	202	187
FLI-CLE	25	9	0	1	4	4	26	22	55	36
FLI-MIL	3	3	1	0	2	1	4	4	10	8

Table 5

ERROR FUNCTIONS

Error	Air	Rail	Bus	Auto	Total
RMS	230	60	94	843	915
ABS	476	131	218	1,718	2,302
10% GAP	11	6	8	16	16
25% GAP	9	4	8	8	13
50% GAP	2	1	4	6	9

Although the overall agreement between calculated and observed values is satisfactory, the demand model severely understates travel between city pairs consisting of one small and one large city. The reason for this understatement is shown clearly in Figure 1, which plots normalized demand as a function of population product. Normalized demand is defined as follows:

$$D_{normalized} = D/W^{\beta(2)}$$

This normalization process removes the effect of travel impedance from the demand term, so that the resulting normalized demand should be a piecewise log-linear function of the trip attraction measure, the income product F_iF_j. Figure 1 shows that the normalized demand between all city pairs except Sault Sainte Marie-Detroit, Houghton-Detroit, Sault Sainte Marie-Flint, and Caro-Detroit clusters closely about the log-linear form defined by the calibration process. It would appear to be impossible to use the chosen model effectively to represent travel between these four cities without destroying the model's ability to reproduce the remainder of Michigan's intercity traffic. There seems to be nothing within the framework of the mathematical model to explain, for instance, why automobile traffic between Detroit and Sault Sainte Marie should be nearly double the combined traffic between Detroit and the larger, closer cities of Pittsburgh and Milwaukee. It is doubtful that even heavy recreation travel to the upper peninsula could account for this inconsistency in the observed data. Such inconsistency, particularly in the case of travel between small and large cities, contributes heavily to the error functions of Table 5.

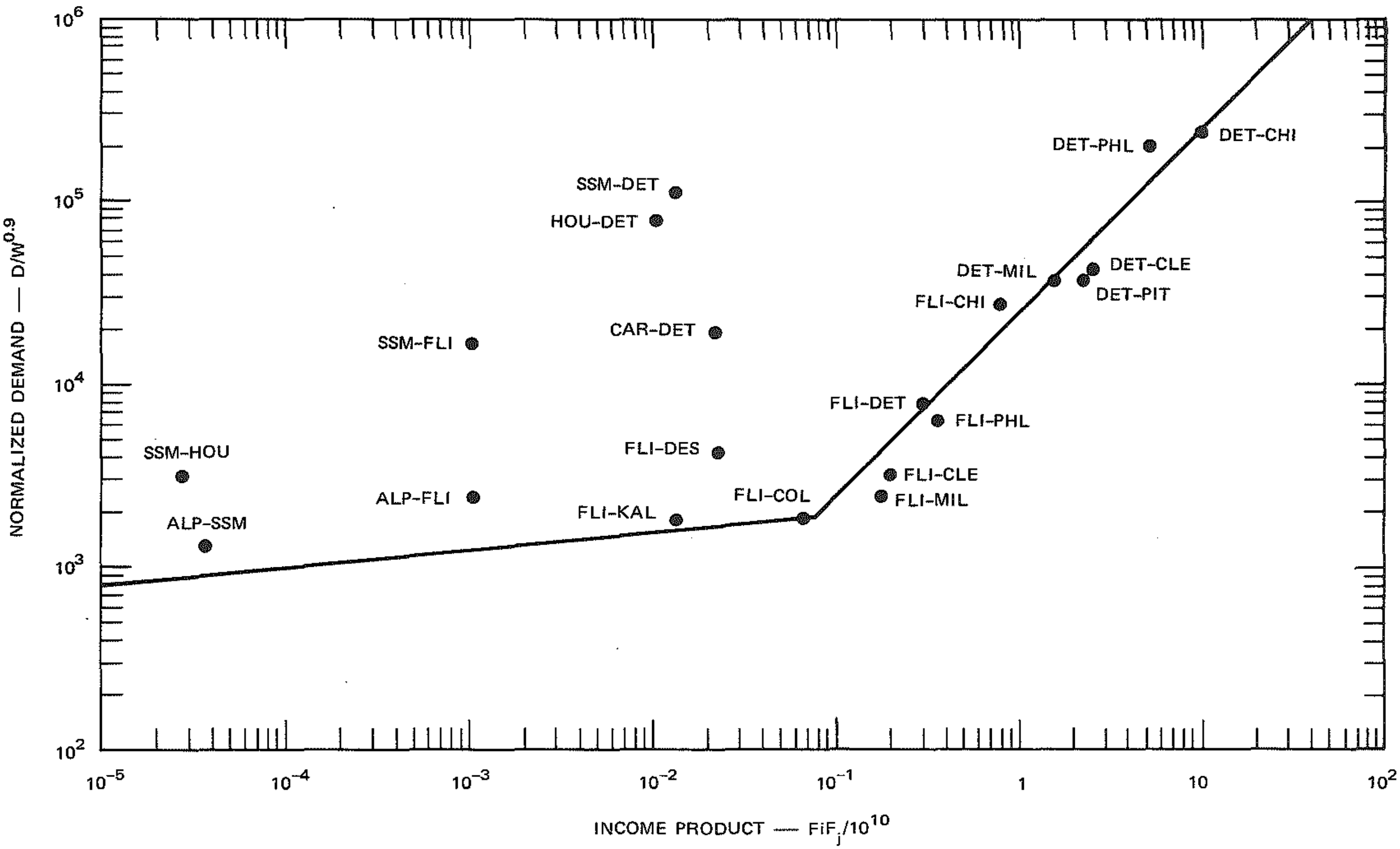

FIGURE 1 NORMALIZED DEMAND VERSUS INCOME PRODUCT

In addition to highlighting data inconsistencies, Figure 1 clearly shows the need for segmenting the Michigan intercity demand model. The data points plotted in this figure make it plain that a single log-linear function cannot reflect travel demand between city pairs of all sizes.

IV MODEL SENSITIVITY

Effect of Variable Changes

One test of the soundness of a demand model is its ability to behave logically in the face of changes in input variables. Since the Michigan intercity passenger demand model is a closed-form mathematical expression, its sensitivity to variable changes may be determined analytically. The first partial derivative of demand with respect to each input variable, $\partial D/\partial V$, provides a measure of this sensitivity, and, by inference, also provides a measure of the impact of each variable on intercity demand.

The value of $\partial D/\partial V$ associated with each model input variable was computed and used to assess the effects of small (10 percent) changes in each variable on model demand and total intercity travel. Table 6 summarizes the results of this assessment. For large city pairs, a 10 percent increase in the number of families in one city earning over $10,000 per year will increase travel demand by 10 percent across all modes. For small cities, an equivalent percentage increase will result in only a 1 percent increase in total travel. Although these differences in the modeled effect of population changes may be valid for extremely large cities and extremely small cities, it is illogical to expect such dichotomous behavior in the case of medium-size city pairs. The abrupt transition from a 1 percent to a 10 percent increase in travel experienced when the income product F_iF_j exceeds $G = 0.075$ might be smoothed by replacing the segmented demand model with a continuous function. The possibility of accomplishing this is discussed in the section devoted to future work.

The effects of small changes in the model input variables time, cost, and frequency vary with the importance of the individual mode in intercity travel. If a mode m dominates intercity travel (i.e., if w_m/W is nearly unity for mode m), the effects of modal changes on total intercity demand are maximized. Conversely, small changes in infrequently used modes (modes for which w_m/W is vanishingly small) will have slight effect on total intercity demand.

Table 6 shows that a 10 percent increase in the cost of travel by common carrier between two cities might cause a decrease of 13.5 percent in the total travel demand between those cities if common carrier is the prevalent mode of intercity travel. If automobile is the prevalent mode of intercity travel, the effect of such a cost increase on total intercity

Table 6

MODEL SENSITIVITY TO INPUT VARIABLES

Variable: Symbol	Variable: Description	Case	Effect of a 10% Variable Increase: Modal Demand (Dm): Minimum % Change	Modal Demand (Dm): Maximum % Change	Total Demand (D): Minimum % Change	Total Demand (D): Maximum % Change
F	Families with income > \$10,000 (families × 10^{-5})	$F_iF_j > 0.075$	10.0	10.0	10.0	10.0
		$F_iF_j \leq 0.075$	1.0	1.0	1.0	1.0
t_m	Time by mode m (hours)	$m \neq$ auto	-13.5	-15.0	0.0	-13.5
		$m =$ auto	-16.2	-18.0	0.0	-16.2
c_m	Cost by mode m (dollars)	$m \neq$ auto	-13.5	-15.0	0.0	-13.5
		$m =$ auto	-16.2	-18.0	0.0	-16.2
f_m	Frequency by mode m (trips per day)	$m \neq$ auto	0.0	3.9	0.0	2.6

travel would be negligible. The effect of the fare increase on travel via the affected mode would be a loss of between 13.5 percent and 15 percent of the mode's pre-increase travel volume. Similar ranges would be expected in the event of a 10 percent increase in travel time. These ranges are shown in Table 6, along with the corresponding ranges for changes in the time and cost of automobile travel. The magnitude of these small changes does not appear to be unreasonable, and, thanks to the constraints imposed in the calibration procedure, the direction of change is proper.

Service frequency is the least effective of the input variables in terms of its ability to influence sizable demand changes. A 10 percent increase in common carrier service frequency can effect no more than a 3.9 percent in modal patronage, and no more than a 2.6 percent increase in total intercity travel.

Effect of Parameter Changes

Just as the effect of variable changes on predicted demand gives a measure of model reasonability, so the effect of parameter variations on demand gives a measure of model stability. If a small parameter change can drastically alter model output, the calibration procedure is complicated and model validity may be suspect. Table 7, which was compiled by evaluating $\partial D/\partial P$, the first derivative of demand with respect to each model parameter, lists the effects of small parameter changes on total intercity travel.

The effect of small parameter changes on total demand is quite complex and may depend on the relative impact of a mode on intercity travel; on the existing population product; on current levels of time, cost, and frequency; or on all of these factors. Table 7 shows that the parameters whose changes have the greatest potential impact on demand are the time and cost exponents $\alpha(1)$, $\alpha(2)$, $\alpha(4)$, and $\alpha(5)$, and the conductance exponent $\beta(2)$. In general, the indicated maximum demand fluctuations caused by changes in these parameters rarely will be approached. The maximum demand change of 40 percent associated with a 10 percent increase in $\alpha(1)$ would be approached only if the dominant mode of travel between two cities were a common carrier mode characterized by relatively high costs (in excess of $50 per one-way trip). Similar provisos may be attached to the maximum demand changes indicated for all modal exponents.

Table 7

MODEL SENSITIVITY TO PARAMETER CHANGES

Parameter Symbol	Parameter Description	Case	Effect of a 10% Parameter Increase — Total Demand D: Minimum % Change	Effect of a 10% Parameter Increase — Total Demand D: Maximum % Change
$\beta(0)$	Demand multiplier	$F_iF_j > 0.075$	10	10
$\beta'(0)$	Demand multiplier	$F_iF_j \le 0.075$	10	10
$\beta(1)$	Income exponent	$F_iF_j > 0.075$	0	±26
$\beta'(1)$	Income exponent	$F_iF_j \le 0.075$	-3	-9
$\beta(2)$	Conductance exponent		-5	-70
a_m	Modal impedance multiplier		0	9
$\alpha(1)$	Modal time exponent	$m \ne$ auto	0	40
$\alpha(4)$	Modal time exponent	$m =$ auto	0	49
$\alpha(2)$	Modal cost exponent	$m \ne$ auto	0	54
$\alpha(5)$	Modal cost exponent	$m =$ auto	0	65
$\alpha(3)$	Modal frequency exponent	$m \ne$ auto	0	-6

The segmenting of the demand model buffers the effect of changes in the income exponent $\beta(1)$. Were it not for this segmentation, a 10 percent change in the parameter $\beta(1)$ could effect a 90 percent change in the demand calculated between small city pairs. This buffering effect suggests that model stability and performance might be improved by similarly segmenting the model with respect to the conductance exponent $\beta(2)$. Such a segmentation would buffer the potentially pronounced effect of changes in the modal time and cost exponents.

V MODEL CHARACTERISTICS

Induced and Diverted Demand

When improvements in a single mode cause an incremental increase in the number of travelers using that mode, these travelers can be assumed to come from one of two sources:

(1) Other modes (diverted demand)

(2) The pool of potential travelers who currently are not included in the total intercity demand (induced demand).

Thus total modal increases are made up of travelers diverted from other modes and travelers induced to make the intercity journey for the first time (or more often). Although the calibrated demand model behaves logically in reproducing the overall impact of variable changes, numerical results of a number of model runs revealed that the model clearly overstates induced demand at the expense of diverted demand.

The reason for this overstatement becomes clear if the sources of incremental demand increases are investigated. Equation (5), repeated here for the sake of convenience, expresses the effect of an incremental cost change in mode m on a competing mode n.

$$\Delta D_n = \frac{\alpha(2)}{c_m} [\beta(2) - 1] \frac{w_m}{W} D_n \Delta c_m \qquad n \neq m$$

The effect of the cost change on the demand for service via mode m is as follows:

$$\Delta D_m = \frac{\alpha(2)}{c_m} D_m \left\{ 1 + \frac{w_m}{W} [\beta(2) - 1] \right\} \Delta c_m \tag{9}$$

Summing the above expression across all modes gives the total intercity demand increment.

$$\Delta D = \Delta D_m + \sum_{n \neq m} \Delta D_n \tag{10}$$

$$\Delta D = \frac{\alpha(2)}{c_m} \beta(2) D_m \Delta c_m \tag{11}$$

In the case of a cost decrease, the constrained calibration procedure forces ΔD_m to be positive and ΔD_n to be negative. Hence, the total demand increment ΔD will represent the total induced demand. The ratio of induced demand to the incremental demand increase via mode m may be found as follows:

$$\frac{\Delta D}{\Delta D_m} = \frac{\beta(2)}{\left\{1 + \frac{w_m}{W} [\beta(2) - 1]\right\}} \tag{12}$$

For the calibrated value of $\beta(2) = 0.9$, this ratio will vary from 0.9 to 1.0 as the ratio w_m/W varies from zero to 1. Thus the induced demand component of traffic increases predicted by the intercity demand model will range between 90 percent and 100 percent. This is not a realistic state of affairs. The model's realism may be improved, however, by defining arbitrarily a more reasonable limit on induced demand and redistributing demand forecasts in accordance with this limit. A simple means of accomplishing this redistribution is to let

$$D_m = (D_o + \gamma \Delta D) \frac{w_m}{W} \tag{13}$$

where D_o represents original intercity demand, and γ represents an arbitrary scaling factor $(0 \leq \gamma \leq 1)$.

Equivalent Cost of Time

The calibrated mode split model may be used to infer the equivalent cost of travel time over a given common carrier mode. An incremental change in travel time over mode m will have a known effect on the demand for intercity travel. This effect is expressed below.

$$\Delta D = \frac{\alpha(1)}{t_m} \beta(2) D_m \Delta t_m \tag{14}$$

The equivalent cost of this incremental change in demand may be considered to be the cost increment Δc^*_m required to cause the same demand change. The effect of a cost change on demand with time held constant has been expressed in Equation (11). Equating the demand increments of Equations (11) and (14) gives the following results.

$$\frac{\alpha(2)}{c_m} \beta(2) D_m \Delta c_m^* = \frac{\alpha(1)}{t_m} \beta(2) D_m \Delta t_m$$

$$\Delta c_m^* = \frac{c_m}{t_m} \frac{\alpha(1)}{\alpha(2)} \Delta t_m \tag{15}$$

Since the calibration process resulted in the relationship $\alpha(1) = \alpha(2)$, it follows that

$$\Delta c_m^* = \frac{c_m}{t_m} \Delta t \quad .$$

For small increments, then, the equivalent cost per unit of travel time by mode m is simply the ratio of the current travel cost, c_m, to the mode's current travel time, t_m. This result is intuitively satisfying, since it is obvious that the current users of the mode have accepted this ratio as a measure of the equivalent cost per unit of their travel time. A similar analysis could be performed for automobile travel.

Table 8 shows the equivalent cost of time by mode for each of the city pairs used in the calibration process. This value was obtained by taking the ratio of the total travel cost by mode to the corresponding travel time. The average over all modes was obtained by weighing each mode's equivalent cost by the number of users of that mode.

Table 8

EQUIVALENT COST OF TIME
(Dollars per Hour)

Case	Air	Rail	Bus	Auto	Average
ALP-SSM	0.00	0.00	1.18	1.83	1.80
SSM-HOU	0.00	0.00	1.48	1.78	1.78
ALP-FLI	0.00	0.00	1.28	1.78	1.75
SSM-FLI	0.00	0.00	1.51	2.10	2.05
CAR-DET	2.62	0.00	0.98	1.80	1.78
SSM-DET	6.12	0.00	1.48	2.13	2.18
HOU-DET	9.17	0.00	1.48	1.97	2.70
FLI-KAL	0.00	0.91	1.59	1.90	1.78
FLI-COL	6.03	1.70	1.32	2.22	2.08
FLI-DES	8.10	0.00	1.50	2.22	3.25
FLI-DET	5.98	0.76	1.77	1.68	1.69
FLI-CHI	5.94	3.62	1.46	1.99	2.89
FLI-PHL	11.36	2.08	1.91	2.32	5.53
DET-CHI	5.57	1.39	1.58	2.04	3.33
DET-PHL	8.69	2.68	2.05	2.22	6.86
DET-CLE	5.54	2.49	1.48	2.02	2.66
DET-PIT	7.32	1.91	2.26	2.15	4.98
DET-MIL	8.18	1.32	1.59	2.20	6.44
FLI-CLE	8.45	0.62	1.23	2.18	3.60
FLI-MIL	8.86	1.27	1.68	2.25	4.66

VI FUTURE WORK

Model Improvements

Future work to improve the accuracy and plausibility of the Michigan Intercity Demand Model might profitably explore the following subjects:

(1) Segmentation over distance

(2) Formulation of a continuous model

(3) Development of an induced demand correction factor

(4) Investigation of the variation of parameter values over time.

Distance Segmentation

The possibility of segmenting the demand model as a function of distance by associating different values of $\beta(2)$ with different conductances has been noted already. The intercity highway traffic model designed for Michigan by Arthur D. Little[4] was segmented in this fashion with good results. Such a segmentation would correct for the tendency of the current model to understate long-distance trips (over 600 miles).

Continuous Model

Certain inconsistencies in model performance might be overcome by developing a continuous demand model having the features of the segmented model. A continuous model having these features is shown below.

$$D = \beta(0)^{(1-S)}(F_i F_j)^{\{\beta(1)+S[\beta'(1)-\beta(1)]\}}\beta'(0)^S W^{\beta(2)} \qquad (17)$$

where $S = \text{EXP}\ [-\mu(F_i F_j) + \tau]$.

The variables μ and τ are calibration constants, and the remaining model variables have the definitions stated in Equations (1), (2), and (4). SRI has achieved some success in calibrating the model of Equation (17), but more experimentation is necessary before this model can replace the current segmented formulation. A similar continuous formulation could be employed to vary the parameter $\beta(2)$ over different intercity distances.

Induced Demand Correction Factor

Historical data regarding induced demand should be gathered in an effort to estimate the value of the parameter γ used in Equation (13) to correct for the model's tendency to overstate induced demand.

Time-Varying Parameters

If the functional form of the demand model is correct, it seems likely that parameter values will change with time. This supposition should be checked by calibrating the model at different points in time and attempting to explain and quantify any differences in the calibration parameters.

Model Application

The true utility of the developed demand model is tested best by applying the model in the investigation of Michigan intercity transportation problems. In the course of SRI's Michigan studies, the model has been applied to the task of predicting potential air traffic from a proposed regional airport and evaluating alternative high speed rail routes between Detroit and Chicago. The model performed creditably in these road tests, both of which are described in separate reports (Reports 3 and 4 of this series). More tests are needed, both to substantiate the model's current capability and to point the way toward future improvements.

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