

DESIGN OF LOAD TRANSFER JOINTS IN CONCRETE PAVEMENTS

By J. W. KUSHING, *Research Engineer*

AND

W. O. FREMONT, *Technical Engineer*
Michigan State Highway Department

SYNOPSIS

The purpose of this paper is to establish a rational procedure of design of load transfer joints in concrete pavements. The data obtained agree favorably with measurements of these variables in actual pavements.

Theories and methods of design advanced by Westergaard (1), Friberg (2), Kushing and Older (3), Fremont and Kushing (6) are reviewed in order to present the limitations of those studies.

In the present study the theoretical method of Westergaard has been extended and adapted, it is hoped, in a more thorough way to the actual conditions of joints in pavements. The analysis is based on two elements, first, the theory of slabs under the action of a load at the edge (4) (5) or on experimental deflection and stress curves obtained from measurements on full size or model slabs. And, secondly, the elastic behavior of individual joint units as determined by laboratory tests or theoretical considerations.

On the basis of this knowledge two continuity equations are established for each joint unit which are sufficient to determine the shear force and bending moment for every unit in the joint.

A case is considered in which nine units are symmetrically distributed with respect to the load and the load placed over one of the units. This problem reduces to ten equations with ten unknowns the solution of which leads to five linear equations with five unknowns. These latter equations are easily solved by several known methods or by methods especially developed for this purpose. This procedure in the solution of the major thesis can be used for semi-infinite or finite slabs, of any shape, with any distribution of joint units and will always give a definite answer.

It is pointed out that charts may be prepared for purposes of practical pavement design on the basis of the analysis described. Such charts would give the maximum permissible spacing of a given joint unit in a given pavement for given subgrade conditions. This spacing being determined so as not to exceed the maximum allowable:

1. Shear force in the joint unit
2. Stress in the slab
3. Pressure of unit on concrete
4. Any other stresses.

It is believed by the authors that the design procedure described is practical and simple.

It is the purpose of this paper to present a theory concerning concrete slab and joint action to the end that a definite outline for the procedure of the design of load transfer joints may be established. Such analysis recognizes the importance of laboratory tests to determine the characteristics of load transfer units and fixes the relationship of these characteristics and those of the slabs to the proper spacing of such units within maximum allow-

able stress and load limits. It is shown that it is possible to prepare charts incorporating these determinations and relationships whereby the design of load transfer joints becomes a practical and simple procedure.

DEFINITIONS

In order clearly to interpret the findings, it is necessary to understand what

is meant by the term "joint", load transfer and joint device.

Joint: A joint in a concrete pavement is understood to be a designed highly localized decrease of rigidity along a horizontal, straight or curved line in the pavement slab. This line is referred to as the "joint line".

The decrease of rigidity at a joint line may be of a total or partial character. In the case of total separation along the joint line no load transfer can take place between the separated parts of the slab. On the other hand, load transfer does take place across the joint, if the separation is partial or special devices connect the separated parts.

Load Transfer: Load transfer refers to the strengthening of joint edges by effective means which provide mechanical interaction between adjoining slabs. In its more strict sense it refers to the inner forces acting at a point of the joint line, which in turn are resolved into three force components and three moment components. Since these components may vary continuously from point to point along the joint line, the term load transfer is defined as any of the above components referred either to a point on the joint line, a joint unit or any part of the joint line. However, it is usually assigned only to the vertical component of the forces.

Joint Devices: The term "joint devices" refers to special devices usually mechanical in character which are intended to provide load transfer between the separated slabs.

Limitations: In this report, the load transfer as treated will consider almost exclusively the transfer effected by transverse forces and bending moments in the joint devices. The horizontal forces will not be considered. Load transfer is regarded as being important for stress relief within the slab and the prevention of undesirable differentials between abutting slabs. Therefore, it is essential to design the load transfer devices and joints in a

proper way so as to be able to select the best and most economical ones among those available for use.

Joint devices may be continuous or discontinuous along the joint line. A theory of load transfer along infinite continuous devices has been developed by Kushing and Fremont (6)¹. If the continuous joint is subdivided into sections it may be treated as a discontinuous joint or vice versa. Many of the "load transfer" joint devices on the market today provide for a discontinuous joint. In order to use these devices effectively an analysis must be made of their functions to the end that they are adequately spaced along the joint. An outline for the analysis and design of discontinuous joints is herein offered.

This outline includes a general theory of load transfer joints in slabs on elastic foundations, a procedure for the design of load transfer joints and the preparation of design charts.

GENERAL THEORY OF LOAD TRANSFER JOINTS

For the purpose of developing a theory of load transfer joints, consider two abutting slabs on elastic foundations (Fig. 1). The horizontal distance between the plane vertical faces of the slabs is W .

The points on the neutral line of the slab faces 0, 1, 2, 3, ... and 0', 1', 2', 3', ... oppose each other and are symmetric with respect to the point 0' at the point of application of the load. The slabs are connected by the joint units 0-0', 1-1', 2-2', 3-3', These joint units react upon the slabs with the forces $T_0, T_1, T_2, T_3, \dots$ and the bending moments $M_0, M_1, M_2, M_3, \dots, M_0', M_1', M_2', M_3', \dots$. No horizontal reaction forces are considered.

Assumptions: To develop the theory, the following assumptions are made:

1. The foundation is elastic in the sense of the assumptions of the elementary

¹ Figures in parentheses refer to list of references at end.

theory of beams on elastic foundations (4).

2. The slabs are elastic; follow the linear law.
3. The solution for one slab loaded by a vertical force at the edge is supposed to be known (4).

8. The system is pre-stressed by pre-stressed joint units. This pre-stressing may be caused by misalignment in the case of dowel joints as illustrated in Figure 3. W' is the original joint opening, $\Delta W'$ is the increase in the joint opening, α is the

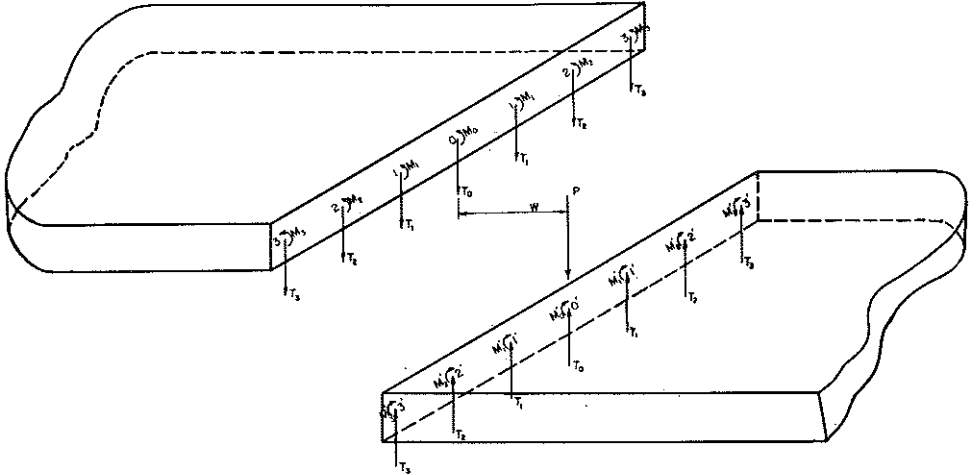
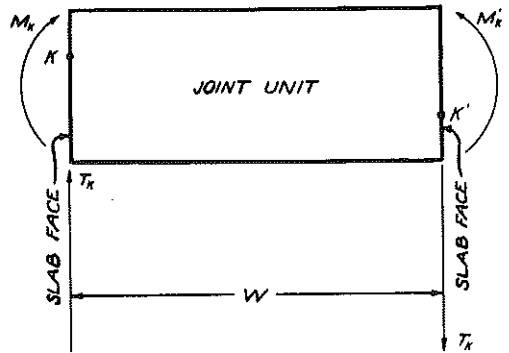


Fig. 1

4. The joint unit 0-0' is directly under the load P.
5. The only forces acting upon any joint unit K-K' or simply K are the shear forces T_K in the slab faces and the bending moments M_K and $M_{K'}$ as shown in Figure 2. M_K and T_K act in the plane of the joint face of the left slab and $M_{K'}$ and T_K act in the plane of the joint face of the right slab. They are interrelated by the equilibrium equation

$$M_{K'} = M_K + WT_K \quad 1$$

6. The joint construction is considered in the broader sense to include the parts of the slabs affected locally by the anchorage of the joint in the slabs.
7. The joint construction may or may not follow the linear law and may be inelastic.



$$M_{K'} = M_K + WT_K$$

Fig. 2

misalignment angle, and e is the eccentricity in the setting of the dowel caused by the increase in joint opening. To adapt Figure 2 to the schemes of further discussion, the

difference in elevation of the unstressed ends of the dowels at a cut is denoted by $(-e)$ as e is < 0 as shown.

9. An imaginary cut pq is made through the joint construction.
10. The joints are so anchored in the concrete that both parts separated by the cut pq are always assuming their initial position in the unloaded condition of the system or may be brought into this position without the application of any forces.

Continuity Equations: The slab of the composite scheme, made up of the two slabs, the elastic foundations and the joint units, will behave theoretically in an elastic manner under forces applied to the slabs, if the system is not pre-stressed, although the joints may be inelastic.

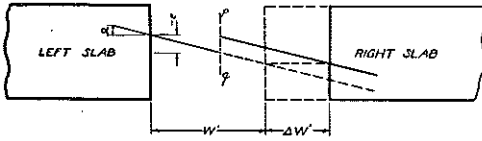


Fig. 3

The slabs on the elastic foundations will follow the linear law when they are not connected by the inelastic joint units, but when connected by these inelastic units, they will not follow the law. Nevertheless, the whole system must obey certain continuity conditions.

As the horizontal forces are not being treated, only the vertical and angular continuity conditions will be derived. For this derivation consider the scheme shown in Figure 4.

Let AB and CD be the neutral axes of the abutting slabs in their unloaded and unstrained conditions; λ and K are the angles they form with the horizontal; H is the superelevation of C over B . Further, let 0 and $0'$ be the corresponding points of the joint construction, which coincided before the cut pq was made.

The position of the points 0 and $0'$ is defined by the distances m , n and the angles ν , μ . The vertical distance between 0 and $0'$ denote by e and it shall be called the eccentricity. It is positive when 0 is higher than $0'$ and negative in the opposite case. Now, consider small elementary plane areas τ and ω of the joint construction at points 0 and $0'$ perpendicular to the plane of the drawing and forming the angles σ and ρ with the horizontal as shown. These planes coincided before the cut pq was made. When points 0 and $0'$ coincide and the planes τ and ω coincide it is assumed that all the corresponding points of the cut coincide and full continuity of the joint is established.

Now suppose that under the action of outer forces applied to the slabs, with shears and moments (equal and opposite) applied at the cut, full continuity at the cut and full equilibrium of the whole system has been established. The system now assumes the position $A'B'0'C'D'$. The slabs have deflected at B and C through the distance δ_u and δ_L vertically and through the angles α_u and α_L . The joint construction has deformed plastically and elastically and has been displaced in addition due to some clearances. As a result the area τ displaces vertically δ' and rotates through γ' with respect to the tangent $C'M$. The area ω displaces δ vertically and rotates through γ with respect to the tangent $B'G$; the area ω displaces vertically over δ and rotates through γ with respect to the tangent $C'M$.

The simple geometric relations follow from Figure 4 and 5.

$$m\nu + n\mu = H + e \quad 2$$

$$\epsilon = \pi - \mu + \nu = B0p + p0'C \quad 3$$

$$\eta = \epsilon - \beta \quad 4$$

$$\delta = m\alpha_u - n\alpha_L + \delta + \delta' \quad 5$$

Further introduce the notation:

$$\beta = \sigma - \rho \quad 6$$

$$\zeta = \kappa - \lambda \quad 7$$

$$\alpha_{r0} = \gamma - \gamma' \quad 8$$

From Figure 4, the following interrelations are obtained:

$$d_r + d_u + H = d_L \quad 9$$

$$d_r + H + e = \delta \quad 10$$

$$\alpha_r = \alpha_L' + \alpha_u' \quad 11$$

$$\alpha_r = \alpha_{r0} + \beta - \zeta \quad 12$$

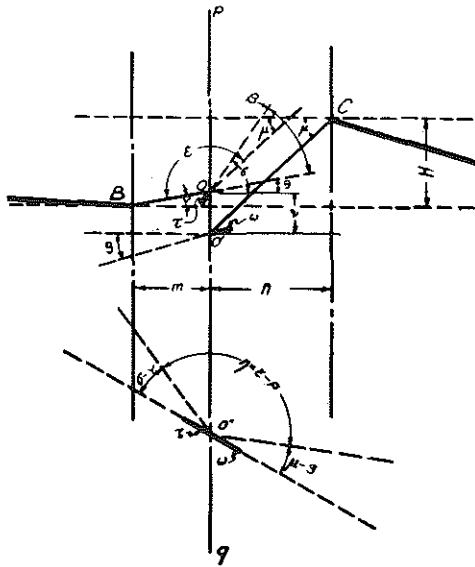


Fig. 5

Equations 9 and 10 give the vertical continuity condition.

$$d_L - d_u = \delta - e \quad 13$$

Equations 11 and 12 give the angular continuity condition.

$$\alpha_L' + \alpha_u' = \alpha_{r0} + \beta - \zeta \quad 14$$

Vertical Continuity Equations: For a definite joint unit k the vertical continuity equation will be

$$d_{Lk} - d_{uk} = \delta_k - e_k \quad 15$$

where d_{Lk} is the deflection of the right loaded slab

d_{uk} is the deflection of the left unloaded slab

δ_k is determined experimentally or analytically

e_k is the eccentricity

d_{uk} may be resolved into two components

$$d_{uk} = d_{kT} + d_{kM} \quad 16$$

where d_{kT} is the vertical deflection due to T_0, T_1, T_2, \dots d_{kM} is the vertical deflection due to M_0, M_1, M_2, \dots d_{Lk} may be resolved into three components

$$d_{Lk} = D_k - d_{kT} + d_{kM}' \quad 17$$

where D_k is the vertical free edge deflection of point k from the force P at point O . d_{kM}' is the vertical deflection due to M_0', M_1', M_2', \dots substituting 16 and 17 into 15 the vertical continuity equation is obtained in the form

$$D_k = \delta_k + 2d_{kT} - e_k + d_{kM} - d_{kM}' \quad 18$$

d_{kM} and d_{kM}' are functions of all the moments M_0, M_1, \dots and M_0', M_1', \dots respectively. These moments are small in the case of rigid slabs and flexible joints; the respective d_{kM} and d_{kM}' will also be small and we shall therefore neglect them.

Then equation 18 reduces to the form

$$D_k = \delta_k + 2d_{kT} - e_k \quad 19$$

In the case of absence of eccentricity we have

$$D_k = \delta_k + 2d_{kT} \quad 20$$

d_{kT} appears in 16 with plus and in 17 with minus as the shears acting on the unloaded and loaded slabs are correspondingly equal and opposite in direction.

Angular Continuity Equations: For a definite joint unit k the angular continuity equation will be

$$\alpha_{Lk} + \alpha_{uk} = \alpha_{r0k} + \beta_k \quad 21$$

where α_{Lk} is the angular deflection of the loaded slab α_{uk} is the angular deflection

of the unloaded slab, α_{r0k} is the angular deflection of the joint unit, β_k is the initial angular deflection. The positive direction of β_k is selected as shown in Figure 4 and defined by the relation δ . It is positive if the right half of the disengaged or cut joint construction is turned counterclockwise with respect to the left half in the initial unstrained state.

α_{uk} may be resolved into two components

$$\alpha_{uk} = \alpha_{kT} + \alpha_{kM} \quad 22$$

where α_{kT} is the angular deflection due to shears α_{kM} is the angular deflection due to b. moments. $\alpha_{T,k}$ may be resolved into three components

$$\alpha_{Lk} = -\alpha_{kT} + \alpha_{kM}' + A_k \quad 23$$

where α_{kM}' is the angular deflection due to b. moments A_k is the angular deflection at the joint k due to P.

Substituting 22 and 23 into 21 we have

$$A_k = \alpha_{r0k} + \beta_k - \alpha_{kM} - \alpha_{kM}' \quad 24$$

Neglecting the influence of b. moments we have

$$A_k = \alpha_{r0k} + \beta_k \quad 25$$

If $\beta_k = 0$ we have

$$A_k = \alpha_{r0k} \quad 26$$

or the angular deflection over the joint α_{r0k} has to be equal to the free edge angular deflection of the slab at the joint.

The d_{kT} may be expressed for the case of nine joint units disposed symmetrically with respect to P and one unit under the force P by

$$d_{0T} = A_0T_0 + 2A_1T_1 + 2A_2T_2 + 2A_3T_3 + 2A_4T_4 \quad 27$$

$$d_{1T} = A_1T_0 + (A_0 + A_2)T_1 + (A_1 + A_3)T_2 + (A_2 + A_4)T_3 + (A_3 + A_5)T_4 \quad 28$$

$$d_{2T} = A_2T_0 + (A_1 + A_3)T_1 + (A_0 + A_4)T_2 + (A_1 + A_5)T_3 + (A_2 + A_6)T_4 \quad 29$$

$$d_{3T} = A_3T_0 + (A_2 + A_4)T_1 + (A_1 + A_5)T_2 + (A_0 + A_6)T_3 + (A_1 + A_7)T_4 \quad 30$$

$$d_{4T} = A_4T_0 + (A_3 + A_5)T_1 + (A_2 + A_6)T_2 + (A_1 + A_7)T_3 + (A_0 + A_8)T_4 \quad 31$$

According to Figure 9 of reference (4) we have in (10^{-6} units) for $h = 7$ in. $E = 4.5 \times 10^6$ p.s.i., $k = 250$ p.c.i., $\mu = 0$, $l = 26.8$ in., $a = 12$ in. = 0.45 l (uniform spacing of joint units a):

$$A_0 = 2.350 \quad A_5 = 0.625$$

$$A_1 = 2.043 \quad A_6 = 0.378$$

$$A_2 = 1.650 \quad A_7 = 0.278$$

$$A_3 = 1.300 \quad A_8 = 0.222$$

$$A_4 = 0.935$$

The values of D_k are determined by the A_k .

In a similar way expressions for α_u and α_L entering the expression for δ might be obtained.

Characteristic Equation of Joints: The state of a joint unit construction depends, according to 1, solely on the shear T and moment M of the same joint unit. Therefore, the δ , δ' , γ , γ' of a joint unit are also functions only of the T and M of the same joint unit.

Therefore we may have with sufficient appreciation

$$\delta = f(T, M) \quad 32$$

$$\alpha_{r0} = \varphi(T, M) \quad 33$$

or δ and α_{r0} will be functions only of the forces T and M.

If $H = e = 0'$ and $\beta = \zeta = 0$ then

$$d_r = \delta = f(T, M) \quad 34$$

$$\alpha_r = \alpha_{r0} = \varphi(T, M) \quad 35$$

On the basis of 10 and 12 or 29 and 30 the $f(T, M)$ and $\varphi(T, M)$ may be determined experimentally for every type and size of joint device unit and may be obtained in the form of either analytical expressions or presented graphically. In some cases $f(T, M)$ and $\varphi(T, M)$ might be derived analytically.

The vertical and angular continuity equations are examined in the form

$$D_k = \delta_k + 2d_{kT} - e_k \quad 36$$

$$A_k = \alpha_{r0k} + \beta_k \quad 37$$

in which only the effect of the joint bending moments upon the slabs has been neglected.

The δ_k and α_{r0k} may be introduced in the way as explained above.

If there is no pre-stressing ($e_k = 0$) and $\beta_k = 0$ and $H = 0$ then we have the equations in the following simple and convenient form

$$D_k = d_{rk} = 2d_{kT} \quad 38$$

$$A_k = \alpha_{r0k} \quad 39$$

This form obtains also if

$$e_k = \beta_k = 0$$

$$\text{or} \quad e_k = \sigma_k = \rho_k = 0$$

Equations 36 37 and 38 39 are good for finite or semi-infinite slabs.

APPLICATION OF THEORY

Equation 38 shows that the relative deflection over the joint unit K plus twice the deflection of the unloaded slab must equal the free edge deflection at k from the load P at 0.

Equation 39 shows that the angular deflection over the joint must be equal to the free edge angular deflection of the loaded slab at K.

In this manner, two equations 36 and 37 or 38 and 39 are obtained for each pair of symmetric units and the unit under the load. There are as many equations as there are unknowns, T_k and M_k .

It is easy to set up these equations and to solve them if a limited number of units is considered. Fortunately, in the case of load transfer joints in concrete pavements the units farther away from the load transmit very little load and the corresponding deflections are very small.

In the case of one concentrated load at the edge of the loaded slab the range of distribution of load transfer will hardly extend over more than πl each side of the load, where l is the radius of relative stiffness and $\pi = 3.14$. Beyond this range the shears transmitted by the units will be very small and will disappear quickly.

Therefore, adopt the following course.

Consider only, say nine joint units, e.g., symmetrically disposed both sides of the load P, with the center unit directly under the load. All other joint units are assumed to be inactive. Ten equations of the type 36 and 37 or 38 and 39 are set up, as the case may be.

Equations 37 and 39 contain, each one, only two unknowns T_k and M_k and can be solved with respect to M_k . This way all the M_k can be easily eliminated from 36 or 38 and our system of 10 equations with 10 unknowns is reduced to 5 equations with 5 unknowns. If the equations are linear the system can easily be solved in a short time.

The problem may also be reduced to 5 equations when δ_k in 36 or d_{rk} in 38 is a function of T_k alone. Then equations 37 or 39 respectively serve for the determination of M_k .

Instead of solving the equations analytically in a direct way by some of the well known algebraic methods the system may also be solved by the method of successive approximations.

If the characteristics of the joint device are available in the form of curves graphically then the solution may be obtained by the method of successive approximations, or by the method of graphical solution of equations.

In the application of the method of successive approximations, to speed up the process, we may adopt the following procedure.

1. Assume functions F and φ in 32 and 33 linear and homogeneous and obtain an approximate solution and then get an exact solution using the method of successive approximations
2. Solve first graphically for 3 or 5 joint units and use these solutions as approximate solutions in some way.
3. There exist some approximate methods yielding approximate solutions.

B. Friberg's method of dowel joints as amended by one of the writers of this study (2) is based on the above given fundamental equations 38 and 39 and the assumptions:

1. d_{r0} is assumed to be a linear homogeneous function of T_0 independent of M_0 and the other M_k and T_k .
2. It is assumed, that 39 may be substituted by a linear homogeneous interrelation between M_0 and T_0 making the moment at the center of the joint equal to zero.
3. The distribution of shears is assumed to be according to the triangular law.

By this procedure all the bending moments are eliminated and all shears are expressed in terms of T_0 . The T_0 is then found by solving the only equation 38

$$D_0 = d_{r0} + 2d_{0T} \quad 40$$

Of course, the other equations 38 and 39 for $k = 1, 2, 3, \dots$ will not be satisfied by these values of $T_0, T_1, \dots, M_0, M_1, M_2, \dots$.

The Older-Kushing method intended by the originators for any type of joints

is based on the fundamental equation 38 for the unit under the load and the assumptions (3).

1. d_{r0} is any function of T_0

$$d_{r0} = \Theta(T_0)$$

independent of M_0 and the other M_k and T_k .

2. It is assumed, that 39 may be substituted by a linear homogeneous interrelation between M_0 and T_0 making the moment at the center of the joint equal to zero.
3. B. Friberg's triangular law of distribution of shears is adopted.

This way all bending moments are eliminated and all shears are expressed in terms of T in equation 38.

The case of load between two joint units may be treated in a similar way and does not present any difficulties as well as the case of non-symmetric distribution of joint units.

The above theory is applicable to finite slabs of any shape. Example 1. Consider 9 joint units, $H = e = 0$

Slab thickness $h = 8$ in.

Modulus of Elasticity $E = 4.5 \times 10^6$ p.s.i.

Modulus of Subgrade $k = 437$ p.c.i.

Poisson's Ratio $\mu = 0$

Load $P = 9000$ lb.

Joint Spacing $\alpha = 20$ in.

then we have: Radius of Rel. Stiffness $l = 25.7$ in.

$$kl^2 = 28.8 \times 10^4$$

$$\frac{1}{kl^2} = 3.47 \times 10^{-6}$$

$$= 0.78$$

$$\frac{a}{l} = \frac{20}{25.7}$$

interrelation (32) we assume in the form

$$d_{rk} = 1.2 \times 10^{-6} \times T_k$$

or d_{rk} is independent of M_k .

Using equations (38) we have five equations

$$D_0 = \delta_0 + 2d_{0T}$$

$$D_1 = \delta_1 + 2d_{1T}$$

$$D_2 = \delta_2 + 2d_{2T}$$

$$D_3 = \delta_3 + 2d_{3T}$$

$$D_4 = \delta_4 + 2d_{4T}$$

Using Figure 9 of reference (4) we have

$$10^6 \times d_{0T} = 1.42 \times T_0 + 2.18 \times T_1 + 1.388 \times T_2 + 0.716 \times T_3 + 0.408 \times T_4$$

$$10^6 \times d_{1T} = 1.42 \times T_1 + 1.09(T_0 + T_2) + 0.694(T_1 + T_3) + 0.358(T_2 + T_4) + 0.204 \times T_3 + 0.139 \times T_4$$

$$10^6 \times d_{2T} = 1.42 \times T_2 + 1.09(T_1 + T_3) + 0.694(T_0 + T_4) + 0.358 \times T_1 + 0.204 \times T_2 + 0.139 \times T_3 + 0.087 \times T_4$$

$$10^6 \times d_{3T} = 1.42 \times T_3 + 1.09(T_2 + T_4) + 0.694(T_1) + 0.358 \times T_0 + 0.204 \times T_1 + 0.139 \times T_2 + 0.087 \times T_3 + 0.052 \times T_4$$

$$10^6 \times d_{4T} = 1.42 \times T_4 + 1.09 \times T_3 + 0.694 \times T_2 + 0.358 \times T_1 + 0.204 \times T_0 + 0.139 \times T_1 + 0.087 \times T_2 + 0.052 \times T_3 + 0.030 \times T_4$$

Substituting these expressions as well as the corresponding values of D_k and δ_k into our five equations, we have the five equations in the form

$$4.04 \times T_0 + 4.36 \times T_1 + 2.776 \times T_2 + 1.432 \times T_3 + 0.816 \times T_4 = 12.8 \times 10^3$$

$$2.18 \times T_0 + 5.428 \times T_1 + 2.896 \times T_2 + 1.796 \times T_3 + 0.994 \times T_4 = 9.81 \times 10^3$$

$$1.388 \times T_0 + 2.896 \times T_1 + 4.448 \times T_2 + 2.458 \times T_3 + 1.562 \times T_4 = 6.25 \times 10^3$$

$$0.716 \times T_0 + 1.796 \times T_1 + 2.458 \times T_2 + 4.214 \times T_3 + 2.284 \times T_4 = 3.22 \times 10^3$$

$$0.408 \times T_0 + 0.994 \times T_1 + 1.562 \times T_2 + 2.284 \times T_3 + 4.100 \times T_4 = 1.84 \times 10^3$$

We solve these equations by the method of direct solution:

$$T_0 = 3.167 \times 10^3 - 1.080 \times T_1 - 0.686 \times T_2 - 0.355 \times T_3 - 0.202 \times T_4$$

$$T_0 = 4.500 \times 10^3 - 2.487 \times T_1 - 1.328 \times T_2 - 0.824 \times T_3 - 0.456 \times T_4$$

$$T_0 = 4.500 \times 10^3 - 2.083 \times T_1 - 3.200 \times T_2 - 1.770 \times T_3 - 1.125 \times T_4$$

$$T_0 = 4.500 \times 10^3 - 2.504 \times T_1 - 3.425 \times T_2 - 5.890 \times T_3 - 3.190 \times T_4$$

$$T_0 = 4.500 \times 10^3 - 2.438 \times T_1 - 3.828 \times T_2 - 5.600 \times T_3 - 10.040 \times T_4$$

$$1.407 \times T_1 + 0.642 \times T_2 + 0.469 \times T_3 + 0.254 \times T_4 = 1.333 \times 10^3$$

$$-0.404 \times T_1 + 1.872 \times T_2 + 0.946 \times T_3 + 0.669 \times T_4 = 0$$

$$0.421 \times T_1 + 0.225 \times T_2 + 4.120 \times T_3 + 2.065 \times T_4 = 0$$

$$-0.066 \times T_1 + 0.403 \times T_2 - 0.290 \times T_3 + 6.850 \times T_4 = 0$$

$$T_1 = 0.456 \times T_2 - 0.333 \times T_3 - 0.180 \times T_4 + 0.948 \times 10^3$$

$$T_1 = 4.640 \times T_2 + 2.340 \times T_3 + 1.654 \times T_4$$

$$T_1 = -0.533 \times T_2 - 0.760 \times T_3 - 4.900 \times T_4$$

$$\begin{aligned}
 T_1 &= 6.100 \times T_2 - 4.400 \times T_3 + \\
 &103.800 \times T_4 \\
 &- 5.096 \times T_2 - 2.673 \times T_3 - \\
 &1.834 \times T_4 + 0.948 \times 10^3 = 0 \\
 &5.173 \times T_2 + 12.100 \times T_3 + \\
 &6.554 \times T_4 = 0 \\
 &- 6.633 \times T_2 - 5.360 \times T_3 - \\
 &108.700 \times T_4 = 0
 \end{aligned}$$

$$T_2 = -0.524 \times T_3 - 0.360 \times T_4 + 0.186 \times 10^3$$

$$T_2 = -2.340 \times T_3 - 1.268 \times T_4$$

$$\begin{aligned}
 T_2 &= -0.808 \times T_3 - 16.380 \times T_4 \\
 &+ 1.816 \times T_3 + 0.908 \times T_4 + \\
 &0.186 \times 10^3 = 0 \\
 &- 1.532 \times T_3 + 15.112 \times T_4 = 0
 \end{aligned}$$

$$T_3 = -0.499 \times T_4 - 0.102 \times 10^3$$

$$\begin{aligned}
 T_3 &= 9.87 \times T_4 \\
 &10.369 \times T_4 + 0.102 \times 10^3 = 0
 \end{aligned}$$

$$T_4 = -9.85;$$

$$T_3 = -9.87 \times 9.85 = -96.2$$

$$\begin{aligned}
 T_2 &= 0.808 \times 96.2 + 16.38 \times 9.85 = \\
 &+ 238.9
 \end{aligned}$$

$$\begin{aligned}
 T_1 &= 6.1 \times 238.9 + 4.4 \times 96.2 - \\
 &103.8 \times 9.85 = +860
 \end{aligned}$$

$$\begin{aligned}
 T_0 &= 4500 - 2.438 \times 860 - 3.828 \times \\
 &238.9 + 5.6 \times 96.2 + 10.04 \times \\
 &9.85 = +2126
 \end{aligned}$$

These solutions are presented graphically on Figure 6.

The theory has made it possible to determine the values of the shear forces and bending moments at the various distributed units. In order to evaluate these data for a definite joint device a knowledge of the deflection curves or characteristic curves of this device is necessary. The information required for each type and size of joint is expressed in general by formulas 32 and 33 or 34 and 35.

These curves determine the elastic, plastic and strength characteristics of the joint device unit.

The test to obtain these characteristics is rather simple. The joint device unit is placed between and embedded into two concrete blocks 7 by 12 by 15 in. The specimen is supported and loaded in such a way that the shear deflection curves are obtained for various values of bending moment. A special machine for the convenient testing of such specimens in the above described way is being built at the present time by the Michigan State Highway Department. This machine is similar, in general, to many which have been used to make shear deflection studies, but differs in the application of loads, methods of measuring deflections, and the accuracy for handling specimens in the test. A report will be made on this machine when tests have been completed on a number of joint devices during the coming winter.

The characteristics of the joint device having been determined, a procedure for the joint design can now be followed.

PROCEDURE OF DESIGN

The following data should be available:

1. Characteristic curves for each type, size and joint opening of joint device.
2. Curves for each of maximum fibre stress, joint unit shear force, or any other stress or force versus spacing of joint units. These curves to be based on standard wheel loads, various subgrade moduli, and slab thicknesses.
3. Maximum permissible shear forces from the characteristic curves.
4. Maximum permissible fibre stresses.

On the basis of these data the maximum allowable spacings for the permissible shear force, fibre stress may be determined. The lowest spacing determined by these various factors should be accepted for the design. The spacing so defined will determine the cost of the

joint construction as far as individual units are concerned.

In order more easily to study the information obtained in the foregoing suggested procedure of design, charts may be prepared for each type of joint device and

modulus. Figure 7 shows an imaginary chart drawn to illustrate its use in design. From the chart it is seen that for a given device and other given values on a subgrade with a modulus value $K = 50$ p.c.i. the fibre stress limit does not permit a

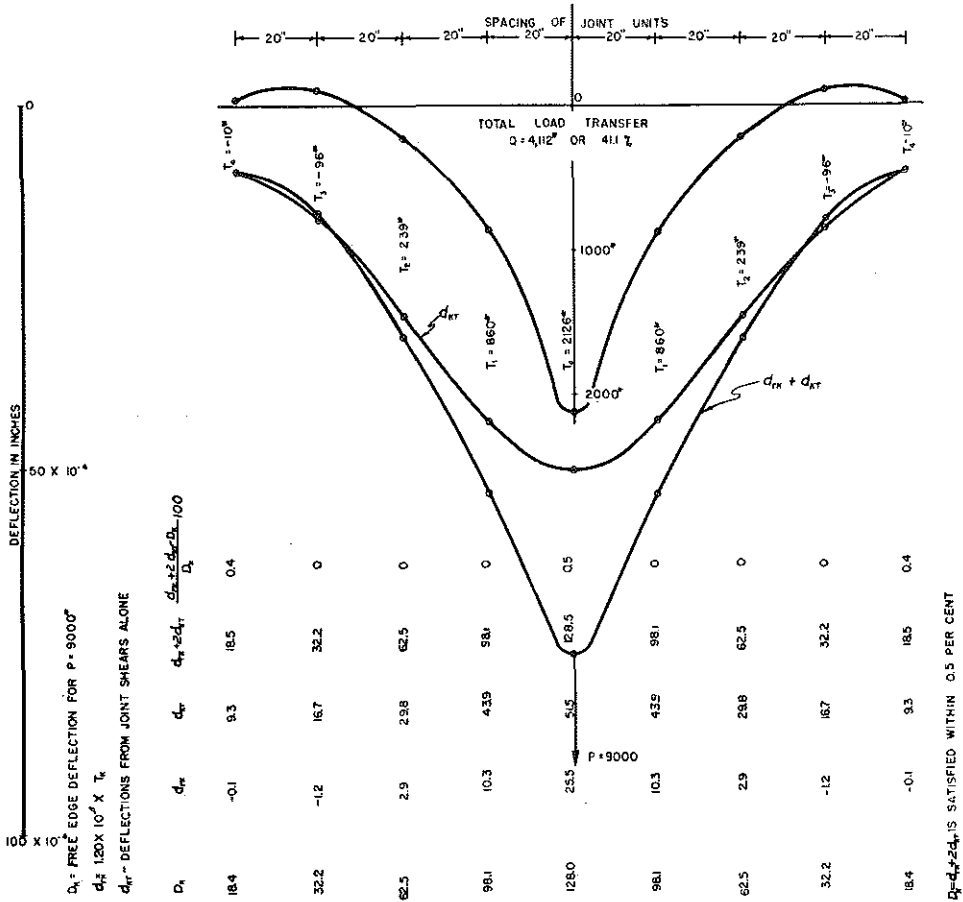


Figure 6. Theoretical Curves

Case of nine load transfer units. $d_{OK} = 1.2 \times 10^{-6} \times T_K$; spacing $A = 20''$; $P = 9000\%$; $h = 8''$; $E = 4.5 \times 10^6$ p.s.i.; $K = 437$ p.c.i.; $\mu = 0, 1 = 25.7''$; $\frac{1}{K\mu} = 3.47 \times 10^{-6}$.

joint opening. From such a design chart the correct spacing may be determined for a definite load at a definite distance from the free edge for given values of slab dimensions, concrete modulus of elasticity, and various values of subgrade

spacing greater than 15.7 in., the shear limit 9.5 in. For other values of subgrade modulus different spacings would be obtained. As has been pointed out previously the least spacing is the value which should be used in the design.

To obtain similar graphs for actual joint design, it is necessary to construct curves as outlined in the procedure for design. Each set of curves must be calculated from values either assumed or obtained from laboratory tests.

signer simple and convenient charts for suitable and satisfactory design. It makes little difference to him by what simple or complicated procedure they have been obtained provided the charts correctly represent actual conditions. The tests to obtain the characteristic curves have to be accurately obtained only once by any given laboratory. The curves have to be prepared only once for any given set of conditions. Having been determined by any one agency they are suitable for the use of any organization requiring their need.

The problem of load transfer design has been under investigation by the authors, over six and one-half years. During this period innumerable tests on joints and joint devices have been conducted. The results of these studies lead them to believe that the theory and procedure for the design of load transfer joints as herein presented is sound and practical. Any other procedure must recognize all the factors which have been discussed and it would seem that only by the application of a like theory and procedure can adequate and proper design of joints be effected.

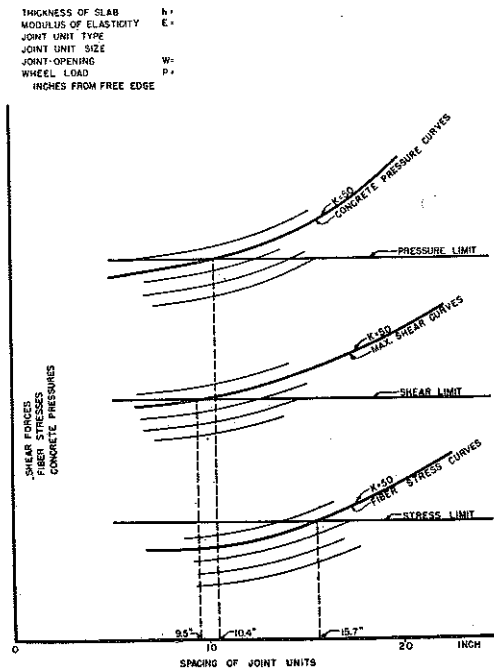


Figure 7. Scheme of Diagram of Shear Forces, Fiber Stress and Concrete Pressure

CONCLUSIONS

Economic considerations in the construction of a pavement slab as well as the joint itself dictate that careful treatment should be given to the joint design. This is particularly true if the effect of the structural efficiency of the joint on the life of pavements is considered of importance. With this in mind, the theory and procedure for design has been developed without unnecessary and dangerous sacrifices in accuracy. Every item has been carefully considered and its effect weighed. At the same time, the solutions have not become too involved.

The results give to the practical de-

REFERENCES

- (1) Westergaard, H. M., "Spacing of Dowels," *Proceedings*, Highway Research Board, Vol. 8, 1928.
- (2) Friberg, B. F., "Dowel Design" *Proceedings* A.S.C.E., November 1939.
- (3) Older and Kushing, "A discussion of the Load Transfer Feature of Transverse Joints." (Letter from Mr. J. W. Kushing to the Michigan State Highway Department of February 8, 1939).
- (4) Westergaard, H. M., "Computation of Stresses in Concrete Roads." *Proceedings* Highway Research Board, Vol. 5, 1925.
- (5) Westergaard, H. M., "Analytical Tools for Judging Results of Structural Tests of Concrete Pavements."
- (6) Kushing and Fremont, "Joint Testing Experiments with Theory of Load Transfer Distribution Along the Length of Joints." *Proceedings*, Highway Research Board, Vol. 15, 1935.

DISCUSSION ON LOAD TRANSFER JOINTS IN CONCRETE PAVEMENTS

MR. E. C. SUTHERLAND, *Public Roads Administration*: In 1928 Westergaard,¹ presented what was perhaps the first rational method of designing dowel joints in concrete pavements. This analysis provided a theoretical means of computing, for different dowel spacings, the reactions in each active dowel and the critical stresses which occur in the slab directly under the load as the load approaches the joint. This analysis is limited to the hypothetical case in which the deflection on the two sides of the joint is equal. Later, the stress conditions in the dowels and the concrete surrounding the dowels were investigated by Grinter² and by Bradbury,³ in independent theoretical studies, using dowel reactions determined by the use of the Westergaard analysis.

Recently, Friberg^{4,5} made a theoretical analysis of flexible dowel joints in concrete pavements in which he attempted to determine, for various conditions, the stresses in the dowels and in the concrete surrounding the dowels. Certain laboratory tests were made in connection with this investigation, to determine the modulus of dowel reaction which is used in the theoretical equations. It was assumed in this analysis that dowels at distances greater than 1.8 times the radius of relative stiffness from the center of the load were inactive and that the effective shear in the respective dowels decreased approximately as a linear function of distance.

Older and Kushing⁶ in a recent unpublished paper presented a method of designing load transmission features for concrete pavement joints. This method was devised to a large degree on the basis of laboratory tests on joint devices which have been made at various laboratories during the past several years. The

reactions on the different units were assumed to be proportional to the deflection of the unit or the relative deflection over each respective unit between the two sides of the joint. Also load transfer units at distances greater than 1.8 times the radius of relative stiffness, from the center of the load, were considered as being inactive.

The paper under discussion is a purely theoretical analysis providing a means for determining the reactions in the different units of a concrete pavement joint. It is intended that these reactions should be used in the Westergaard formulas for determining the critical stresses in the concrete slab as the load approaches the joint.

Several assumptions were necessarily made in this analysis. It is evident that investigators have been greatly handicapped in their efforts to find some rational means for designing joints, by the lack of experimental data on full-size concrete pavement slabs. Also, engineers whose duty it is to design joints are handicapped in the selection of a method by the lack of experimental data to support any one of the various methods which have been advanced. It has been stated that the State of Michigan is planning to make some joint tests on full-size concrete pavement slabs. The data obtained from such tests should be very valuable.

The Public Roads Administration during the past several years has tested a limited number of dowel joints, in which the spacing of the dowels and the widths of the joints were varied. The first series of these tests was reported in *Public Roads*⁷ in 1936. Two widths of joints ($\frac{1}{2}$ and $\frac{3}{4}$ in., respectively) and three dowel spacings (18, 27 and 36 in., respectively) were investigated. The dowels were of $\frac{3}{4}$ -in. diameter and 36-in. length in all cases. It was found

¹ Superior figures refer to list of references at end of this discussion.

that these joints were rather ineffective in controlling the stresses directly under the loads, when the load was at a point some distance from the edge of the pavement, but that they were fairly effective in controlling the critical stresses caused by a load acting in the vicinity of a corner formed by the transverse joint and the pavement edge.

Later a joint was tested in which dowels of $\frac{3}{4}$ -in. diameter and 24-in. length were spaced at 12-in. intervals. The width

two dowels near the center of the 10-ft. slab width and measurements were made of the strains on the loaded side of the joint, and of the relative deflections between the two sides of the joint across the width of the slab. These measurements were first made with all dowels intact and then repeated after each pair of dowels, symmetrically spaced with respect to the load, was cut through. The pairs of dowels were cut in the order of their distance from the load, the dowels

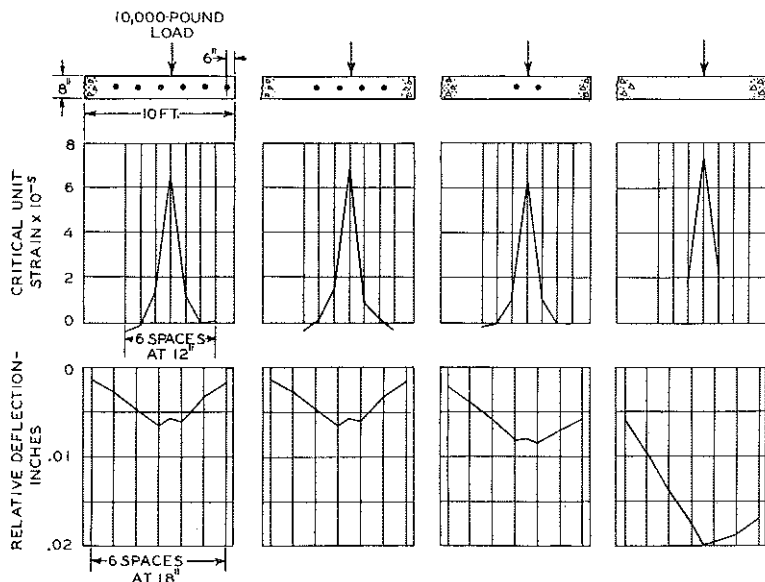


Fig. 1. Strains and Relative Deflections as the Number of Active Dowels Was Decreased. Load between Dowels

of the joint opening was $\frac{3}{4}$ in. in this case. It was found that the efficiency of this joint was quite high in controlling both the corner stresses and the stresses which occur directly under the load when it acts at the joint but away from the edge of the pavement.

Another limited series of tests that has not been previously reported, was made on two joints with $\frac{1}{2}$ -in. openings in which dowels of $\frac{3}{4}$ -in. diameter and 36-in. length were spaced at 18-in. intervals. On one joint a load was placed between

nearest the load being severed last. The second joint was tested in the same manner except that in this case the load was placed directly over one of the dowels near the center of the joint.

The data obtained from these tests are shown in Figures 1 and 2. The number of dowels that were active in each test is indicated on the figure.

It is indicated by the data of Figure 1 that with the load centered between dowels, the critical strains remain practically constant as the number of active

dowels is decreased until after the last pair of dowels is cut, when an appreciable increase occurs. The relative deflections are approximately the same with six and with four active dowels, but increase as the number of active dowels is reduced below four.

The data of Figure 2, for the case of a load active over a dowel, indicate that there is actually a slight reduction in the critical stresses as the number of active dowels is reduced until the last dowel is cut when a noticeable increase occurs.

under a load placed at the edges and the interior of a pavement slab are not directly related to the total deflections caused by the load or to the apparent deflections in the vicinity of the load. For example, the deflection curve, for a point loading at the edge of a pavement slab, has the same apparent shape and the magnitude of the total deflection is the same as that for an equivalent load applied on the bearing area of considerable size, but the magnitude of the stresses occurring directly under the

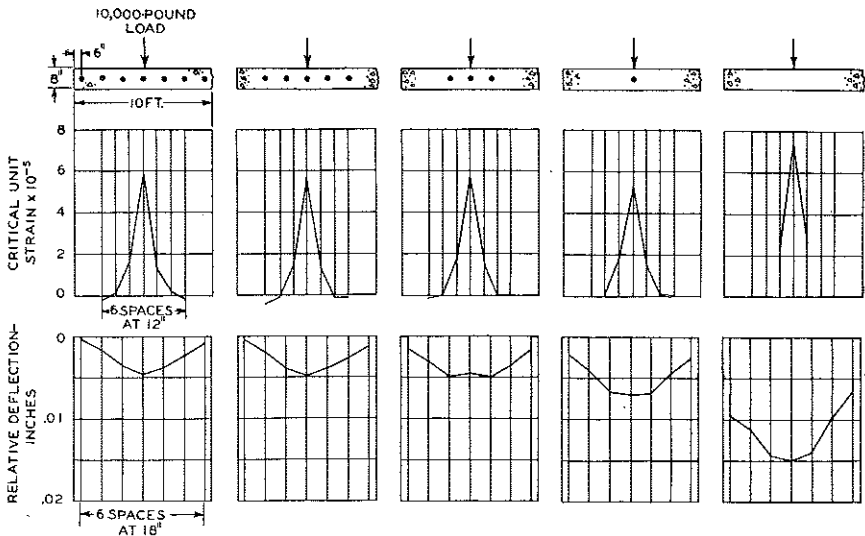


Fig. 2. Strains and Relative Deflections as the Number of Active Dowels Decreased. Load over Dowels

The relative deflections are approximately the same with 7, 5 and 3 active dowels, but increase as the number of active dowels is reduced below three. It will be noted in both Figures 1 and 2 that changes in the relative deflections are not necessarily accompanied by a corresponding change in the critical strains. The data of these two figures appear to support the conclusions of Westergaard, in his analysis of rigid dowel joints, that only the dowels near the load are effective in stress control.

The magnitude of the stresses directly

load for the two load conditions will be appreciably different. The magnitude of the stress directly under the load is determined by the degree or rate of curvature at this point and differences in the rate of curvature under the load are not detected by present methods of measuring deflections.

In the studies of structural behavior made at Arlington a number of butt type longitudinal joints with bonded tie bars spaced at intervals that varied from 24 to 60 in. were tested. In a number of the tests the deflection data showed that

the two sides of the joint deflected the same amount indicating that 50 percent of the load was being transferred.

In spite of the apparent effectiveness of this type of connection, when judged by the deflection data, it was found that a load placed midway between two of the bonded dowels or tie bars produced essentially the same critical stress under the load as was caused by the same load acting at a free edge of the same slab. Since the load transfer units are introduced to control critical stresses, it is apparent that their effectiveness cannot be accurately estimated from deflection data.

In order to effectively control critical load stresses at a slab edge, it is believed that the mechanical connection between the two slab ends should be as nearly continuous as possible so that the elastic curve of the slab edge to which the load is applied will be smoothed out and its maximum rate of curvature will be reduced by the support offered from the unloaded slab by the joint connection.

Any method of testing joints, or of determining their efficiency by theoretical means should give consideration to the localized effect or influence of the dowels on the rate of curvature of the slab in the region directly under the load.

REFERENCES

1. H. M. Westergaard, "Spacing of Dowels". *Proceedings*, Highway Research Board, Vol. 8, 1928.
2. L. E. Grinter, "Design of the Reinforced Concrete Road Slab". Bul. no. 39, Texas Engineering Experiment Station, March 1931.
3. R. D. Bradbury, "Design of Joints in Concrete Pavements". *Proceedings*, Highway Research Board, Vol. 12, 1932.
4. Bengt Friberg, "Design of Dowels in Transverse Joints of Concrete Pavements". *Proceedings* A. S. C. E., November 1938.
5. Bengt Friberg, "Load and Deflection Characteristics of Dowels in Transverse Joints of Concrete Pavements". *Proceedings*, Highway Research Board, Vol. 18, 1938.
6. Clifford Older and John W. Kushing, "The Design of the Load Transmission Feature of Transverse Joints". (Unpublished)

7. L. W. Teller and Earl C. Sutherland, "The Structural Design of Concrete Pavements".

Part 4—A Study of the Structural Action of Several Types of Transverse and Longitudinal Joint Designs. *Public Roads*, September and October 1936.

MR. W. O. FREMONT, *Michigan State Highway Department*: The data presented by Mr. Sutherland are very interesting. They are limited only to the action of the so-called $d = \frac{3}{4}$ in. standard dowel joints. The theory and procedure presented by the authors has no such limitation. They can be applied to the standard dowels and to dowels reinforced by sleeves of any rigidity. They can be used in connection with many other types of joint units. It is pointed out that according to the tests in the laboratory and in the field on pavements conducted by the Michigan State Highway Department the $d = \frac{3}{4}$ in. dowel joints have proved to be the most inefficient joint units in use. The conclusion has been reached, that they are of very little use for stress relief. This behavior of the $d = \frac{3}{4}$ in. dowel joints has been repeatedly verified experimentally and if the proposed procedure were applied to the conditions described by Mr. Sutherland the same stresses and the same deflections could have been calculated as those obtained experimentally and described by Mr. Sutherland.

The intent and purpose of the development and presentation of the report was the improvement of the design and construction of joints in rigid pavements. The recommended procedure enables us to analyse in advance, on the basis of only preliminary laboratory tests, more satisfactory types of joints and to predict their behavior in pavements without going into extensive, costly and time-consuming field tests on actual pavements. On the basis of such analysis different types of joint units may be compared as to their structural and economic values in a very convenient way.