DESIGN CONSIDERATIONS FOR REINFORCEMENT AND JOINT SPACING FOR PAVEMENTS WITH SMALL-RADIUS CURVES

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At the request of W. W. McLaughlin, Testing and Research Engineer, the Research Laboratory Division inspected a transverse crack in a 17-ft pavement slab, located on a 100-ft radius curve intersection ramp at M 46 and US 31 near Muskegon.

The crack occurred approximately halfway between two joints which were spaced about 105 ft apart. The maximum width of this crack measured at the surface was 0.2 in.

Figure 1 shows the ramp with the approximate joint spacings, crack location, and surface width variation of the crack. Photos, including a close-up view of the crack, are presented in Figure 2.

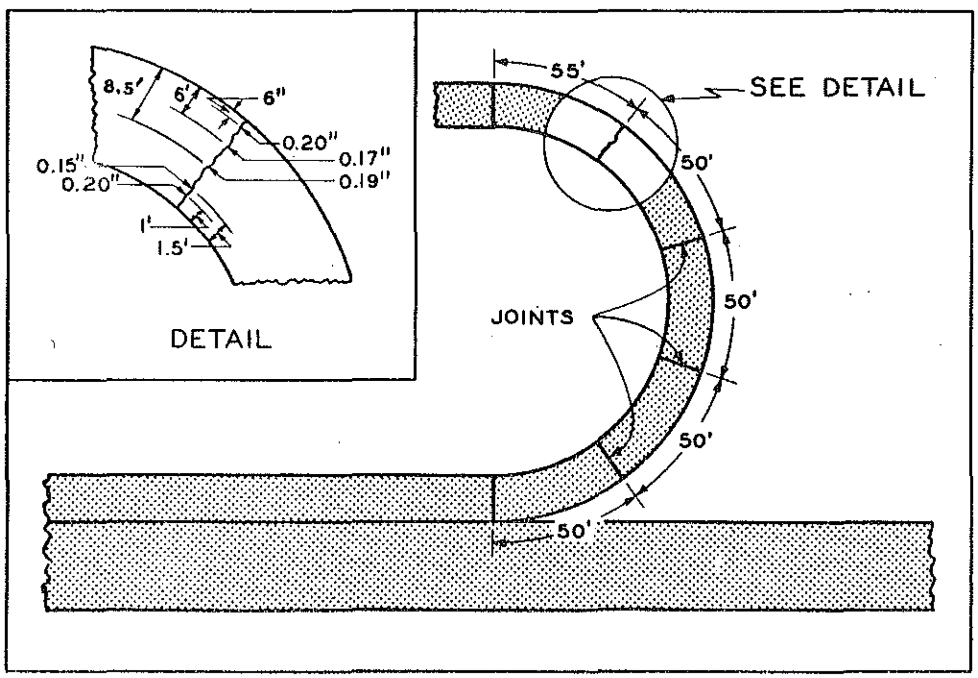

Figure 1. Location of crack and approximate joint spacing on interchange ramp.

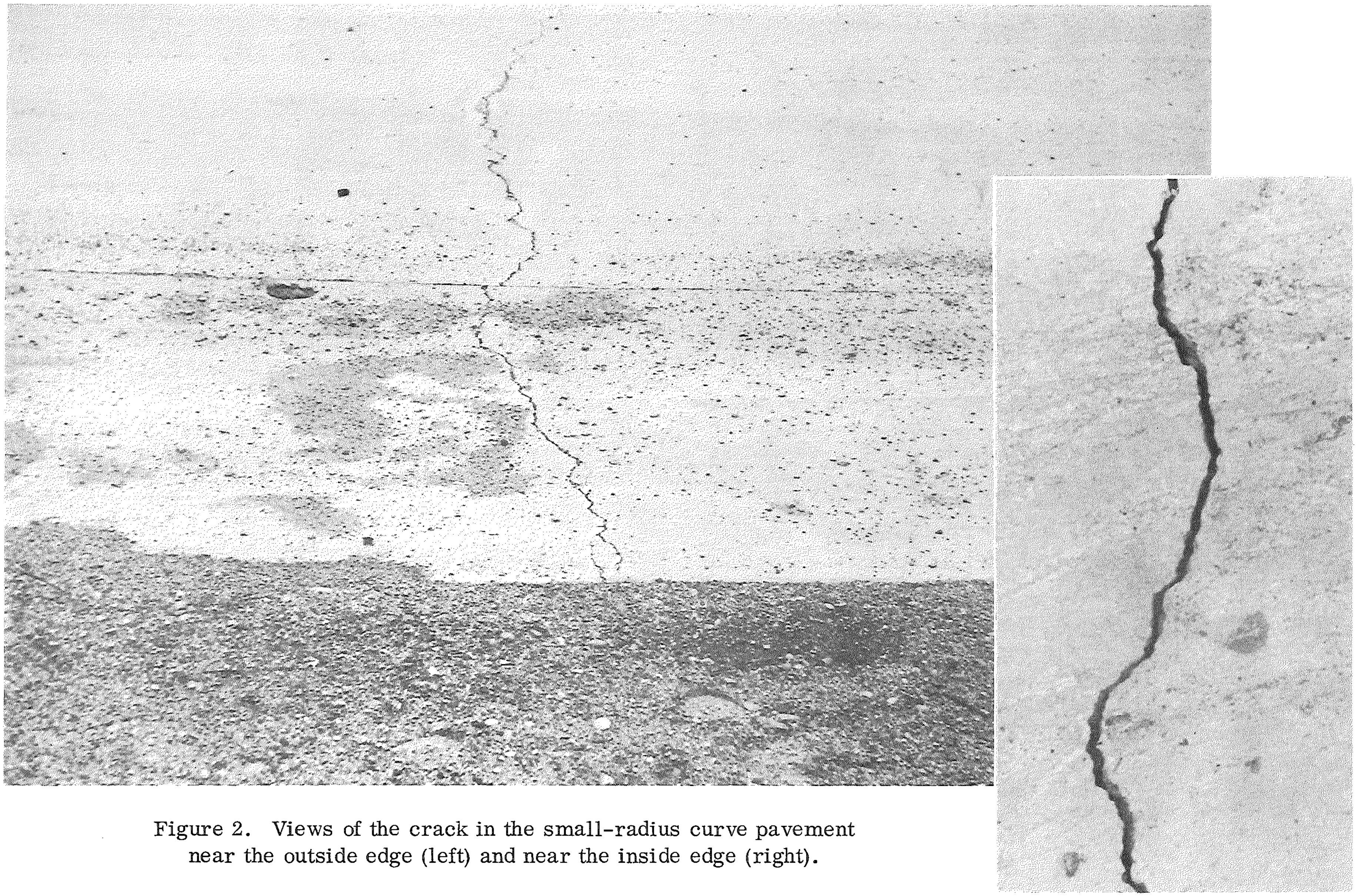

Figure 2. Views of the crack in the small-radius curve pavement near the outside edge (left) and near the inside edge (right).

The following analysis is concerned with the determination of an appropriate joint spacing design for pavements with small-radius curves. Equations are developed and a graph presented showing the joint spacing variation with the radius of curvature. This analysis is based on comparable reinforcing steel stresses found for straight pavements with joints spaced at 99-ft intervals, the current Departmental design practice.

In this study, it is assumed that a slab segment of length L, is free at each end (at the joints), and is partially restrained from deformations caused by temperature change, shrinkage, etc., by the frictional resistance of the subgrade. It is further assumed that the volume changes which take place are uniformly distributed throughout the depth of the slab, and the coefficient of subgrade friction is constant and equal to 1.5.

First, consider a straight segment of length L, 1 ft wide and h in. thick, subjected to the friction force f per foot of length as shown in Figure 3.

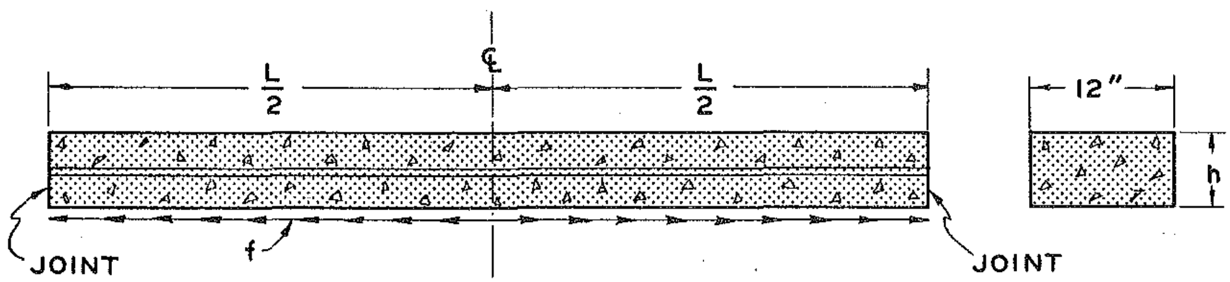

Figure 3. Longitudinal cross-section of a straight pavement subjected to subgrade friction.

If a crack occurs in the concrete at the center of the segment, assuming the tensile stress in the steel reinforcement resists the pull of the subgrade friction forces, then $f_S A_S = \frac{fL}{2} = \frac{wh}{12} \times 1 \times \frac{L}{2} \times 1.5$, where f_S is the steel stress, A_S the steel area, and w the density of the concrete. Taking w as 144 lb per cu ft, one gets

$$f_S = \frac{9hL}{A_S} = \frac{9hL}{pA_C} = \frac{9hL}{p \times 12h} = \frac{.75L}{p},$$

where A_C is the concrete area and p is the steel ratio $\frac{A_S}{A_C}$.

For Michigan standard pavement reinforcement, $L = 99$ ft, and $p = 0.18$ percent for deformed bar mats or 0.16 percent for welded wire fabric. The corresponding steel stresses would be

$$f_s \text{ (bar mat)} = \frac{.75(99)}{.0018} = 41{,}300 \text{ psi}$$

and

$$f_s \text{ (wire fabric)} = \frac{.75(99)}{.0016} = 46{,}400 \text{ psi.}$$

Now consider a circular segment of length L, and width b, subjected to the tangential friction force f per foot of length as shown in Figure 4. These forces are balanced by the radially distributed force q per foot of length, set up by subgrade friction and the resistance of the adjacent shoulder material.

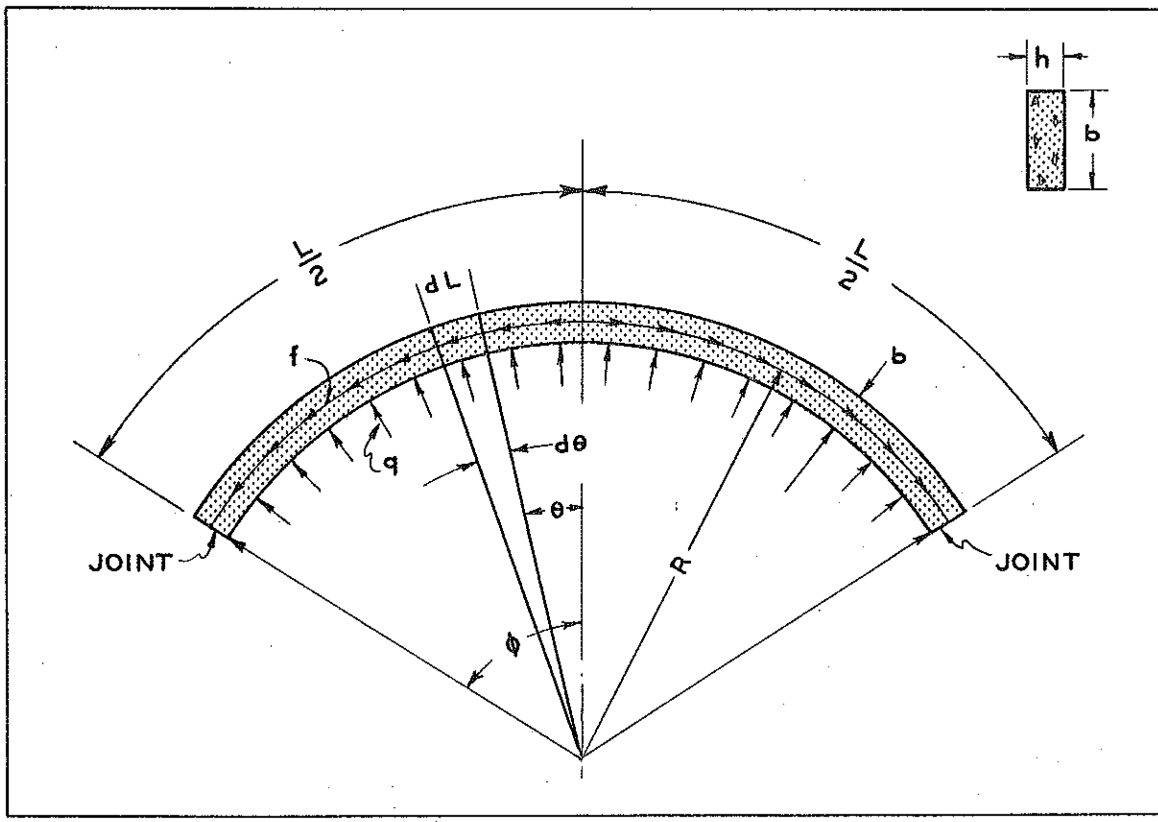

Figure 4. Circular pavement subjected to subgrade friction.

For equilibrium

$$\int_0^{\frac{L}{2}} f \sin \theta \, dL = \int_0^{\frac{L}{2}} q \cos \theta \, dL.$$

Since

$$dL = R d\theta,$$

and

$$f = \frac{144 \times h \times b \times 1.5}{12} = 18 \, hb$$

$$q = \frac{18 \, hbR \int_0^{\phi} \sin \theta \, d \theta}{R \int_0^{\phi} \cos \theta \, d \theta}.$$

Integrating, one finds

$$q = 18 \, hb \, \frac{(1 - \cos \phi)}{\sin \phi}.$$

Now consider a free body of half of the segment (Fig. 5), again assuming the concrete has cracked at the center. The steel resists the tension caused by the normal force and moment set up by the subgrade frictional forces f and q.

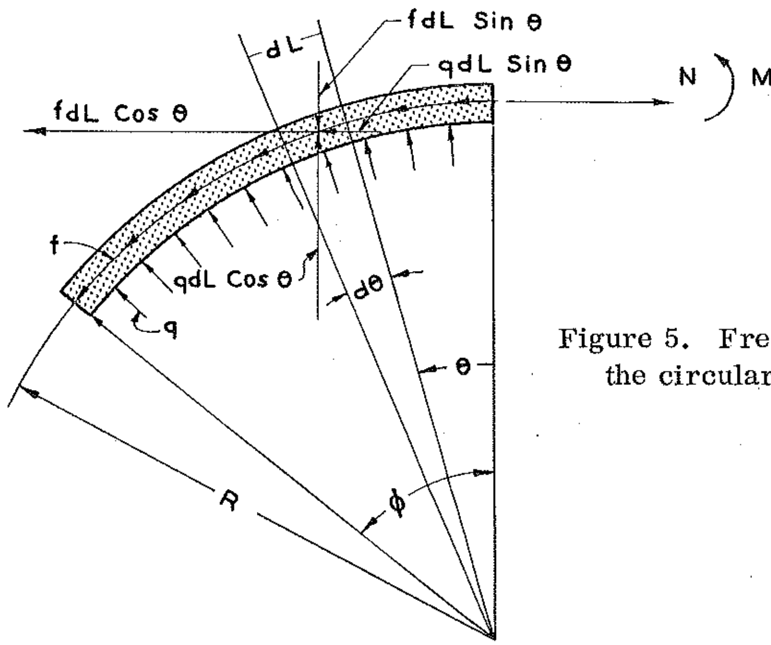

Figure 5. Free body of half the circular segment.

For equilibrium

$$N = \int_0^{\frac{L}{2}} f dL \cos\theta + \int_0^{\frac{L}{2}} q dL \sin\theta \quad \text{or}$$

$$N = 18 hbR \int_0^{\phi} \cos\theta \, d\theta + 18 hbR \frac{(1-\cos\phi)}{\sin\phi} \int_0^{\phi} \sin\theta \, d\theta.$$

Integrating, one finds

$$N = 36 \, hbR \frac{(1-\cos\phi)}{\sin\phi} \text{ lb}$$

also

$$M = \int_0^{\frac{L}{2}} (f dL \cos\theta + q dL \sin\theta) R (1-\cos\theta)$$

$$+ \int_0^{\frac{L}{2}} (q dL \cos\theta - f dL \sin\theta) R \sin\theta,$$

or

$$M = 18 hbR^2 \int_0^{\phi} \left[\cos\theta - \cos^2\theta + \left(\frac{1-\cos\phi}{\sin\phi}\right)(\sin\theta - \sin\theta\cos\theta)\right] d\theta$$

$$+ 18 hbR^2 \int_0^{\phi} \left[\frac{(1-\cos\phi)}{\sin\phi}(\sin\theta \cos\theta) - \sin^2\theta\right] d\theta.$$

Integrating and reducing, one gets

$$M = 18 \, hbR^2 \left(\frac{2 - 2\cos\phi - \phi\sin\phi}{\sin\phi}\right) \text{ lb-ft},$$

or

$$M = 216 \, hbR^2 \left(\frac{2 - 2\cos\phi - \phi\sin\phi}{\sin\phi}\right) \text{ lb-in.}$$

For standard bar mat reinforcement, consisting of 3/8-in. deformed bars spaced at 7 in., the section modulus for a width of 17 ft would be

$$\frac{I}{C} = \frac{2 \times (.11) 49 \sum (1)^2 + (2)^2 + (3)^2 + (4)^2 + \ldots + \ldots (14)^2}{98} = 112 \text{ in.}^3$$

For standard wire mesh reinforcement, consisting of 00 gage wires spaced at 6 in., the section modulus for a width of 17 ft would be

$$\frac{I}{C} = \frac{2\times(.086)(36)\sum(.5)^2+(1.5)^2+(2.5)^2+\ldots+\ldots(16.5)^2}{99} = 102\,\text{in.}^3$$

The maximum tensile steel stress will occur on the first bar or wire at the inside edge of the curve. This stress will be

$$f_s\ (\text{bar mat}) = \frac{N}{pA_c} + \frac{M}{I/C} = \frac{36\,hbR\,(1-\cos\phi)}{(.0018)(12bh)\sin\phi}$$

$$+\frac{216\,hbR^2\,(2-2\cos\phi-\phi\sin\phi)}{112\sin\phi},$$

or

$$f_s\ (\text{bar mat}) = 1667\,R\,\frac{(1-\cos\phi)}{\sin\phi}$$

$$+1.9\,hbR^2\,\frac{(2-2\cos\phi-\phi\sin\phi)}{\sin\phi};$$

and

$$f_s\ (\text{wire mesh}) = \frac{36\,hbR\,(1-\cos\phi)}{(.0016)(12bh)\sin\phi}$$

$$+\frac{216\,hbR^2\,(2-2\cos\phi-\phi\sin\phi)}{102\sin\phi},$$

or

$$f_s\ (\text{wire mesh}) = 1875\,R\,\frac{(1-\cos\phi)}{\sin\phi}$$

$$+2.1\,hbR^2\,\frac{(2-2\cos\phi-\phi\sin\phi)}{\sin\phi}.$$

Considering the average steel stresses for bar mat and wire mesh reinforcement, the steel stress would be given by

$$f_s = 1770R\,\frac{(1-\cos\phi)}{\sin\phi} + 2\,hbR^2\,\frac{(2-2\cos\phi-\phi\sin\phi)}{\sin\phi}.$$

In order for this stress to be comparable with the average steel stress found for the straight pavement $f_s = 43{,}800$ psi.

For the case where $b = 17$ ft and $h = 9$ in., f_s would be given by

$$f_s = 1770R\,\frac{(1-\cos\phi)}{\sin\phi} + 306R^2\,\frac{(2-2\cos\phi-\phi\sin\phi)}{\sin\phi}.$$

For $R = 100$ ft, $L = 105$ ft, $\phi = \frac{105}{200} = .5250$, $\sin\phi = .5012$, $\cos\phi = .8653$, and

$$f_s = \frac{1770(100)(1-.8653)}{(.5012)} + \frac{306(10000)[2-2(.8653)-(.525)(.5012)]}{.5012},$$

or

$$f_s = 47{,}400 + 38{,}400 = 85{,}800 \text{ psi}$$

which is greater than the ultimate tensile strength of the steel.

For $R = 100$ ft, $L = 70$ ft, $\phi = \frac{70}{200} = .3500$, $\sin\phi = .3429$, $\cos\phi = .9394$, and

$$f_s = \frac{1770(100)(1-.9394)}{.3429} + \frac{306(10000)[2-2(.9394)-(.35)(.3429)]}{.3429},$$

or

$$f_s = 31{,}300 + 10{,}700 = 42{,}000 \text{ psi}.$$

Thus for a 100 ft radius curve, with $h = 9$ in. and $b = 17$ ft, and using standard pavement reinforcement, the joint spacing should be about 70 ft. For the intersection ramp with a joint spacing of about 105 ft., the maximum steel stress could possibly exceed the ultimate tensile strength of the steel.

For $h = 9$ in., $b = 17$ ft, and standard pavement reinforcement, a graph based on the above equations, showing the appropriate joint spacing and curve radius variation, is presented in Figure 6. This graph shows that the minimum radius of curvature, for which a standard joint spacing of 99 ft could be used, is around 500 ft, corresponding to a degree of curvature of about 11.5°. As the curve radius decreases below this value, the joint spacing should decrease also, to maintain the comparable steel stresses occurring in straight pavements.

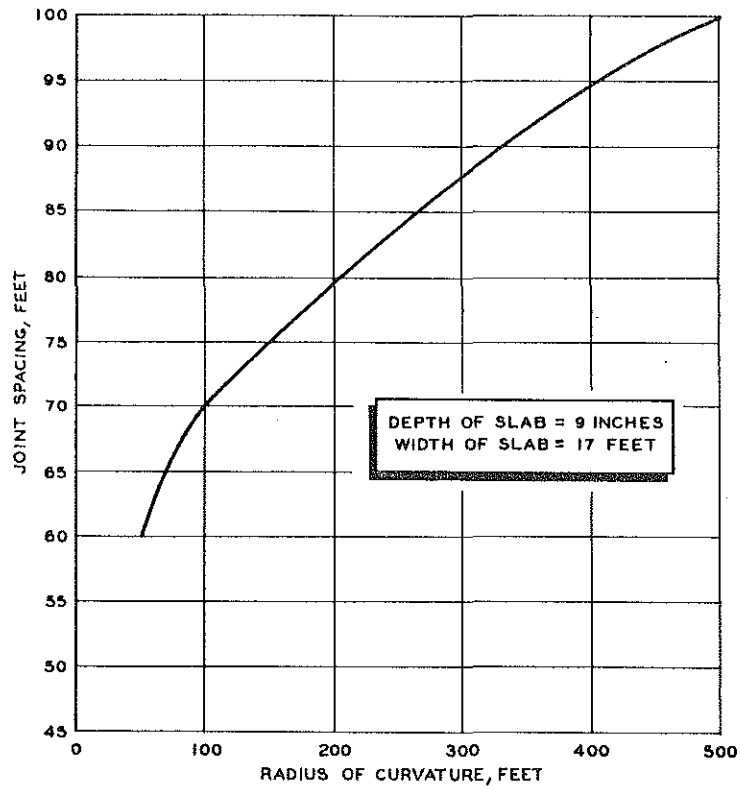

Figure 6. Joint spacing and curve radius variation.

This analysis is not intended as complete design criteria for reinforcement on small-radius curve pavements. The formulation of the equations presented here is based on average stresses occurring in the slab as representative of a beam rather than a plate. Also, the assumption concerning the constant coefficient of friction is conservative, as this value actually varies with the slab movement. This analysis is rather a simplifying comparison between straight and curved pavement segments, and points out the increased steel stresses occurring on the inside edge of the curve as a result of the curvature and resulting frictional forces.

On the basis of this analysis, for intersection ramp curves of 100-ft radius, 17 ft wide, and 9 in. thick, provided standard reinforcement is used with a constant steel ratio across the width of the slab, the joint spacing should be about 70 ft. If it would be desirable for the joint spacing to be an integral number of reinforcement mat lengths, a joint spacing of 71 ft 2 in. corresponding to five standard 15-ft mat lengths should be satisfactory.