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SUBBASE DRAINAGE CRITERIA

MICHIGAN

RESEARCH LABORATORY SECTION

MICHIGAN DEPARTMENT OF
STATE HIGHWAYS AND TRANSPORTATION

SUBBASE DRAINAGE CRITERIA

F. T. Hsia

Research Laboratory Section
Testing and Research Division
Research Project 75 E-56
Research Report No. R-991

Michigan State Highway Commission
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Lansing, March 1976

INTRODUCTION

Gradation, and to some extent permeability, have been used as criteria for subbase drainability in Michigan. Novak (1) noted that effective porosity, as well as permeability, should also be incorporated in drainage analysis. An approximate theoretical analysis method, based on Darcy's Law and the concept of effective porosity, was developed by Casagrande (2). Casagrande's analytical method, originally developed for airfield base course drainage, is only applicable to very simple pavement cross sections and is therefore considered too imprecise to be used for drainage analysis of Michigan's highway subbases, which are much more complex in geometry than those of airfield base courses. This report presents an extension of Casagrande's work to establish a theoretical analytical method for the analysis of drainability which can distinguish between the acceptable and unacceptable subbase materials.

Although the finite element method might be considered as another approach to this problem, its accuracy is still questionable and the development and application of this method would be cumbersome for a practical engineering problem.

CASAGRANDE'S METHOD OF ANALYSIS

Subbase drainage occurs under either steady or transient flow conditions. Steady flow occurs in a pavement subbase that is beneath the groundwater table and where seepage water can only be carried away by edge drains. Transient flow occurs when the quantity of water seeping through the subbase varies during a given time interval. For example, water entering the subbase during a rainstorm would be drained away under transient flow conditions. Since normal subbase drainage occurs under transient flow conditions, the suitability of subbase materials will be determined on the basis of this type of flow.

Subbase drainage is affected by three factors:

1) Properties of the subbase soil; such as effective porosity and permeability.

2) Geometric layout of the subbase; such as thickness, length, superelevation, and location of the plan grade.

3) Boundary conditions of the subbase; such as length and location of the open boundaries and impervious boundaries.

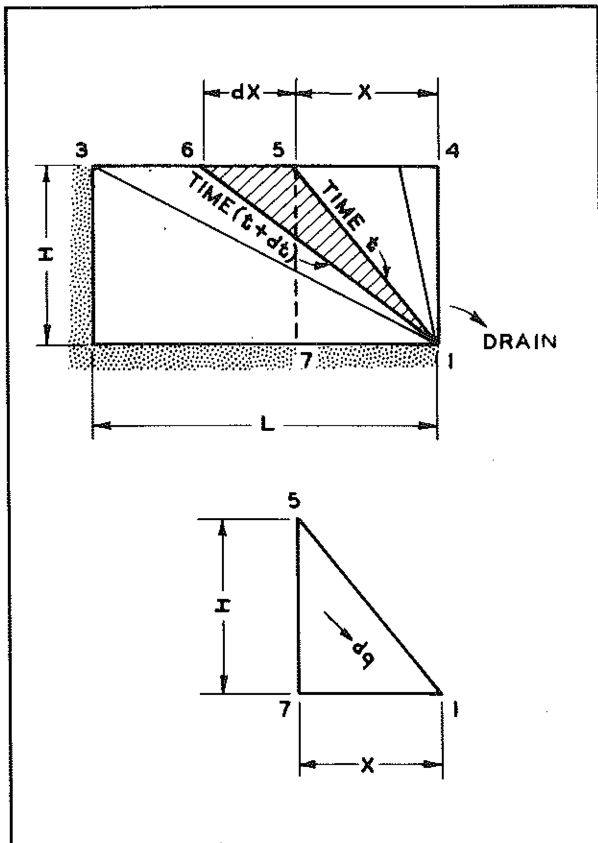


Figure 1. Phreatic surface changes from 1-4 to 1-3.

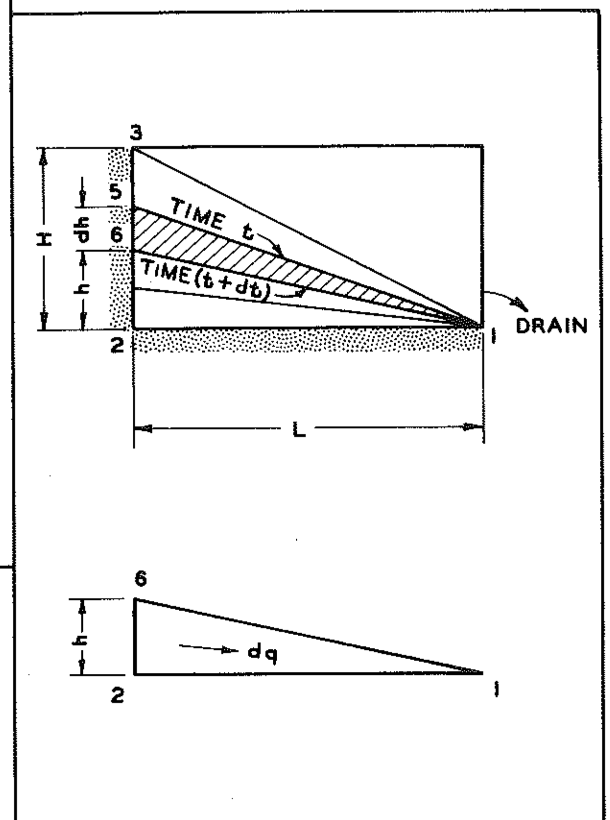


Figure 2. Phreatic surface changes from 1-3 to 1-2.

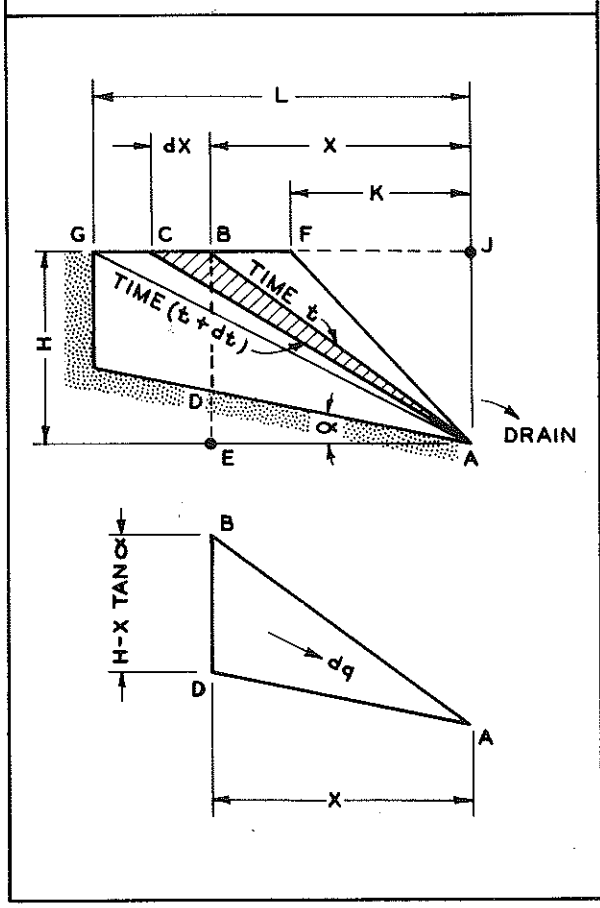


Figure 3. Shape I

A correlation of the above factors was first formulated by Casagrande through the application of Darcy's Law. Laboratory experiments proved the validity of his basic formulations of the differential equations used for the analysis (2).

Casagrande's analysis is based upon several simplified assumptions:

1) The bottom and one side of the subbase are impervious boundaries (cross-hatched areas in Fig. 1).

2) The subbase is assumed to be 100 percent water-saturated at the time drainage starts.

3) No further water enters the subbase once drainage begins.

4) Open discharge is assumed at one side which is suddenly opened for free drainage.

5) The phreatic surface is assumed to be a straight line (Fig. 1).

6) The effective porosity, n_e, is assumed to be independent of height above the impervious boundary.

Figure 1 shows the phreatic surface as it gradually changes from position 1-4 to 1-3. Differential equations can be established by considering the position of the phreatic surface at elapsed time t, then at time (t + dt). In the time element dt, the quantity, dq, discharged per unit width is equal to the area of the narrow, shaded triangle 1-5-6, multiplied by the effective porosity n_e. The effective porosity is the ratio of the volume of voids that can be drained under gravity flow to the total volume of soil mass. The quantity discharged can be expressed:

$$dq = \frac{Hn_e}{2}\,dx \qquad (1a)$$

Darcy's Law ($\frac{dq}{dt} = k\frac{dh}{dl}A$) states that rate of discharge $\frac{dq}{dt}$ can be computed as the product of permeability k, hydraulic gradient $\frac{dh}{dl}$, and the total cross-sectional area of the flow. Since the total head dh is the only head that determines flow, it is used in Darcy's Law to compute gradients.

The flow through volume 1-5-7 in Figure 1 is computed by Darcy's Law. The simplest assumption is to use $\frac{H}{2}$ as the average area per unit of width through which flow takes place, and to assume an average effective gradient of $\frac{H}{x}$. Then the rate of flow is expressed as:

$$\frac{dq}{dt} = k\frac{H}{2}\frac{H}{x} = k\frac{H^2}{2x} \qquad (1b)$$

By applying the chain rule $\frac{dx}{dt} = \frac{dx}{dq}\frac{dq}{dt}$, Eq. (1a) and Eq. (1b) can be combined as:

$$dt = \frac{n_e}{k}\frac{x}{H}\,dx \qquad (1c)$$

This equation is to be solved as:

$$t = \frac{n_e}{k}\left(\frac{x^2}{2H}\right) + C \qquad (1d)$$

where C is the integration constant.

The boundary condition of this differential equation is that x = 0 when t = 0. Hence C = 0 when the boundary condition is substituted into Eq. (1d). The final form of Eq. (1d) is now:

$$t = \frac{n_e}{k}\left(\frac{x^2}{2H}\right) \qquad (2)$$

Eq. (2) represents the time required to remove a certain amount of water from the subbase which is generally defined as the drainability of the subbase.

Figure 2 shows the phreatic surface changes from position 1-3 to position 1-2. The variable triangle 1-5-6 has a constant base length L and a variable height h. The starting equations are:

$$dq = \frac{Ln_e}{2}\,dh \qquad (3a)$$

$$\frac{dq}{dt} = -k\frac{h^2}{2L} \qquad (3b)$$

The negative sign in Eq. (3b) indicates that total head, h, is decreasing with increasing time.

With the boundary condition h = H when t = 0, Eq. (3a) and Eq. (3b) can be solved as:

$$t = \frac{n_e}{k} \left[\frac{L^2}{hH} \quad (H - h) \right] \tag{4}$$

Eq. (2) and Eq. (4) represent the formulation of the drainability for the simplest geometrical shapes; however, Michigan's pavement subbases are much more complex. Therefore, drainabilities for various other shapes must be developed. Casagrande established the criteria for satisfactory drainability as a 10 day maximum to reach the 50 percent degree of drainage. The degree of drainage U is defined as the ratio of the drained area to the total area ($U = \frac{\text{drained area}}{\text{total area}}$). This criterion was also adopted by the Highway Research Board (3) and recommended by Ref. (1) for the drainage design of subbase layers.

DRAINABILITY FOR BASIC GEOMETRIC SHAPES

Drainability for nine basic geometric shapes, other than shown in Figures 1 and 2, are to be developed in order to estimate drainability of various Michigan highway subbases. It is seen that the basic equations to develop drainability are Eqs. (1a) and (1b). For different geometric shapes, the content of these two equations varies, but the basic form remains unchanged. Specifically, the mathematical involvement is no more than solving the first order, first degree ordinary differential equations. Although the formulas for the drainability of some of the shapes may be lengthy, their calculation is greatly facilitated by the electronic pocket calculators available today.

Drainability for Shape I

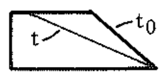
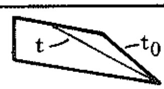
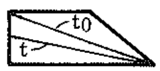
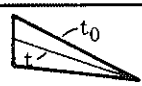
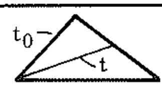
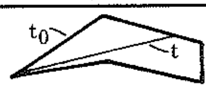
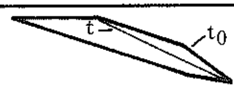
As shown in Figure 3 for Shape I, the impervious base slopes at an angle α . The open side boundary slopes up to a horizontal distance k from point A (the top boundary G-F is horizontal). The phreatic surface gradually changes from position AF to AG.

In the time element dt, the quantity dq discharged per unit width is equal to triangle ABC, multiplied by n_e.

$$dq = \frac{n_e}{k} H\,dx \tag{5a}$$

The flow through volume ABD is computed by Darcy's Law. To do this it

TABLE 1
DRAINABILITY FOR BASIC GEOMETRIC SHAPES

No.	Shape	Drainability Equation (see Appendix A for symbols)
I		$t = \frac{n_e}{k} \left(\frac{K - x}{\tan \alpha} + \frac{H}{\tan^2 \alpha} \ln \frac{H - K \tan \alpha}{h - x \tan \alpha} \right)$
II		$t = \frac{n_e}{k} \left[\frac{1}{2H} (x^2 - K^2) \right]$
III		$t = \frac{n_e}{k} \left(\frac{x - K}{\tan \alpha} + \frac{H}{\tan^2 \alpha} \ln \frac{H + K \tan \alpha}{H + x \tan \alpha} \right)$
IV		$t = \frac{n_e}{k} \left[L^2 \left(\frac{H - h}{Hh} \right) \right]$
V		$t = \frac{n_e}{k} \left[\frac{L}{\tan \alpha} \ln \frac{(H - L \tan \alpha)(h + L \tan \alpha)}{Hh} \right]$
VI		$t = \frac{n_e}{k} \left\{ \frac{L}{\tan \alpha} \left[\frac{L(H - L \tan \alpha + x \tan \alpha)}{(L - x)H} + \ln \frac{(L - x) \tan \alpha}{H} \right] \right\}$
VII		$t = \frac{n_e}{k} \left[\frac{H + \ell \sin \gamma}{\tan \gamma \tan \alpha} \left(\ln \frac{h}{H} \right) - \frac{H + \ell \tan \gamma}{\tan \alpha (\tan \gamma + \tan \alpha)} \ln \frac{h(\tan \alpha + \tan \gamma) - (L \tan \gamma + h_1) \tan \alpha}{H(\tan \alpha + \tan \gamma) - (L \tan \gamma + h_1) \tan \alpha} \right]$
VIII		$t = \frac{n_e}{k} \frac{H + \ell \tan \gamma}{L \tan^2 \gamma \tan \alpha} \left\{ H_1 + L(\tan \gamma + \tan \alpha) \ln \frac{H_1 + L(\tan \alpha + \tan \gamma) - x \tan \gamma}{H_1 + L \tan \alpha} - (H_1 + \tan \gamma) \ln \frac{H_1 + L \tan \gamma - x \tan \gamma}{H_1} \right\}$
IX		$t = \frac{n_e}{k} \left\{ \frac{H \cos^2 \gamma - L \sin(\gamma - \alpha)}{H \cos^2 \gamma \tan \gamma} \left[x - L - \frac{H + L(\tan \alpha - \tan \gamma)}{\tan \gamma} \ln \frac{H + L(\tan \alpha - \tan \gamma) + x \tan \gamma}{H + L \tan \alpha} \right] \right.$

is necessary to assume that the average area per unit of width through which flow takes place, and the average effective gradient $\frac{H}{x}$, is a ratio of area ABD and area AJBE expressed in terms of H. Then flow can be expressed as:

$$\frac{dq}{dt} = k\frac{H}{x}\left[\left(\frac{\text{area ABD}}{\text{area AJBE}}\right)(H)\right]$$

$$= k\frac{H}{x}\left[\frac{x(H - x\tan\alpha)}{2Hx}\,H\right]$$

$$= \frac{kH(H - x\tan\alpha)}{2x} \qquad (5b)$$

Eq. (5a) and Eq. (5b) can be combined as:

$$dt = \frac{n_e}{k}\,\frac{x}{H - x\tan\alpha}\,dx \qquad (5c)$$

The solution of Eq. (5c) is:

$$t = \frac{n_e}{k}\left[-\frac{x}{\tan\alpha} - \frac{H}{\tan^2\alpha}\ln(H - x\tan\alpha)\right] + C \qquad (5d)$$

where C is the integration constant which can be determined by the boundary condition.

With the boundary condition x = K when t = 0, Eq. (5d) is solved as:

$$t = \frac{n_e}{k}\left(\frac{k - x}{\tan\alpha} + \frac{H}{\tan^2\alpha}\ln\frac{H - K\tan\alpha}{h - x\tan\alpha}\right) \qquad (6)$$

Eq. (6) expresses the drainability of Shape I as functions of geometry, effective porosity, and permeability.

Drainability for All Basic Shapes

Investigation into all of the Michigan standard typical pavement cross-sections indicates that subbases are composed of various combinations of nine basic geometric shapes. Using the procedure as shown for Shape I, the development of drainabilities for all other basic shapes are included in Appendix A. Table 1 summarizes these shapes and their drainability equations. In the sketch, t_0 indicates the beginning position of the phreatic surface at t = 0, and t indicates the position of the phreatic surface at time t.

Development of Drainage Requirements for Michigan Subbases - The Superposition Principle

It is now known that the drainability of the various geometric shapes are functions of permeability, effective porosity, and shape geometry. For a certain subbase section, geometry is constant, while only the material characteristic factor $\frac{k}{n_e}$ will determine the time required for drainage.

The subbase section is a combination of various basic geometric shapes summarized in Table 1. Because of the complexity of the geometry of the subbase section, its drainability has to be estimated on the basis of the drainability of each individual basic geometric shape, and a superposition principle is then used to calculate the combined effects. The superposition principle will be stated as: The total time required to drain to a particular point in the subbase is equivalent to the sum of the drainage time required to drain each individual geometric shape composing that section to that particular point.

A subbase section has both of its sides open for drainage at the beginning. Gradually, the phreatic surfaces of the two drainage sides will meet at a common point at the bottom of the slab. Due to the difference in geometry of the two sides, drainage rates will be different but drainage times required for the two phreatic surfaces to reach that common point are the same. The location of this point and the time required to reach it can be calculated by trial and error. An example was worked out in Appendix B to illustrate the Superposition Principle, the trial and error solution of the drainability, and the minimum drainage requirement.

Drainage Requirements for Michigan Subbases

For a given subbase cross-section, the drainage requirement is a function of the material characteristic $\frac{k}{n_e}$ only. Although any typical cross-section has several different superelevations, it is not hard to determine the most critical case among them. In fact, detailed calculations for several selected typical cross-sections revealed that values of the material characteristic factor vary within 15 percent among the different superelevations for a certain cross-section. It is also known that the back slope and the front slope of each cross-section are either 1 on 2, 1 on 4, or 1 on 6, depending upon the local relief features. But 1 on 6 is the most common case and its drainage requirement has control over that for the 1 on 2 or the 1 on 4 cases. To represent the most general case, and to take the greatest safety measure, 1 on 6 slope and the most critical superelevation for drainage requirement are presented for each typical cross-section.

Figures 4 through 11 represent all the common normal and superelevated typical Michigan highway cross-sections. The minimum required material characteristic factors ($\frac{k}{n_e}$) for the corresponding cross-sections as calculated by the procedure used in Appendix B are shown in Table 2.

It may be seen from Table 2 that the minimum required material characteristic factor ranges from 34.2 ft/day for a 16-ft concrete ramp to 62.4 ft/day for 9-in. dual 36-ft concrete pavement. Since some of the values are very close, Table 2 is reassembled into four groups and presented in a more informative and convenient manner as Figure 12.

To determine the suitability of an in-place subbase material the engineer identifies the pavement cross-section from those shown in Figures 4 to 11, and then by referring to Figure 12 determines the minimum required $\frac{k}{n_e}$ ratio for that cross-section. The $\frac{k}{n_e}$ ratio can easily be determined in the field using the field permeability test described in Ref. (7). The subbase layer must have a $\frac{k}{n_e}$ ratio larger than the minimum required. Should it be less, the same material can only be accepted under the condition that supplementary drains be installed.

An additional benefit to be derived from methods described in this report is that drainage requirements for newly developed subbase sections, in the future, can be evaluated by the procedure used in Appendix B and the equations listed in Table 1.

Additional Requirements

It is theoretically possible for a subbase material to meet the drainage criteria as represented by the material characteristic factor and still remain nearly 100 percent saturated because its effective porosity, n_e, approaches zero. A limitation on the capillary water content of a subbase material is then required. As indicated in reports by Mullis (5) and Novak (6) there is justification for requiring all base and subbase materials to be less than 90 percent saturated if they may be subjected to freezing. Therefore, it should be required that all subbase materials, when gravity drained, be less than 90 percent saturated, and this factor should be controlled by the inspector at the time of construction.

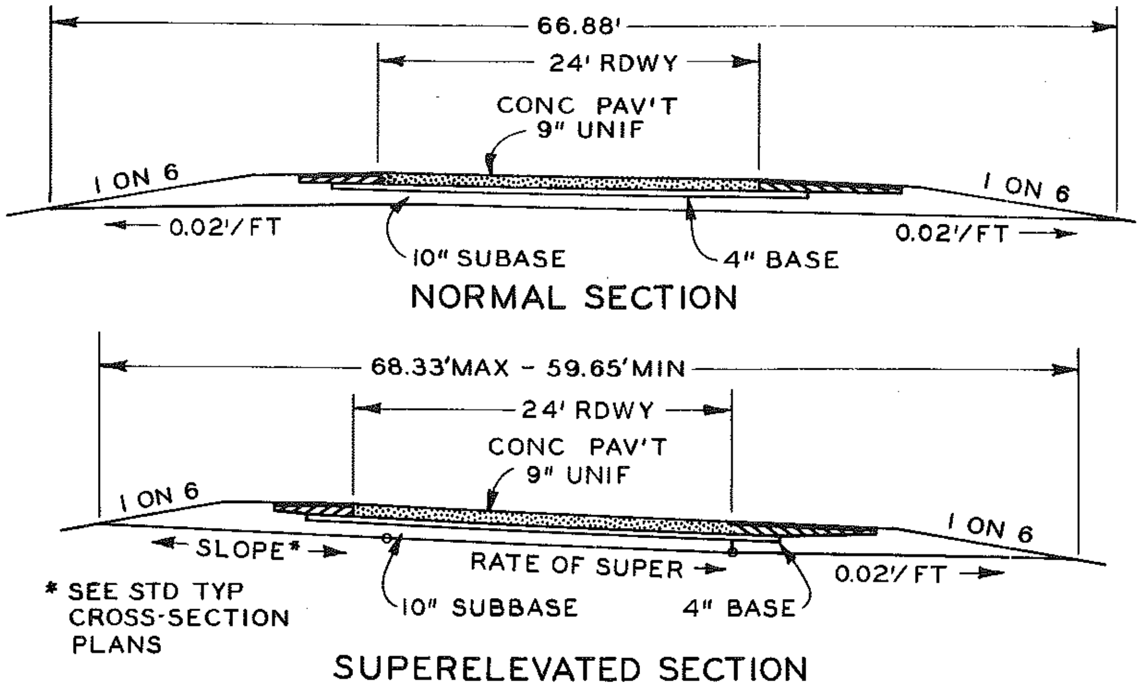


Figure 4. Dual 24-ft concrete roadway (9-in. uniform), Type A.

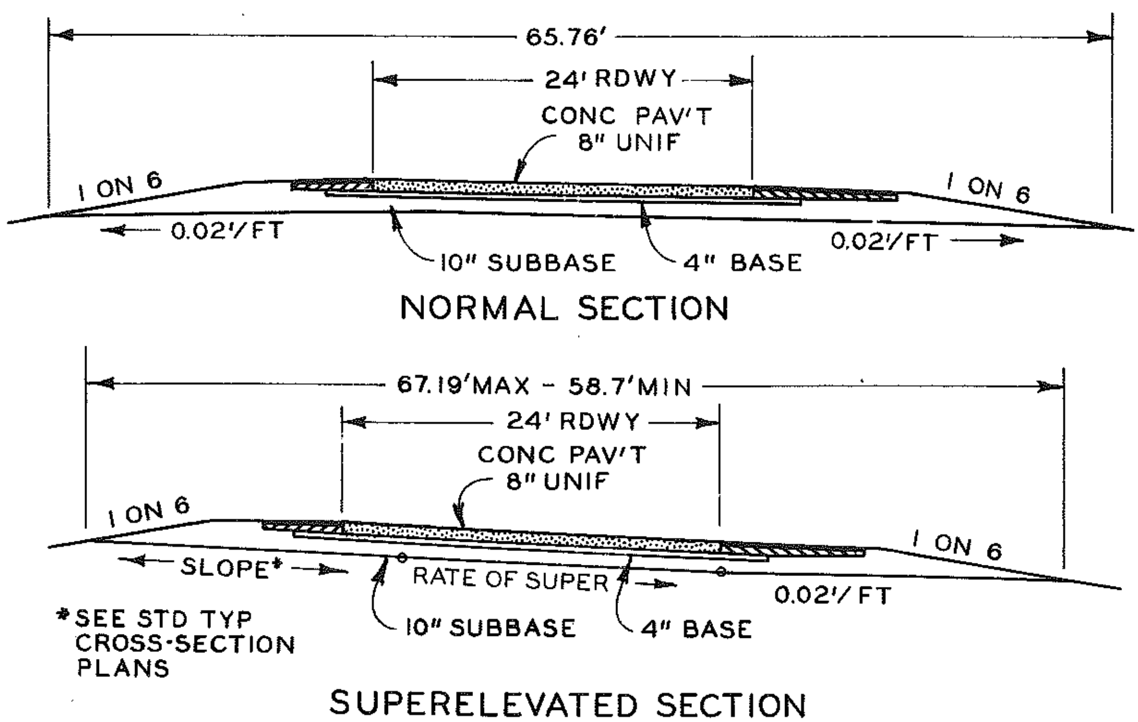


Figure 5. Dual 24-ft concrete roadway (8-in. uniform), Type B.

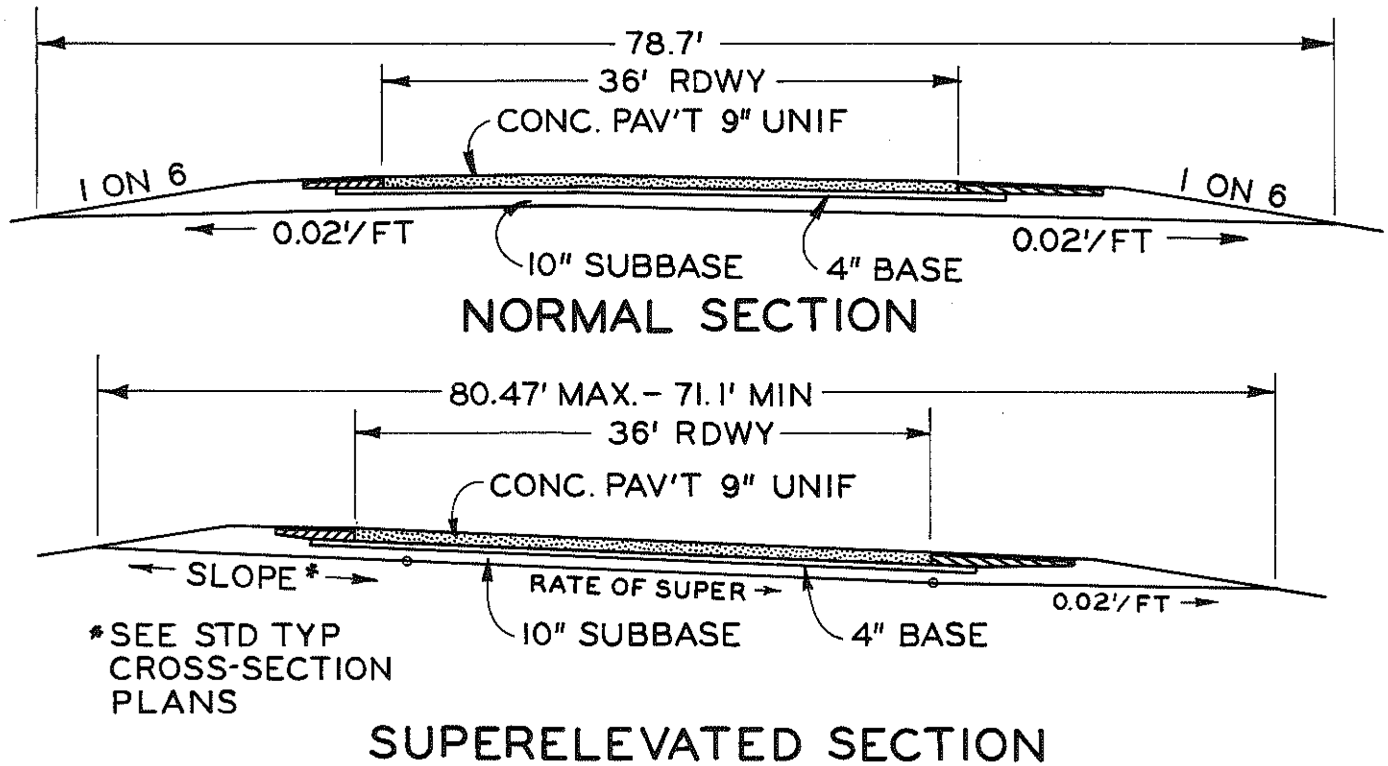


Figure 6. Dual 36-ft concrete roadway (9-in. uniform), Type C.

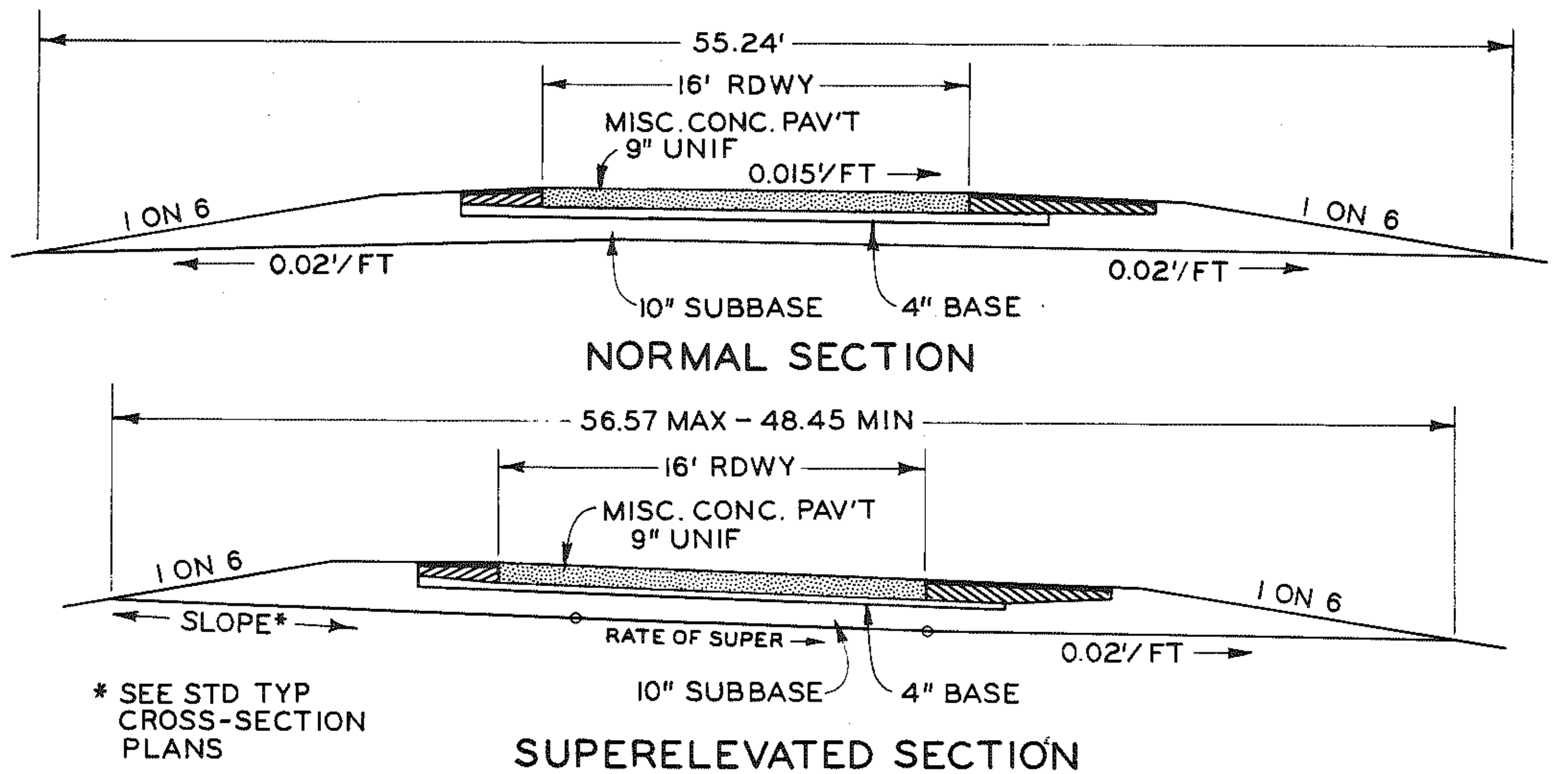


Figure 7. Sixteen ft concrete ramp (9-in. uniform), Type D.

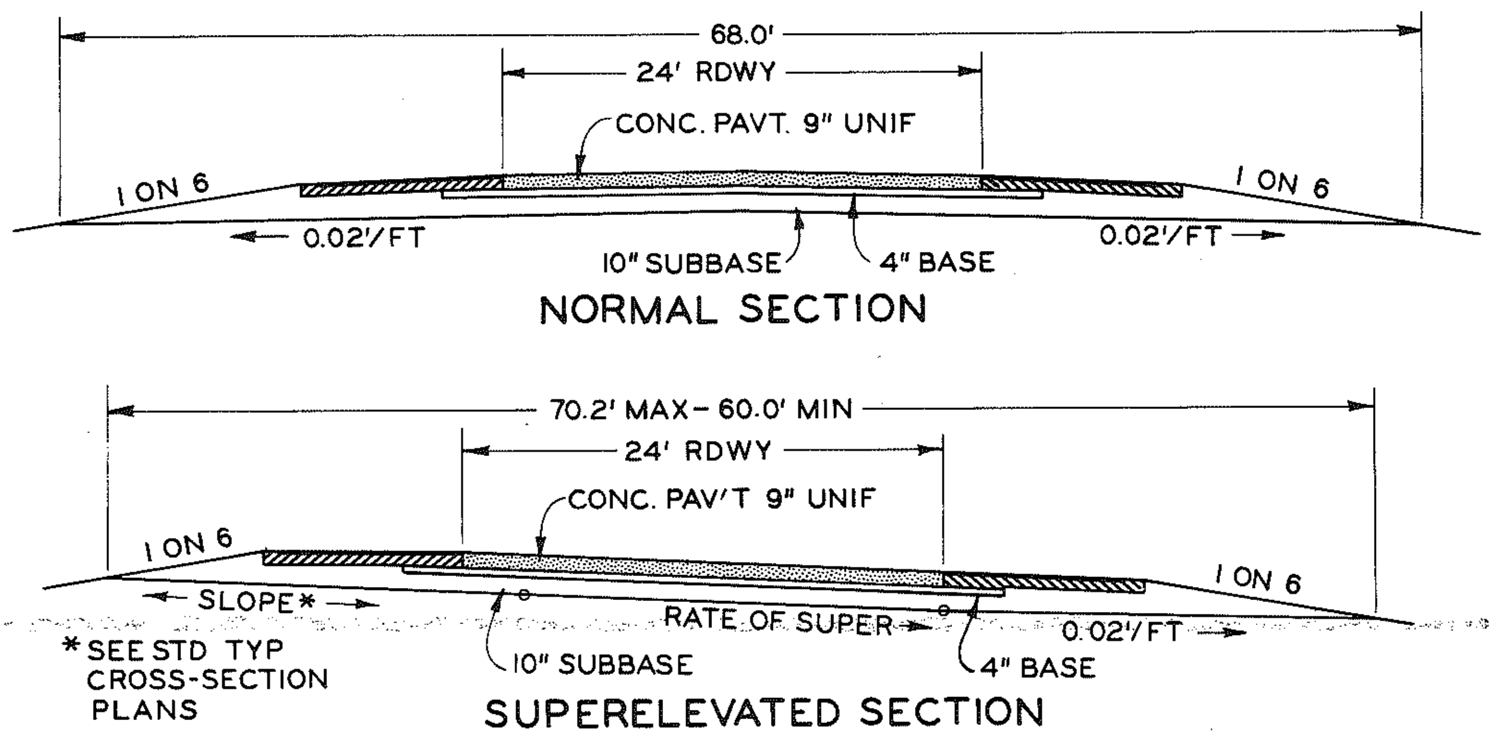


Figure 8. Two-way 24-ft concrete roadway (9-in. uniform), Type E.

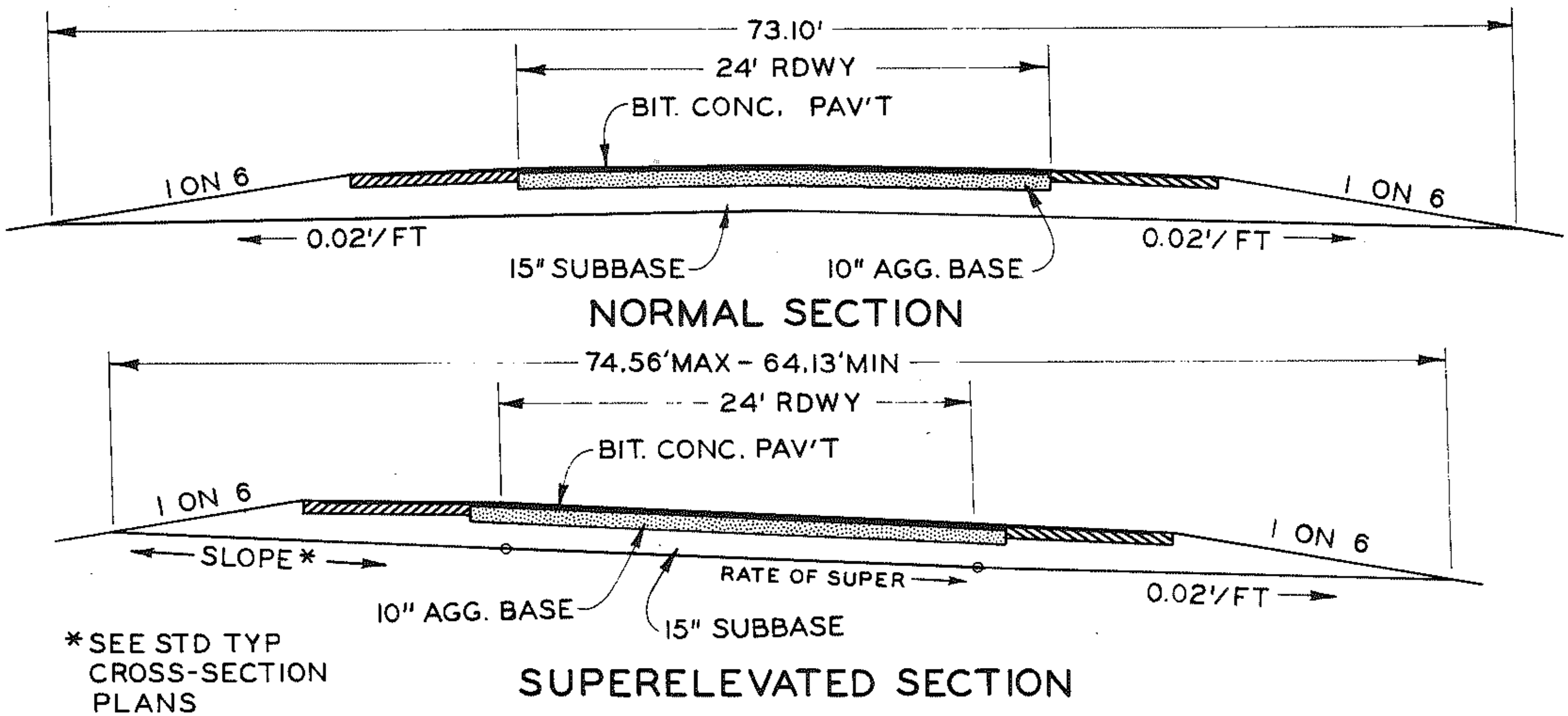


Figure 9. Two-way 24-ft bituminous concrete roadway (ADT 500-2,000), Type F.

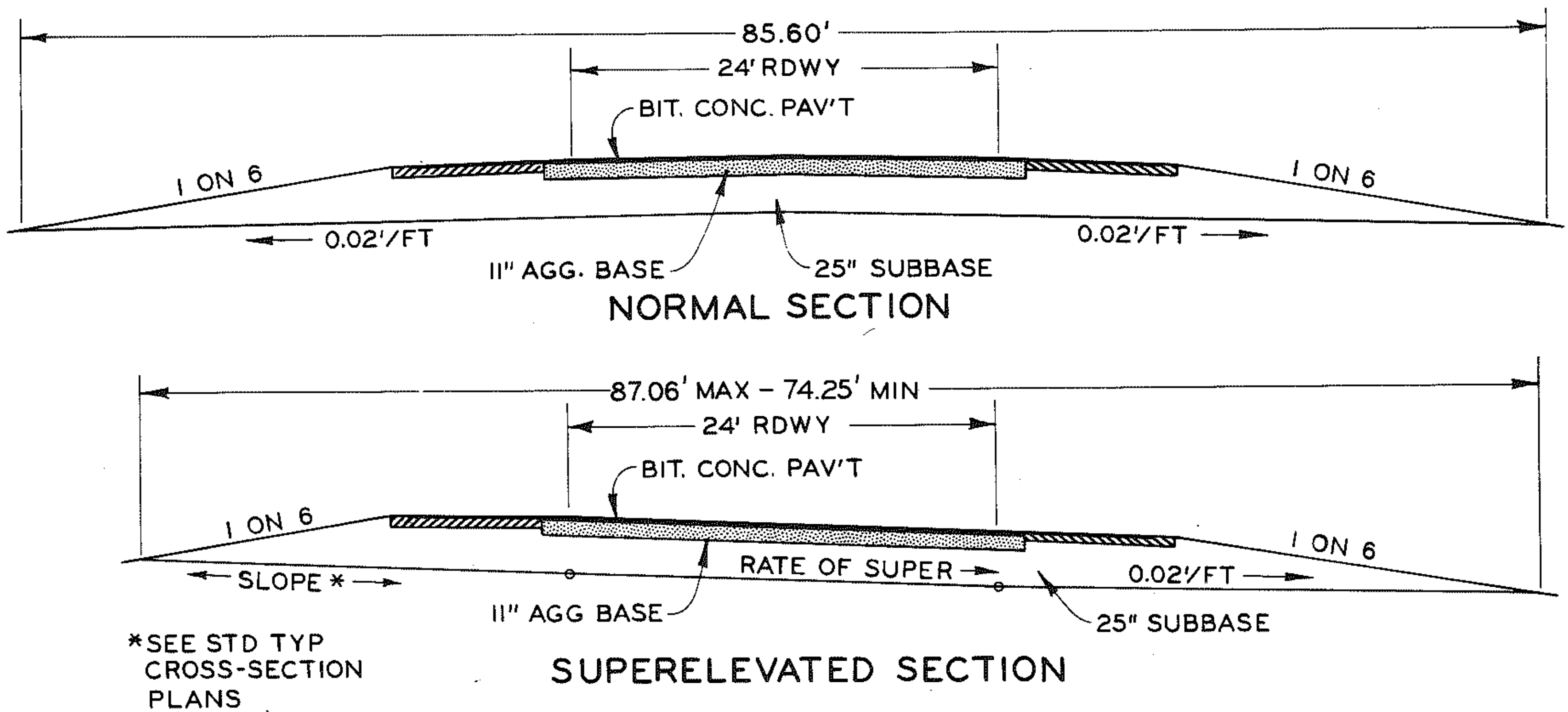


Figure 10. Two-way 24-ft bituminous concrete roadway (ADT 2,000-3,000), Type G.

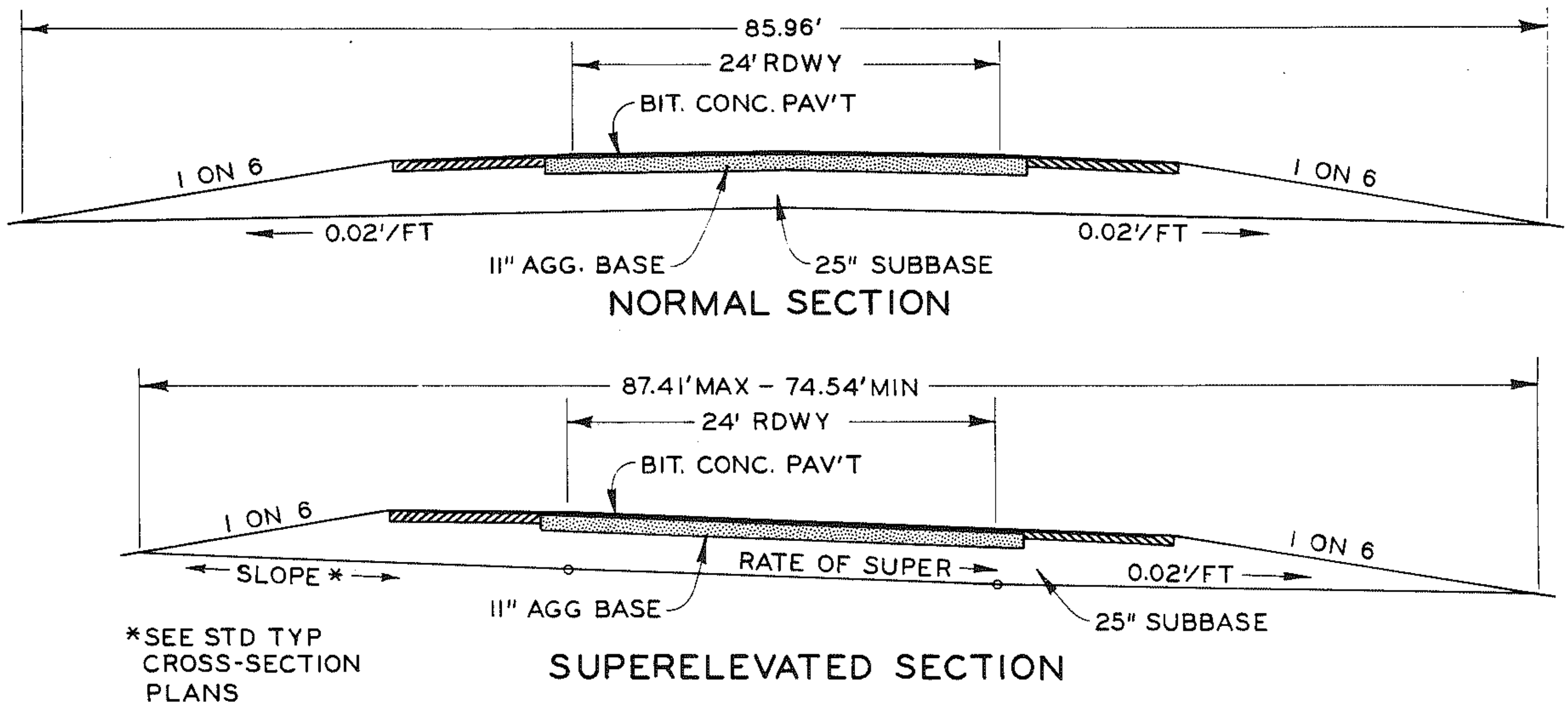


Figure 11. Two-way 24-ft bituminous concrete roadway (ADT 3,000-6,000), Type H.

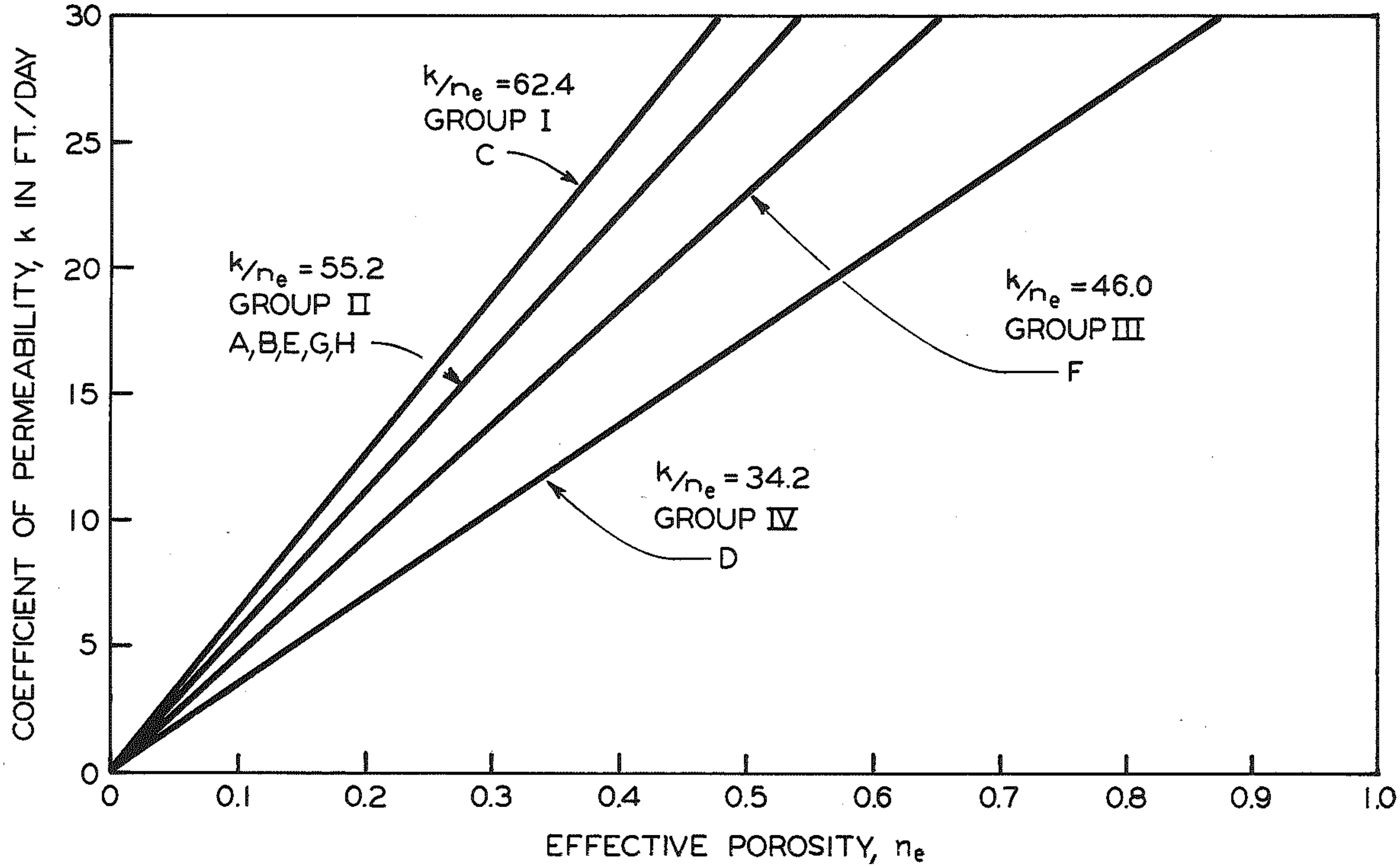


Figure 12. Material characteristic factor $\frac{k}{n_e}$ requirements for standard typical cross-sections.

TABLE 2
SUBBASE DRAINAGE REQUIREMENT FOR
STANDARD TYPICAL CROSS-SECTIONS AT t_{50} = 10 DAYS

Cross-Section Type	Pavement Type [1]	Subbase Thick-ness, in.	Maximum Length of Subbase, ft	Minimum k/n_e, ft/day
A	Dual 24 ft Concrete (9-in.)	10	68.33	55.2
B	Dual 24 ft Concrete (8-in.)	10	67.19	55.2
C	Dual 36 ft Concrete (9-in.)	10	80.47	62.4
D	16 ft Concrete Ramp (9-in.)	10	56.57	34.2
E	Two Way 24 ft Concrete (9-in.)	10	70.20	53.5
F	Two Way 24 ft Bituminous Concrete	15	74.56	46.0
G [2]	Two Way 24 ft Bituminous Concrete	25	87.06	52.5
H [3]	Two Way 24 ft Bituminous Concrete	25	87.41	52.5

[1] Figures in parentheses indicate the thickness of pavement.
[2] ADT 2,000-3,000
[3] ADT 3,000-6,000

CONCLUSIONS

1) When supplemented with a maximum allowable percent saturation requirement, Casagrande's 10 day for 50 percent drainage requirement appears to be a practical criterion for determining the acceptability of subbase materials for drainage requirements.

2) Subbase drainage requirement is expressed by a minimum material characteristic factor, $\frac{k}{n_e}$. Figure 12 specifies the minimum requirements for various standard typical cross-sections of Michigan pavements.

3) Material characteristic factors can either be field determined or laboratory determined. If field determined, Figure 12 can serve as the fastest reference for the engineer to make his determination.

4) Subbase material rejected by the conventional gradation evaluation method should be re-evaluated based upon its drainability characteristic factor, which may result in acceptance.

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2. Casagrande, A., Shannon, W. L., "Base Course Drainage for Airport Pavements," Proceedings ASCE, Vol. 77, Separate No. 75, June 1951.

3. Barber, E. S., "Subsurface Drainage of Highways and Airports," Highway Research Board Bulletin 209, January 1958.

4. Strohm, W. E., Nettles, E. H., Calhoun, C. C., "Study of Drainage Characteristics of Base Course Materials," Highway Research Record 203, 1967.

5. Mullis, I. B., "Properties of Roadbeds," ASCE Journal of the Highway Division, Vol. 89, No. HW1, April 1963.

6. Novak, E. C., "Study of Frost Action in Class AA Shoulders Near Pontiac," MDSHT Research Report R-671, April 1968.

7. Novak, E. C., "A Field Permeability Test Method," Unpublished Research report (in progress).

APPENDIX A

DEVELOPMENT OF DRAINABILITY EQUATIONS

Drainability for Shape II

As shown in Figure A-1, Shape II is essentially similar to that shown in Figure 1, except that the vertical open boundary is replaced by a slope line AF. Phreatic surface changes gradually from AF to AE.

The starting equations, similar to those in Figure 1, are:

$$dq = \frac{n_e H}{2} dx \qquad \text{(A-1a)}$$

$$\frac{dq}{dt} = k\frac{H^2}{2x} \qquad \text{(A-1b)}$$

Together with the boundary condition x = K when t = 0, this set of equations is solved as:

$$t = \frac{n_e}{k}\left[\frac{1}{2H}(x^2 - K^2)\right] \qquad \text{(A-2)}$$

One should note that as long as Darcy's Law calls for the implementation of the total head instead of the pressure head in its calculation of the gradient, the open boundaries can either be open air or attached to some kind of drained media.

Drainability for Shape III

Shape III is similar to Shape I except that the horizontal top boundary in Shape I is replaced by an inclined boundary at the same slope as that of the bottom boundary. As shown in Figure A-2, phreatic surface changes gradually from AF to AE.

In the time element dt, the quantity dq discharged per unit width is equal to:

$$dq = n_e \frac{Hdx}{2} \qquad \text{(A-3a)}$$

Flow through volume ABD is again computed by Darcy's Law with $\frac{H + x\tan\alpha}{x}$ as gradient and ratio of area ABD and area AGBD expressed

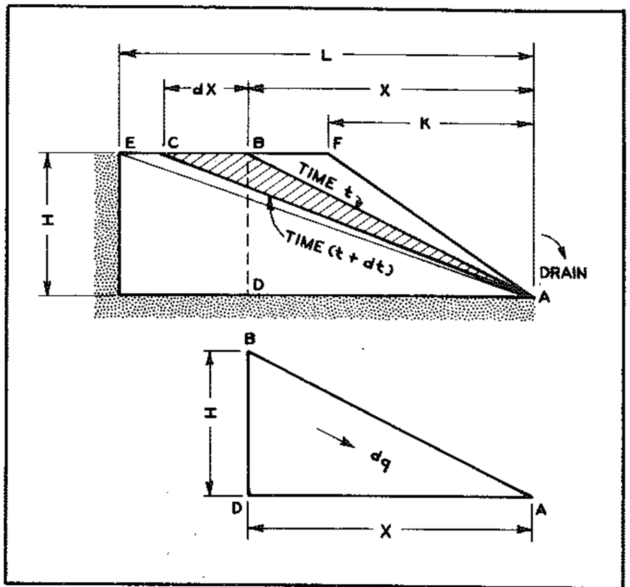
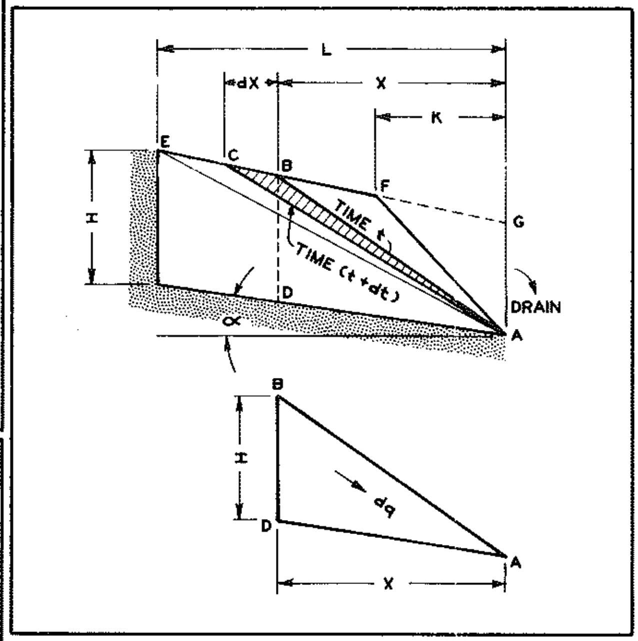


Figure A-1. Shape II

Figure A-2. Shape III



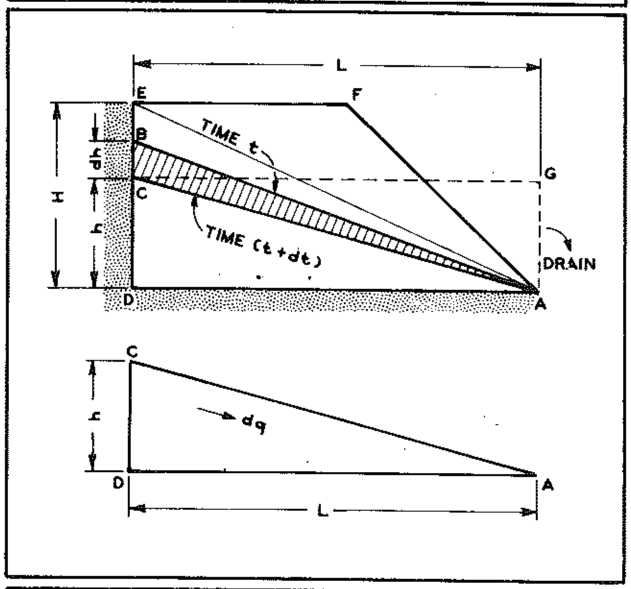


Figure A-3. Shape IV

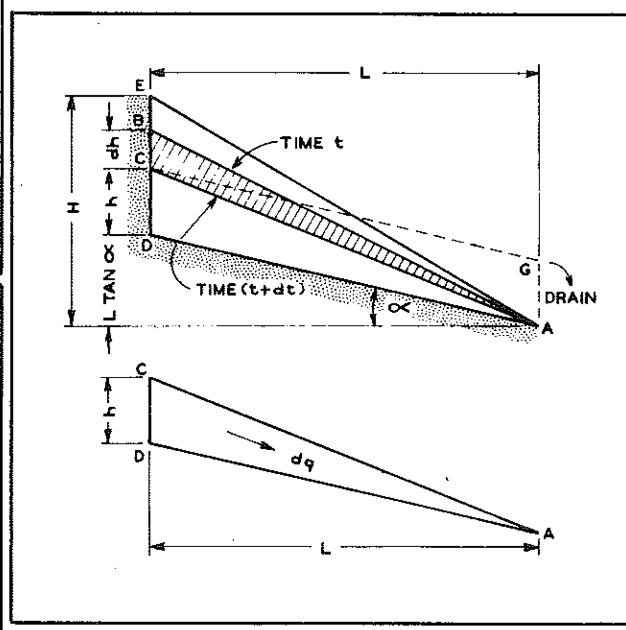


Figure A-4. Shape V

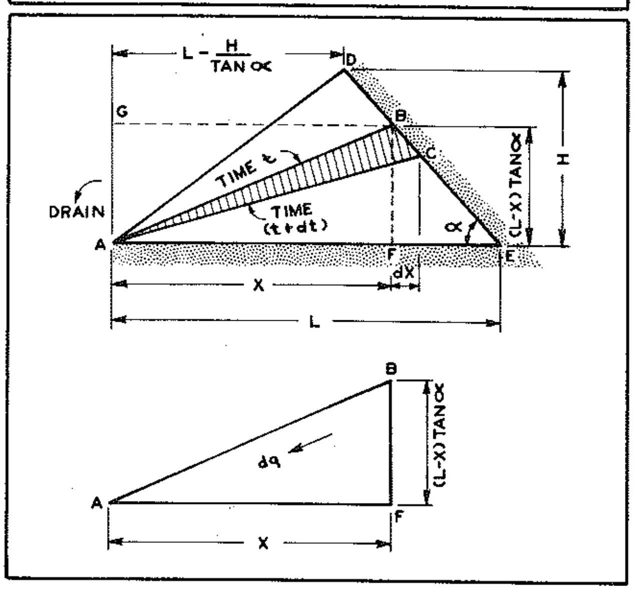


Figure A-5. Shape VI

in terms of H as the average area per unit of width to assume this gradient.

$$\frac{dq}{dt} = k\frac{H + x\tan\alpha}{x}\left[\frac{(\text{area ABD})}{(\text{area AGBD})}(H)\right]$$

$$= k\frac{H + x\tan\alpha}{x}\;\frac{H}{2} \qquad \text{(A-3b)}$$

With the boundary condition x = k when t = 0, Eq. (A-3a) and Eq. (A-3b) can be solved as:

$$t = \frac{n_e}{k}\left(\frac{x - k}{\tan\alpha} + \frac{H}{\tan^2\alpha}\ln\frac{H + k\tan\alpha}{H + x\tan\alpha}\right) \qquad \text{(A-4)}$$

Drainability for Shape IV

Shape IV (Fig. A-3) is exactly the same as Shape II. However, phreatic surface changes gradually downward, instead of horizontally, from AE to AD. It has to be noted here that as long as the drainability is desired in the vertical direction, only the undrained area, such as triangle AED in Figure A-3, is accounted for. Consequently, the position of F, beyond AE, is irrelevant. The same drainability would be obtained even if F coincided E in the figure.

In the time element dt, the quantity dq discharged per unit width is equal to the product of n_e and the triangle area ABC.

$$dq = n_e\frac{Ldh}{2} \qquad \text{(A-5a)}$$

Flow through volume ACD is computed by Darcy's Law with $\frac{h}{L}$ as the gradient and ratio of area ACD to the rectangular area AGCD expressed in terms of h as the average area per unit of width. The negative sign indicates that total head h is decreasing with increasing time.

$$\frac{dq}{dt} = -k\frac{h}{L}\left[\frac{(\text{area ACD})}{(\text{area AGCD})}(h)\right] \qquad \text{(A-5b)}$$

With the boundary condition h = H when t = 0, Eq. (A-5a) and Eq. (A-5b) are solved as:

$$t = \frac{n_e}{k}\left[L^2\left(\frac{H - h}{Hh}\right)\right] \qquad \text{(A-6)}$$

Drainability of Shape V

The difference between Shape IV and Shape V is that one has a horizontal base, while the other has a sloping base, with an angle α from the horizon. Otherwise, both drain gradually downward from the top of the impervious side boundary.

As Figure A-4 shows, the flow through volume ACD has a gradient $\frac{h + L\tan\alpha}{L}$ and an average area per unit width as the ratio of triangle ACD and parallelogram AGCD, expressed in terms of h. The starting equations are:

$$dq = n_e \frac{Ldh}{2} \qquad \text{(A-7a)}$$

$$\frac{dq}{dt} = -k \frac{h + L\tan\alpha}{L} \frac{h}{2} \qquad \text{(A-7b)}$$

With boundary condition $h = H - L\tan\alpha$ when $t = 0$, the final solution of Eq. (A-7a) and Eq. (A-7b) is:

$$t = \frac{n_e}{k} \left[\frac{L}{\tan\alpha} \ln \frac{(H - L\tan\alpha)(h + L\tan\alpha)}{Hh} \right] \qquad \text{(A-8)}$$

Drainability for Shape VI

The impervious side boundary of Shape VI is a slope with an angle α. As shown in Figure A-5, phreatic surface changes gradually from AD to AE. Flow through volume AFB had a gradient $\frac{(L - x)\tan\alpha}{x}$ and an average area per unit width as the ratio of triangle AFB and rectangle AFBG as expressed in terms of height $(L - x)\tan\alpha$. The starting equations are:

$$dq = n_e \frac{L\tan\alpha \; dx}{2} \qquad \text{(A-9a)}$$

$$\frac{dq}{dt} = k \left[\frac{(L - x)\tan\alpha}{x} \right] \left[\frac{1}{2} (L - x)\tan\alpha \right] \qquad \text{(A-9b)}$$

Boundary condition for this set of equations is $x = L - \frac{H}{\tan\alpha}$ when $t = 0$, and the solution to them is:

$$t = \frac{n_e}{k} \left\{ \frac{L}{\tan \alpha} \left[\frac{L(H - L\tan \alpha + x\tan \alpha)}{(L - x)H} + \ln \frac{(L - x)\tan \alpha}{H} \right] \right\} \tag{A-10}$$

Drainability for Shape VII

Drainability is desired to change the phreatic surface gradually from AD to AE in Figure A-6, Shape VII has a slope base with an angle α . In the time element dt, the quantity dq discharged per unit width is the product of the triangular area ABC and the effective porosity n_e.

$$dq = n_e \frac{\frac{dh}{\sin \gamma} (H + \ell \tan \gamma) \cos \gamma}{2} \tag{A-11a}$$

The flow through volume AFC, computed by Darcy's Law, has $\dfrac{h}{L - \dfrac{h - h_1}{\tan \gamma}}$ as the gradient and the average area per unit width is the ratio of area AFC to area AFCG expressed in terms of height CF.

$$\frac{dq}{dt} = -k \frac{h}{L - \dfrac{h - h_1}{\tan \gamma}} \left[\frac{\text{(area AFC)}}{\text{(area AFCG)}} \quad \text{(CF)} \right]$$

$$= -k \frac{h}{L - \dfrac{h - h_1}{\tan \gamma}} \left(\frac{1}{2}\right) \left[h - (L - \frac{h - h_1}{\tan \gamma}) \tan \alpha \right]$$

$$= k \frac{h(L\tan \gamma + h_1)\tan \alpha - h^2 (\tan \alpha + \tan \gamma)}{2(L\tan \gamma - h + h_1)} \tag{A-11b}$$

Eq. (A-11a) and Eq. (A-11b), together with the boundary condition $h = H$ when $t = 0$, can be solved as:

Figure A-6. Shape VII

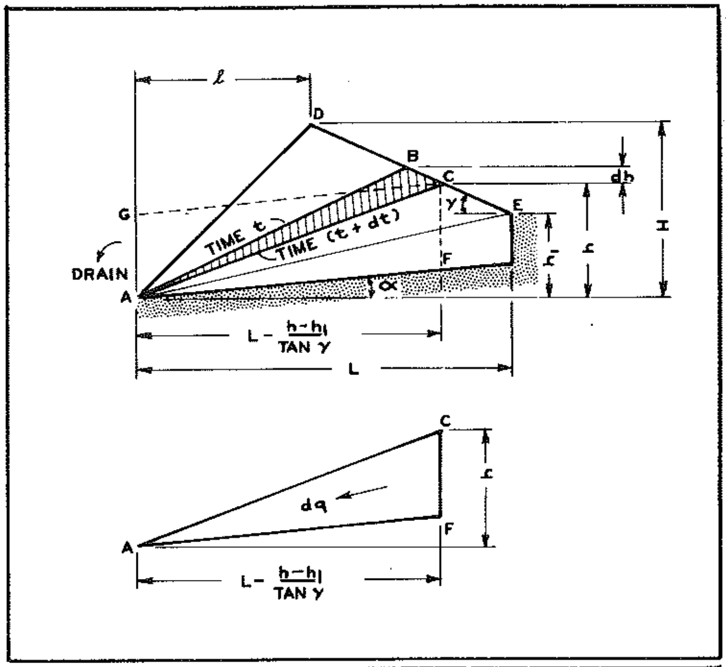
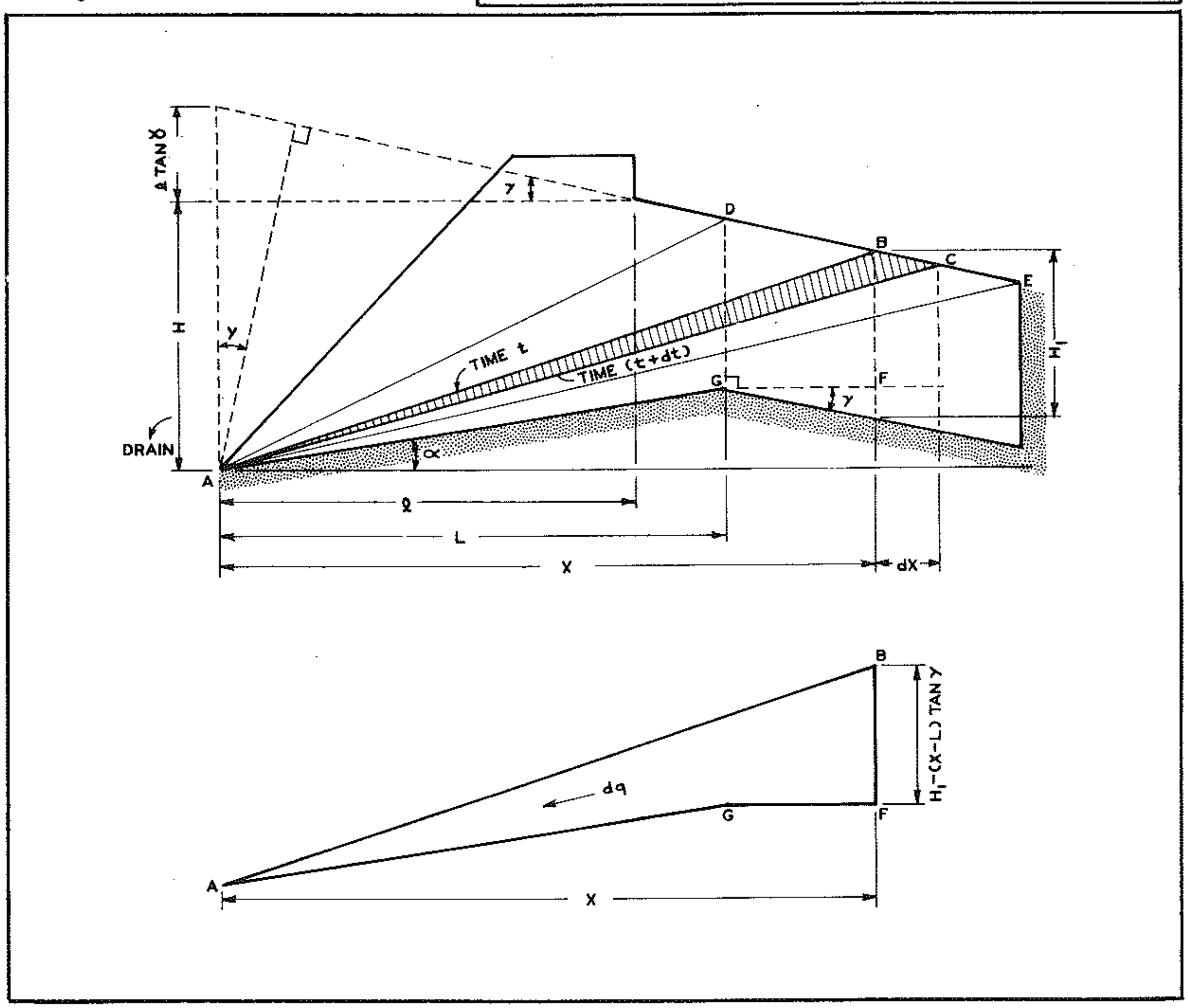


Figure A-7. Shape VIII



$$t = \frac{n_e}{k} \left[\frac{H + \ell \sin \gamma}{\tan \gamma \quad \tan \alpha} \left(\ln \frac{h}{H}\right) \right.$$

$$- \frac{H + \ell \tan \gamma}{\tan \alpha \; (\tan \gamma + \tan \alpha)}$$

$$\left. \ln \frac{h (\tan \alpha + \tan \gamma) - (L \tan \gamma + h_1) \tan \alpha}{H (\tan \alpha + \tan \gamma) - (L \tan \gamma + h_1) \tan \alpha} \right] \qquad \text{(A-12)}$$

Drainability for Shape VIII

Shape VIII, as shown in Figure A-7, has a phreatic surface that changes gradually from AD to AE. In the time element dt, the quantity dq discharged per unit width is:

$$dq = n_e \cdot \Delta ABC = n_e \cdot \frac{1}{2} \frac{dx}{\cos \gamma} (H + \ell \tan \gamma) \cos \gamma$$

$$= \frac{n_e (H + \ell \tan \gamma)}{2} dx \qquad \text{(A-13a)}$$

The flow through volume ABFG (water below GF cannot drain toward A), computed by Darcy's Law, has a gradient $\frac{H_1 - (x - L) \tan \gamma + L \tan \alpha}{x}$.

Since both α and γ are small ($\tan \gamma = 0.035$ max, and $\tan \alpha = 0.02$ max in all cases) average area per unit width can be estimated as $\frac{BF}{2}$ or $\frac{H_1 (x - L) \tan \gamma}{2}$.

$$\frac{dq}{dt} = k \frac{H_1 - (x - L) \tan \gamma + L \tan \alpha}{x} \; \frac{H_1 - (x - L) \tan \gamma}{2} \qquad \text{(A-13b)}$$

With the boundary condition x = L when t = 0, Eq. (A-13a) and Eq. (A-13b) are solved as:

$$t = \frac{n_e}{k} \frac{H + \ell \tan \gamma}{L \tan^2 \gamma \; \tan \alpha} \left\{ \left[H_1 + L (\tan \gamma + \tan \alpha) \right] \right.$$

$$\ln \frac{H_1 + L (\tan \alpha + \tan \gamma) - x \tan \gamma}{H_1 + L \tan \alpha}$$

$$\left. - (H_1 + \tan \gamma) \ln \frac{H_1 + L \tan \gamma - x \tan \gamma}{H_1} \right\} \qquad \text{(A-14)}$$

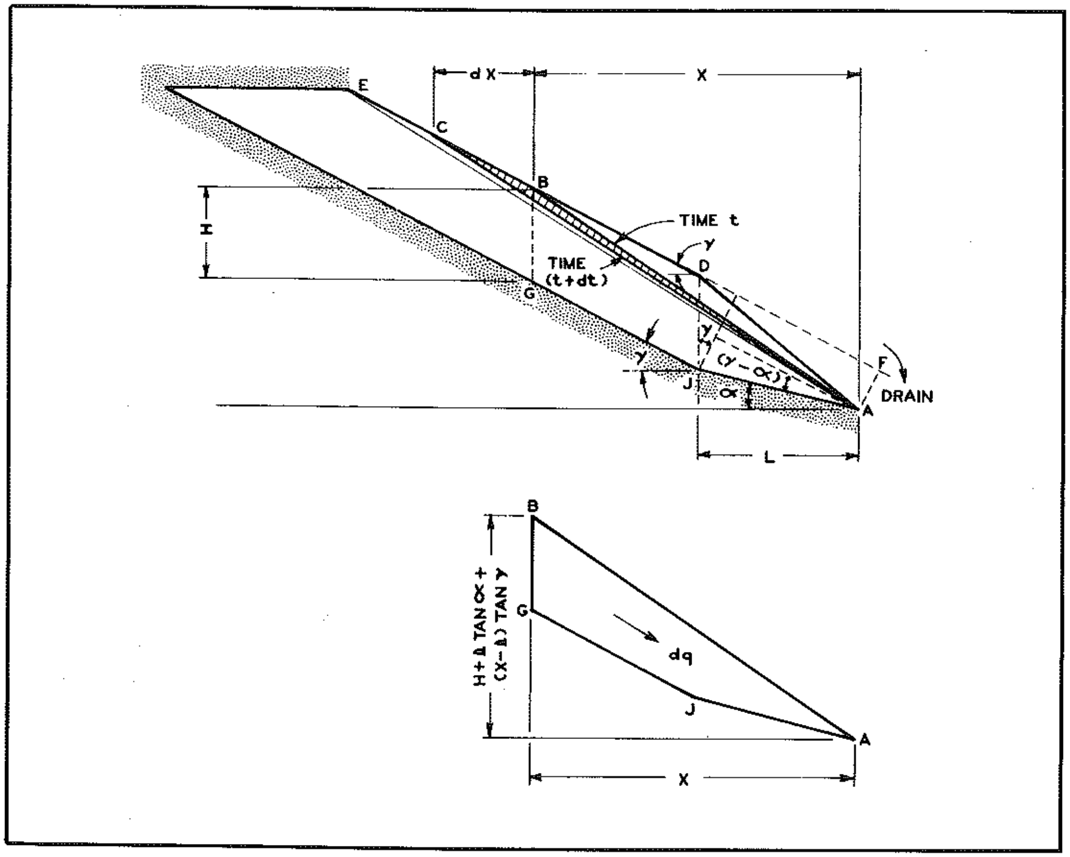


Figure A-8. Shape IX

Drainability for Shape IX

Phreatic surface in Shape IX, shown in Figure A-8, changes gradually from AD to AE. In the time element dt, the quantity dq discharged per unit width is:

$$dq = n_e \cdot \Delta ABC = n_e \left(\frac{1}{2} BC \cdot AF\right)$$

$$= n_e \left(\frac{1}{2}\right) \left(\frac{dx}{\cos \gamma}\right) \left[H \cos \gamma - \frac{L}{\cos \gamma} \sin(\gamma - \alpha) \right] \quad \text{(A-15a)}$$

The flow through volume ABGJ, computed by Darcy's Law, had a gradient $\frac{H + L \tan \alpha + (x - L) \tan \gamma}{x}$. Since both α and γ are small ($\tan \alpha = 0.02$

max, $\tan \gamma = 0.07$ max in all cases), average area per unit width to assume this gradient can be estimated as $\frac{H}{2}$.

$$\frac{dq}{dt} = k \frac{H + L \tan \alpha + (x - L) \tan \gamma}{x} \frac{H}{2} \tag{A-15b}$$

With the boundary condition $x = L$ when $t = 0$, Eq. (A-15a) and Eq. (A-15b) can be solved as:

$$t = \frac{n_e}{k} \left\{ \frac{H \cos^2 \gamma - L \sin(\gamma - \alpha)}{H \cos^2 \gamma \tan \gamma} \left[x - L - \frac{H + L(\tan \alpha - \tan \gamma)}{\tan \gamma} \ln \frac{H + L(\tan \alpha - \tan \gamma) + x \tan \gamma}{H + L \tan \alpha} \right] \right\} \tag{A-16}$$

APPENDIX B

AN ILLUSTRATIVE EXAMPLE TO CALCULATE THE DRAINAGE REQUIREMENT

Figure B-1a represents the subbase of a typical cross-section of a 24-ft concrete pavement roadway with a superelevation of 0.035 ft per lineal foot. The particular point in question is point P. Phreatic surface at the left portion of the section changes gradually from AB to AC to AD to AE, and then to AF. In the same manner, the phreatic surface at the right portion changes gradually from A'B' to A'E'. The two phreatic surfaces finally meet at point P. In other words, time required for drainage from AB to AP is the same as that from A'B' to A'P. According to the superposition principle:

$$t_{(AB - AP)} = t_{(AB - AC)} + t_{(AC - AD)} + t_{(AD - AE)} + t_{(AE - AF)} + t_{(AF - AP)} \qquad \text{(B-1a)}$$

$$t_{(A'B' - A'P)} = t_{(A'B' - A'C')} + t_{(A'C' - A'D')} + t_{(A'D' - A'E')} + t_{(A'E' - A'P)} \qquad \text{(B-1b)}$$

$t_{(AB - AC)}$ is the time required to relocate the phreatic surface from AB to AC (or to drain the triangle ABC). Other elements in these two equations are designated in a similar manner. Figures B-1b to B-1j represent every individual basic geometric shape, along with Figure B-1a, associated with these two equations. Also, for both sides to drain to P at the same time, the following relationship has to exist:

$$t_{(AB - AP)} = t_{(A'B' - A'P)} \qquad \text{(B-2)}$$

The example was worked out numerically here for demonstration purposes. Since the exact location of P is unknown, it is assumed to be x distance from point A, and x' from A'. They are then solved by trial and error. All pertinent data are read directly from "Typical Cross-Sections" published by the Department.

for $t_{(AB - AC)}$, Shape V (Fig. B-1b):

From plan data: L = 14.5 ft, $\tan\alpha = 0.018$, H = 2.42 ft, h = 1.52 ft. Using Eq. (A-8):

$$t_{(AB - AC)} = \frac{n_e}{k}\left[\frac{L}{\tan\alpha} \ln \frac{(H - L\tan\alpha)(h + L\tan\alpha)}{Hh}\right]$$

$$= 36\,\frac{n_e}{k}$$

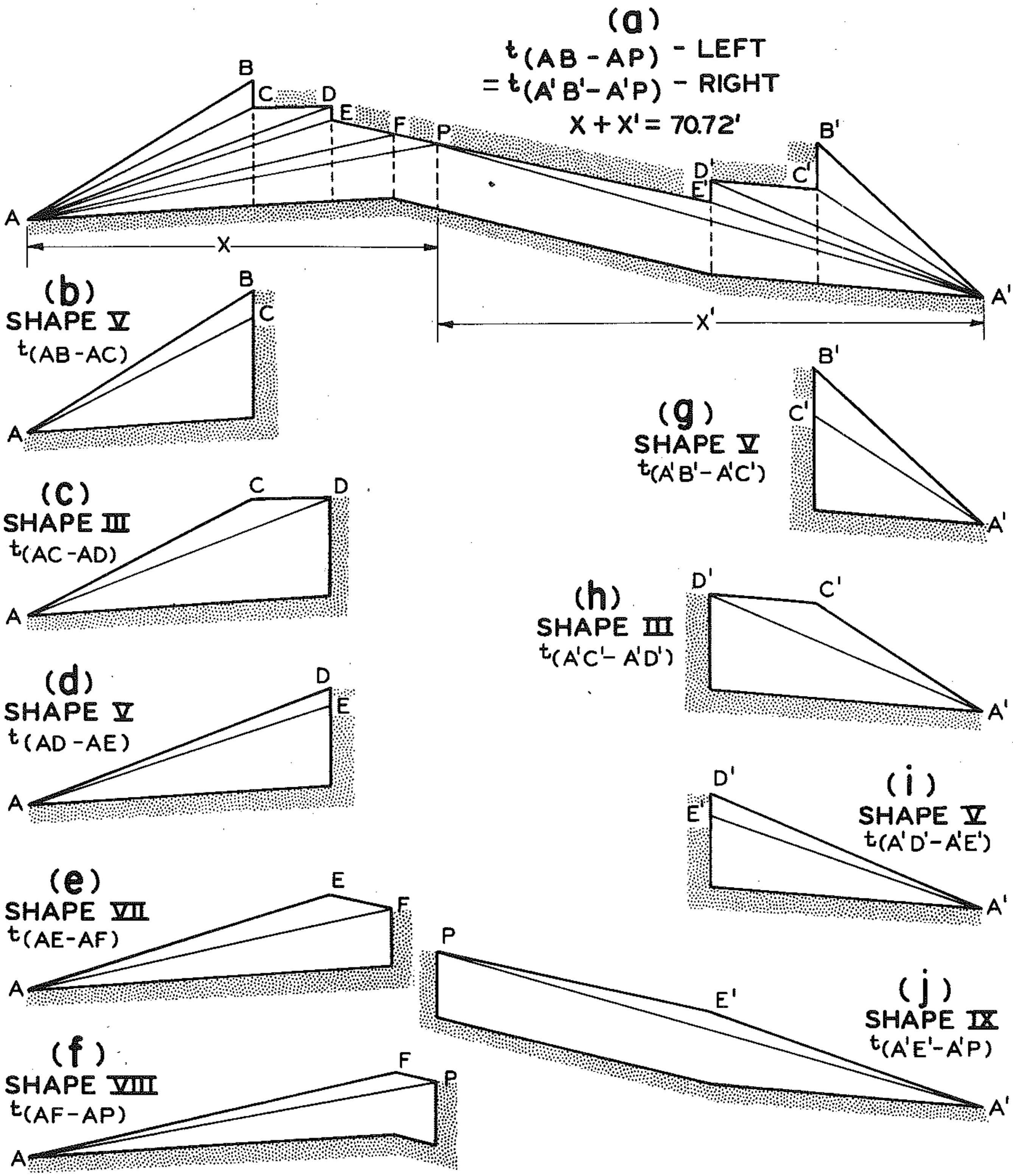


Figure B-1. Subbase for a two-way concrete pavement.

for $t_{(AC - AD)}$, Shape III (Fig. B-1c):

From plan data: $x = L = 2.51$ ft, $K = 14.5$ ft, $H = 1.52$ ft, $\tan \alpha = 0.018$. Using Eq. (A-4):

$$t_{(AC - AD)} = \frac{n_e}{k} \left(\frac{L - K}{\tan \alpha} + \frac{H}{\tan^2 \alpha} \ln \frac{H + K \tan \alpha}{H + L \tan \alpha} \right)$$

$$= 70 \frac{n_e}{k}$$

for $t_{(AD - AE)}$, Shape V (Fig. B-1d):

From plan data: $L = 21.5$ ft, $\tan \alpha = 0.018$, $H = 1.9$ ft, $h = 1.15$ ft. Using Eq. (A-8):

$$t_{(AD - AE)} = \frac{n_e}{k} \left[\frac{L}{\tan \alpha} \ln \frac{(H - L \tan \alpha)(h + L \tan \alpha)}{Hh} \right]$$

$$= 75 \frac{n_e}{k}$$

for $t_{(AE - AF)}$, Shape VII (Fig. B-1e):

From plan data: $L = 27.5$ ft, $\ell = 21.5$ ft, $H = 1.5$ ft, $h = h_1 = 1.32$ ft, $\tan \alpha = 0.018$, $\tan \gamma = \sin \gamma = 0.035$. Using Eq. (A-12):

$$t_{(AE - AF)} = \frac{n_e}{k} \left[\frac{H + \ell \sin \gamma}{\tan \gamma \quad \tan \alpha} \ln \frac{h}{H} - \frac{H + \ell \tan \gamma}{\tan \alpha \; (\tan \gamma \quad \tan \alpha)} \right.$$

$$\left. \ln \frac{h(\tan \alpha + \tan \gamma) - (L \tan \gamma + h1) \tan \alpha}{H(\tan \alpha + \tan \gamma) - (L \tan \gamma + h_1) \tan \alpha} \right]$$

$$= 209 \frac{n_e}{k}$$

for $t_{(AF - AP)}$, Shape VIII (Fig. B-1f):

From plan data: $L = 27.5$ ft, $\ell = 21.5$ ft, $H = 1.5$ ft, $H_1 = 0.83$ ft, $\tan \alpha = 0.018$, $\tan \gamma = 0.035$. Using Eq. (A-14):

$$t_{(AF - AP)} = \frac{n_e}{k} \frac{H + \ell \tan \gamma}{L \tan^2 \gamma \quad \tan \alpha} \left\{ \left[H_1 + L(\tan \gamma \quad + \tan \alpha) \right] \right.$$

$$\ln \frac{H_1 + L(\tan\alpha + \tan\gamma) - x\tan\gamma}{H_1 + L\tan\alpha}$$

$$\left. - (H_1 + L\tan\gamma) \ln \frac{H_1 + L\tan\gamma - x\tan\gamma}{H_1} \right\}$$

$$= \left[8496 \ln (1.7264 - 0.0264x) - 6657 \ln (2.1596 - 0.0422x) \right] \frac{n_e}{k}$$

for $t_{(A'B' - A'C')}$, Shape V (Fig. B-1g):

From plan data: L = 11.7 ft, H = 1.95 ft, H = 1.1 ft, $\tan\alpha = 0.02$. Using Eq. (A-8):

$$t_{(A'B' - A'C')} = \frac{n_e}{k} \left[\frac{L}{\tan\alpha} \ln \frac{(H - L\tan\alpha)(h + L\tan\alpha)}{Hh} \right]$$

$$= 38 \frac{n_e}{k}$$

for $t_{(A'C' - A'D')}$, Shape III (Fig. B-1h):

From plan data: x = L = 18.7 ft, K = 11.7 ft, H = 1.1 ft, $\tan\alpha = 0.02$. Using Eq. (A-4):

$$t_{(A'C' - A'D')} = \frac{n_e}{k} \left[\frac{L - K}{\tan\alpha} + \frac{H}{\tan^2\alpha} \ln \frac{H + K\tan\alpha}{H + L\tan\alpha} \right]$$

$$= 75 \frac{n_e}{k}$$

for $t_{(A'D' - A'E')}$, Shape V (Fig. B-1i):

From plan data: L = 18.7 ft, H = 1.5 ft, h = 0.75 ft, $\tan\alpha = 0.02$. Using Eq. (A-8):

$$t_{(A'D' - A'E')} = \frac{n_e}{k} \left[\frac{L}{\tan\alpha} \ln \frac{(H - L\tan\alpha)(h + L\tan\alpha)}{Hh} \right]$$

$$= 110 \frac{n_e}{k}$$

for $t_{(A'E' - A'P)}$, Shape IX (Fig. B-1j):

From plan data: L = 21.5 ft, H = 0.83 ft, tan α = 0.02, tan γ = 0.035, sin (γ - α) = 0.015, cos γ = 1. It is also known that x + x' = 70.22 ft, thus x' = 70.22 ft - x. Using Eq. (A-16):

$$t_{(A'E' - A'P)} = \frac{n_e}{k} \left\{ \frac{H\cos^2\gamma - L\sin(\gamma - \alpha)}{H\cos^2\gamma \tan\gamma} \left[x' - L - \frac{H + L(\tan\alpha - \tan\gamma)}{\tan\gamma} \ln \frac{H + L(\tan\alpha - \tan\gamma) + x'\tan\gamma}{H + L\tan\alpha} \right] \right\}$$

$$= \left[851 - 17.47x - 253.3 \ln(2.3533 - 0.0278x) \right] \frac{n_e}{k}$$

All the values obtained are now to be substituted into Eq. (B-1a) and Eq. (B-1b).

$$t_{(AB - AP)} = \left[36 + 70 + 75 + 209 + 8496 \ln(1.7264 - 0.0264x) - 6657 \ln(2.1596 - 0.0422x) \right] \frac{n_e}{k}$$

$$= \left[390 + 8496 \ln(1.7264 - 0.0264x) - 6657 \ln(2.1596 - 0.0422x) \right] \frac{n_e}{k}$$

$$t_{(A'B' - A'P)} = \left[38 + 75 + 110 + 851 - 17.47x - 253.3 \ln(2.3533 - 0.0278x) \right] \frac{n_e}{k}$$

$$= \left[1074 - 17.47x - 253.3 \ln(2.3533 - 0.0278x) \right] \frac{n_e}{k}$$

Eq. (B-2) is used for the final solution. Owing to the complicated nature of the equation, a trial and error method is used by assuming a reasonable value of x and substituting this value into the left hand side (LHS) and right hand side (RHS) of Eq. (B-2).

First trial: assume x = 31 ft

$$LHS = t_{(AB - AP)} = \left\{ 390 + 8496 \ln \left[1.7264 - (0.0264)(31) \right] - 6657 \ln \left[2.1596 - (0.0422)(31) \right] \right\} \frac{n_e}{k}$$

$$= 633 \frac{n_e}{k}$$

$$RHS = t_{(A'B' - A'P)} = \left\{ 1074 - (17.47)(31) - 253.3 \ln -253.3 \ln \left[2.3533 - (0.0278)(31) \right] \right\} \frac{n_e}{k}$$

$$= 430 \frac{n_e}{k}$$

Second trial: assume x = 29 ft

$$LHS = t_{(AB - AP)} = 496 \frac{n_e}{k}$$

$$RHS = t_{(A'B' - A'P)} = 456 \frac{n_e}{k}$$

Third trial: assume x = 28.5 ft

$$LHS = t_{(AB - AP)} = 462 \frac{n_e}{k}$$

$$RHS = t_{(A'B' - A'P)} = 462 \frac{n_e}{k} = LHS$$

Therefore, P is at a horizontal distance of 28.5 ft from A, and the time required for both sides to drain P is 462 $\frac{n_e}{k}$.

The degree of drainage for this example, calculated as the ratio of the area above APA' to the total area, is 53.6 percent. Since the criterion was established as 10 days for U = 50 percent, drainability calculated other than to this degree of drainage has to be converted to the U = 50 percent condition. The simplest assumption that can be made is to use a linear relationship between the degree of drainage and drainability.

$$\frac{U\ 50\%}{U\ x\%} = \frac{t_{50}}{t_x} \tag{B-3}$$

Therefore, the corresponding time for U = 50 percent is calculated as:

$$t_{50} = \frac{U\ 50\%}{U\ x\%} \quad t_x = \left(\frac{50}{53.6}\right) \left(462\ \frac{n_e}{k}\right) = 430\ \frac{n_e}{k}$$

To meet the criterion of 10 days for 50 percent drainage, the material characteristic factor, $\frac{k}{n_e}$ should have a minimum value of

$$\frac{k}{n_e} = \frac{430}{t_{50}}$$

$$= \frac{430}{10}$$

$$= 43$$

It should be noted that conformity of units is imperative. In this example, the unit of permeability is to be ft/day. As long as the effective porosity is dimensionless, the material characteristic factor, $\frac{k}{n_e}$, assumes the same unit as ft/day.