AGGREGATE GRADATION QUALITY CONTROL

MICHIGAN

RESEARCH LABORATORY SECTION

MICHIGAN DEPARTMENT OF
STATE HIGHWAYS AND TRANSPORTATION

AGGREGATE GRADATION QUALITY CONTROL

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ABSTRACT

The techniques of multivariate statistical analysis are used to study the segregation pattern of a flatted stockpile. This study leads to the design of a stratified random sampling method for sampling stockpiled aggregates. The stratified random sampling method is then compared statistically with the current stockpile sampling method in estimating aggregate composition for the final selection of stockpile sampling methods. Based on the final suggested sampling method, we designed several potential variables plans for stockpile aggregate inspection.

Since stockpiled aggregate is not the end-product of aggregate used for highway construction, the feasibility of stockpiled aggregate inspection is carefully investigated. The major interest in such an investigation is the effect of aggregate handling and compaction on aggregate composition. Models for estimating the degradation rate of each aggregate size due to handling and compaction, and for estimating the composition of in-place aggregate after compaction from that before compaction or stockpile aggregate composition, are presented. Moreover, several plans of so-called "acceptance sampling by attributes" to control the fraction of failing spots at the construction site are presented and recommended for in-place aggregate inspection.

It is apparent that aggregate inspection can be performed at any of the following three locations: the stockpile, and the construction site before and after compaction. The question is, "Which is the most beneficial location for the Department to perform aggregate inspection?" To answer this question, factors which should be considered for choosing the inspection locations situated between the stockpile and the construction site are discussed.

Because the current aggregate gradation testing method is time consuming, and therefore costly, a more efficient aggregate gradation testing method is proposed. Samples are collected and tested by the current and proposed test methods in a statistical experiment. The data are then analyzed by the techniques of multivariate statistical analysis to see whether the two test methods give comparable results in measuring aggregate composition. An estimation procedure for converting the test results of the proposed test method to those obtainable with the current test method is also proposed. The estimation precision of the estimation procedure is statistically evaluated. Based on the results of this experiment, we recommend that the Department conduct a large-scale similar experiment to obtain much wider information before introducing the proposed test method into field operation.

Computational formulas for some multivariate statistical experiments are summarized in an Appendix to this report.

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INTRODUCTION

The work described in this report is the result of a research program carried out by the Research Laboratory of the Michigan Department of State Highways and Transportation in cooperation with the Federal Highway Administration. The contents of this report reflect the views of the author who is responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

The current practice of the Michigan Department of State Highways and Transportation is to inspect aggregate at the production site (stockpile). Since stockpiled aggregate will undergo a remixing process when transported to the construction site, it is natural to ask whether the accepted stockpiled aggregate is still acceptable after it is in place.

Aggregate composition changes from the stockpile to the construction site could be due to the handling process (transporting stockpiled aggregate to the construction site) or the compaction process (obtaining the required aggregate density). Presumably, the handling process improves the aggregate uniformity, a desirable feature, but also degrades aggregate to some extent. Moreover, the compaction process definitely degrades aggregate. Thus, the practice of stockpile inspection makes sense only if the desirable characteristics of in-place aggregate are preserved by stockpiled aggregate control. That is, a specification used for stockpile inspection must not be compromised by the degradation factors of the handling and compaction processes. Moreover, can the aggregate uniformity of in-place aggregate be maintained at a satisfactory level through stockpile aggregate control?

The major purpose of this study is to develop statistically sound quality assurance programs for stockpiled and in-place aggregate inspection. In the process of developing quality assurance programs, the remaining objectives originally proposed are also accomplished.

a. To develop practical and meaningful sampling procedures for sieve analysis of 22A aggregate;

b. to determine the extent of aggregate degradation due to handling and compaction; and

c. to estimate variability introduced into screening results by changes in aggregate materials, and sampling and testing procedures.

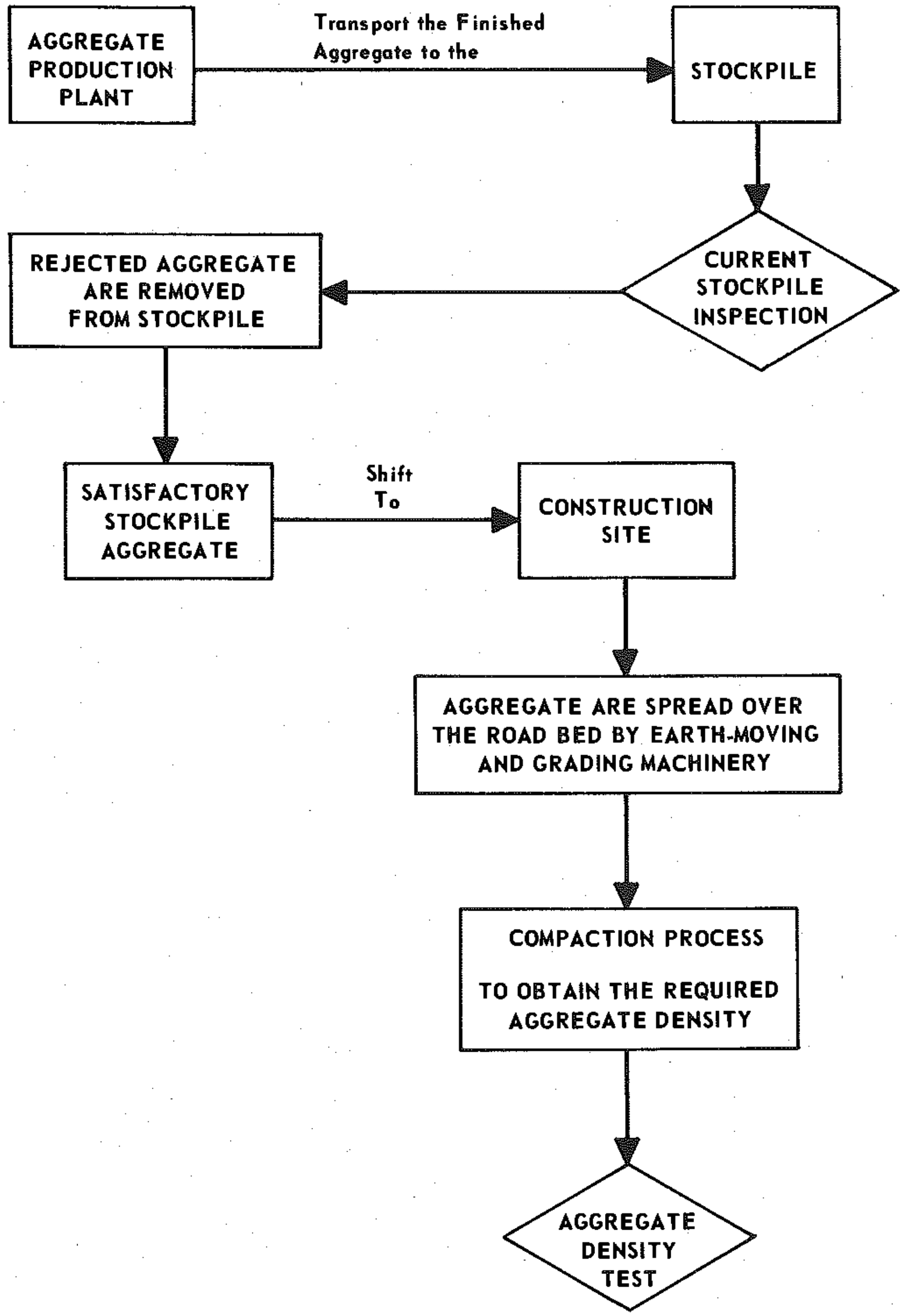

Figure 1. The Aggregate Production Process.

The research procedures required to achieve the above objectives are closely related to the current aggregate production process which is outlined in the flow chart in Figure 1.

It is apparent from the flow chart that there are two other locations, besides the stockpile, for inspecting aggregate. These are at the construction site, before and after compaction. Thus, aggregate quality assurance programs for stockpiled and in-place aggregate inspection are discussed and presented in Chapter I-3 and Chapter I-5, respectively. Factors that should be considered for the choice of inspection locations situated between the stockpile and the construction site are discussed in Chapter I-6.

The first step in the development of aggregate quality assurance programs for stockpile inspection is to design a sampling procedure for collecting representative samples from stockpiles. For this purpose, an investigation of the stockpile segregation pattern was conducted to facilitate the stockpile stratification sampling scheme. These matters are discussed in Chapters I-1 and I-2. In order to discuss the feasibility of stockpiled aggregate inspection, information concerning aggregate degradation and uniformity due to handling and compaction is needed. This is discussed in Chapter I-4.

There were three production pits, Stillman, Pifke, and Anderson, available at the time that this research was undertaken. Since the stockpile formation of the Stillman pit is different from that of the other two pits, we discuss the Stillman pit in Part I and the other two pits in Part II.

The major defect of the current aggregate gradation testing method is that the length of time needed to complete a gradation test is too long. Consequently, it is very difficult for aggregate inspection to keep up with production. This could be a very serious problem whenever there is need to increase testing sample size, e.g., for reducing the risk of accepting poor quality aggregates and rejecting high quality aggregates, and when extra manpower is either costly or unavailable. This brings our attention to the range of testing procedure choices for aggregate. A more efficient gradation testing method is proposed in Part III in which the statistical evaluation of the feasibility of this proposed testing method is included.

For a sample of 22A aggregate, the current gradation test measures the percent passing the 1-in., 3/4-in., 3/8-in., and No. 8 sieves, the percent loss-by-washing, and the percentage of aggregates retained on the 3/8-in. sieve crushed according to the current 22A aggregate specification. It is obvious that these measurements are intercorrelated; therefore, the techniques of multivariate statistical analysis are used to analyze the data obtained from the conducted experiments.

Findings and Recommendations

1) Based on information obtained from statistical experiments described in this report, we propose systematic sampling procedures for sampling aggregate from flatted stockpiles (Chapter I-2).

2) The current sampling method for sampling aggregate from trucks is equally as good as the stratified random sampling procedure, and, therefore, is recommended because of its simplicity (Part II, "Pifke and Anderson pits").

3) The current acceptance sampling plan for stockpiled aggregate inspection is not appropriate in the sense that the aggregate composition of an accepted stockpile might not meet the desired quality level. Moreover, the probability of accepting any layer of aggregate produced under a manufacturing process with aggregate composition set at the upper specification limit is 0.4945. That is, the Department runs a high risk of accepting poor quality aggregate (Chapter I-3).

4) Due to the reason stated in 3, we propose several acceptance sampling plans for stockpiled aggregate inspection, based on the systematic sampling procedures for the flatted stockpile and the current sampling method for trucks (Chapter I-3 and Part II, "Pifke and Anderson Pits").

5) Stockpiled aggregate inspection would not be appropriate if the handling process did not improve the uniformity of in-place aggregate up to the acceptable level. Under this circumstance, the Department would have to practice aggregate inspection at the construction site. Due to this possibility, we designed a "non-composite" sampling method for sampling in-place aggregate. Based on this sampling method, we propose several acceptance sampling plans for in-place aggregate inspection (Chapter I-5).

6) Critical information needed to evaluate the cost-benefit analysis concerning the choice of inspection location is not available at this time. Therefore, we are not in the position of recommending the location at which the aggregate inspection should be performed. However, the general considerations on the choice of inspection locations situated between the stockpile and the construction site are discussed in Chapter I-6.

7) The handling and compaction processes do affect the aggregate composition in the sense of degrading aggregate grain size and improving the aggregate uniformity.

8) We present a statistical model (Chapter I-4) for the purpose of estimating the degradation rate of each aggregate size due to handling and compaction. Based on this model, we conclude that each aggregate size degrades only to the next smaller size. Consequently, the composition of in-place aggregate after compaction can be estimated from that before compaction or from stockpiled aggregate.

9) Based on the model for estimating aggregate degradation rate, we obtain the following estimates:

For the Stillman pit: about 33.6, 25.6, and 20.5 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade.

For the Pifke pit: about 15.0, 8.0, and 6.8 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade.

For the Anderson pit: about 8.9, 4.0, and 5.5 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade.

As one can see from the above estimates, the degradation rate of each aggregate size differs very much from one pit to another. That is, the degradation rate depends on aggregate (source) type.

10) If aggregate inspection is not performed at the construction site after compaction, the test results must be adjusted by the degradation factors to estimate the composition of in-place aggregate after compaction. Since the aggregate degradation rate depends on aggregate type, an acceptance sampling plan for stockpile inspection or in-place inspection before compaction is a function of that type's degradation rate. For this reason, we recommend a further study to determine the degradation rate of each type of aggregate so that specific acceptance sampling plans can be designed.

11) Since the current aggregate testing method is quite time consuming, and, therefore, costly, a more efficient aggregate testing method is

proposed in Part III. The feasibility of the proposed testing method is statistically evaluated. Based on this evaluation, we recommend that the Department conduct a larger scale study to obtain more information about this matter. The research procedures are outlined in Appendix I.

PART I

THE STILLMAN PIT

The Stillman pit supplied the required 22A aggregate for a constructed section of M 57. Samples were taken from the stockpile and the construction site according to statistical experiments designed for those purposes described in the introduction of this report.

Information obtained from the Stillman pit regarding the aggregate quality assurance programs is described as follows: Chapter I-1 describes the stockpile formation and the current stockpile aggregate sampling procedure. Discussion of the current sampling method leads to the investigation of the stockpile aggregate segregation pattern. This investigation facilitates the stockpile stratification scheme, and consequently a stratified random sampling method is designed. In Chapter I-2 the stratified random sampling method is then compared with the current one in estimating the stockpile aggregate composition. The statistical comparison leads to selection of the stockpile sampling method. In Chapter I-3 comment is made on the current stockpile aggregate inspection practice, and the further development of more meaningful and practical stockpile aggregate inspection plans based on the suggested sampling method. Chapter I-4 is devoted to the study of the aggregate degradation due to the handling and compaction processes. The information obtained from this study is valuable for the justification of practicing aggregate inspection at the stockpile or at the construction site before compaction. Following this chapter, we discuss the sampling method and the inspection plans for in-place aggregate. And finally, factors that should be considered in the choice of inspection locations situated between the stockpile and the construction site are discussed in the last chapter.

Figures 2 and 3 are flow charts describing research procedures.

I-1. STOCKPILE FORMATION AND SAMPLING METHODS

At the Stillman pit, a 10-ft wide scraper-hauler (Tournapull) was used to stockpile the finished aggregate. Consequently, a layer about 12 ft wide was made by each pass of the scraper-hauler. This process was continued until a stockpile was formed of dimensions of about either 24 by 150 ft, or 36 by 150 ft as shown in Figure 4.

Since stockpiled aggregates will undergo a remixing process when transported to the construction site, our major concern is to obtain a "good" estimate of stockpile aggregate composition. This purpose can be achieved if every sample taken from a layer is a representative sample of that layer. This suggests that many scoops of aggregate should be taken from various locations of a layer to form a composite sample. On the other hand, the

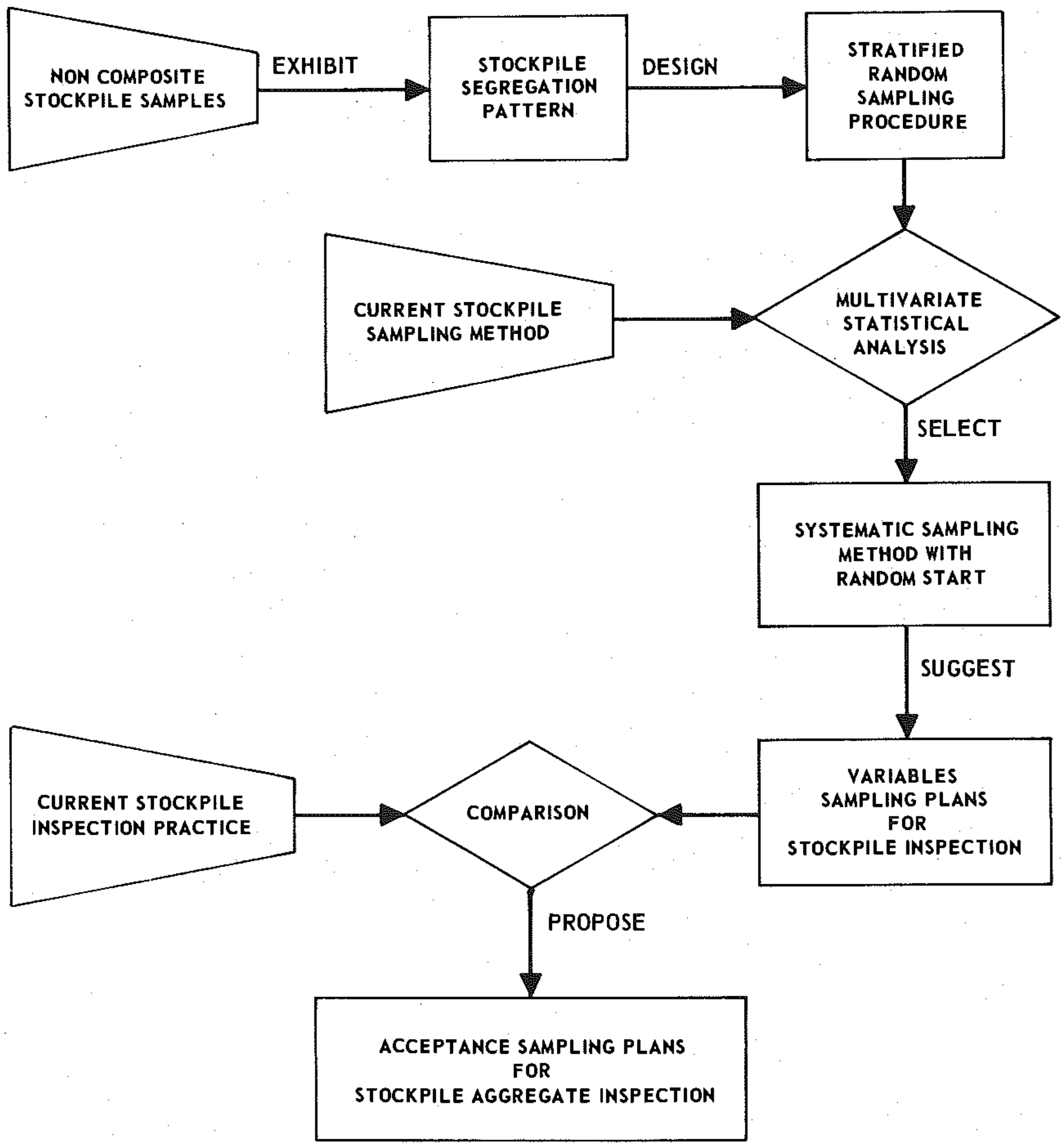

Figure 2. Research procedures for designing stockpile aggregate inspection practice.

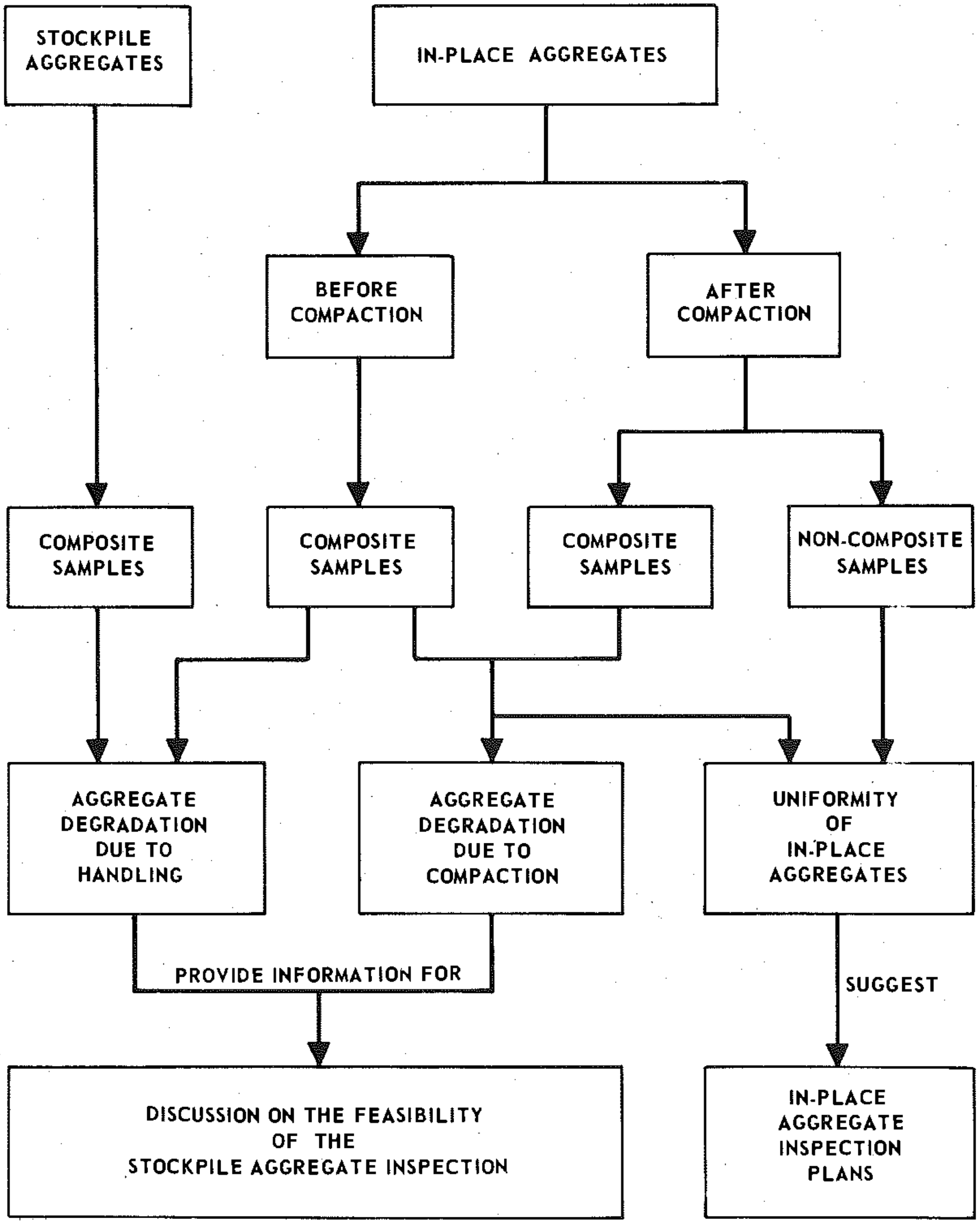

Figure 3. Research procedures for designing in-place aggregate inspection practice.

number of scoops should be kept as low as possible for practical reasons. The question is how many scoops are sufficient to form a representative composite sample? Moreover, at what locations these numerous scoops should be taken? These subjects are discussed in the following sections.

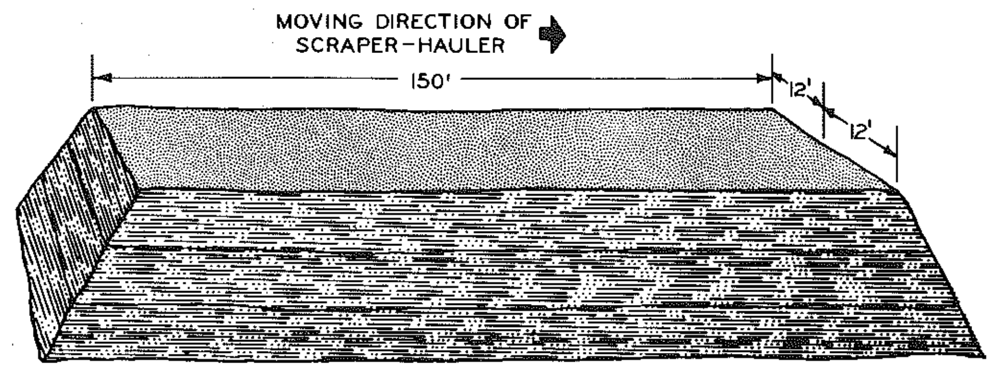

Figure 4. Stockpile formation.

Current Sampling Method (CSM) for Stockpiled Aggregate

Much coarse aggregate was visible on the surface of the outside edges of stockpiles; therefore, it was claimed by the most experienced aggregate inspectors that aggregate is coarser toward these outside edges. This assertion needs to be carefully investigated, and it will be discussed in the next section. However, this observation could have been the major reason behind Michigan's adoption of the current sampling method (CSM) which is shown in Figure 5.

The CSM specifies that each testing sample be composed of five or six scoops of aggregate which are diagonally taken from a layer of the stockpile. If we longitudinally and transversely stratify each layer and number each stratum as shown in Figure 6, the CSM can be interpreted as a so-called "systematic sampling method with fixed beginning," i.e., each sample is composed of five scoops (ignoring the optional scoop) taken at stratum No. 1 from each group which is composed of six strata numbered as 1, 2, . . . 6.

We note that the above sampling method might give a good estimate of the aggregate composition of a layer since each sample is basically composed of aggregate taken from the outside edges, the middle, the front end, and the rear end of a layer. But, if aggregate in each transverse stratum exhibits large variations or special segregation patterns, this sampling method will give a biased estimate of the aggregate composition since not every location in each transverse stratum has an equal chance of being sampled. Because of this, we introduce the following stratified random sampling method.

Stratified Random Sampling Method (SRSM) for Stockpiled Aggregate

As mentioned before, it was conjectured that aggregate is coarser toward the outside edges of stockpiles. If this is so, it becomes apparent that each layer should be stratified transversely as shown in Figure 7. That is, each sample should be composed of many scoops of aggregate taken randomly from each transverse stratum. To verify this, we conducted an experiment as follows:

Twenty layers (lots) were randomly chosen, each representing an area 12 ft by 150 ft by 6 in. in depth. Each layer was transversely stratified into five strata as shown in Figure 7. From each stratum, one location (spot) was randomly chosen at which a sample of about 30 lb of aggregate was taken. This sample is called a non-composite sample. These five non-composite samples taken from the same layer were then randomly ordered and tested in the field by an aggregate inspector who used one set of standard sieves to measure the percent passing the 1-in., 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing. The test results are presented in Table 1, Appendix B. The statistical analyses of the test results are also presented in Appendix B for readers who are interested in the statistical details. Here, we simply conclude, based on this experiment, that despite inspector observations, aggregate at the outside edge is not significantly (at the 0.05 level) coarser than that in the middle of the same layer. When aggregate is pushed over the pile's edges, coarse aggregate naturally emerges on the surface. Therefore, there appears to be surface and deep segregation phenomena at the outside edges. If, however, a sample from an outside edge is composed of aggregate taken from the full depth of a layer, the composition of this sample should not differ much from that of the sample taken from the middle of the same layer. In this circumstance, it is not really necessary to stratify each layer as shown in Figure 7.

Estimates of the aggregate composition and its covariance matrix are presented in Tables 1 and 2 for later reference.

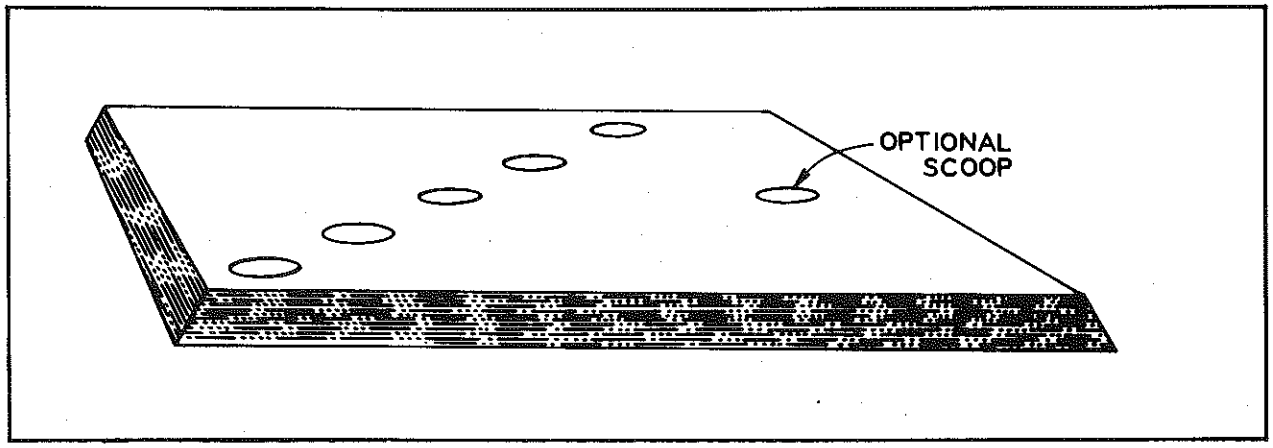

Figure 5. The layout of the current sampling method (CSM).

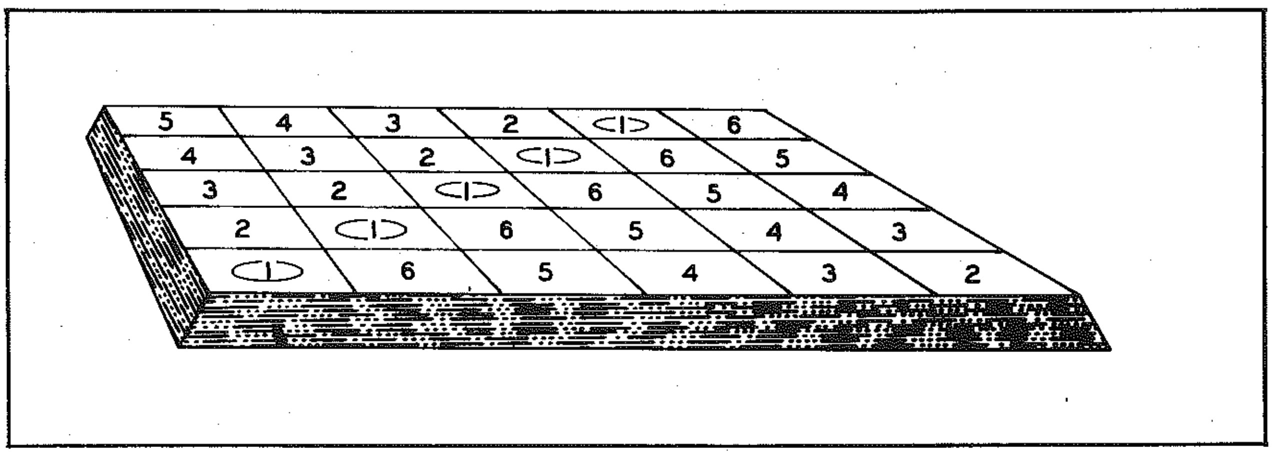

Figure 6. The layout of the current sampling method as a systematic sampling method.

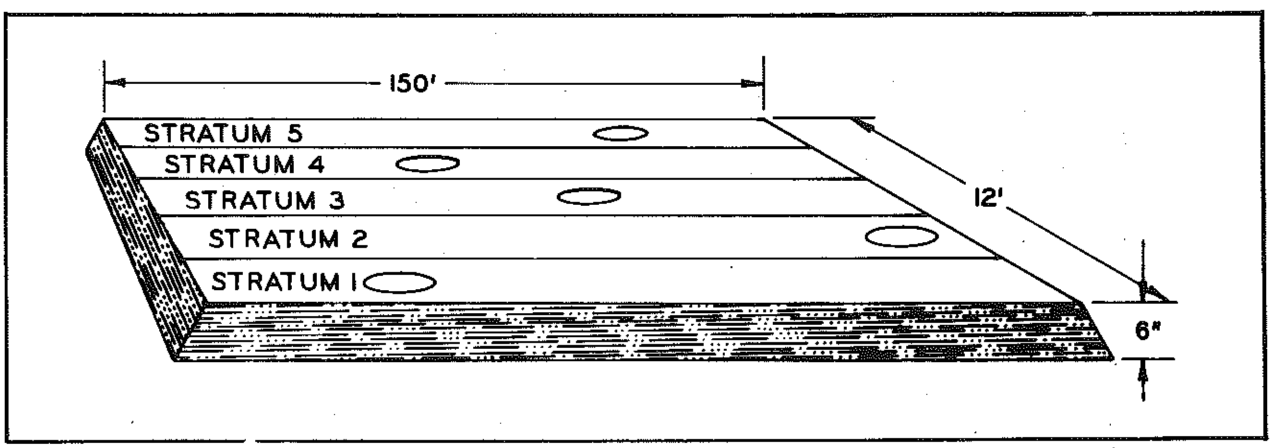

Figure 7. Transverse stratification of a layer.

TABLE 1
ESTIMATES OF AGGREGATE COMPOSITION OF EACH STRATUM BASED ON "NON-COMPOSITE" SAMPLES

Stratum No.	Percent Passing Sieves				Percent Loss-By-Washing (L.B.W.)
	1-in.	3/4-in.	3/8-in.	No. 8	
1	100	94.79	71.59	44.55	5.04
2	100	96.31	77.11	50.11	4.98
3	100	95.10	76.52	49.60	5.15
4	100	96.54	78.04	50.39	5.01
5	100	95.61	74.34	46.41	4.90

TABLE 2
ESTIMATES OF THE COVARIANCE MATRIX OF THE AGGREGATE COMPOSITION OF EACH STRATUM BASED ON "NON-COMPOSITE" SAMPLES

	Size Combination	Stratum No.				
		1	2	3	4	5
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	20.63	3.82	7.22	3.41	5.21
	3/4-in. and 3/8-in.	45.36	4.57	12.80	7.49	13.59
	3/4-in. and No. 8	36.87	4.09	7.67	6.51	9.31
	3/4-in. and L.B.W.	1.48	0.42	-0.15	-0.57	0.93
	3/8-in. and 3/8-in.	142.11	18.72	32.95	32.41	63.40
	3/8-in. and No. 8	116.52	11.92	24.19	31.48	54.59
	3/8-in. and L.B.W.	4.03	-0.93	-0.30	-1.08	3.82
	No. 8 and No. 8	100.70	12.62	26.94	35.62	61.25
	No. 8 and L.B.W.	3.30	0.24	1.53	-0.13	5.86
	L.B.W. and L.B.W.	1.06	0.84	1.46	1.64	2.25

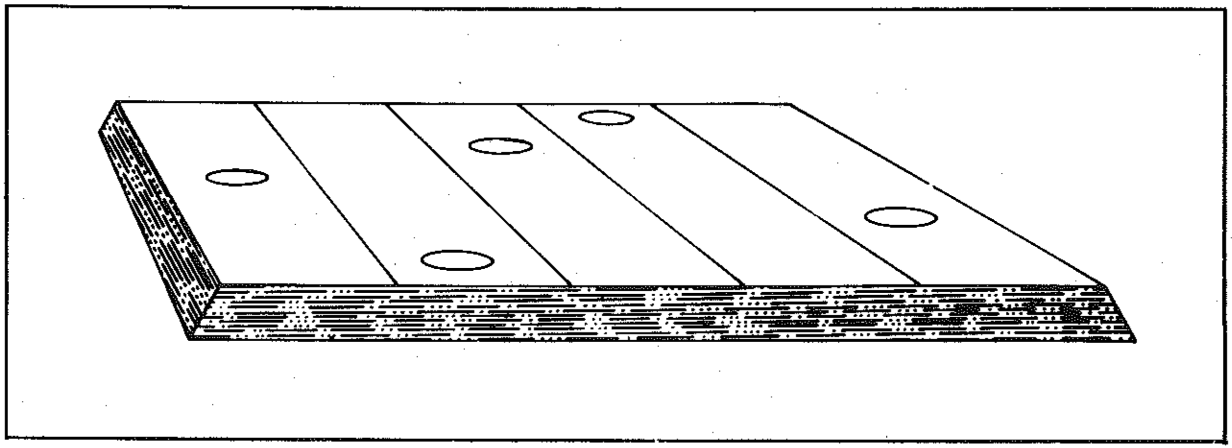

Figure 8. Longitudinal stratification of a layer.

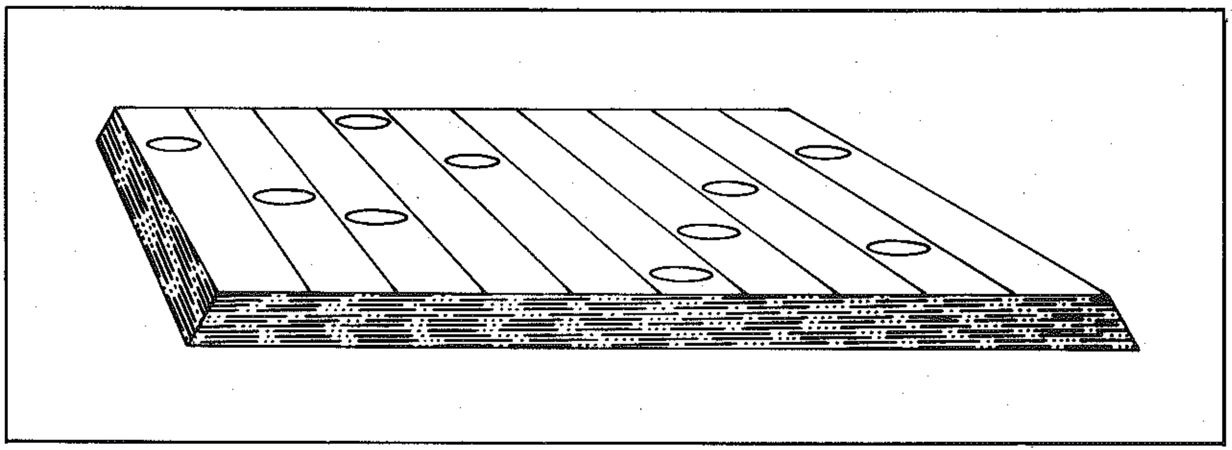

Figure 9. Sampling pattern for a 12 by 150-ft stockpile.

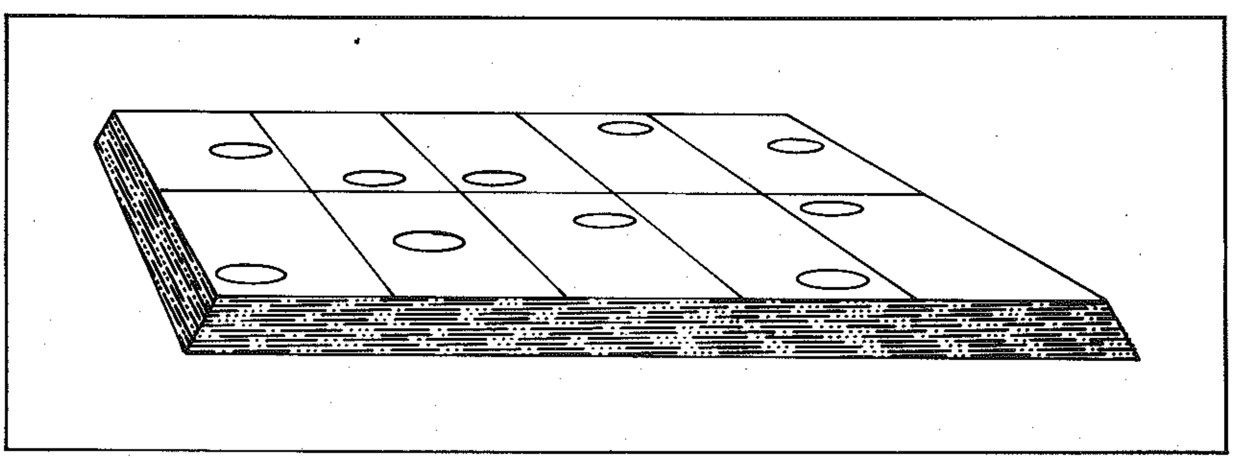

Figure 10. Sampling pattern for a 24 by 150-ft stockpile.

If we consider coarse aggregate as the material which does not pass through the No. 8 sieve, we see from Table 1 that aggregate in Strata 1 and 5 (outside edges) is slightly, but not significantly, coarser than that from the middle three strata. We also see from Table 2 that aggregate composition in each stratum varies greatly, particularly in the outside strata (1 and 5). This indicates that there is a need to longitudinally stratify each layer of a stockpile. That is, each layer of a stockpile should be stratified in the way shown in Figure 8. Thus, each sample is composed of scoops of aggregate taken randomly from each longitudinal stratum. The above discussion indicates the stratification pattern of a layer. The question arises as to how many strata (scoops) should be taken to make a representative composite sample? The answer to the question depends on the marginal estimation errors that can be afforded. This is analyzed in Appendix C. Based on the result obtained in Appendix C, i.e., 10 scoops to form a composite sample at the stockpile, we adopt the following sampling procedures for different size stockpiles.

1) For a 12 by 150-ft stockpile, we longitudinally stratify each layer into 10 strata and randomly take one scoop from each of the 10 strata to form a composite sample as shown in Figure 9.

2) For a 24 by 150-ft stockpile, we longitudinally and transversely stratify each layer into 10 strata as shown in Figure 10. One scoop of aggregate is randomly taken from each of the 10 strata to form a composite sample.

3) For a 36 by 150-ft stockpile, we first stratify each layer into three long transverse strips of 12 by 150 ft each. Two strips are chosen randomly and further stratified longitudinally into five strata; thus we have a total of 10 strata. Again, one scoop of aggregate is randomly taken from each of the 10 strata to form a composite sample.

4) For 48 by 150-ft, 60 by 150-ft, etc., stockpiles, we first stratify each layer into numerous long transverse strips of 12 by 150 ft each. Then, two strips are chosen randomly and further stratified longitudinally into five strata; thus, we have 10 strata in total. One scoop of aggregate is randomly taken from each of the 10 strata to form a composite sample.

The above sampling methods, along with the current sampling method, shall be used to take samples at the Stillman pit to study aggregate characteristics.

TABLE 3
ESTIMATES OF AGGREGATE COMPOSITION
OF THE STOCKPILED AGGREGATE UNDER
THE CSM AND THE SRSM

Sampling Method	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
CSM	100	94.43	72.19	46.14	4.77
SRSM	100	94.54	72.76	46.43	4.79

TABLE 4
ESTIMATES OF THE COVARIANCE MATRIX
OF THE AGGREGATE COMPOSITION
UNDER THE CSM AND SRSM PLANS

	Size Combination	Sampling Method	
		CSM	SRSM
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	3.82	3.99
	3/4-in. and 3/8-in.	3.78	4.92
	3/4-in. and No. 8	2.06	2.84
	3/4-in. and L.B.W.	0.12	-0.08
	3/8-in. and 3/8-in.	13.11	15.32
	3/8-in. and No. 8	8.83	10.55
	3/8-in. and L.B.W.	0.34	2.87
	No. 8 and No. 8	8.88	10.47
	No. 8 and L.B.W.	0.74	0.60
	L.B.W. and L.B.W.	0.47	0.49

I-2. PROPOSED STOCKPILE SAMPLING METHOD (SYSTEMATIC SAMPLING METHOD WITH RANDOM START)

Many factors can affect aggregate screening results, such as testing location, sieving equipment, inspector, and sampling method. In general, testing procedures are more controllable in the laboratory than in the field. Additionally, non-standard sieves may produce bias in the screening results. Finally, an inspector's experience in taking and testing samples may also affect the screening results. On the other hand, the sampling method is a very important factor in determining aggregate composition. Therefore, our major concern in this chapter is to compare the current sampling method (CSM) with the stratified random sampling methods (SRSM) defined in Chapter I-1. The results of the comparison should provide a final suggestion for stockpile sampling procedures.

Comparison of the CSM and the SRSM

In order to compare the SRSM with the CSM, we conducted a statistical experiment taking samples using both methods. The descriptions and the statistical analyses of the experiment are presented in Appendix D for readers who are interested in these details. Here we simply state, based on the conducted experiment, the conclusion that the two sampling methods are statistically the same at the 0.05 level in estimating stockpile aggregate composition. We present the aggregate composition estimates and their covariance matrices under the CSM and the SRSM in Tables 3 and 4.

Equality of the CSM and SRSM covariance matrices (Table 4) can be tested statistically; but, each sample obtained by using the SRSM is comprised of 10 scoops of aggregate forming a composite sample, while it only takes five or six scoops of aggregate to form a composite sample under the CSM. This indicates that the transverse stratification does reduce some variability. Moreover, the SRSM is certainly much more difficult than the CSM in a field operation. Thus, we favor systematic sampling procedures. Since 10 scoops are shown to be sufficient to form a representative composite sample and because of possible defects in the CSM mentioned in Chapter I-1, we suggest the following stockpile sampling procedures, to be called "Systematic Sampling Method with Random Start."

Systematic Sampling Method with Random Start

1) For a 12 by 150-ft stockpile, we stratify each layer longitudinally and transversely into 60 strata and number them as shown in Figure 11.

150'

5	4	3	2	1	6	5	4	3	2	1	6
4	3	2	1	6	5	4	3	2	1	6	5
3	2	1	6	5	4	3	2	1	6	5	4
2	1	6	5	4	3	2	1	6	5	4	3
1	6	5	4	3	2	1	6	5	4	3	2

12'

Figure 11. Stratification layout of each layer of a 12 by 150-ft stockpile.

The procedures for making a composite sample are described as follows:

a. Select a number from one to six by rolling a fair die.

b. Take one scoop of aggregate randomly from each of the 10 strata corresponding to the selected random number. For example, if "three" was chosen, the strata sampled would be those indicated in Figure 11.

c. The 10 scoops of aggregate together form a composite sample.

Note that the location at which a scoop of aggregate will be taken within each stratum is determined by other criteria. For example, one might choose to take the scoop from the center of each stratum.

Therefore, each sample is composed of 10 scoops of aggregate taken from a layer according to one of the six patterns in Figure 12.

2) Consider each layer of a 24 by 150-ft stockpile as composed of two long transverse strips, 12 by 150-ft each. We stratify each strip longitudinally and transversely as shown in Figure 13.

The procedures for making composite samples are the same as those described before. That is, each sample is composed of 10 scoops of aggregate taken from a layer according to the six patterns in Figure 14.

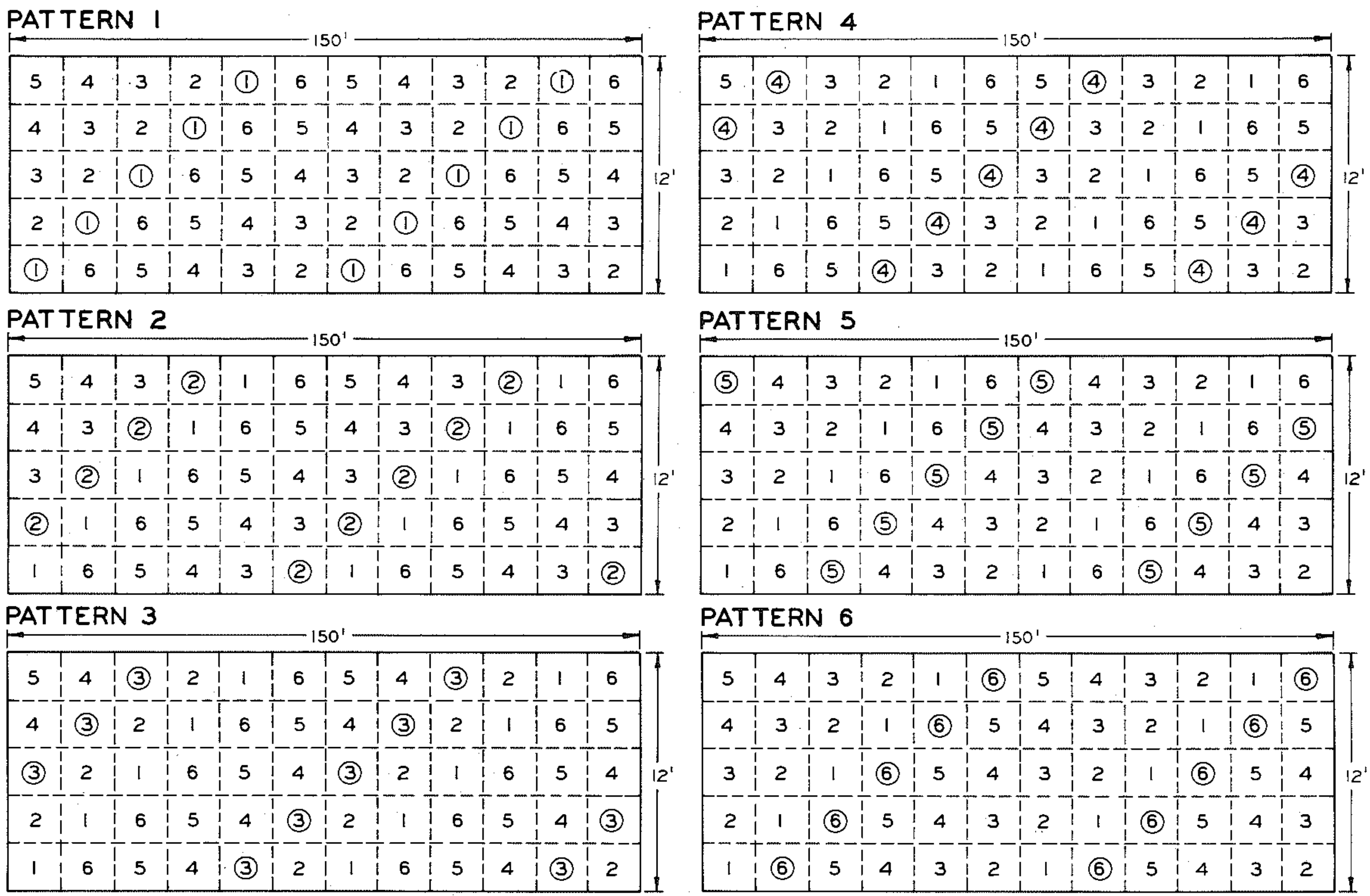

Figure 12. Sampling patterns for a 12 by 150-ft stockpile.

3) For a 36 by 150-ft, 48 by 150-ft, etc., stockpile, we consider each layer as composed of numerous long transverse strips, 12 by 150 ft each. We choose two strips at random. For these two chosen strips, the sampling procedures are the same as those described in 2).

We shall assume that the systematic sampling method with random start defined above is the one selected for stockpiled aggregate. Based on this sampling method, we shall discuss and design potential acceptance sampling plans for stockpile aggregate inspection in the next chapter.

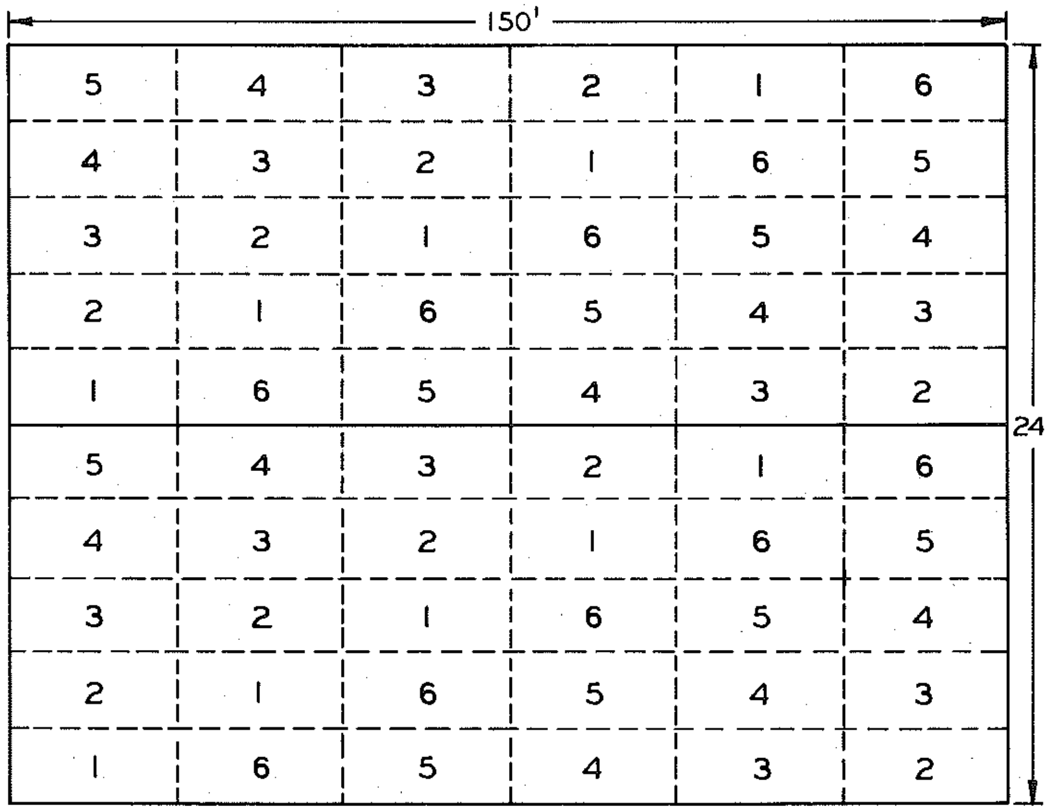

Figure 13. Stratification layout of each layer of a 24 by 150-ft stockpile.

PATTERN 1

150' (length) × 24' (width)

5	4	3	2	(1)	6
4	3	2	(1)	6	5
3	2	(1)	6	5	4
2	(1)	6	5	4	3
(1)	6	5	4	3	2
5	4	3	2	(1)	6
4	3	2	(1)	6	5
3	2	(1)	6	5	4
2	(1)	6	5	4	3
(1)	6	5	4	3	2

PATTERN 2

150' (length) × 24' (width)

5	4	3	(2)	1	6
4	3	(2)	1	6	5
3	(2)	1	6	5	4
(2)	1	6	5	4	3
1	6	5	4	3	(2)
5	4	3	(2)	1	6
4	3	(2)	1	6	5
3	(2)	1	6	5	4
(2)	1	6	5	4	3
1	6	5	4	3	(2)

PATTERN 3

150' (length) × 24' (width)

5	4	(3)	2	1	6
4	(3)	2	1	6	5
(3)	2	1	6	5	4
2	1	6	5	4	(3)
1	6	5	4	(3)	2
5	4	(3)	2	1	6
4	(3)	2	1	6	5
(3)	2	1	6	5	4
2	1	6	5	4	(3)
1	6	5	4	(3)	2

PATTERN 4

150' (length) × 24' (width)

5	(4)	3	2	1	6
(4)	3	2	1	6	5
3	2	1	6	5	(4)
2	1	6	5	(4)	3
1	6	5	(4)	3	2
5	(4)	3	2	1	6
(4)	3	2	1	6	5
3	2	1	6	5	(4)
2	1	6	5	(4)	3
1	6	5	(4)	3	2

PATTERN 5

150' (length) × 24' (width)

(5)	4	3	2	1	6
4	3	2	1	6	(5)
3	2	1	6	(5)	4
2	1	6	(5)	4	3
1	6	(5)	4	3	2
(5)	4	3	2	1	6
4	3	2	1	6	(5)
3	2	1	6	(5)	4
2	1	6	(5)	4	3
1	6	(5)	4	3	2

PATTERN 6

150' (length) × 24' (width)

5	4	3	2	1	(6)
4	3	2	1	(6)	5
3	2	1	(6)	5	4
2	1	(6)	5	4	3
1	(6)	5	4	3	2
5	4	3	2	1	(6)
4	3	2	1	(6)	5
3	2	1	(6)	5	4
2	1	(6)	5	4	3
1	(6)	5	4	3	2

Figure 14. Sampling patterns for a 24 by 150-ft stockpile.

I-3. ACCEPTANCE SAMPLING PLANS FOR STOCKPILE INSPECTION

The proposed sampling method for stockpiled aggregate described in Chapter I-2 will give us a good estimate of the aggregate composition of a stockpile. Based on this sampling method, we need a decision rule which describes criteria for accepting or rejecting an aggregate lot submitted for acceptance inspection. In the field of quality control, this kind of decision rule is termed an "acceptance sampling plan" which merely prescribes a procedure that, if applied to a series of lots, will give a specified risk of accepting lots of given quality. That is, an acceptance sampling plan provides quality assurance. It is the purpose of this chapter to design aggregate quality assurance programs for stockpiled aggregate inspection.

The Current Acceptance Sampling Plan

The current acceptance sampling plan specifies that one sample is taken and tested according to the current sampling and testing method. If the aggregate composition of this sample meets the specification, the stockpiled material represented by this sample is accepted. If it fails to meet the specification, the testing inspector notifies the producer and tests another sample from the next production layer. If the second sample meets the specification, the stockpiled material represented by these two samples is accepted. Otherwise, the stockpiled material represented by two failing samples is rejected and removed from that stockpile. Under this plan, each layer for which aggregate composition does not meet the specification is considered a "defective" layer. Thus, the product quality, P, of the stockpiled aggregate is defined as the proportion of defective layers. Statistically, P can be estimated as the probability that a representative sample randomly taken from a layer does not meet the specification. The mathematical expression for estimating P is presented in Section 1, Appendix E. For a given value of P, the probability P_a of accepting any layer under the current plan is computed by Eq. (3), Appendix E, and is plotted in Figure 15.

Discussion of the Current Plan

Since stockpiled aggregate will undergo a "remixing" process when transported to the construction site, the major purpose of stockpile inspection is to ensure that the composition of the accepted stockpiled aggregate meets the specification. To this end, we make the following comments about the current stockpile inspection practice.

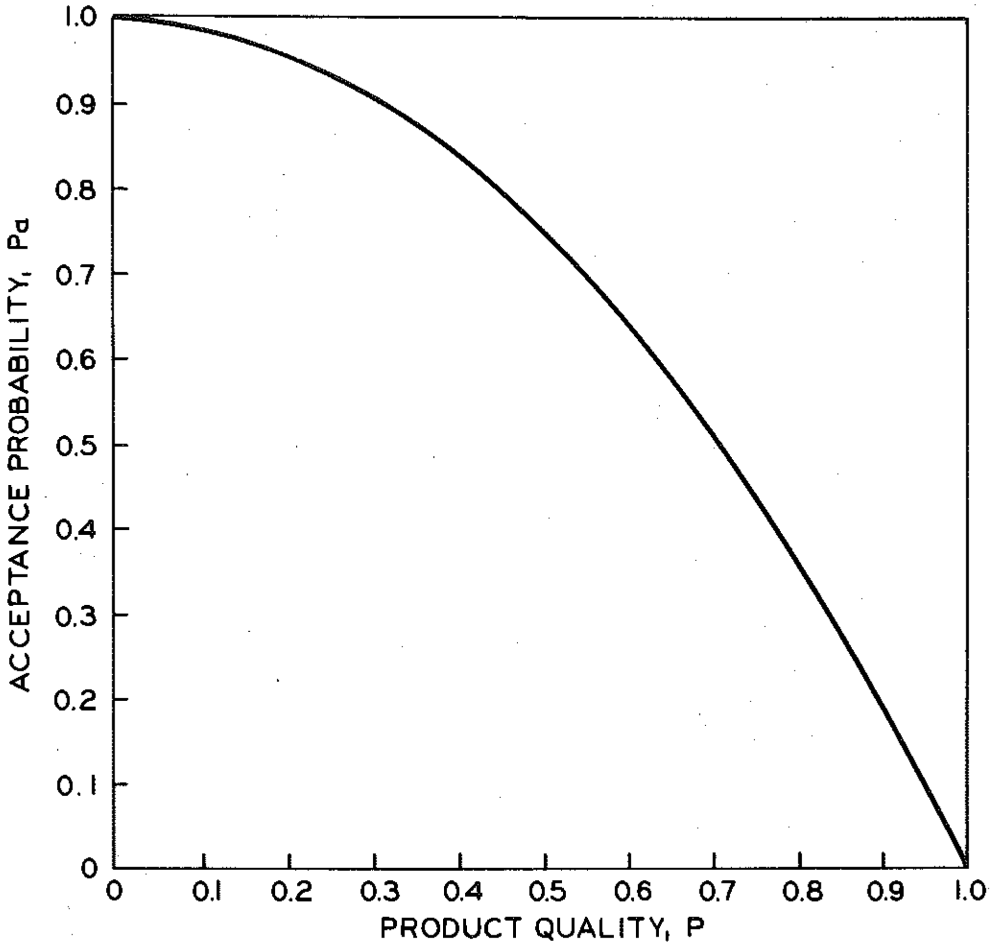

Figure 15. OC curve of the current plan.

Under the current plan, the aggregate quality of a stockpile is judged by the proportion of defective layers, P. That is, the aggregate quality of a stockpile is considered to be poor if the value of P is large. Let us examine the above statement. Suppose that a stockpile of 100 layers is totally tested. The test results show that measurements on every sieve of these 100 layers meet the specification. But, there are 50 layers whose measurements of loss-by-washing are above the upper specification limit with an average of 10. The average percent loss-by-washing of the 50 non-defective layers is 7. Then, the average percent loss-by-washing of this stockpile is 8.5 which is still above the upper specification limit. That is, the composition of this stockpile does not meet the specification. However, if the average loss-by-washing of 50 defective and non-defective layers are

10 and 4, respectively, the average percent loss-by-washing of this stockpile meets the specification. The above example indicates that P does not provide the full information needed.

For the purpose of discussion, let us assume that a stockpile with P = 0.3 is not desirable. We see from Figure 15 that, on the average, if 50 of 100 layers do not meet the specification (P = 0.5), the current plan would require the Department to accept 75. Under these circumstances, 25 of the accepted 75 layers would not meet the specification. That is, the product quality of the accepted aggregate is 0.33. This shows that the Department runs a high risk of accepting poor quality aggregate (as judged by P).

Under the current stockpile inspection, when the producer is informed that a failing sample has been found, he would normally take immediate action to ensure that the second sample from the next production layer would meet the specification. This would ensure that both production layers would be accepted according to the current plan. Will, however, the aggregate composition of two layers taken together meet the specification? Let us look at the following example.

Example 1 — Suppose that the percent passing the No. 8 sieve for the first and second sample are 55 and 50, respectively. Further, assume that measurements on other sieves of both samples fall inside the specification limits. Since the lower and upper limits for the No. 8 sieve are 30 and 50, respectively, the first sample fails to meet the specification; but the second sample meets the standard. Thus, the aggregate quality of both layers is judged to be satisfactory, and therefore, is accepted according to the current plan. However, the average percent passing the No. 8 sieve of these two samples is 52.5 which is the estimate of the percent passing the No. 8 sieve of both layers combined. Since 52.5 falls outside the specification limits, the aggregate quality of both layers is judged to be unsatisfactory based on this estimate and the relevant specification. This example indicates that the presumed aggregate quality of both layers judged by the current plan might be in error. Let us extend this example to cover the entire stockpile.

Example 2 — If only one of each of two consecutive samples (layers in a stockpile) meets the specification, the entire stockpile would be accepted according to the current plan. Therefore, the aggregate quality of the stockpile is judged to be satisfactory using the decision rules of the current inspection plan. However, the true aggregate composition of the entire stockpile might fall outside the specification limits as demonstrated in Example 1.

The above two examples suggest that the second sample of the current plan gives little additional aggregate quality assurance. Rather, it gives the producer a second chance to pass a defective product as defined by the specification.

Now we suppose that a manufacturing process produces aggregate with average composition 100, 85, 50, 8 (the upper specification limits) and percent crushed 46.57 (estimated from the Stillman pit). That is, the composition of a stockpile produced under this production process meets the specification. But, the probability that the composition of a representative sample randomly taken from a layer will meet the specification is 0.2889. That is, the product quality of this manufacturing process is 0.7111 (see Table 1, Appendix E). Thus, the probability of accepting any layer is only 0.4945 under the current plan. This means that, on the average, 49.45 percent of the layers will be rejected under the current plan. But, we know that the average composition of the rejected and accepted layers meets the specification. This shows that the current plan can reject acceptable aggregate (judged by the composition) with high probability.

The above discussions point out the following three defects of the current plan.

1) The proportion of defective layers does not provide full information on the composition of stockpiled aggregate.

2) The Department runs a high risk of accepting poor quality aggregate judged by the proportion of defective layers.

3) There is a good chance that the composition of the accepted aggregate might not be desirable and the rejected aggregate is acceptable judged by its composition.

This calls our attention to the possible improvement of the current acceptance sampling plan.

Proposed Acceptance Sampling Plans for Stockpile Inspection

As mentioned before, stockpile inspection is sensible only if the desired characteristics of in-place aggregate can be maintained at the acceptance level through stockpile gradation control. Ideally, the aggregate composition of every spot at the construction site meets the quality standards. We know that this is highly unlikely. However, the desired degree of uniformity of in-place aggregate could be obtained at the acceptable level through control of stockpile aggregate composition and proper aggregate handling procedures. The latter subject will be discussed in the next chapter. We devote this section to the discussion of stockpile gradation control.

For the purpose of controlling stockpile aggregate composition, we consider a plan of so-called "acceptance sampling by variables" to give assurance regarding the composition of aggregate lots.

For an aggregate lot submitted for acceptance inspection, this type of plan basically specifies the following:

1. Sample size, N

2. The lower and/or upper acceptance limits for each component of aggregate composition and percent crushed.

Let U_i and L_i, $i = 1, \ldots, 4$, be the upper and lower acceptance limits for the percent passing the 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing, respectively, and L_5 be the lower acceptance limit for the percent crushed. We desire an acceptance sampling plan such that the lot acceptance probability is no more than β for aggregate falling outside the specification limits (poor quality aggregate). If the aggregate composition is midway between the specification limits and the percent crushed is at least 40 (good quality aggregate), we will want the lot acceptance probability to be at least $1 - \alpha$. The α and β are called the "producer's" and "consumer's" risks, respectively.

Equations for the determination of the required sample size, the upper and lower acceptance limits, based on the concept of simultaneous confidence intervals (see Appendix A), are presented in the third section of Appendix E. Various acceptance sampling plans derived through these equations are presented in Table 5.

The difference between Plan A_i and Plan B_i is that Plan A_i was derived holding α at the desired level, while Plan B_i was derived holding β at the desired level. Plan A_i is interpreted as follows: For each lot, say a half day or one day's production, two samples are taken according to the systematic sampling procedures stated in Chapter I-2. If the average percent passing each sieve, the average percent loss-by-washing and the average percent crushed fall within the acceptance intervals of Plan A_i specified in Table 5, the lot represented by these two samples is accepted; otherwise, the lot is rejected. The other plans can be interpreted in the same way.

TABLE 5

ACCEPTANCE SAMPLING PLANS BY VARIABLES TO CONTROL STOCKPILE AGGREGATE COMPOSITION AND PERCENT CRUSHED AGGREGATE

Plan	Risk		Sample Size	Acceptance Interval For Each Measurement										
				1-in.		3/4-in.		3/8-in.		No. 8		Loss-By-Washing		Crushed Aggregate
	α	β		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower
A_1	0.05	0.05	3	100	100	93.28	96.72	70.38	79.62	35.82	44.18	5.27	6.73	34.63
A_2	0.01	0.01	4	100	100	93.26	96.74	70.33	79.67	35.77	44.23	5.26	6.74	34.40
A_3	0.05	0.01	3	100	100	93.28	96.72	70.38	79.62	35.82	44.18	5.27	6.73	34.63
B_1	0.05	0.05	3	100	100	91.58	98.42	69.22	80.78	33.82	46.18	4.67	7.33	30.37
B_2	0.01	0.01	4	100	100	91.64	98.36	69.40	80.60	33.98	46.02	4.70	7.30	30.60
B_3	0.05	0.01	3	100	100	91.90	98.10	70.08	79.92	34.60	45.40	4.80	7.20	30.60

TABLE 6

THE ACCEPTANCE PROBABILITY, P_a, OF EACH PLAN SPECIFIED IN TABLE 5 WITH 60 PERCENT CRUSHED

Mean Aggregate Composition, μ				Acceptance Probability of Plan					
3/4-in.	3/8-in.	No. 8	L.B.W.	A_1	A_2	A_3	B_1	B_2	B_3
100.00	85.0	50.0	8.0	0.000	0.000	0.000	0.000	0.000	0.000
98.25	82.5	47.5	7.5	0.000	0.000	0.000	0.045	0.019	0.005
97.50	80.0	45.0	7.0	0.013	0.008	0.013	0.715	0.714	0.428
96.25	77.5	42.5	6.5	0.669	0.758	0.669	0.993	0.999	0.979
95.00	75.0	40.0	6.0	1.000	1.000	1.000	1.000	1.000	1.000
93.75	72.5	37.5	5.5	0.657	0.745	0.657	0.998	0.999	0.976
92.50	70.0	35.0	5.0	0.011	0.006	0.011	0.720	0.712	0.441
91.25	67.5	32.5	4.5	0.000	0.000	0.000	0.029	0.014	0.006
90.00	65.0	30.0	4.0	0.000	0.000	0.000	0.000	0.000	0.000

We again must use computer simulation methods, based on 1,000 observation vectors to compute the acceptance probability defined in Eq. (8), Appendix E for various acceptance sampling plans in Table 5. The results are presented in Table 6.

Note that the percent crushed was set at 60 which is high enough to ensure that the acceptance probability will not be affected by this variable. That is, the acceptance probability in Table 6 can be considered as the acceptance probability based on aggregate composition only. Since the percent crushed is almost uncorrelated with the aggregate composition as indicated in Matrix (4), the acceptance probability when the mean vector is $u = (u_1, u_2, u_3, u_4, u_5)$ can be estimated as the product of the acceptance probability based on aggregate composition with mean vector $(u_1, \ldots, u_4)$ and the acceptance probability based on the percent crushed aggregate with mean u_5. For example, under Plan A, the acceptance probability, when the mean vector is (95, 75, 40, 6, 35) is $P(\bar{x}_5 \geq L_5) = 0.579$, while the acceptance probability, when the mean vector is (95, 75, 40, 6, 40) becomes $P(\bar{x}_5 \geq L_5) = 0.998$.

What has been accomplished in this section is the determination that a variables type of plan should be adopted for stockpiled aggregate inspection. Also, some examples, based on data from the Stillman pit, of this type of plan were provided. Since a proper acceptance sampling plan depends upon the covariance matrix, the choice of the tolerance limits, α and β which are related to other factors such as the sensitivity of the pavement's performance to the aggregate composition and the availability of manpower (cost consideration), etc., which are beyond the scope of this study, we are not in the position of recommending a more specific plan to be used in stockpile inspection. One can see, however, that any particular plan can easily be developed by following the same steps that have been outlined in this Chapter.

We shall end this Chapter with the following remarks:

1) The choice of the upper and lower tolerance limits in Table 2, Appendix E should reflect the aggregate degradation factor due to the handling and compaction process and the degree of aggregate uniformity due to handling. We shall discuss these subjects in the next Chapter.

2) A composite sample obtained by using the proposed systematic sampling method described in Chapter I-2 could weigh about 30 lb. But, the weight of a test sample is limited to 10 lb due to the capacity of testing sieves. Instead of testing the sample three times (about 10 lb for each

time), the conventional method is to use a Gilson Sample Splitter to reduce the sample to test size. Does, however, the reduced sample still represent the original one in terms of aggregate composition? The answer to this question depends on the splitting ability of the Gilson Sample Splitter in dividing one sample into two equal samples in the sense of aggregate composition. It is apparent that the splitting ability of the splitter is a function of the weight of the split sample. We do not know whether or not the splitting ability of the Gilson Sample Splitter has been scientifically evaluated. Since usage of the splitter has to do with the testing efficiency (time) and the accuracy of the estimation of aggregate composition, we recommend the Department conduct experiments to determine the splitting ability of these devices.

I-4. AGGREGATE CHANGE AND DEGRADATION DUE TO HANDLING AND COMPACTION

We pointed out in the previous Chapter that the justification of stockpiled aggregate inspection depends on the effect of the handling process on the degree of in-place aggregate uniformity. Moreover, if the handling process could improve the degree of uniformity up to the acceptable level, the determination of the stockpile acceptance sampling plans would be a function of the aggregate degradation rate. Thus, the purpose of this Chapter is to determine the effects of the handling and compaction processes on the aggregate composition and on the degree of the in-place aggregate uniformity. This information would be a valuable reference for justifying the stockpile aggregate inspection and for designing the in-place aggregate inspection plans if needed.

Effects of the Compaction Process on Aggregate Composition

The compaction process is used to obtain the required aggregate density in the roadbed. It is expected that the 'quality' of aggregate material will degrade after compaction. In order to see whether this degradation is significant, an experiment was conducted taking composite samples from the roadbed before and after compaction. The descriptions of the sampling method and statistical analyses are presented in Appendix F. Here, we simply state the conclusion that the compaction process significantly effects the aggregate composition at the 0.05 level. We present estimates of the composition of in-place aggregate before and after compaction in Table 7.

TABLE 7
ESTIMATES OF IN-PLACE AGGREGATE COMPOSITION
BEFORE AND AFTER COMPACTION

Sieve Size	Percent Passing Each Sieve		
	Before Compaction	After Compaction	Difference
1-in.	100.0000	100.0000	0
3/4-in.	94.8622	95.2548	0.3926
1/2-in.	84.6136	85.1779	0.5643
3/8-in.	77.2384	77.6124	0.3740
No. 4	62.0874	62.3241	0.2367
No. 8	50.4697	50.6500	0.1803
No. 16	39.2419	39.5009	0.2590
No. 30	27.8024	28.2652	0.4628
No. 50	14.3943	15.1879	0.7936
No. 100	8.1735	8.9093	0.7358
No. 200	6.1921	6.8069	0.6148
Loss-By-Washing	5.8193	6.5626	0.7433

We see from Table 7 that "differences" in aggregate composition before and after compaction are about the same. But, from previous experiments, the coarse aggregate measurement error is generally larger than the fine aggregate measurement error. Thus, the compaction process statistically affects fine aggregate more than coarse aggregate. The above statement can be seen more clearly from Table 8 which presents the 95-percent simultaneous confidence intervals (SCI) for the difference in the aggregate composition before and after compaction. The concept of SCI is described in Appendix A.

Table 8 shows that the only confidence interval that does not contain zero is the one which measures the loss-by-washing. We also see that confidence intervals for the measurement with fine sieves barely covers zero. This shows that the compaction process particularly affects fine aggregate (in the statistical sense).

TABLE 8
NINETY-FIVE PERCENT SCI FOR THE
DIFFERENCE IN AGGREGATE COMPOSITION
BEFORE AND AFTER COMPACTION

Sieve Size	Lower Confidence Limit	Upper Confidence Limit
1-in.	0	0
3/4-in.	-1.18	1.86
1/2-in.	-1.82	2.94
3/8-in.	-2.24	2.98
No. 4	-2.30	2.78
No. 8	-2.14	2.50
No. 16	-1.86	2.18
No. 30	-1.34	2.26
No. 50	-0.39	1.97
No. 100	-0.16	1.63
No. 200	-0.18	0.14
Loss-By-Washing	0.14	1.34

Further investigation of the data obtained before and after compaction provides the following information:

1) The composition of in-place aggregate after compaction can be estimated with reasonable accuracy from samples taken before compaction. Statistical models for this procedure are presented in Appendix G.

2) The degradation rate of each aggregate size due to compaction can also be accurately estimated. Statistical models for this purpose are also presented in Appendix G. For the Stillman pit, we estimate that about 5.1, 2.5, and 2.9 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade due to compaction.

The above information is needed if for economic reasons aggregate inspection is to be practiced at the construction site before compaction. Details will be discussed in Chapter I-6.

Effects of the Handling Process on Aggregate Composition

The aggregate manufacturing process places the first day's aggregate on the ground to form the first layer of a stockpile, and places the second day's aggregate on the top of the first layer to form the second layer of that stockpile, and so on. When stockpiled aggregate is ready to be transported to the construction site, aggregate at the front end of the stockpile will be removed first, and removal operations will continue toward the tail of that stockpile as shown in Figure 16.

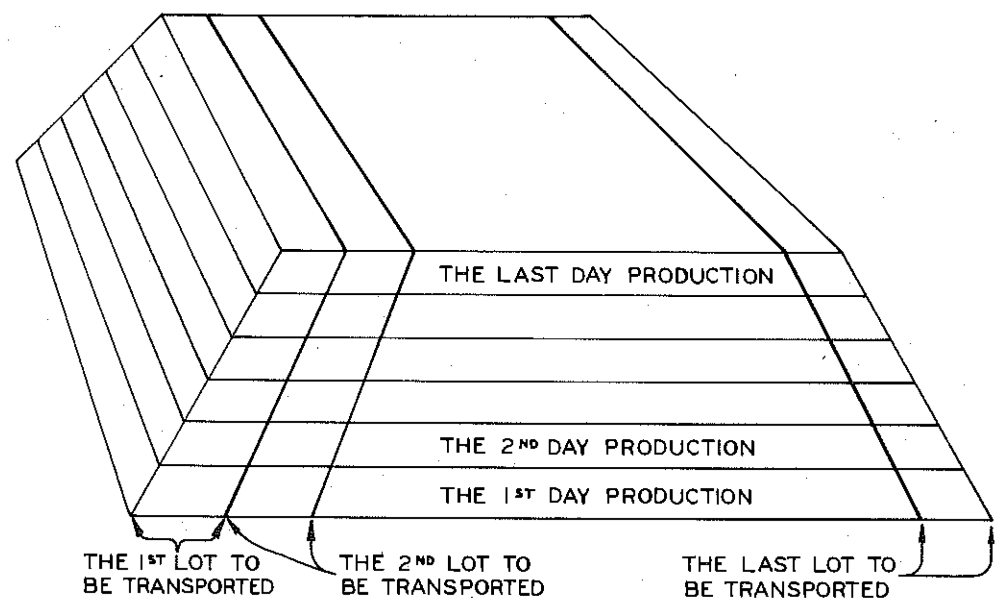

Figure 16. Stockpile formation and removal operations.

That is, aggregate on a stockpile undergoes the first stage of a remixing process when transported to the construction site. Aggregate at the construction site will be further spread over the roadbed by earthmoving and grading machines; this is the second stage of the remixing process. Do these remixing processes change the aggregate composition or degrade the aggregate in a manner detectable with our sampling methods? Does this remixing process provide aggregate uniformity for the roadbed?

To answer the above questions, we conducted an experiment taking composite samples from the roadbed before compaction and from the stockpile. The descriptions and the analyses of this experiment are presented in Appendix F. Here, we simply state the conclusion that the handling process significantly affects the aggregate composition especially fine aggregates at the 0.05 level. Estimates of the aggregate composition are presented in Table 9.

TABLE 9
ESTIMATES OF AGGREGATE COMPOSITION OF THE STOCKPILE AND IN-PLACE AGGREGATE BEFORE COMPACTION

Sieve Size	Percent Passing Each Sieve		
	Before Compaction	Stockpile	Difference
1-in.	100.00	100.00	0
3/4-in.	94.84	92.85	1.99
1/2-in.	84.59	81.19	3.40
3/8-in.	77.19	73.15	4.04
No. 4	62.02	57.99	4.03
No. 8	50.41	46.58	3.83
No. 16	39.19	35.67	3.52
No. 30	27.78	24.66	3.12
No. 50	14.39	11.92	2.47
No. 100	8.19	6.41	1.78
No. 200	6.22	4.70	1.52
Loss-By-Washing	5.84	4.56	1.28

The differences in aggregate composition of the stockpile and the construction site before compaction could be due to:

1) difference in the two sampling methods used at the stockpile and at the construction site, respectively, in estimating the aggregate composition,

2) aggregate degradation during the remixing stage,

3) mud from truck traffic.

TABLE 10
ESTIMATES OF THE COMPOSITION OF IN-PLACE AGGREGATE AFTER COMPACTION BASED ON NON-COMPOSITE SAMPLES

Percent Passing Sieves	1-in.	100.00
	3/4-in.	94.93
	3/8-in.	76.34
	No. 8	49.31
Percent Loss-By-Washing		6.53

TABLE 11
ESTIMATES OF THE COVARIANCE MATRIX OF AGGREGATE COMPOSITION AT THE CONSTRUCTION SITE BASED ON NON-COMPOSITE SAMPLES

Size Combination	Covariance
3/4-in. and 3/4-in.	4.17
3/4-in. and 3/8-in.	4.00
3/4-in. and No. 8	2.62
3/4-in. and L.B.W.	0.26
3/8-in. and 3/8-in.	9.88
3/8-in. and No. 8	7.80
3/8-in. and L.B.W.	0.96
No. 8 and No. 8	10.76
No. 8 and L.B.W.	1.36
L.B.W. and L.B.W.	0.59

We believe that the differences in Table 8 due to sampling methods are negligible in the sense that the two sampling methods used have the same degree of precision in estimating aggregate composition. Thus, the differences in Table 9 are the amount due to 2) and 3) above which can be termed as the amount of aggregate degradation due to the handling process.

Since there is no way to pair samples taken from the construction site and the stockpile, the ratio estimating method (Model 1, Appendix G) is the only one that can be used to estimate the composition of in-place aggregate before compaction from that of the stockpile. The estimating results are presented in Table 7, Appendix G. The degradation rate of each aggregate size due to handling cannot be determined unless we assume that each aggregate size degrades only to the next size. If this is assumed, we estimate by using Eq. (14), Appendix G that 27.83, 20.51, and 14.42 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade due to handling.

So far, we have demonstrated that the composition of in-place aggregate after compaction can be estimated either from that before compaction or from that of the stockpile. We have also shown that the degradation rate of each aggregate size due to handling and compaction can be estimated with reasonable accuracy under the proper assumptions. All of this information is needed to adjust the current aggregate specification if the inspection practice is not to be performed at the construction site after compaction.

If the inspection practice is going to be performed at the stockpile, the degree of in-place aggregate uniformity obtainable with the handling process should be carefully examined.

Uniformity of In-Place Aggregate

As mentioned before, our major concern with in-place aggregate is its uniformity which is determined by the handling process and possibly by stockpile formation as well. To study these characteristics, we conducted an experiment taking non-composite samples from the construction site after compaction (see Appendix F). Estimates of aggregate composition and its covariance matrix, based on non-composite samples, are presented in Tables 10 and 11.

It can be shown statistically that the above covariance matrix is not equal to the covariance matrix in Eq. (7), Appendix C, which is the covariance matrix of the composition of the stockpiled aggregate based on non-composite sampling methods. The variances of the percent passing each sieve and the percent loss-by-washing measured at various locations are summarized in Table 12.

TABLE 12
VARIANCES OF THE PERCENT PASSING EACH SIEVE MEASURED AT VARIOUS LOCATIONS

Sieve Size	Stockpile		In-Place	
	Non-Composite	Composite	Non-Composite	Composite
3/4-in.	7.29	2.87	4.17	3.28
3/8-in.	57.26	13.49	9.88	6.73
No. 8	47.15	8.67	10.76	6.38
Loss-By-Washing	0.80	0.30	0.59	0.49

The difference between the composite and non-composite sampling method is that, for a given lot, a composite sample is composed of aggregate taken randomly from 10 spots in that lot according to a stratification scheme, while a non-composite sample is composed of aggregate taken from a randomly determined spot of that lot. Consequently, the variance of each measurement under the non-composite sampling method is expected to be much larger than that under the composite sampling method. This is precisely the case for stockpiled aggregate, but much less so for in-place aggregate. This can occur only if the aggregate gradation variation within the stockpile is larger than that of in-place aggregate. All of this indicates that the handling process has improved the aggregate uniformity of in-place aggregate.

The extent of desirable aggregate uniformity depends on the purpose served by this characteristic. We shall briefly mention this subject in the next Chapter.

Findings

1) The handling and compaction processes do degrade the aggregate.

2) The degradation rate of each aggregate size due to compaction can be accurately estimated. Moreover, the aggregate degrades only to the

next size aggregate when degradation is caused by the compaction process. For the Stillman pit, we estimate that about 5.1, 2.5, and 2.9 percent of the total percentage retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade due to compaction. Statistical models for the above estimates are presented in Appendix G.

3) With reasonable assumptions, we estimate that about 27.83, 20.51, and 14.42 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade due to handling.

4) The degradation rate due to handling is much larger than that due to compaction.

5) Combining both degradation rates, we estimate that about 33.63, 25.58, and 20.47 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade due to handling and compaction taken together.

6) The composition of in-place aggregate after compaction can be estimated from either that before compaction or that of the stockpile. Statistical models for these purposes are presented in Appendix G. Good estimatibility is needed to justify the practice of aggregate inspection either at the construction site before compaction or at the stockpile.

7) The handling process does improve the in-place aggregate uniformity. This information is needed for investigating the feasibility of practicing stockpile aggregate inspection.

I-5. ACCEPTANCE SAMPLING PLANS FOR IN-PLACE AGGREGATE INSPECTION

It was demonstrated in Chapter I-4 that the composition of in-place aggregate can be estimated from that of the stockpile. Moreover, the handling process also improves the uniformity of in-place aggregate to some extent. Therefore, it might seem that there is no need to practice aggregate inspection at the construction site. However, we might still have to perform the aggregate inspection at the construction site for the following reasons:

1) The handling process might not improve the aggregate uniformity to the desired degree so that controlling the stockpile aggregate composition would not guarantee the desired quality of in-place aggregate.

2) From the administrative point of view, it would be better not to mix the inspection practice with the aggregate production.

3) Since we have to perform the aggregate density test of in-place aggregate, it would be a good practice from an administrative and economic point of view to also perform the aggregate gradation test of in-place aggregate.

Thus, our major task in this chapter is to explore suitable acceptance sampling plans for aggregate inspection at the construction site.

Preliminary Set-Up for Designing Inspection Plans

The ideal distribution of in-place aggregate is such that the composition of every spot meets the specification. To ensure that the accepted project has a high degree of uniformity, we should test in-place aggregates based on the "non-composite" sampling method which was mentioned in the previous chapter. Moreover, we want to adopt an inspection plan of so-called "acceptance sampling by attributing," such as the single sampling fraction defective sampling plan (SSFD) and Wald's truncated sequential probability ratio plan (TSPR), as a decision rule to accept or reject in-place aggregate. These plans require that we choose the producer's risk (α), the consumer's risk (β), the acceptable product quality level (P_α) and the rejected product quality level (P_β).

Each non-composite sample taken from the construction site is classified as defective if it fails to meet the specification. For such a sampling method, the covariance matrix of the composition of in-place aggregate was estimated in Table 11 in the last chapter. First, we consider Table 11 as the true covariance matrix of aggregate composition based on a non-composite sampling method. Second, we use simulation procedures to calculate the product quality, P, defined in Eq. (1), Appendix E. The results are presented in Table 13.

Note that each P in Table 13 was obtained based on 1,000 simulation points. As one can see from Table 13, the probability that a random sample is defective is 0.018 if the aggregate composition of the aggregate production process is set at the center of the upper and the lower specification limits. The product quality, of course, becomes poor (the P value gets larger) as the composition deviates farther from the centerline.

Note also that the product quality P in Table 13 was obtained based on aggregate composition only. If the measurement x_5, the percent crushed aggregates, shall also be included in the computation of the product quality, then the true value corresponding to the mean vector $(u_1, \ldots, u_5)$ is approximately equal to $1 - (1 - P) \cdot P_r (x_5 \geq 25 \quad u_5)$, where P is the product quality corresponding to $(u_1, \ldots, u_4)$ in Table 13 and $P_r (x_5 \geq 25 \quad u_5)$ can be found from normal probability table. If u_5 is set at 40, the P in Table 13 remains unchanged. However, if u_5 is set at 25, the actual value becomes $\frac{1 + P}{2}$.

TABLE 13
RELATIONSHIP OF AGGREGATE COMPOSITION AND PRODUCT QUALITY

Aggregate Composition (u_1, u_2, u_3, u_4)				Product Quality, P
Percent Passing Sieve			Loss-By-Washing	
3/4-in.	3/8-in.	No. 8		
100.00	85.0	50.0	8.0	0.671
98.25	82.5	47.5	7.5	0.402
97.50	80.0	45.0	7.0	0.149
96.25	77.5	42.5	6.5	0.043
95.00	75.0	40.0	6.0	0.018
93.75	72.5	37.5	5.5	0.064
92.50	70.0	35.0	5.0	0.224
91.25	67.5	32.5	4.5	0.503
90.00	65.0	30.0	4.0	0.810

For demonstration purposes, we shall set α at the 0.05 level and β at the 0.10 level. Based on Table 13 we choose four pairs (P_α, P_β). That is, we shall have four plans for SSFD and TSPR, respectively. These are coded in Table 14.

TABLE 14
SPECIFICATION RISK OF EACH ACCEPTANCE SAMPLING PLAN

Plan Name		Specification			
SSFD	TSPR	α	P_α	β	P_β
SSFD 1	TSPR 1	0.05	0.01	0.10	0.10
SSFD 2	TSPR 2	0.05	0.02	0.10	0.20
SSFD 3	TSPR 3	0.05	0.03	0.10	0.30
SSFD 4	TSPR 4	0.05	0.02	0.10	0.30

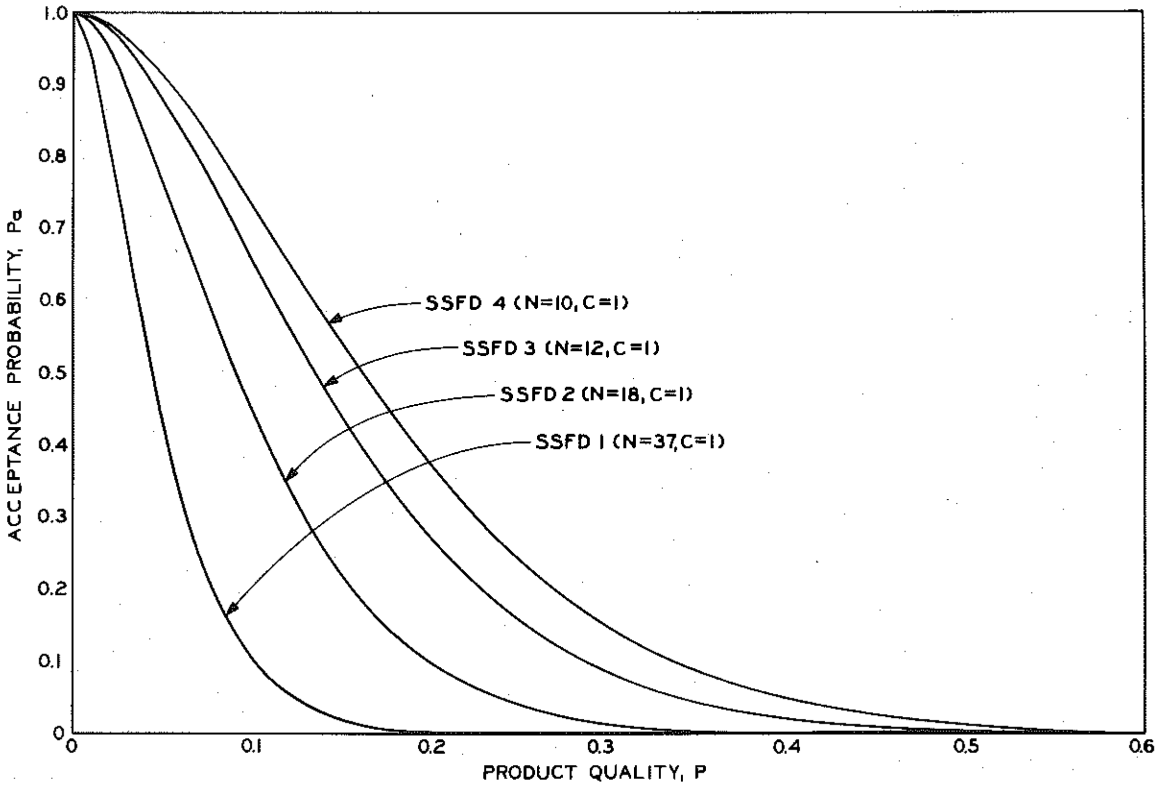

Figure 17. OC curves of SSFD 1, 2, 3, and 4.

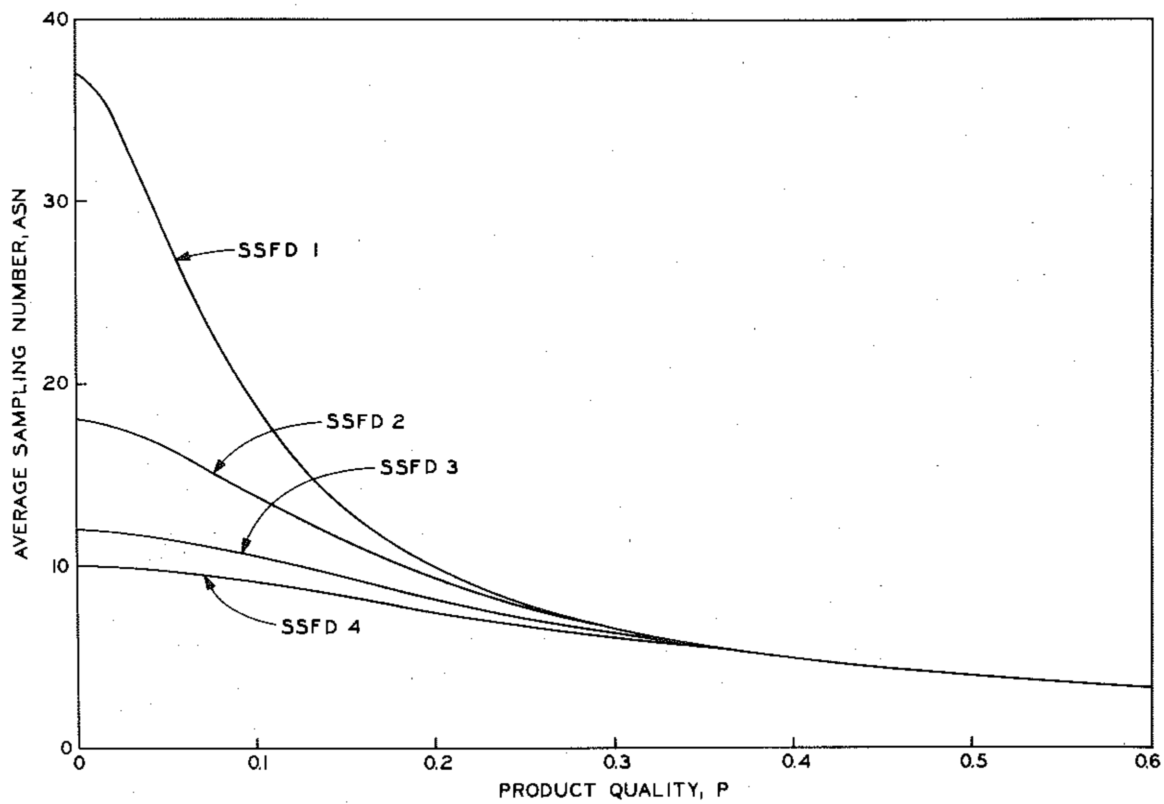

Figure 18. ASN curves of SSFD 1, 2, 3, and 4.

The details of each type of acceptance sampling plan are discussed in the following sections.

Single-Sampling Fraction Defective Sampling Plan (SSFD)

This type of plan specifies the sample size, N, that should be taken and the acceptance number, C, of the defective items that are allowed for lot acceptance. A lot here could be 1,000-ft or one mile of road length. That is, we accept a lot if the total number of defective samples among N samples taken from that lot is less than or equal to C; otherwise, we reject that lot. We desire to have a plan such that the acceptance probability is $1-\alpha$ for material of P_α quality and is β for material of P_β quality. Equations for the determination of the required sample size N and the acceptance number C are presented in Appendix H. Using Eq. (1) of Appendix H and the techniques of Grubbs (3), we obtain the plans given in Table 15.

TABLE 15
SINGLE-SAMPLING FRACTION DEFECTIVE ACCEPTANCE SAMPLE PLANS UNDER THE SPECIFICATION SET IN TABLE 14

Plan	Sample Size, N	Acceptance Number, C
SSFD 1	37	1
SSFD 2	18	1
SSFD 3	12	1
SSFD 4	10	1

For a given value of product quality P, the acceptance probability (OC curve) is now computed by Eq. (3), Appendix H. The results are plotted in Figure 17.

The number of defective samples taken during the inspection might be greater than C before all N samples are tested. If this situation occurs, there is no need to examine the remaining samples unless the estimate of the fraction defective is desired. Thus, the actual sample size needed to reach a decision (acceptance or rejection) is less than N. For a given value of P, the average sampling size ASN of each plan in Table 15 is computed by Eq. (3), Appendix H. The results are plotted in Figure 18. As one can see from Figure 18, the required average sampling number drops substantially as the product quality declines.

We combine Table 13, Figure 17, and Figure 18 to get an overall picture of the relation of aggregate composition to the acceptance probability and the average sampling size to reach a decision under SSFD 1, 2, 3, and 4. This is presented in Table 16.

TABLE 16
RELATION OF AGGREGATE COMPOSITION
TO THE ACCEPTANCE PROBABILITY
AND THE AVERAGE SAMPLING NUMBER UNDER SSFD 1, 2, 3, AND 4

Mean Aggregate Composition				SSFD 1		SSFD 2		SSFD 3		SSFD 4	
3/4-in.	3/8-in.	No. 8	L.B.W.	P_a	ASN	P_a	ASN	P_a	ASN	P_a	ASN
100.00	85.0	50.0	8.0	0.00	3.0	0.00	3.0	0.00	3.0	0.00	3.0
98.25	82.5	47.5	7.5	0.00	5.0	0.00	5.0	0.02	5.0	0.05	5.0
97.50	80.0	45.0	7.0	0.02	13.1	0.23	11.4	0.45	9.4	0.55	8.3
96.25	77.5	42.5	6.5	0.54	29.5	0.82	16.9	0.91	11.6	0.94	9.8
95.00	75.0	40.0	6.0	0.85	35.0	0.96	17.7	0.98	11.9	0.99	9.9
93.75	72.5	37.5	5.5	0.34	25.0	0.71	15.8	0.84	11.3	0.86	9.6
92.50	70.0	35.0	5.0	0.00	8.9	0.07	8.5	0.22	7.6	0.36	7.1
91.25	67.5	32.5	4.5	0.00	4.0	0.00	4.0	0.00	4.0	0.01	4.0
90.00	65.0	30.0	4.0	0.00	2.5	0.00	2.5	0.00	2.5	0.00	2.5

From Table 16 it seems either SSFD 2, SSFD 3, or SSFD 4 would give satisfactory protection to both producer and consumer. In addition, these three plans do not require large sample sizes to reach a final decision, which is a very important factor to be considered in adopting an acceptance sampling plan, especially if manpower is in short supply. Since reducing sample size is of great concern, we shall also consider Wald's sequential probability ratio plan which requires smaller sample sizes than SSFD on the average under the same specification of (α, P_α, β, P_β).

Wald's Sequential Probability Ratio Plan (SPR)

We shall describe SPR by using the graphical method, one sample taken at a time. The cumulated sample results are plotted consecutively in the chart. For each point the abscissa, n, is the total number of samples drawn up to that time and the ordinate, x, is the total number of these samples that are defective. If the plotted points stay within a zone marked by two parallel lines, the sampling is continued without a decision. As soon as a point falls outside that zone, we accept the lot if that point is on or below the lower line, and reject the lot if that point is on or above the upper line. The above concept is presented in Figures 19 through 21.

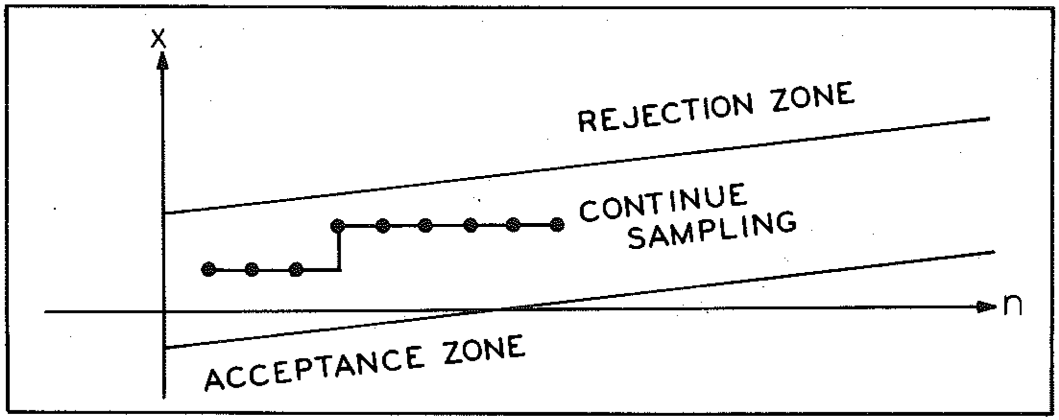

Figure 19. Decision has not been reached yet.

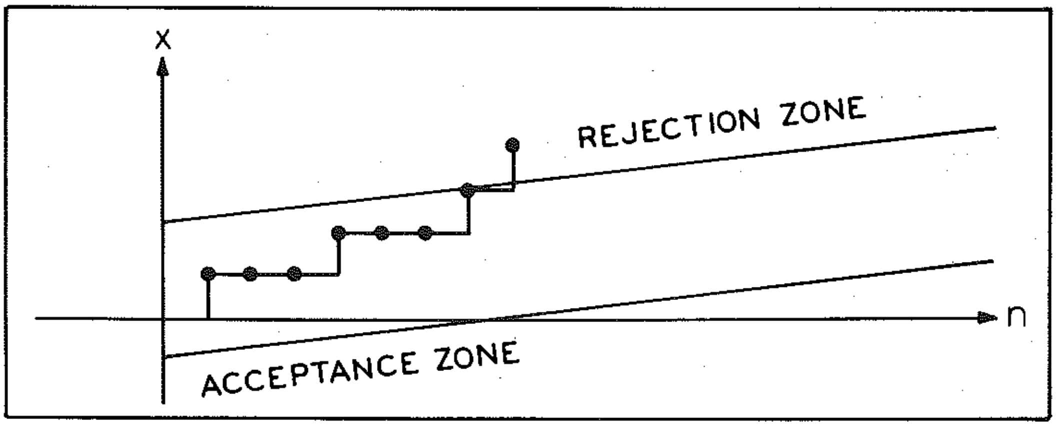

Figure 20. The lot is rejected after inspecting eight samples.

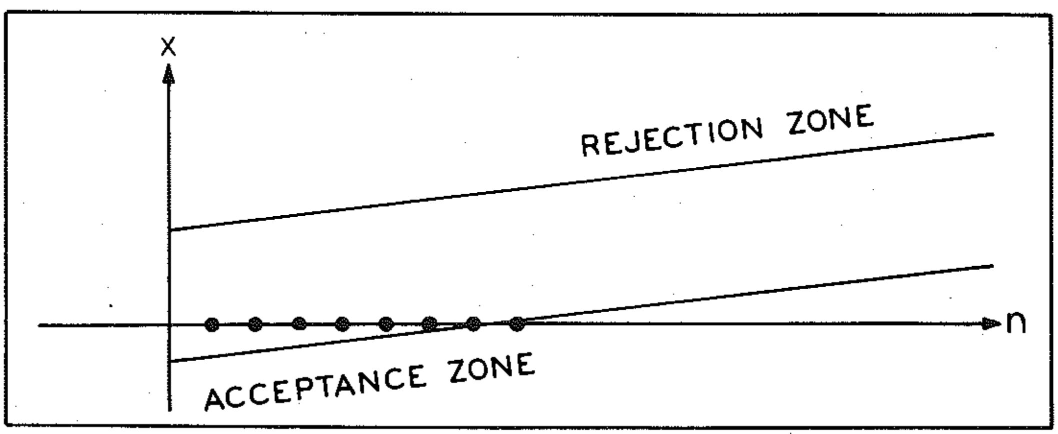

Figure 21. The lot is accepted after inspecting eight samples.

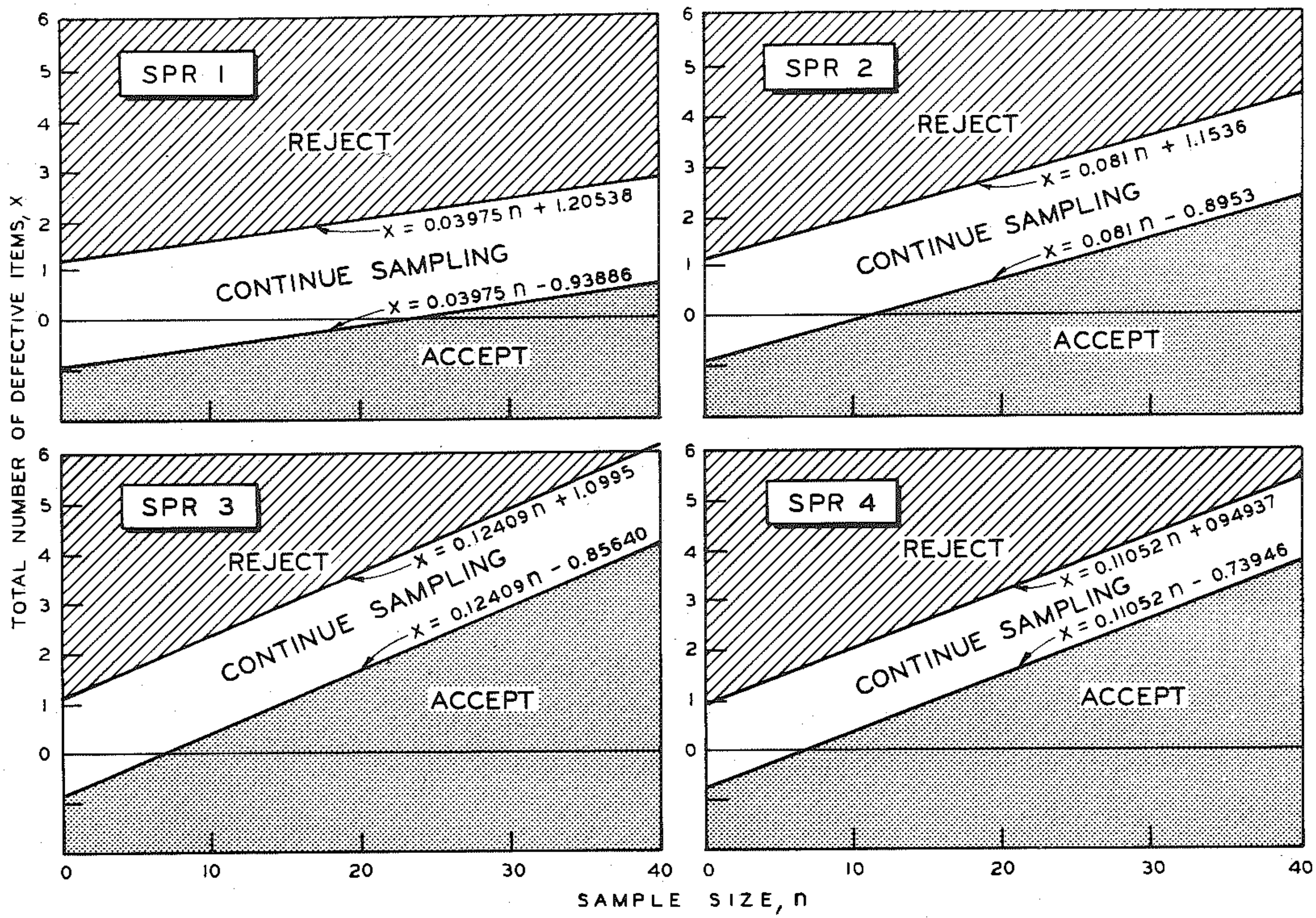

Figure 22. Acceptance, rejection and continuing sample zones of SPR 1, 2, 3, and 4.

Equations of these two parallel lines derived by Wald (5) are summarized in Appendix H. Using Eqs. (6), (7), and (8), Appendix H, we obtain the plans in Table 17. We also graph and illustrate the four plans in Figure 22 and Tables 18 and 19. The symbol * in Tables 18 and 19 means that no acceptance decision will be made for that sample size. That is, no acceptance decision will be made until 23, 11, 6, and 6 samples, respectively, for SPR 1, 2, 3, and 4 have been inspected. The acceptance probability (OC curve) and ASN curve of SPR 1, 2, 3, and 4 are computed by using Eqs. (9), (10) and (11), Appendix H. The results are presented, respectively in Figures 23 and 24. Comparing Figure 24 with Figure 18, we see that the average sampling size of the SPR plan is less than that of the SSFD plan.

TABLE 17
WALD'S SEQUENTIAL PROBABILITY RATIO ACCEPTANCE SAMPLING PLANS UNDER THE SPECIFICATION SET IN TABLE 14

Plan	h_1	h_2	s
SPR 1	0.93886	1.20538	0.03975
SPR 2	0.89853	1.15360	0.08100
SPR 3	0.85640	1.09950	0.12409
SPR 4	0.73946	0.94937	0.11052

Again, we tie the above results to Table 13 to get the relation of aggregate composition to the acceptance probability and the average sampling size under the SPR plan. These results are presented in Table 20.

Although Table 16 and Table 20 indicate that the SPR plan is superior to the SSFD plan as to average sampling cost, the actual sample size needed to reach a decision with the SPR plan might sometimes be much larger than that with the SSFD plan because of the continuation zone. Since reducing the sample size is our major concern, we shall revise the SPR plan by adding an extra termination rule. The revised plan will be called "Wald's Truncated Sequential Probability Ratio Plan."

TABLE 18

ILLUSTRATION OF ACCEPTANCE AND REJECTION NUMBERS FOR SPR 1

n	A	R	n	A	R	n	A	R	n	A	R
2	*	2	20	*	3	38	0	3	56	1	4
3	*	2	21	*	3	39	0	3	57	1	4
4	*	2	22	*	3	40	0	3	58	1	4
5	*	2	23	*	3	41	0	3	59	1	4
6	*	2	24	0	3	42	0	3	60	1	4
7	*	2	25	0	3	43	0	3	61	1	4
8	*	2	26	0	3	44	0	3	62	1	4
9	*	2	27	0	3	45	0	3	63	1	4
10	*	2	28	0	3	46	0	4	64	1	4
11	*	2	29	0	3	47	0	4	65	1	4
12	*	2	30	0	3	48	0	4	66	1	4
13	*	2	31	0	3	49	1	4	67	1	4
14	*	2	32	0	3	50	1	4	68	1	4
15	*	2	33	0	3	51	1	4	69	1	4
16	*	2	34	0	3	52	1	4	70	1	4
17	*	2	35	0	3	53	1	4	71	1	5
18	*	2	36	0	3	54	1	4	72	1	5
19	*	2	37	0	3	55	1	4	73	1	5

TABLE 19

ILLUSTRATION OF ACCEPTANCE AND REJECTION NUMBERS FOR SPR 2, 3, AND 4

n	SPR 2		SPR 3		SPR 4		n	SPR 2		SPR 3		SPR 4	
	A	R	A	R	A	R		A	R	A	R	A	R
2	*	2	*	2	*	2	27	1	4	2	5	2	4
3	*	2	*	2	*	2	28	1	4	2	5	2	5
4	*	2	*	2	*	2	29	1	4	2	5	2	5
5	*	2	*	2	*	2	30	1	4	2	5	2	5
6	*	2	*	2	*	2	31	1	4	2	5	2	5
7	*	2	0	2	0	2	32	1	4	3	6	2	5
8	*	2	0	3	0	2	33	1	4	3	6	2	5
9	*	2	0	3	0	2	34	1	4	3	6	3	5
10	*	2	0	3	0	3	35	1	4	3	6	3	5
11	*	3	0	3	0	3	36	2	5	3	6	3	5
12	0	3	0	3	0	3	37	2	5	3	6	3	6
13	0	3	0	3	0	3	38	2	5	3	6	3	6
14	0	3	0	3	0	3	39	2	5	3	6	3	6
15	0	3	1	3	0	3	40	2	5	4	7	3	6
16	0	3	1	4	1	3	41	2	5	4	7	3	6
17	0	3	1	4	1	3	42	2	5	4	7	3	6
18	0	3	1	4	1	3	43	2	5	4	7	4	6
19	0	3	1	4	1	4	44	2	5	4	7	4	6
20	0	3	1	4	1	4	45	2	5	4	7	4	6
21	0	3	1	4	1	4	46	2	5	4	7	4	7
22	0	3	1	4	1	4	47	2	5	4	7	4	7
23	0	4	1	4	1	4	48	2	6	5	8	4	7
24	1	4	2	5	1	4	49	3	6	5	8	4	7
25	1	4	2	5	2	4	50	3	6	5	8	4	7
26	1	4	2	5	2	4	51	3	6	5	8	4	7

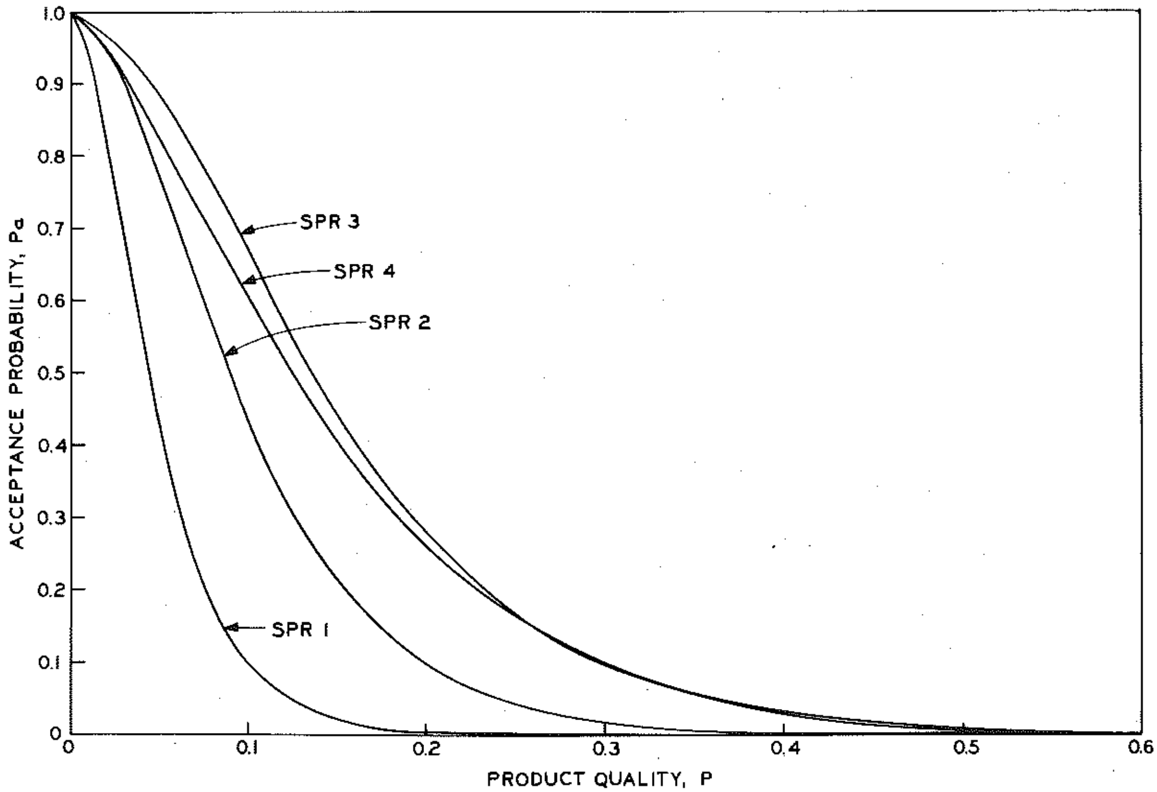

Figure 23. OC curves of SPR 1, 2, 3, and 4.

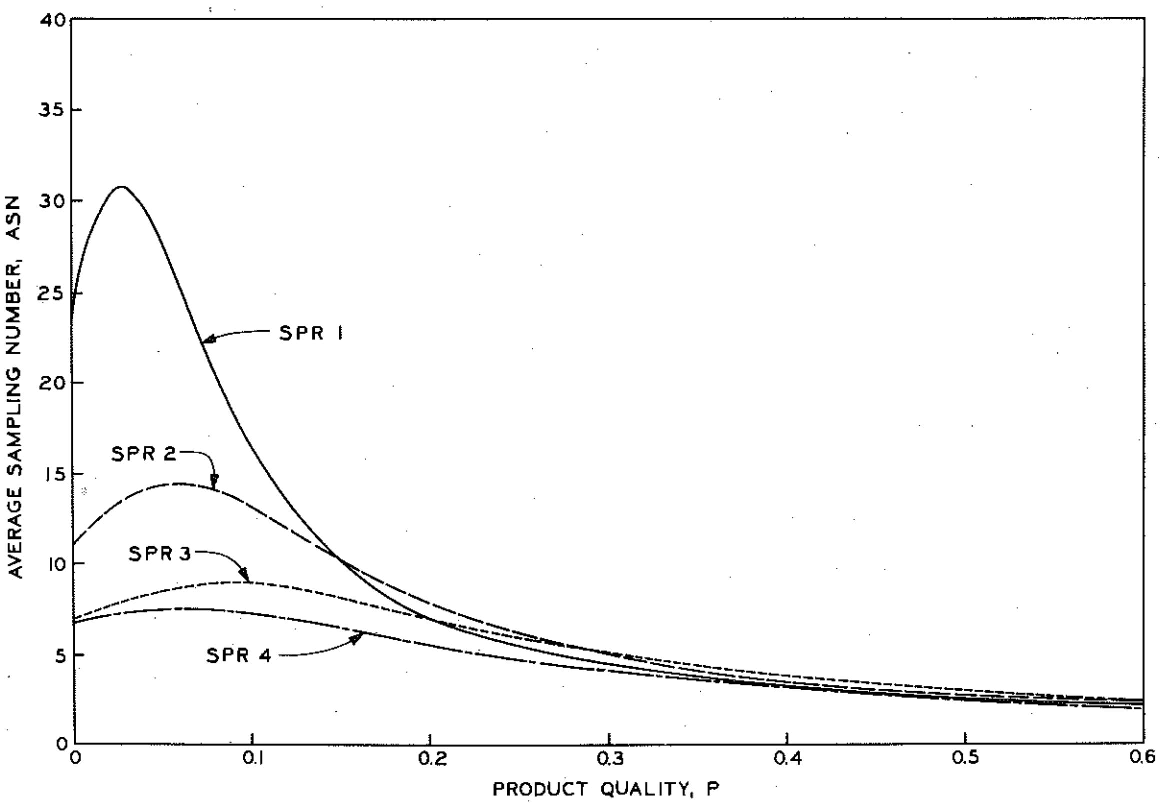

Figure 24. ASN curves of SPR 1, 2, 3, and 4.

TABLE 20
RELATION OF AGGREGATE COMPOSITION
TO THE ACCEPTANCE PROBABILITY
AND THE AVERAGE SAMPLING NUMBER UNDER SPR 1, 2, 3, AND 4

Mean Aggregate Composition				SSFD 1		SSFD 2		SSFD 3		SSFD 4	
3/4-in.	3/8-in.	No. 8	L.B.W.	Pa	ASN	Pa	ASN	Pa	ASN	Pa	ASN
100.00	85.0	50.0	8.0	0.00	1.9	0.00	2.0	0.00	2.0	0.00	1.7
98.25	82.5	47.5	7.5	0.00	3.2	0.00	4.4	0.03	4.3	0.04	3.1
97.50	80.0	45.0	7.0	0.02	10.5	0.21	10.0	0.44	8.1	0.44	6.6
96.25	77.5	42.5	6.5	0.52	29.2	0.83	14.2	0.86	8.3	0.91	7.5
95.00	75.0	40.0	6.0	0.86	30.0	0.96	13.1	0.98	7.7	0.95	7.2
93.75	72.5	37.5	5.5	0.30	24.4	0.68	14.4	0.76	8.7	0.83	7.5
92.50	70.0	35.0	5.0	0.00	6.5	0.07	7.0	0.22	6.6	0.15	5.2
91.25	67.5	32.5	4.5	0.00	2.9	0.00	2.6	0.01	2.4	0.02	2.2
90.00	65.0	30.0	4.0	0.00	1.6	0.00	1.6	0.00	1.7	0.00	1.4

Wald's Truncated Sequential Probability Ratio Plan (TSPR)

This type of plan (TSPR) is the same as the SPR plan except that it includes a new rule for the acceptance or rejection at the N_0^{th} trial if the SPR procedures did not lead to a final decision for $N \leq N_0$. By truncating the sequential process at the N_0^{th} trial we shall change the producer's and consumer's risk. The effect of the truncation on the producer's and consumer's risk will depend on the value of N_0. The larger the N_0, the smaller the effect of truncation on both risks. A reasonable choice is N_0 equal to 37 for TSPR 1, 18 for TSPR 2, 12 for TSPR 3 and 10 for TSPR 4. Next, we determine the acceptance number C_0 at the N_0^{th} trial. One method for the determination of C_0 suggested by Wald is presented in Eq. (13), Appendix H. The other method is to simply set C_0 as the acceptance number obtained under the SSFD plan. It turns out that C_0 is equal to 1 for TSPR 1, 2, 3, and 4 as determined by either method. These plans are presented in Figure 25 and illustrated in Table 21. For a given value of product quality P, the acceptance probability (OC curve) and the ASN curve are computed by Eqs. (14) through (23), Appendix H. The results are presented in Figures 26 and 27.

Comparing the OC and ASN curve of the TSPR plan with those of the SPR plan, we see that they do not differ by much. But, the TSPR plan will always lead to a decision on or before the N_0^{th} trial. This is the superiority of the TSPR plan over SPR plan especially when the testing procedures for each sample are time-consuming and costly.

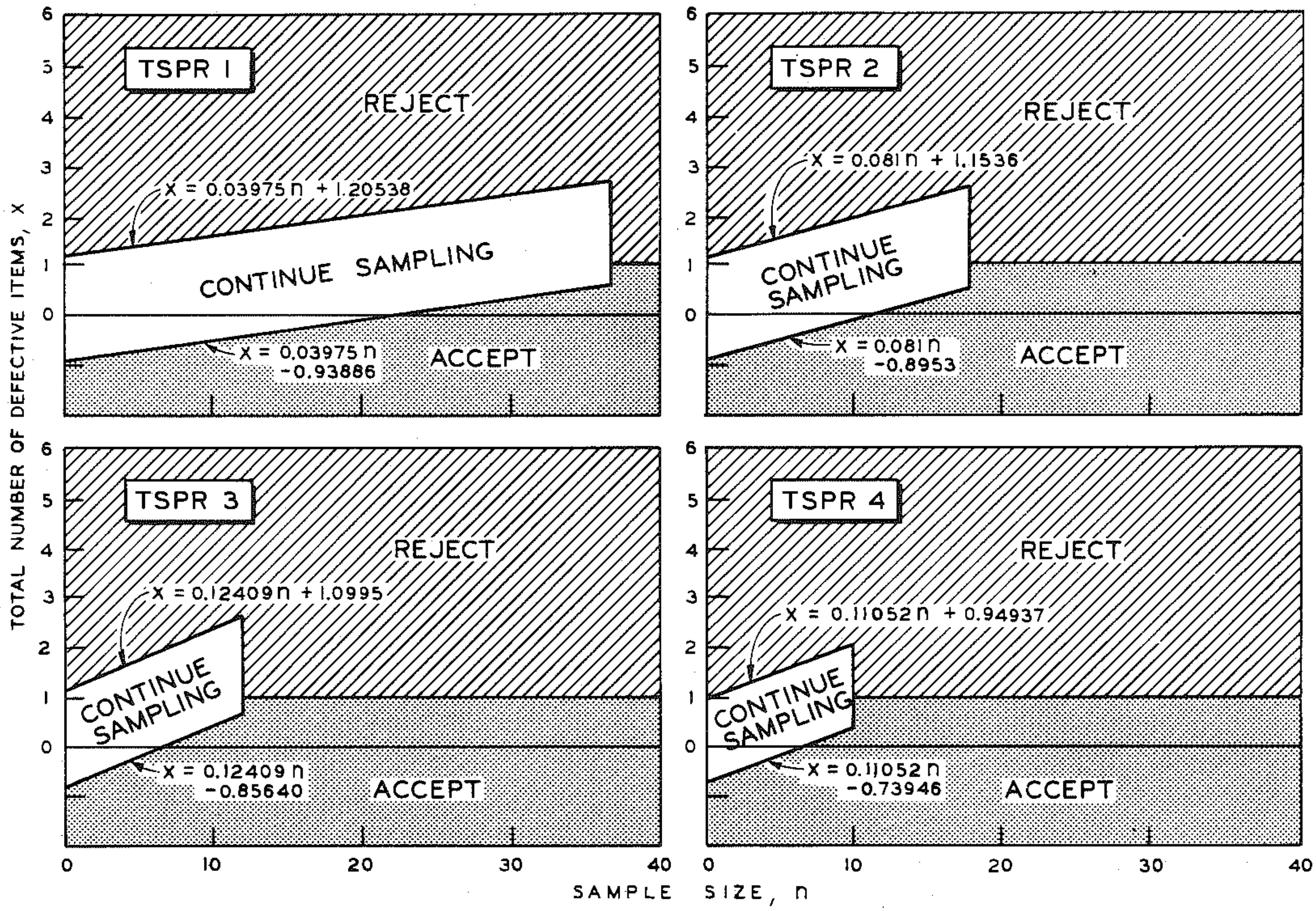

Figure 25. Acceptance, rejection and continuing sample zones of TSPR 1, 2, 3, and 4.

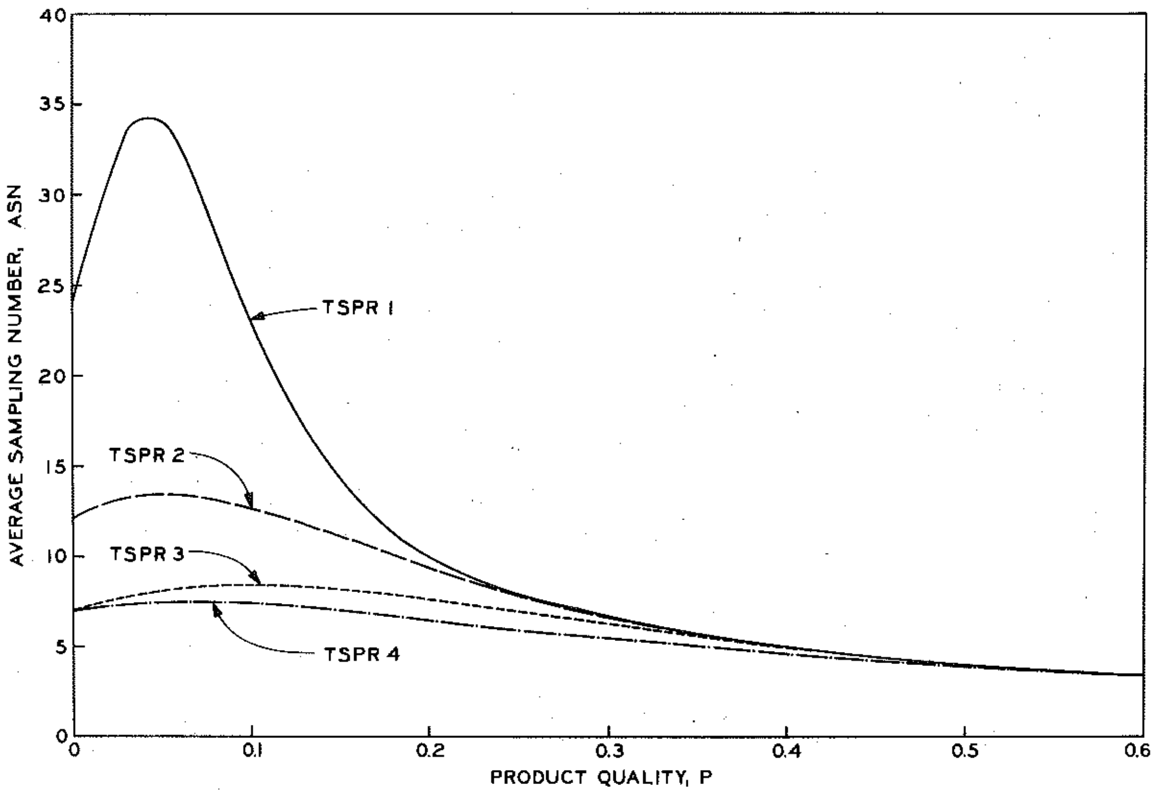

Figure 26. ASN curves of TSPR 1, 2, 3, and 4.

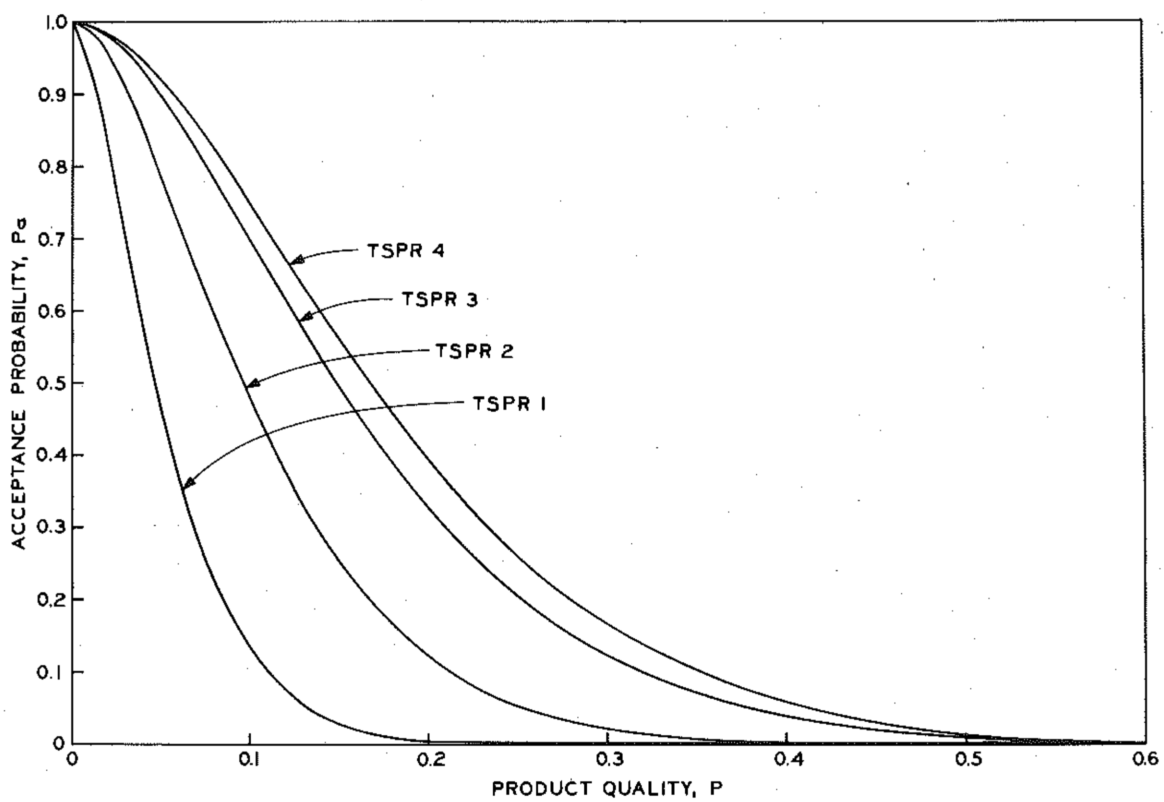

Figure 27. OC curves of TSPR 1, 2, 3, and 4.

TABLE 21
ILLUSTRATION OF ACCEPTANCE
AND REJECTION NUMBERS FOR
TSPR 1, 2, 3, AND 4

n	TSPR 1		TSPR 2		TSPR 3		TSPR 4		n	TSPR 1	
	A	R	A	R	A	R	A	R		A	R
2	*	2	*	2	*	2	*	2	20	*	3
3	*	2	*	2	*	2	*	2	21	*	3
4	*	2	*	2	*	2	*	2	22	*	3
5	*	2	*	2	*	2	*	2	23	*	3
6	*	2	*	2	*	2	*	2	24	0	3
7	*	2	*	2	0	2	0	2	25	0	3
8	*	2	*	2	0	3	0	2	26	0	3
9	*	2	*	2	0	3	0	2	27	0	3
10	*	2	*	2	0	3	1	2	28	0	3
11	*	2	*	3	0	3			29	0	3
12	*	2	0	3	1	2			30	0	3
13	*	2	0	3					31	0	3
14	*	2	0	3					32	0	3
15	*	2	0	3					33	0	3
16	*	2	0	3					34	0	3
17	*	2	0	3					35	0	3
18	*	2	1	2					36	0	3
19	*	2							37	1	2

Remarks

1) A variable plan can be used to control the fraction of defective material, provided that adequate aggregate uniformity has been achieved by the handling process. The advantage of the variable plan over the attribute plan is that a smaller sample size is needed to obtain the same risks. The designs of such a plan are very similar to those described in Chapter I-3, and will not be discussed here.

2) The determination of an acceptance sampling plan depends on the choices of α, P_α , β and P_β which are related to other factors such as the availability of manpower, the sensitivity of the aggregate composition to the pavement's performance and the Department's policies, etc. Discussions of these factors are beyond the scope of this project. Therefore, we are not in the position of recommending a particular plan that should be

used for in-place aggregate inspection practice. However, the material presented in this Chapter should be sufficient to design a plan for a particular need.

I-6. GENERAL CONSIDERATIONS OF THE CHOICE OF INSPECTION LOCATIONS SITUATED BETWEEN THE STOCKPILE AND THE CONSTRUCTION SITE

Since stockpiled aggregate is not the end-product of aggregate used in highway construction, and is significantly affected by the handling and compaction processes as shown in the previous Chapter, we naturally should consider shifting the aggregate inspection from the stockpile to the construction site.

It is clear that aggregate inspection can be performed at any one of the following three locations: stockpile, and construction site before and/or after compaction. Critical information needed to evaluate the cost-beneficial advantages of each potential inspection location are not available at the present time. Therefore, in this chapter, we shall only provide a general discussion concerning cost-benefit matters.

Policies for Handling Rejected In-Place Aggregates

In general, there are two policies for handling rejected in-place aggregate:

1) Return the Rejected Aggregate. The average aggregate quality level at the site under this policy will be at the desired level as specified by the Department's acceptance sampling plan. However, the aggregate price will be much higher than the usual price due to the extra cost of the compaction procedures and removing the rejected aggregate from the site.

2) Accept the Rejected Aggregate at a Reduced Price. The average aggregate quality level under this policy would probably be lower than the desired level specified by the Department's inspection plan.

The policy to be used depends both upon the sensitivity of the pavement's performance to aggregate gradation and cost considerations. Unless proper aggregate gradation is so critical to the pavement's performance, i.e., the cost of pavement damage due to improper aggregate gradation is higher than that mentioned in 1) above, the second policy should be adopted up to the point that the in-place aggregate is too poor to be used. At the present time, the relationship between aggregate gradation and pavement performance is not clearly understood. Thus, we do not have an accounting base for the choice of the two policies. In this circumstance, we would probably be better off in adopting the second policy and handle the rejected aggregate at the construction site.

Aggregate Inspection Before or After Compaction

If the aggregate inspection shall be performed at the construction site, should the Department inspect the in-place aggregate before or after compaction? Since the degradation rate of each type of aggregate due to compaction can be estimated, we certainly should inspect the in-place aggregate before the road is compacted to avoid the extra cost of the compaction procedures if rejected aggregate is to be removed and returned to the producer.

Even if the rejected aggregate is accepted at a reduced price, we might still want to inspect the in-place aggregate before compaction for the following reasons:

1) It is much easier to take samples from the site before compaction.

2) If there is a plan for correcting the unsatisfactory sections, it is advisable to know as soon as possible the aggregate composition and distribution.

Based on the above discussion, we recommend inspection of in-place aggregate before compaction if aggregate inspection is to be performed at the construction site.

Stockpile or In-Place Aggregate Inspection

The main advantage of in-place aggregate inspection is knowledge of the actual aggregate composition and the aggregate uniformity at the construction site. But, this information is not without cost. We either pay a high price for aggregate, or sacrifice the aggregate quality level in exchange for the above information. If this information can be inexpensively and accurately obtained from the stockpile, then stockpile inspection is certainly justified as a substitute for the in-place aggregate inspection. This is a very general statement of the choice between the stockpile and in-place inspection. In order to decide which one would be of most benefit, a cost and benefit analysis should be performed. Unfortunately, much information required in cost benefit analysis is not available. Therefore, we do not have the cost figures to judge the "best" location for aggregate inspection. This is why we discussed acceptance sampling plans for both stockpile and construction sites.

Discussion in this section offered general thoughts on the choice between stockpile and in-place aggregate inspection. Once the decision is made, the required inspection plan can be developed by following those steps described in this report.

Remarks and Suggestions

Based on the above discussion, a further investigation of the advantages or disadvantages of stockpile inspection versus in-place inspection in terms of costs should be conducted.

If the inspection practice is not to be performed at the construction site after compaction, the aggregate degradation rate of the handling and compaction processes for each type of aggregate should be estimated in advance.

PART II

PIFKE AND ANDERSON PITS

The Pifke pit was originally scheduled to supply all of the required aggregate for a constructed section of M 189. After this pit had furnished about 60 percent of the required aggregate, production was terminated for reasons not related to this study. The remaining aggregate was then supplied by a new pit, the Anderson pit, which is located about 1.25 miles to the south of the Pifke pit. Aggregate produced by the Pifke pit was later transported to the south end of the construction site, while aggregate from the Anderson pit was shifted to the other end. Thus, samples taken from the roadbed can be traced to their pit source.

The stockpile formation of these two pits is different from that of the Stillman pit studied in Part I. However, those statistical experiments conducted at the Stillman pit can be used without change since the objectives envisioned for these two pits are the same as those for the Stillman pit. Therefore, the same type of statistical techniques are used to analyze the data. To avoid redundancy, the conducted experiment and its analysis will not be fully described unless necessary.

Sampling Methods Used at the Production Site

The produced aggregate was first loaded and trucked to the stockpile site. Each dumping by a truck formed a cone shaped mass. At the end of the day, a scraper was used to flatten those cone shaped masses into a flat layer. Samples could thus be taken from that flattened layer. The problem with this sampling procedure is that sampling inspection cannot keep up with the production. In order to keep up with production, samples had to be taken from the formed cone hills. The current sampling method (CSM) for taking a sample from a truck is shown in Figure 28. As one can see, this constitutes a systematic sampling method (with fixed beginning). Again, the first step in studying these two pits is to design a random sampling scheme to compare with the CSM in estimating the aggregate composition of a stockpile. Since the covariance matrix of the aggregate composition in a truck was not known, and time did not permit conducting a study of the segregation pattern in a truck, 10 scoops from a truck, based on the experience gained from the Stillman pit, were used to form a composite sample according to a stratified random sampling scheme (SRSM) as shown in Figure 29. That is, a truck-load of aggregate is first stratified into two layers, basal and upper. Each layer is further stratified transversely into five strata as shown in Figure 29. Thus, 10 strata in total are formed in each sampled truck. One scoop of aggregate is randomly taken from each of the 10 strata. These 10 scoops of aggregate taken from a truck are then mixed together and reduced by a Gilson Sample Splitter to the test size to form a composite sample.

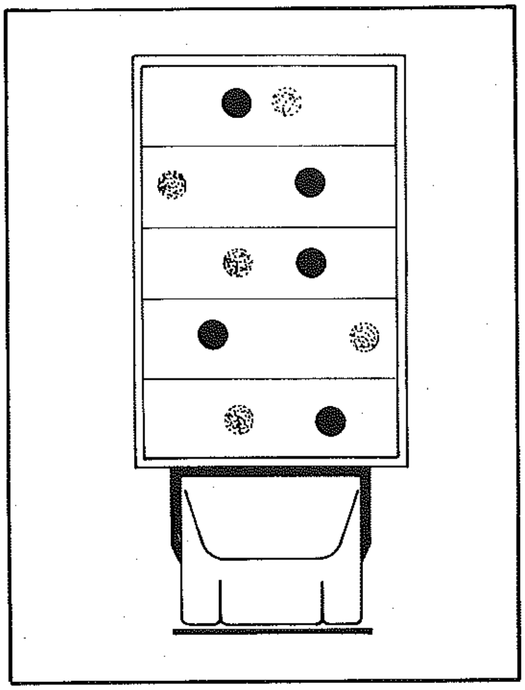

Figure 28. Sampling layout of the current sampling method for a truck.

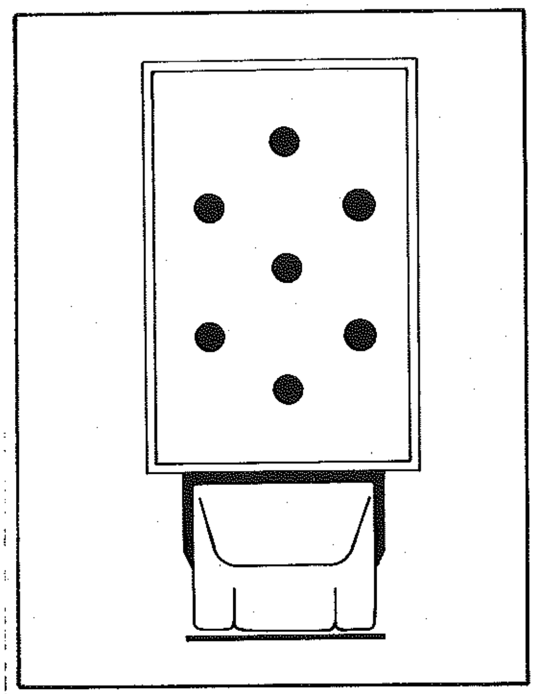

Figure 29. Sampling layout of the stratified random sampling method for a truck.

Both the CSM and the SRSM were used to collect samples from the Pifke and Anderson pits according to the experiment described in Chapter I-3. The data are analyzed in the next section.

Comparison of the CSM and the SRSM

The major conclusion, based on the statistical analysis of the above data, is that the two sampling methods are equally good in estimating the aggregate composition of stockpiled material. Therefore, because of its simplicity, the current sampling method should still be used for sampling aggregate from trucks.

The statistical work required to arrive at the above conclusion is described as follows. Readers who are not interested in statistical details can skip the remaining part of this section.

In continuous production run, every four or five hours produced about 50 truckloads of aggregate. Thus, every 50 trucks of aggregate were considered as one block. Eight trucks were randomly selected according to a stratified scheme for the eight combinations of three factors: testing location, inspector, and sampling method. The same type of difficulties in conducting such a statistical experiment as mentioned in Chapter I-3 force us again to analyze the data according to two designs; one for the field test and the other for the laboratory test. The results are described as follows:

1) Field Test: Two inspectors used the same set of sieves to measure the percent passing the 1-in., 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing, according to the designed experiment which is a two-way completely randomized block design. Each block in this case is a combination of production period, pit, and the sieve wearing process. The statistical analyses of the data show, at the 0.05 significance level, the following results:

a. There is no interaction between inspection and sampling method.

b. The two inspectors performed identically in the aggregate testing. This result was expected because the two inspectors had the same amount of working experience in this area.

c. The two sampling methods are equivalent in estimating the aggregate composition.

2) Laboratory Test: The same two inspectors for the field test again performed the laboratory test. Each inspector, however, used his own new set of sieves to measure the percent passing the 1-in., 3/4-in., 1/2-in., 3/8-in., No. 4, No. 8, No. 16, No. 30, No. 50, No. 100, and No. 200, and the percent loss-by-washing. The statistical analyses of the data show, at the 0.05 significance level, the following results:

a. There is no interaction between inspector-sieve and sampling method.

b. Two levels of the factor, inspector-sieve, have the same effect on the sample testing. Since the two inspectors performed the same in the aggregate testing as established by the field test, we conclude that two sets of sieves perform the same in measuring the aggregate composition.

c. As it should be, the two sampling methods are the same with regard to estimating the aggregate composition of a stockpile and agree with the field test.

It is indicated from the field and the laboratory tests that the CSM and the SRSM show no difference in estimating the mean aggregate composition. Since two inspectors are statistically the same in performing aggregate testing, the test results done by these two inspectors can be pooled to measure the aggregate composition of that block where samples were taken. The resultant data is of a one-way completely randomized block design. Each block is a combination of production period, pit, testing location, and sieve wearing process. Of course, the statistical analyses of this revised set of data again show, at the 0.05 significance level, that two sampling methods are the same in estimating the aggregate composition of a stockpile, thus agreeing with the previous analyses.

The estimates of the parameters relative to these two pits are presented in the following tables for later use. Tables 22 and 23 show estimates of the aggregate composition and its covariance matrix according to the various combinations of pit source, sampling method, and testing location.

TABLE 22
ESTIMATES OF THE AGGREGATE COMPOSITION
OF TWO PITS UNDER THE VARIOUS
SAMPLING METHODS AND TESTING LOCATIONS

Pit Source	Sampling Method	Testing Location	Percent Passing Sieves				Percent Loss-By-Washing
			1-in.	3/4-in.	3/8-in.	No. 8	
Pifke	CSM	Field	100	94.24	67.42	47.26	6.65
		Lab	100	95.08	68.34	47.11	6.98
	SRSM	Field	100	94.25	67.21	47.22	6.62
		Lab	100	94.47	65.79	44.77	6.87
Anderson	CSM	Field	100	95.56	68.71	44.84	5.23
		Lab	100	96.51	68.75	44.25	5.03
	SRSM	Field	100	95.28	68.23	44.62	5.14
		Lab	100	95.89	68.56	43.18	4.76

Tables 24 and 25 are obtained from Tables 22 and 23, respectively, by pooling the test results of the field and laboratory tests.

TABLE 23

ESTIMATES OF THE COVARIANCE MATRIX OF THE AGGREGATE COMPOSITION UNDER THE VARIOUS COMBINATIONS OF PIT SOURCE, SAMPLING METHOD AND TESTING LOCATION

	Size Combination	Pifke				Anderson			
		CSM		SRSM		CSM		SRSM	
		Field	Lab	Field	Lab	Field	Lab	Field	Lab
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	2.98	3.63	3.08	4.28	3.64	3.57	3.88	4.92
	3/4-in. and 3/8-in.	2.27	3.80	2.74	3.37	2.84	4.95	3.06	5.70
	3/4-in. and No. 8	1.35	1.84	1.60	1.65	1.59	3.28	1.62	2.52
	3/4-in. and L.B.W.	0.27	0.09	0.37	0.50	-0.34	0.57	-0.03	-0.30
	3/8-in. and 3/8-in.	10.21	15.70	10.50	13.45	7.62	18.18	9.44	16.68
	3/8-in. and No. 8	9.20	11.70	9.74	11.40	6.13	12.99	5.65	6.84
	3/8-in. and L.B.W.	1.07	1.00	1.10	0.97	-0.01	0.90	-0.05	-0.29
	No. 8 and No. 8	10.96	12.53	11.83	12.31	7.17	11.87	6.53	4.24
	No. 8 and L.B.W.	1.47	1.16	1.57	1.00	1.25	1.24	0.73	0.22
	L.B.W. and L.B.W.	0.66	0.72	0.66	0.62	1.74	1.63	1.72	1.16

TABLE 24
ESTIMATES OF THE AGGREGATE COMPOSITION OF TWO PITS UNDER THE VARIOUS SAMPLING METHODS

Pit Source	Sampling Method	Percent Passing Sieves				Percent Loss-By-Washing
		1-in.	3/4-in.	3/8-in.	No. 8	
Pifke	CSM	100	94.63	67.85	47.19	6.81
	SRSM	100	94.24	66.47	46.06	6.86
Anderson	CSM	100	96.04	68.73	44.54	5.13
	SRSM	100	95.59	68.39	43.90	4.95

TABLE 25
ESTIMATES OF THE COVARIANCE MATRIX OF THE AGGREGATE COMPOSITION UNDER THE VARIOUS COMBINATIONS OF PIT SOURCE AND SAMPLING METHOD

	Size Combination	Pifke		Anderson	
		CSM	SRSM	CSM	SRSM
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	3.29	4.57	3.60	4.39
	3/4-in. and 3/8-in.	2.99	2.78	3.89	4.38
	3/4-in. and No. 8	1.58	1.43	2.44	2.07
	3/4-in. and L.B.W.	0.46	4.23	0.12	-0.17
	3/8-in. and 3/8-in.	12.79	10.42	12.90	13.06
	3/8-in. and No. 8	10.38	8.93	9.56	6.25
	3/8-in. and L.B.W.	1.04	7.28	0.45	-0.17
	No. 8 and No. 8	11.70	10.28	9.52	5.38
	No. 8 and L.B.W.	1.33	0.78	1.25	0.48
	L.B.W. and L.B.W.	0.69	0.69	1.58	1.44

The test results of the two pits are further pooled together to estimate aggregate composition under the current and suggested sampling methods. The results are presented in Tables 26 and 27 to be used in later sections.

TABLE 26
ESTIMATES OF AGGREGATE COMPOSITION
OF THE PIFKE AND ANDERSON PITS
JOINTLY UNDER THE CSM AND THE SRSM

Sampling Method	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
CSM	100	95.22	68.22	46.08	6.11
SRSM	100	94.82	67.30	45.12	6.04

TABLE 27
ESTIMATES OF THE COVARIANCE MATRIX
OF THE PIFKE AND ANDERSON PITS
JOINTLY UNDER THE CSM AND SRSM

	Size Combination	CSM	SRSM
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	3.42	4.50
	3/4-in. and 3/8-in.	3.36	3.47
	3/4-in. and No. 8	1.94	1.71
	3/4-in. and L.B.W.	0.32	0.17
	3/8-in. and 3/8-in.	12.84	11.55
	3/8-in. and No. 8	10.04	7.78
	3/8-in. and L.B.W.	0.79	0.34
	No. 8 and No. 8	10.80	8.18
	No. 8 and L.B.W.	1.29	0.65
	L.B.W. and L.B.W.	1.06	1.00

We end this section by noting again that the current sampling method for sampling aggregate from trucks is recommended because of its simplicity.

Aggregate Change Due to Handling and Compaction

For this part of the experiment, we did not take samples from the construction site before compaction as in Part I. Therefore, the aggregate degradation rate due to compaction cannot be estimated. However, composite samples taken from the construction site after compaction, along with samples taken from its pit source, can be used to analyze the aggregate change due to handling and compaction. With those facts and the assumptions stated in Chapter I-4, the aggregate degradation rate due to handling and compaction can still be estimated.

First, we recall that every 50 trucks of aggregate in the production site was considered as one block. Thus, a "section" at the construction site was defined to be the rectangular area containing the same volume of aggregate as one truck-load of aggregate and every 50 consecutive sections was therefore defined as one block. The above definition is the same as the one used in Part I. Two trucks at the production site and two sections at the construction site from each block were randomly chosen and one composite sample was taken from each chosen truck and section according to the designated sampling methods. The sampling method used to take composite samples from trucks was the SRSM described above, while the sampling method used to take composite samples from the construction site was the one described in Chapter I-4. The test results of the two samples from each block were pooled to measure the aggregate composition of that block. The collected samples were tested in the laboratory according to two one-way completely randomized designs to study the effects of the handling and compaction processes on the aggregate composition; one for the Pifke pit and the other one for the Anderson pit as shown in Table 28. Although 12 measurements were obtained from each sample, four measurements (the percent passing the 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing) will be used to study the effects of handling and compaction on the aggregate composition. This is because that sample size is not large enough to give reliable results for analyzing the above data of 12 component observations, the percent passing the 1-in. sieve is always 100, the above four measurements are the required ones in the specification. The statistical analyses of the above data show, at the 0.05 significance level, that the handling and compaction processes significantly affect the aggregate composition of both pits.

The estimates of aggregate composition of the stockpile and the construction site after compaction for both pits are presented in Table 29. The 95 percent simultaneous confidence materials (SCI) for the mean differences on each component of the composition of stockpile and in-place aggregate are also presented in Table 29. As one can see from the table the only interval that does not cover zero for both pits is the one for the measurement on the percent loss-by-washing. This indicates that the handling and compaction processes only significantly affect the measurement on fine sieves.

TABLE 28
LAYOUT OF ONE-WAY COMPLETELY RANDOMIZED DESIGN FOR THE STUDY OF THE HANDLING AND COMPACTION PROCESSES EFFECTS ON AGGREGATE COMPOSITION

Pit	Pit Sample	In-Place Sample
Pifke	N = 25	N = 32
Anderson	N = 16	N = 20

TABLE 29
ESTIMATES OF THE AGGREGATE COMPOSITION AND THE 95 PERCENT SCI'S FOR THE MEAN DIFFERENCES ON EACH COMPONENT OF THE COMPOSITION OF STOCKPILE AND IN-PLACE AGGREGATE

		Percent Passing the Sieve			Percent Loss-By-Washing
		3/4-in.	3/8-in.	No. 8	
Pifke Pit	Construction Road	94.6672	68.6397	47.1763	7.72469
	Stockpile	93.7276	66.9172	46.1856	6.45400
	Difference	0.9396	1.7225	0.9907	1.27069
	LCL	-0.8065	-0.5661	-1.2687	0.75233
	UCL	2.6857	4.0111	3.2500	1.78910
Anderson Pit	Construction Road	96.1015	68.7330	44.0020	5.78250
	Stockpile	95.7206	67.8438	42.0931	4.75313
	Difference	0.3809	0.8892	1.9089	1.02940
	LCL	-1.2278	-2.0190	-0.0736	0.21510
	UCL	1.9897	3.7975	3.8913	1.84360

Since samples obtained from the stockpile and the construction site after compaction cannot be matched in pairs, the aggregate degradation rate of each aggregate size cannot be determined as was done in Chapter I-4. However, based on the experience gained from those experiments conducted at the Stillman pit, it is reasonable to assume that a significant portion of the aggregate change due to handling and compaction is because of aggregate degradation, and that the aggregate only degrades to the next smaller size. With these assumptions, the degradation rate of both pits can be determined by Eq. (14), Appendix G. The values of $\overline{A_i}$ and $\overline{B_i}$ can be found from Table 29 and are substituted into that equation, we obtain the following results (Table 30).

TABLE 30
AGGREGATE DEGRADATION RATE DUE TO THE HANDLING AND COMPACTION PROCESS

Aggregate Size	Degradation Rate	
	Pifke Pit	Anderson Pit
3/4-in.	0.149799	0.089008
3/8-in.	0.080468	0.039498
No. 8	0.067878	0.054899

Table 30 indicates that, for aggregate produced from the Pifke pit, 14.98, 8.05, and 6.79 percent of the total percentage retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade, and, for aggregate produced from the Anderson pit, 8.90, 3.95, and 5.49 percent of the total percentage retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade.

Thus, we conclude that:

1) The handling and compaction processes significantly change aggregate composition. Based on the previous experience, the significant portion of aggregate change is the aggregate degradation.

2) Under reasonable assumptions, the degradation rate of each aggregate size due to handling and compaction can be estimated and are presented in Table 30.

Uniformity of In-Place Aggregate

To study uniformity of in-place aggregate, one non-composite sample was taken from each section chosen for the previous study of aggregate degradation. Estimates of the covariance matrix of aggregate composition at the construction site based on non-composite and composite samples are presented in Table 31.

TABLE 31
ESTIMATES OF THE COVARIANCE MATRIX OF AGGREGATE COMPOSITION OF THE CONSTRUCTION SITE UNDER VARIOUS COMBINATIONS OF PIT SOURCE AND SAMPLING METHOD

	Size Combination	Pifke		Anderson	
		Composite	Non-Composite	Composite	Non-Composite
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	3.93	4.33	3.02	5.42
	3/4-in. and 3/8-in.	2.68	2.50	2.07	3.24
	3/4-in. and No. 8	1.92	1.29	0.90	1.41
	3/4-in. and L.B.W.	0.25	0.05	0.09	0.12
	3/8-in. and 3/8-in.	5.80	4.28	4.80	8.48
	3/8-in. and No. 8	4.18	2.75	3.21	4.64
	3/8-in. and L.B.W.	0.40	0.49	0.33	0.64
	No. 8 and No. 8	4.06	2.63	3.49	3.87
	No. 8 and L.B.W.	0.53	0.50	0.53	0.52
	L.B.W. and L.B.W.	0.32	0.41	0.37	0.30

The test results of the two pits are further pooled together to estimate the covariance matrix of aggregate composition based on non-composite and composite samples. The results are presented in Table 32.

The covariance matrix of aggregate composition based on composite samples presented in Table 32 is obviously different (as can be statistically shown) from that of aggregate composition based on the SRSM presented in Table 27. Since the variance of each aggregate component based on the SRSM is larger than that based on the composite sampling method used at the construction site, and because these two sampling methods are very similar, we conclude that the handling process has significantly improved the aggregate uniformity. The above conclusion is also supported by the arguments that follow.

TABLE 32
ESTIMATES OF THE COVARIANCE MATRIX
OF AGGREGATE COMPOSITION AT THE
CONSTRUCTION SITE BASED ON COMPOSITE
AND NON-COMPOSITE SAMPLES

	Size Combination	Composite	Non-Composite
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	3.56	4.77
	3/4-in. and 3/8-in.	2.43	2.80
	3/4-in. and No. 8	1.51	1.34
	3/4-in. and L.B.W.	0.18	0.08
	3/8-in. and 3/8-in.	5.40	5.99
	3/8-in. and No. 8	3.79	3.52
	3/8-in. and L.B.W.	0.37	0.55
	No. 8 and No. 8	3.83	3.14
	No. 8 and L.B.W.	0.53	0.51
	L.B.W. and L.B.W.	0.34	0.37

For the measurement on the sieve corresponding to the index i, denote α_{ci}^2 and α_{Ni}^2 to be the true variances based on composite and non-composite sampling methods, respectively. We also denote α_1^2 and α_2^2 to be the errors introduced by the sample size (weight) corresponding to the composite and non-composite sampling methods, respectively. The other errors introduced by the Gilson Sample Splitter, testing method, inspector, etc., are denoted as α_i^2. Then, the sample variance of each component of the aggregate composition based on composite and non-composite sampling methods in Table 32 can be expressed as $\alpha_{ci}^2 + \alpha_1^2 + \alpha_i^2$ and $\alpha_{Ni}^2 + \alpha_2^2 + \alpha_i^2$, respectively. Since each composite sample is made up of 10 scoops, α_{Ni}^2 should be 10 times α_{ci}^2, theoretically. In this circumstance, the difference between these two numbers is small if α_{Ni}^2 is relatively small with respect to $\alpha_1^2 + \alpha_i^2$ and $\alpha_2^2 + \alpha_i^2$, and large if α_{Ni}^2 is relatively large with respect to $\alpha_1^2 + \alpha_i^2$ and $\alpha_2^2 + \alpha_i^2$. As one can see from

Table 32, the difference between the two variances of each aggregate component is quite small (statistically insignificant). This indicates that the true variance of each component of the aggregate composition based on the non-composite sampling method is small relative to the error introduced by the Gilson Sample Splitter and other random errors. With the presence of all of these random errors, we must conclude that the handling process has improved the aggregate uniformity up to the point that the precision of the estimation cannot be increased significantly by means of the composite method.

Thus, the major conclusion of this section is that the handling process does improve the uniformity of in-place aggregate.

Acceptance Sampling Plans

The acceptance sampling plans for in-place aggregate have been discussed in detail in Chapter I-5. Therefore, in this section we shall only discuss the acceptance sampling plans for stockpile (truck) inspection. As explained in Chapter I-3, the proper type of the acceptance sampling plan for truck inspection is again the so-called "acceptance sampling by variables" which gives assurance regarding the mean aggregate composition of a stockpile.

The covariance matrix of aggregate composition and percent crushed under the CSM was estimated (see Table 27) to be:

$$\hat{\Sigma} = \begin{bmatrix} 3.42 & 3.36 & 1.94 & 0.32 & -2.95 \\ & 12.84 & 10.04 & 0.79 & -5.14 \\ & & 10.80 & 1.29 & 4.14 \\ & & & 1.06 & 2.32 \\ & & & & 20.26 \end{bmatrix}$$

Using Equations presented in Appendix E, we obtain the following acceptance sampling plans for stockpile inspection (Table 33). The difference between Plan A_i and Plan B_i in Table 33 was explained in Chapter I-3.

Plan A_i is interpreted as follows: For each lot, say 50 or 100 trucks, 10 samples are taken according to the current sampling method. If the average percent passing each sieve and the average percent loss-by-washing of these samples fall within the acceptance intervals of Plan A_i specified in Table 33 the lot represented by these samples is accepted; otherwise, the layer made up by those trucks is rejected.

TABLE 33

ACCEPTANCE SAMPLING PLANS BY VARIABLES TO CONTROL THE MEAN AGGREGATE COMPOSITION UNDER THE CSM FOR STOCKPILE (TRUCK) INSPECTION

Plan	Risk		Sample Size	Acceptance Interval For Each Measurement										
				1-in.		3/4-in.		3/8-in.		No. 8		Loss-By-Washing		Crushed Aggregate
	α	β		LCL	UCL	LCL	UCL	LCL	UCL	LCL	UCL	LCL	UCL	LCL
A_1	0.05	0.05	11	100	100	93.14	96.86	71.41	78.59	36.70	43.30	4.97	7.03	35.88
A_2	0.01	0.01	15	100	100	93.14	96.86	71.41	78.59	36.70	43.30	4.97	7.03	35.75
A_3	0.05	0.01	13	100	100	93.29	96.71	71.69	78.31	36.97	43.03	5.05	6.95	36.20
B_1	0.05	0.05	11	100	100	91.70	98.31	66.09	83.92	33.01	46.99	4.94	7.06	29.12
B_2	0.01	0.01	15	100	100	91.75	98.25	68.39	81.61	33.10	46.90	4.97	7.03	29.57
B_3	0.05	0.01	13	100	100	91.88	98.12	63.64	81.36	33.34	46.66	5.04	6.96	28.79

The other plans can be interpreted in the same way.

The acceptance probability, P_a, of each plan in Table 33 is computed by using Eq. (12), Appendix E, with the help of a simulation computer program. The results are presented in Table 34.

We end this section with Table 35 which specifies the product quality of in-place aggregate in terms of the aggregate composition based on the non-composite sampling method, assuming that the aggregate covariance matrix of non-composite samples in Table 32 is the true covariance matrix.

Summary

1) The current method used for truck sampling is recommended.

2) Estimates of aggregate composition and their covariance matrices are presented in Tables 22 through 27.

3) The potential candidates for acceptance sampling plans for stockpile (truck) aggregate inspection are presented in Table 33.

4) The product quality of in-place aggregate in terms of the aggregate composition based on the non-composite sampling method is presented in Table 34. This table can be used as a reference when choosing the specification risks for the determination of an acceptance sampling plan for the in-place aggregate inspection discussed in Chapter I-5.

5) The handling and compaction processes significantly affect the aggregate composition. With suitable assumptions, the degradation rate of each aggregate size due to handling and compaction is estimated and presented in Table 30.

6) The handling process does improve the uniformity of in-place aggregate.

TABLE 34
ACCEPTANCE PROBABILITY OF EACH PLAN SPECIFIED IN TABLE 12 WITH THE MEAN PERCENT CRUSHED AGGREGATE SET AT 60

Mean Aggregate Composition, μ				Acceptance Probability of Plan					
3/4-in.	3/8-in.	No. 8	L.B.W.	A_1	A_2	A_3	B_1	B_2	B_3
100.00	85.0	50.0	8.0	0.000	0.000	0.000	0.000	0.000	0.000
98.25	82.5	47.5	7.5	0.000	0.000	0.000	0.035	0.011	0.003
97.50	80.0	45.0	7.0	0.012	0.004	0.004	0.547	0.518	0.382
96.25	77.5	42.5	6.5	0.670	0.726	0.587	0.966	0.980	0.950
95.00	75.0	40.0	6.0	1.000	1.000	1.000	1.000	1.000	1.000
93.75	72.5	37.5	5.5	0.661	0.722	0.570	0.950	0.968	0.930
92.50	70.0	35.0	5.0	0.013	0.010	0.008	0.525	0.507	0.381
91.25	67.5	32.5	4.5	0.000	0.000	0.000	0.012	0.004	0.003
90.00	65.0	30.0	4.0	0.000	0.000	0.000	0.000	0.000	0.000

TABLE 35
RELATION OF THE AGGREGATE COMPOSITION AND THE PRODUCT QUALITY

Mean Aggregate Composition				Product Quality, P
Percent Passing Sieve			Loss-By-Washing	
3/4-in.	3/8-in.	No. 8		
100.00	85.0	50.0	8.0	0.679
98.25	82.5	47.5	7.5	0.276
97.50	80.0	45.0	7.0	0.058
96.25	77.5	42.5	6.5	0.010
95.00	75.0	40.0	6.0	0.005
93.75	72.5	37.5	5.5	0.050
92.50	70.0	35.0	5.0	0.176
91.25	67.5	32.5	4.5	0.453
90.00	65.0	30.0	4.0	0.823

PART III

THE FEASIBILITY OF A MORE EFFICIENT AGGREGATE GRADATION TESTING METHOD

Whenever there is a need to increase testing sample size, e.g., for reducing the risk of accepting poor quality aggregate and rejecting good quality aggregate, testing efficiency becomes very important if extra manpower is either costly or unavailable. This brings our attention to the range of testing procedure choices for aggregate. We would like the most efficient testing procedure available consistent with practical field and laboratory concerns. The current Michigan 22A aggregate testing procedure is shown in Figure 30.

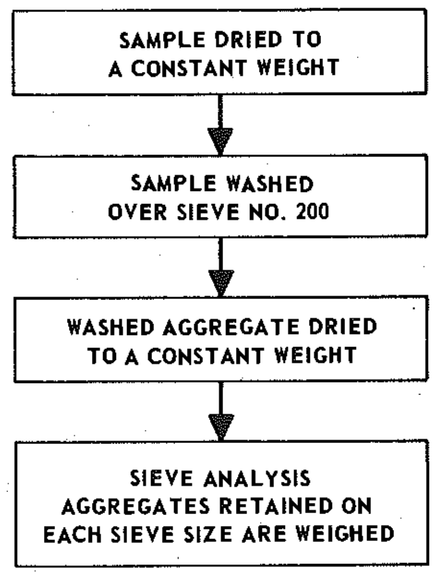

Figure 30. Current 22A aggregate testing procedure.

In this method, the washing process is followed by an extra drying, thus requiring two dryings per sample. However, the drying process is time consuming and the water supply is often troublesome and expensive in the field. Therefore, the simplified testing procedure shown in Figure 31 is proposed as a substitute for the current testing procedure.

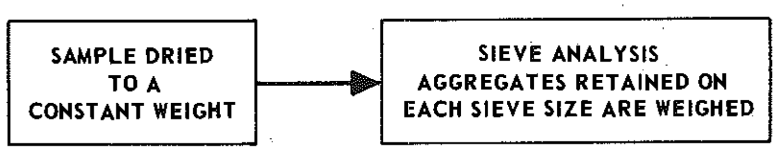

Figure 31. The proposed testing method.

This proposed testing method requires only one drying and no washing. Consequently, considerable increase in testing efficiency and considerable reduction in cost should result. However, we must consider whether or

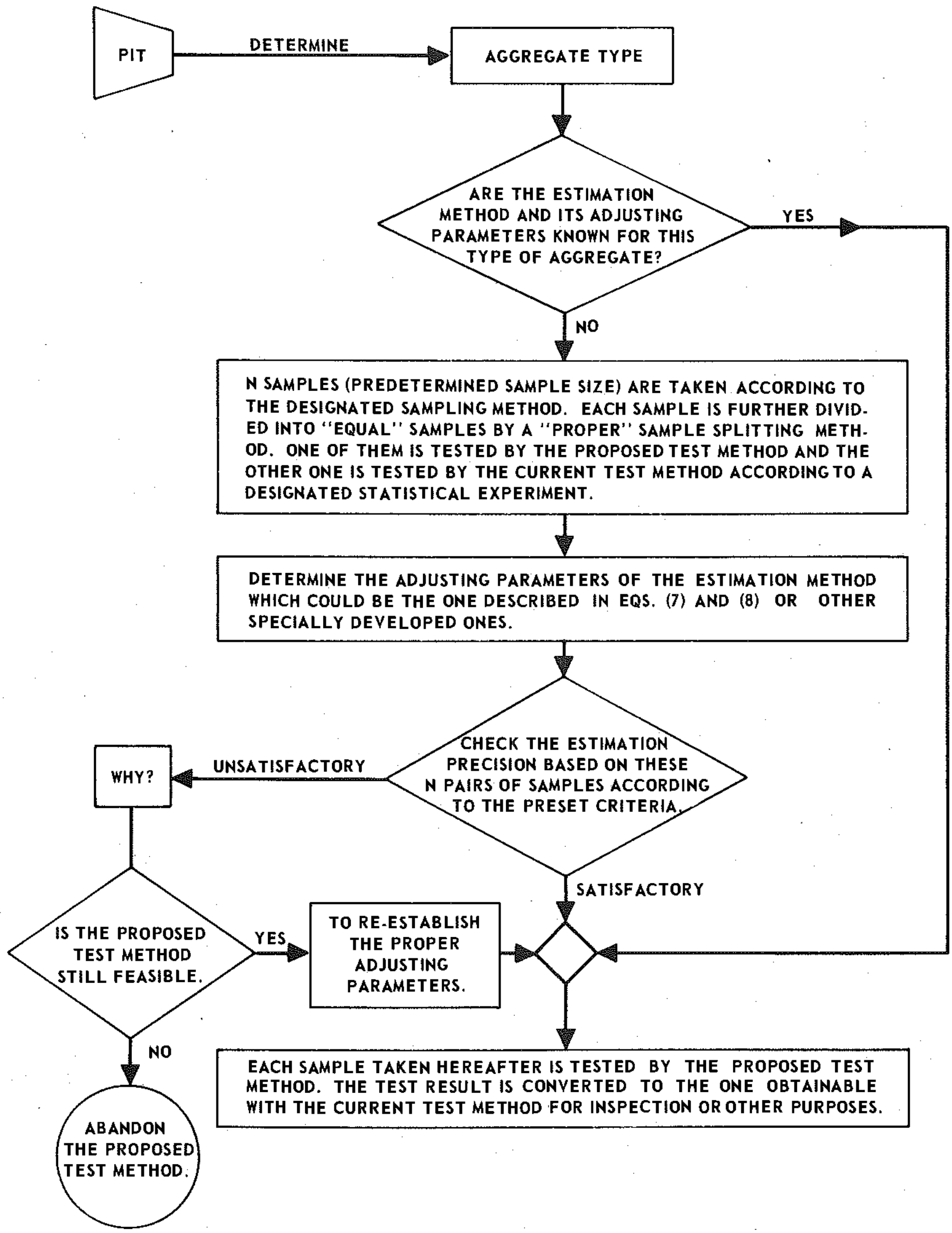

Figure 32. The procedures required in using the proposed test method.

not the proposed test method is as good as the current one for measuring aggregate composition. That is, whether or not the aggregate composition can be as accurately estimated using the proposed test method. A statistical experiment was conducted for this purpose. The description of the experiment and the statistical evaluation of the proposed testing method are presented in Appendix I. Based on this experiment, the following conclusions and recommendations are made.

Conclusions

The proposed testing method, together with the estimation procedures, appear to be as good as the current testing method for measuring aggregate composition. However, after taking into account the factors of cost reduction and time efficiency, the proposed testing method is certainly recommended.

Implementation of the Proposed Test Method

The proposed test method procedures are described in Figure 32.

The work plan presented in Figure 32 would be subject to modification as practical consideration requires. For example, implementing a check plan to update the adjusting parameters might be a good operating system for pits involving large quantities of aggregate production.

Recommendations

If the adjusting parameters are not known for a pit producing small quantities of aggregate, the work plan presented in Figure 32 might not be feasible since the total amount of time needed to establish the initial adjusting parameters might be a sizable fraction of the total production time. Moreover, considerable information needs to be established such as the sample size, N, required to establish reliable adjusting parameters, a sample splitting method, criteria to evaluate the estimation method, etc. This must be known prior to the implementation of the proposed test method. We suggest that a large scale study for such purposes should be carried out. A brief outline for such a study would be:

a) Study the necessity for categorization of aggregate types

b) Collect samples from various production pits (aggregate types) under some statistical rules of practice

c) Develop a proper sample splitting method

d) Test the collected samples by both the current and proposed test methods

e) Use multivariate statistical analysis to analyze the effects of the testing methods on various types of aggregate

f) Develop all feasible estimation procedures based on the collected data from various aggregate types; the best method among the developed methods should be pointed out for future reference

g) Prepare a handbook guiding the use of the proposed test method. The handbook should include sufficient information such as the estimation method and its adjusting parameters for each type of aggregate, the sample size required to establish the initial adjusting parameters or to update the adjusting parameters if needed, the evaluation criteria, etc.

h) Thoroughly compare the test efficiency of the two test methods; reduction in cost using the proposed test method should be carefully estimated to facilitate a cost benefit analysis

i) Overall advantages and disadvantages of the two test methods should be reviewed and recommendations presented.

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APPENDICES

GLOSSARY

α:	Producer's Risk
β :	Consumer's Risk
CSM:	Current Sampling Method
LCL:	Lower Confidence Limit
P_α:	Acceptable Quality Level
P_β :	Rejected Quality Level
SPR:	Wald's Sequential Probability Ratio Plan
SRSM:	Stratified Random Sampling Method
SSFD:	Single Sampling Fraction Defective Sampling Plan
TSPR:	Wald's Truncated Sequential Probability Ratio Plan

APPENDIX A

COMPUTATIONAL FORMULAS FOR MULTIVARIATE STATISTICAL ANALYSIS METHODS USED IN THIS REPORT

Frequently, an experiment requires the simultaneous measurement of a group of interrelated variables. For example, the gradation test of 22A aggregate measures the percent passing the 1-in., 3/4-in., 3/8-in., and No. 8 sieves and the percent loss-by-washing to describe the aggregate composition of a sample. The proper statistical techniques for analyzing the data from this kind of experiment are called "multivariate statistical analysis."

It is the purpose of this appendix to summarize the computational formulas of the proper statistical analysis for experiments conducted in this report. The materials presented in this appendix can be found in standard multivariate statistical analysis texts such as Ref. (1) and (4).

Throughout this appendix, the bold letter stands for either a column vector or a matrix. The superscript t of a vector or a matrix means the transpose of that vector or matrix. As in the example cited previously, $\mathbf{X} = (x_1, \cdots, x_5)^t$ is the observation vector of the composition of a sample of aggregates.

Estimates of the Mean Vector and Covariance Matrix of a Multinormal Population

A random vector $\mathbf{X} = (X_1, \cdots, X_P)^t$ is considered to be distributed according to the (p-dimensional) multinormal law with mean vector $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_P)^t$ and non-singular covariance matrix $\boldsymbol{\Sigma} = [\sigma_{ij}]$ IF $\sum_{i=1}^{P} b_i X_i$ for every non-null vector $\mathbf{b} = (b_1, \cdots, b_P)^t$ is normally distributed. Arrange the N observation vectors collected from this multinormal population in the following matrix form. The $\mathbf{X}_i$, $i = 1, \cdots, N$, , in Equation (A-1).

$$\mathbf{X} = \begin{bmatrix} x_{11}, \cdots, x_{1P} \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ x_{N1}, \cdots, x_{NP} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^t \\ \cdots \\ \cdots \\ \mathbf{x}_N^t \end{bmatrix} \qquad \text{(A-1)}$$

is the i^{th} observation vector. Then, the maximum-likelihood estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ based on these N observation vectors are the sample average vector $\bar{\mathbf{X}} = (\bar{x}_1, \cdots, \bar{x}_P)^t$ and the sample covariance matrix $\hat{\boldsymbol{\Sigma}} = [\hat{\sigma}_{ij}]$, respectively, where

$$\bar{x}_i = \frac{1}{N} \sum_{K=1}^{N} x_{Ki}, \quad i = 1, \cdots\cdots, P \qquad \text{(A-2)}$$

and

$$\hat{\sigma}_{ij} = \frac{\sum_{K=1}^{N}(x_{Ki}-\bar{x}_i)(x_{Kj}-\bar{x}_j)}{N} = \frac{1}{N}\sum_{K=1}^{N} x_{Ki}\, x_{Kj} - \bar{x}_i\,\bar{x}_j\,,\quad i,j = 1,\cdots,P \qquad \text{(A-3)}$$

It can be easily shown that $\hat{\Sigma}$ is a biased estimate of Σ, and we shall adopt instead the unbiased modification $S = \left[s_{ij} \right]$, where

$$s_{ij} = \frac{\sum_{K=1}^{N}(x_{Ki}-\bar{x}_i)(x_{Kj}-\bar{x}_j)}{N-1}\,,\quad i,j = 1,\cdots,P \qquad \text{(A-4)}$$

The S will be called throughout the appendix, the sample covariance matrix.

Frequently, samples are collected on K independent groups with N_i observation vectors in the i^{th} group. In this situation, computing the sample average vector $\bar{\mathbf{x}}_h = (\bar{x}_{1h},\cdots,\bar{x}_{Ph})$ and the sample covariance matrix $S_h = \left[s_{ij}^{(h)} \right]$ of the h^{th} data group using Eqs. (A-2) and (A-4), then the unbiased estimates of the mean vector and the covariance matrix, are respectively:

$$\bar{x}_i = \frac{1}{N}\sum_{h=1}^{K} N_h\,\bar{x}_{ih} \qquad \text{(A-5)}$$

and

$$s_{ij} = \frac{1}{N-K}\sum_{h=1}^{K}(N_h - 1)\, s_{ij}^{(h)} \qquad \text{(A-6)}$$

where

$$N = \sum_{h=1}^{K} N_h \qquad \text{(A-7)}$$

Test for the Assigned Mean Vector

Based on K groups of data with N_i observation vectors in the i^{th} group, we would like to test whether the population mean vector $\boldsymbol{\mu} = (\mu_1,\cdots,\mu_P)^t$ is equal to a particular vector $\boldsymbol{\mu}_0 = (\mu_{01},\cdots,\mu_{0P})^t$. That is, the null hypothesis to be tested is

$$H_0 : \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_P \end{bmatrix} = \begin{bmatrix} \mu_{01} \\ \vdots \\ \mu_{0P} \end{bmatrix}$$

and its alternative hypothesis is

$$H_0 : \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_P \end{bmatrix} \neq \begin{bmatrix} \mu_{01} \\ \vdots \\ \mu_{0P} \end{bmatrix}$$

The test procedures are as follows:

1) Compute $\bar{\mathbf{x}}$ and $\mathbf{S}$ by using Eqs. (A-2), (A-4), (A-5), and (A-6).
2) Compute T^2 defined as

$$T^2 = N\,(\bar{\mathbf{x}} - \boldsymbol{\mu}_0)^t\, \mathbf{S}^{-1} (\bar{\mathbf{x}} - \boldsymbol{\mu}_0) \qquad \text{(A-8)}$$

where $\mathbf{S}^{-1}$ is the inverse matrix of $\mathbf{S}$.

3) Compute F^* defined as

$$F^* = \frac{(N-K)\,P}{N-K-P-1}\, F_{\alpha\,;\,P,\,N-K-P+1} \qquad \text{(A-9)}$$

where α is the significant level and $F_{\alpha\,;\,P,\,N-K-P+1}$ is the $100\,(1-\alpha)$ percent point of the F-distribution with degrees of freedom P and $N-K-P+1$.

4) The decision for a test of level α is to accept the null hypothesis H_0 that $\boldsymbol{\mu} = \boldsymbol{\mu}_0$ if $T^2 \leq F^*$ and to reject H_0, otherwise.

If the null hypothesis is rejected, it would be desirable to know which components or combination of components might have led to the rejection of the null hypothesis. This is done by computing the simultaneous confidence intervals for all linear compounds of means, $\sum_{i=1}^{P} a_i \cdot \mu_i$. Any interval that does not cover the point $\sum_{i=1}^{P} a_i \cdot \mu_i$ can be considered as a criterion for rejecting the null hypothesis. Denote $\mathbf{a} = (a_1, \cdots\cdots, a_P)^t$. Then, the $100\,(1-\alpha)$ percent simultaneous confidence intervals for all linear compounds of means are

$$\mathbf{a}^t \bar{\mathbf{x}} - \sqrt{\frac{1}{N}\mathbf{a}^t \mathbf{S} \mathbf{a}\, F^*} \leq \mathbf{a}^t \boldsymbol{\mu} \leq \mathbf{a}^t \bar{\mathbf{x}} + \sqrt{\frac{1}{N}\mathbf{a}^t \mathbf{S} \mathbf{a}\, F^*} \qquad \text{(A-10)}$$

where F^* was defined in Eq. (A-9).

The above computations were made by assuming that the population covariance matrix Σ is unknown and was estimated by S. Occasionally, the population covariance matrix Σ is known through past experience. In this situation, the previous test procedures can be still used with the following modifications.

1) The S in step (2) is replaced by Σ

2) The F^* in step (3) is the $100(1-\alpha)$ percent point of the chi-square distribution with P degrees of freedom.

The Multivariate Analysis of Variance

As in the case of the univariate analysis of variance, an experimental model can be expressed as

$$X = A\xi + e \tag{A-11}$$

where X is the observation matrix of dimension $N \times P$, N is the total number of p-dimensional observation vectors, A is the appropriate design matrix of dimension $N \times q$ with rank r, ξ is the parameter matrix of dimension $q \times P$ and e is the matrix of residual variates with dimension $N \times P$.

The null hypothesis to be tested is

$$H_0: C\xi M = 0$$

and its alternative is

$$H_1: C\xi M \neq 0$$

where C is the hypothesis matrix of dimension $g \times q$ with rank $g \leq r$ and M has dimension $P \times u$ with rank $u \leq P$. The testing procedures for the above null hypothesis are as follows:

1) Find matrices H and E defined as

$$H = M^t X^t A_1 (A_1^t A_1)^{-1} C_1^t [C_1 (A_1^t A_1)^{-1} C_1^t]^{-1} C_1 (A_1^t A_1)^{-1} A_1^t X M \tag{A-12}$$

and

$$E = M^t X [I - A_1 (A_1^t A_1)^{-1} A_1^t] X M \tag{A-13}$$

where A_1 has r columns from A and is a basis of A and C_1 is a submatrix of C with dimension $q \times r$ in conformance with the partition of A into A_1 and the other matrix.

2) Find the greatest root c_s of the determinantal equation

$$|H - \lambda E| = 0 \tag{A-14}$$

3) Compute the following numbers:

$$\theta_s = \frac{c_s}{1 + c_s} \tag{A-15}$$

and

$$\begin{cases} s = \text{MIN}(q, u) \\ m = \dfrac{|q-u|-1}{2} \\ n = \dfrac{N-r-u-1}{2} \end{cases} \tag{A-16}$$

4) The decision is to accept the null hypothesis at the α significance level if

$$\theta_s \leq x_{\alpha; s,m,n}$$

where $x_{\alpha; s,m,n}$ is the upper $100\,\alpha$ percentage point obtained from the appropriate Heck chart and to reject, otherwise.

In the case that $s = 1$, the test statistic for the above null hypothesis is defined as

$$F = \frac{n+1}{m+1} \, \frac{\theta}{1-\theta} \tag{A-17}$$

where θ is the single eigenvalue of the matrix $H(H+E)^{-1}$. The null hypothesis is accepted if F is less than or equal to the $100(1-\alpha)$ percent point of the $F-$ distribution with degrees of freedom $2m+2$ and $2n+2$, and rejected otherwise.

As before, when the null hypothesis is rejected, it is desirable to determine which components of the observation or component treatment combinations may have led to the rejection. This is done again by computing the $100(1-\alpha)$ percent simultaneous confidence bounds on all function $b^t C \xi M a$ of the parameters, where a is a $u-$ dimensional non-null vector and b is a $q-$ dimensional vector whose elements have been standardized so that

$$b^t C_1 (A_1^t A_1)^{-1} C^t b = 1 \qquad \text{(A-18)}$$

For such a structure, Roy and Bose have shown that the $100(1-\alpha)$ percent simultaneous confidence bounds on $b^t C \xi M a$ are

$$b^t C_1 (A_1^t A_1)^{-1} A_1^t X M a - \sqrt{\frac{x_{\alpha;s,m,n}}{1-x_{\alpha;s,m,n}} a^t E a}$$

$$\leq b^t C \xi M a \leq \qquad \text{(A-19)}$$

$$b^t C_1 (A_1^t A_1)^{-1} A_1^t X M a + \sqrt{\frac{x_{\alpha;s,m,n}}{1-x_{\alpha;s,m,n}} a^t E a}$$

Now, we shall convert the above computational formulas in terms of the various experimental designs conducted in this report.

A. One-Way Completely Randomized Design

In this design, we assume that there are K treatments with N_i observation vectors under the i^{th} treatment and P components on each observation vector. Denote x_{ijh} to be the i^{th} observation on component h under the j^{th} treatment, $i = 1, \cdots, N_j$, $j = 1, \cdots, K$ and $h = 1, \cdots, P$. Then, Eq. (A-11) can be written as

$$x_{ijh} = \mu_h + \tau_{jh} + e_{ijh} \qquad \text{(A-20)}$$

where

μ_h = general level effect for component h
τ_{jh} = effect of the j^{th} treatment on the h^{th} component
e_{ijh} = random effect of the ijh^{th} combination

The null hypothesis H_0 to be tested is the equal effect among treatments, i.e.,

$$H_0 = \begin{bmatrix} \tau_{11} \\ \vdots \\ \tau_{1P} \end{bmatrix} = \cdots\cdots = \begin{bmatrix} \tau_{K1} \\ \vdots \\ \tau_{KP} \end{bmatrix} \qquad \text{(A-21)}$$

For such a model, it can be shown that the rs^{th} element of matrices H and E defined in Eqs. (A-12) and (A-13) are, respectively:

$$h_{rs} = \sum_{j=1}^{K} \frac{1}{N_j} T_{jr} T_{js} - \frac{1}{N} G_r G_s \tag{A-22}$$

and

$$e_{rs} = \sum_{j=1}^{K} \sum_{i=1}^{N_j} x_{ijr} x_{ijs} - \sum_{j=1}^{K} \frac{1}{N_j} T_{jr} T_{js} \tag{A-23}$$

where

$T_{jr} = \sum_{i=1}^{N_j} x_{ijr}$ = sum of all observations on the r^{th} component under the j^{th} treatment

$G_r = \sum_{j=1}^{K} T_{jr}$ = grand total of all observations on the r^{th} component

It also follows from Eq. (A-19) that the $100(1-\alpha)$ percent simultaneous confidence intervals for all non-null compounds of the differences of the i^{th} and the j^{th} treatment effect vectors are

$$\sum_{h=1}^{P} a_h(\bar{x}_{ih} - \bar{x}_{jh}) - \sqrt{\frac{x_{\alpha;s,m,n}}{1-x_{\alpha;s,m,n}} \, a^t E a \left(\frac{1}{N_i} + \frac{1}{N_j}\right)}$$

$$\leq \sum_{h=1}^{P} a_h(\tau_{ih} - \tau_{jh}) \leq \tag{A-24}$$

$$\sum_{h=1}^{P} a_h(\bar{x}_{ih} - \bar{x}_{jh}) + \sqrt{\frac{x_{\alpha;s,m,n}}{1-x_{\alpha;s,m,n}} \, a^t E a \left(\frac{1}{N_i} + \frac{1}{N_j}\right)}$$

where

$$\bar{x}_{ih} = \frac{1}{N_i} \sum_{n=1}^{N_i} x_{nih}$$

The above intervals are used for multiple comparisons of the effects of every two treatments. For example, if the vector a is set to be $(1,0,\cdots,0)^t$ then the above interval is used to detect whether the effect of the i^{th} and the j^{th} treatments on the first component are equal or not. Other types of the multiple comparisons can be obtained from Eq. (A-19) through the specification of hypothesis matrix $\mathbf{C}$.

B. One-Way Completely Randomized Block Design

In this design, we assume that there are K treatments with one observation vector in each of b blocks and P components on each observation vector. Denote x_{ijr} to be the observation on the r^{th} component under the i^{th} treatment in block j, $i=1,\cdots,K$, $j=1,\cdots,b$, and $r=1,\cdots,p$. Then, Eq. (A-11) can be written as

$$x_{ijr} = \mu_r + \tau_{ir} + \beta_{jr} + e_{ijr} \tag{A-25}$$

where

μ_r = general level effect for component r
τ_{ir} = effect of the i^{th} treatment on the r^{th} component
β_{jr} = effect of the j^{th} block on the r^{th} component
e_{ijr} = random effect of the ijr^{th} combination

The null hypothesis to be tested is H_0 specified in Eq. (A-21). For such a model, it can be shown that the rs^{th} element of matrices $\mathbf{H}$ and $\mathbf{E}$ defined in Eqs. (A-12) and (A-13) are, respectively, to be:

$$h_{rs} = \frac{1}{b} \sum_{i=1}^{K} T_{ir} T_{is} - \frac{1}{bK} G_r G_s \tag{A-26}$$

and

$$e_{rs} = \sum_{i=1}^{K} \sum_{j=1}^{b} x_{ijr} x_{ijs} - b_{rs} - h_{rs} - \frac{1}{bK} G_r G_s \tag{A-27}$$

where

$B_{jr} = \sum_{i=1}^{K} x_{ijr}$ = total of the observations on the r^{th} component in block j

$T_{ir} = \sum_{j=1}^{b} x_{ijr}$ = total of the observations on the r^{th} component under treatment i

$G_r = \sum_{j=1}^{b} B_{jr}$ = grand sum of the r^{th} component

b_{rs} = the rs^{th} element for the matrix due to block effect

$$= \frac{1}{K} \sum_{j=1}^{b} B_{jr} B_{js} - \frac{1}{bK} G_r G_s$$

The $100(1-\alpha)$ percent simultaneous confidence intervals for all non-null linear compounds of the differences of the i^{th} and the j^{th} treatment effects can be shown to be:

$$\sum_{h=1}^{p} a_h(\bar{x}_{ih} - \bar{x}_{jh}) - \sqrt{\frac{x_{\alpha;s,m,n}}{1-x_{\alpha;s,m,n}} \, a^t E a \, \frac{2}{b}}$$
$$\leq \sum_{h=1}^{p} a_h(\tau_{ih} - \tau_{jh}) \leq \tag{A-28}$$
$$\sum_{h=1}^{p} a_h(\bar{x}_{ih} - \bar{x}_{jh}) + \sqrt{\frac{x_{\alpha;s,m,n}}{1-x_{\alpha;s,m,n}} \, a^t E a \, \frac{2}{b}}$$

where

$$\bar{x}_{ih} = \frac{1}{b}\sum_{h=1}^{b} x_{ihh}$$

The major purpose of the above equation has been explained before and will not be repeated here.

C. Two-Way Completely Randomized Design

In this design, we have two factors, Row and Column factors, with r and c treatments, respectively. We also assume that there are n observation vectors in each treatment combination and p components in each observation vector. Denote x_{ijkl} to be the k^{th} observation on component l under the $(i,j)^{th}$ treatment combination, $i=1,\cdots,r$, $j=1,\cdots,c$, $k=1,\cdots,n$, and $l=1,\cdots,p$. Then, Eq. (A-11) can be written as

$$x_{ijkl} = \mu_l + \alpha_{il} + \beta_{jl} + \eta_{ijl} + e_{ijkl} \qquad \text{(A-29)}$$

where

μ_l = general level effect for component l

α_{il} = effect of the i^{th} row treatment on the l^{th} component

β_{jl} = effect of the j^{th} column treatment on the l^{th} component

η_{ijl} = effect of the interaction of the i^{th} row treatment and the j^{th} column treatment on the l^{th} component

e_{ijkl} = random effects of the $(ijkl)^{th}$ combination

There are three hypotheses to be tested in this design, which are

H_{01} : Equal effects among row treatments
H_{02} : Equal effects among column treatments (A-30)
H_{03} : There is no interaction between row and column treatments.

For such a model, it can be shown that the rs^{th} element of matrix $\mathbf{E}$ defined in Eq. (A-13) is

$$e_{rs} = \sum_{i=1}^{s}\sum_{j=1}^{c}\sum_{k=1}^{n} x_{ijkr}\, x_{ijks} - \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{c} C_{ijr}\, C_{ijs} \qquad \text{(A-31)}$$

and the rs^{th} element of matrix $\mathbf{H}$ defined in Eq. (A-12) for each hypothesis in Eq. (A-30) are respectively, to be

$$h^{(1)}_{rs} = \frac{1}{cn} \sum_{i=1}^{r} R_{ir} R_{is} - \frac{G_r G_s}{rcn} \qquad \text{(A-32)}$$

$$h^{(2)}_{rs} = \frac{1}{rn} \sum_{j=1}^{c} T_{jr} T_{js} - \frac{G_r G_s}{rcn} \qquad \text{(A-33)}$$

and

$$h^{(3)}_{rs} = t_{rs} - h^{(1)}_{rs} - h^{(2)}_{rs} - e_{rs} \qquad \text{(A-34)}$$

where

$$C_{ijh} = \sum_{K=1}^{n} x_{ijKh}$$

$$T_{jh} = \sum_{i=1}^{r} \sum_{K=1}^{n} x_{ijKh}$$

$$R_{ih} = \sum_{j=1}^{c} \sum_{K=1}^{n} x_{ijKh}$$

$$G_h = \sum_{i=1}^{r} \sum_{j=1}^{c} \sum_{K=1}^{n} x_{ijKn}$$

and

$$t_{rs} = \sum_{i=1}^{r} \sum_{j=1}^{c} \sum_{K=1}^{n} x_{ijKr}\, x_{ijKs} - \frac{G_r G_s}{rcn}$$

It follows from Eq. (A-19) that the $100(1-\alpha)$ percent simultaneous confidence intervals for the linear compound of the differences of the i^{th} row effects and column effects are, respectively

$$\sum_{h=1}^{p} a_h (\bar{x}_{i \cdot h} - \bar{x}_{j \cdot h}) - \sqrt{\frac{2x_{\alpha;s,m,n}}{cn(1-x_{\alpha;s,m,n})}\, a^t E a}$$

$$\leq \sum_{h=1}^{p} a_h (\alpha_{ih} - \alpha_{jh}) \leq \qquad \text{(A-35)}$$

$$\sum_{h=1}^{p} a_h (\bar{x}_{i \cdot h} - \bar{x}_{j \cdot h}) + \sqrt{\frac{2x_{\alpha;s,m,n}}{cn(1-x_{\alpha;s,m,n})}\, a^t E a}$$

and

$$\sum_{h=1}^{p} a_h (\bar{x}_{\cdot ih} - \bar{x}_{\cdot jh}) - \sqrt{\frac{x_{\alpha;s,m,n}}{rn(1-x_{\alpha;s,m,n})}\, a^t E a}$$

$$\leq \sum_{h=1}^{p} a_h (\beta_{ih} - \beta_{jh}) \leq \qquad \text{(A-36)}$$

$$\sum_{h=1}^{p} a_h (\bar{x}_{\cdot ih} - \bar{x}_{\cdot jh}) + \sqrt{\frac{x_{\alpha;s,m,n}}{rn(1-x_{\alpha;s,m,n})}\, a^t E a}$$

where

$$\bar{x}_{i \cdot h} = \frac{1}{cn} \sum_{j=1}^{c} \sum_{K=1}^{n} x_{ijKh}$$

and

$$\bar{x}_{\cdot jh} = \frac{1}{r n} \sum_{i=1}^{r} \sum_{K=1}^{n} x_{ijKh}$$

D. Two-Way Completely Randomized Block Design

In this design, we have two factors, Row and Column factors, with r and c treatments, respectively, and n blocks. Denote x_{ijKL} to be the observation on component L under the $(i,j)^{th}$ treatment combination in block K, $i=1,\cdots,r, j=1,\cdots,c$, $K=1,\cdots,r$, and $L=1,\cdots,p$. Then, Eq. (A-11) can be written as

$$x_{ijKL} = \mu_L + \alpha_{iL} + \beta_{jL} + \rho_{KL} + \eta_{ijL} + e_{ijKL} \tag{A-37}$$

where ρ_{KL} is the effect of block K on the L^{th} component and the meaning of the other parameters in Eq. (A-37) can be seen in Eq. (A-29). The three hypotheses to be tested are the same as those stated in Eq. (A-30) of the previous design. Define,

$B_{KL} = \sum_{i=1}^{r} \sum_{j=1}^{c} x_{ijKL} =$ sum of the K^{th} block on component L

$C_{ijL} = \sum_{K=1}^{n} x_{ijKL} =$ sum of the $(i,j)^{th}$ treatment combination on component L

$R_{iL} = \sum_{j=1}^{c} \sum_{K=1}^{n} x_{ijKL} =$ sum of the i^{th} row treatment on component L

$T_{jL} = \sum_{i=1}^{r} \sum_{K=1}^{n} x_{ijKL} =$ sum of the j^{th} column treatment on component L

$G_L = \sum_{i=1}^{r} \sum_{j=1}^{c} \sum_{K=1}^{n} x_{ijKL} =$ grand sum on component L

Then, the rs^{th} element of matrix $\mathbf{E}$ defined in Eq. (A-13) is

$$e_{rs} = \sum_{i=1}^{r} \sum_{j=1}^{c} \sum_{K=1}^{n} x_{ijKr}\, x_{ijKs} - \frac{1}{rc} \sum_{K=1}^{n} B_{Kr} B_{Ks} - \frac{1}{n} \sum_{i=1}^{r} \sum_{j=1}^{c} C_{ijr}\, C_{ijs} + \frac{G_r G_s}{rcn} \tag{A-38}$$

and the rs^{th} element of matrix $\mathbf{H}$ defined in Eq. (A-12) for each hypothesis in Eq. (A-30) are, respectively

$$h_{rs}^{(1)} = \frac{1}{cn} \sum_{i=1}^{r} R_{ir} R_{is} - \frac{G_r G_s}{rcn} \quad \text{(A-39)}$$

$$h_{rs}^{(2)} = \frac{1}{rn} \sum_{j=1}^{c} T_{jr} T_{js} - \frac{G_r G_s}{rcn} \quad \text{(A-40)}$$

and

$$h_{rs}^{(3)} = \frac{1}{n} \sum_{i=1}^{r} \sum_{j=1}^{c} C_{ijr} C_{ijs} - h_{rs}^{(1)} - h_{rs}^{(2)} - \frac{G_r G_s}{crn} \quad \text{(A-41)}$$

The $100(1-\alpha)$ simultaneous confidence intervals for the linear compound of the differences of the i^{th} and j^{th} row effects and column effects are respectively stated in Eqs. (A-35) and (A-36).

APPENDIX B

A STATISTICAL EXPERIMENT TO STUDY AGGREGATE SEGREGATION PATTERNS

The test results of samples taken according to the statistical experiment in the third section of Chapter I-1 are presented in Table B-1. Note that the percent passing the 1-in. sieve is not included in Table B-1 since this measurement is always 100 in this experiment. That experiment is a so-called "one-way completely randomized block design which can be mathematically described as follows. Let X_{ijK} $K = 1, 2, 3,$ and 4, be the percent passing the 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing, respectively, of the sample taken from the i^{th} stratum of the j^{th} layer, $i = 1, \ldots, 5$, and $j = 1, \ldots, 20$. The statistical model for the observation X_{ijK} is

$$X_{ijK} = \mu_K + \tau_{iK} + \beta_{jK} + e_{ijK} \tag{B-1}$$

where:

μ_K = general level effect for the measurement corresponding index k

τ_{iK} = effect of the i^{th} stratum on the measurement corresponding to index k

β_{jK} = effect of the j^{th} layer on the measurement corresponding to index k

e_{ijK} = random effect specific to the ijK^{th} combination of stratum, layer and sieve size.

We would like to know whether or not the five transversal strata have the same mean aggregate composition. Statistically, we are testing the hypothesis

$$H_0 : \begin{bmatrix} \tau_{11} \\ \vdots \\ \tau_{14} \end{bmatrix} = \begin{bmatrix} \tau_{21} \\ \vdots \\ \tau_{24} \end{bmatrix} = \cdots\cdots = \begin{bmatrix} \tau_{51} \\ \vdots \\ \tau_{54} \end{bmatrix} \tag{B-2}$$

of equal strata effects. A multivariate analysis of variance program was used to obtain the following results:

$$\text{Treatment Matrix} = H = \begin{bmatrix} 45.59 & 123.61 & 113.34 & -2.28 \\ & 533.42 & 528.06 & 2.07 \\ & & 539.22 & 4.49 \\ & & & 0.64 \end{bmatrix} \tag{B-3}$$

TABLE B-1
TEST RESULTS OF NON-COMPOSITE SAMPLES TAKEN FROM THE STOCKPILE

Lot	Strata	Percent Passing Sieves			Percent L.B.W.
		3/4-in.	3/8-in.	No. 8	
1	1	80.87	47.38	24.37	4.33
	2	93.91	77.67	45.94	3.30
	3	92.09	65.82	37.25	3.57
	4	96.37	79.05	51.40	5.03
	5	95.28	69.17	39.17	4.44
2	1	97.29	80.34	50.85	5.09
	2	96.81	79.42	49.27	4.06
	3	97.53	78.09	49.07	4.32
	4	97.27	83.33	59.29	4.10
	5	94.44	77.25	54.50	3.97
3	1	91.59	60.58	35.34	4.09
	2	98.08	83.41	53.13	5.77
	3	98.58	83.52	48.58	3.69
	4	95.92	78.26	47.28	3.53
	5	92.23	52.18	24.76	2.91
4	1	95.55	73.77	44.03	3.75
	2	96.22	79.43	50.59	4.25
	3	97.27	84.90	54.04	4.04
	4	98.85	73.85	41.06	3.44
	5	93.38	76.56	49.37	3.04
5	1	96.21	75.32	48.45	7.88
	2	97.26	79.04	50.48	4.81
	3	95.83	78.65	54.70	7.66
	4	98.02	78.27	53.00	9.07
	5	93.58	66.14	41.14	4.06
6	1	99.30	85.34	54.92	6.32
	2	96.27	76.94	50.05	4.96
	3	93.76	75.89	50.25	4.21
	4	94.73	68.54	39.90	4.29
	5	93.77	75.01	48.00	5.37
7	1	96.57	80.46	54.75	4.10
	2	96.68	78.54	50.35	2.92
	3	90.85	67.84	41.37	4.82
	4	100.00	86.19	57.83	3.94
	5	95.71	81.37	49.57	3.00
8	1	97.70	80.51	55.86	5.15
	2	99.33	78.65	50.11	5.26
	3	97.10	85.63	57.13	4.02
	4	97.70	86.20	58.69	4.60
	5	99.03	77.63	46.40	3.93
9	1	85.49	53.71	30.67	4.32
	2	94.91	77.18	48.88	4.76
	3	98.17	81.40	52.23	4.45
	4	98.49	82.79	53.44	5.11
	5	92.96	66.97	39.87	3.44
10	1	97.98	80.22	54.61	4.36
	2	94.65	78.79	53.57	4.71
	3	94.74	74.93	53.03	5.26
	4	98.45	79.14	54.17	4.05
	5	100.00	86.49	45.22	3.67
11	1	95.92	72.12	41.79	5.53
	2	98.84	82.14	55.63	5.18
	3	96.23	81.87	56.39	6.69
	4	96.17	77.69	49.32	4.86
	5	95.63	79.88	54.04	5.52
12	1	95.89	75.62	49.78	4.86
	2	93.41	70.30	42.50	5.30
	3	96.76	77.22	45.60	5.66
	4	97.22	81.96	50.75	5.22
	5	97.17	74.11	43.45	5.28
13	1	95.18	64.99	37.74	4.77
	2	97.33	77.00	51.51	5.91
	3	94.79	76.23	50.68	7.47
	4	94.54	75.87	47.95	5.25
	5	96.23	78.38	51.26	6.59
14	1	91.36	42.88	23.46	4.32
	2	91.65	73.48	49.57	5.27
	3	93.89	76.89	53.03	3.73
	4	96.16	82.76	56.17	4.78
	5	97.22	77.14	53.21	5.78
15	1	98.79	73.86	43.62	4.44
	2	97.38	79.25	55.89	4.31
	3	99.17	79.69	51.57	5.72
	4	96.38	81.71	52.93	6.58
	5	95.55	69.17	42.09	5.14
16	1	96.49	74.38	48.35	6.34
	2	95.11	73.51	47.29	5.54
	3	90.57	73.54	49.47	6.18
	4	93.74	73.91	47.32	5.57
	5	95.98	74.79	49.04	6.72
17	1	93.68	75.96	48.87	5.93
	2	95.11	63.93	42.47	5.48
	3	89.81	66.43	43.48	5.02
	4	95.29	68.63	42.49	4.78
	5	92.21	67.36	39.04	5.60
18	1	98.57	84.37	55.37	5.43
	2	97.53	79.22	51.50	5.06
	3	95.21	72.60	50.74	5.95
	4	92.44	66.20	40.91	6.40
	5	94.73	69.32	43.08	5.75
19	1	95.82	69.91	38.85	5.79
	2	98.37	74.59	52.51	6.86
	3	94.25	70.89	42.57	5.32
	4	95.73	75.03	48.71	5.69
	5	97.94	85.26	59.84	8.94
20	1	95.56	80.15	49.34	4.04
	2	97.42	79.69	50.98	5.95
	3	95.38	78.26	50.85	5.14
	4	97.37	81.37	55.23	3.98
	5	99.10	82.62	54.22	4.85

and

$$\text{Error Matrix} = E = \begin{bmatrix} 537.88 & 1,072.06 & 773.34 & 30.64 \\ & 4,047.81 & 3,287.77 & 140.72 \\ & & 3,233.54 & 169.03 \\ & & & 63.49 \end{bmatrix} \quad \text{(B-4)}$$

The observed test statistic which is the largest eigenvalue of the matrix $H(H+E)^{-1}$ was computed to be 0.16. The distribution parameters (see Appendix A) are:

$$s = \text{Min}\ (5-4,\ 4) = 4$$

$$m = (|5-1-4|-1)/2 = -1/2$$

and

$$n = (5(20-1)-20-4)/2 = 35.5$$

If we choose to test H_0 at the 0.05 level, the critical value for the test statistic obtained from Heck's chart is 0.2075, which is larger than the observed test statistic. So, we do not reject the hypothesis of equal strata effects. That is, aggregates at the outside edges are not necessarily coarser than those in the middle of a layer.

It is interesting to note, based on the historical data, that the measurement x_i, the percent passing the sieve corresponding to the index i, $i = 1, \ldots, 4$, is approximately distributed according to normal law. That is, the distribution of x_i takes the following form,

$$F_i(x) = \begin{cases} N(x; \mu_i, \sigma_i^2), & 0 \leq x \leq 100 \\ 0 & \text{, otherwise} \end{cases} \quad \text{(B-5)}$$

where $N(x; \mu_i, \sigma_i^2)$ is the normal distribution with mean μ_i and variance σ_i^2. Moreover, other linear combinations of x_i's, such as $100 - x_1$, $x_1 - x_2$, and $x_2 - x_3$, are also approximately normal distributed taking the form specified in Eq. (B-5). This suggests that the aggregate composition ($x_1, \ldots\ldots, x_4$) is approximately distributed according to multinormal law, taking the following distribution form.

$$F(x_1, \ldots, x_4) = \begin{cases} \mathbf{N}(x_1, \ldots, x_4 ; \boldsymbol{\mu}, \boldsymbol{\Sigma}), & 0 \leq x_4 \leq x_3 \leq x_2 \leq x_1 \leq 100, \\ 0 & , \text{otherwise} \end{cases} \tag{B-6}$$

where $\mathbf{N}$ is a multinormal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

Although we did not use Eq. (B-6) to fit the historical data, the chi-square test did not reject the null hypothesis, at the 0.05 significant level, that those observation vectors in the historical data came from the population with the distribution specified in Eq. (B-6), where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ were estimated from the data.

In general, slight violation of the multinormality assumption required by the multivariate analysis of variance would not affect the conclusions if the sample size is large enough to justify the multidimensional central limit law. This was why we assumed in the previous analysis that $(x_{ij1}, \ldots\ldots, x_{ij4})$ was distributed according to multinormal law so that the techniques presented in Appendix A could be used. Based on the above facts, we shall assume throughout this report that the aggregate composition is distributed according to multinormal law.

APPENDIX C

DETERMINATION OF THE NUMBER OF SCOOPS TO MAKE A COMPOSITE SAMPLE AT THE STOCKPILE

Let X_{ij}, $j = 1, 2, 3$, and 4, be the percent passing the 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing, respectively, of the i[th] random sample, where $i = 1, \ldots, N$. Denote

$$\bar{X}_j = \frac{1}{N}\sum_{i=1}^{N} X_{ij} \qquad \text{(C-1)}$$

$\bar{X}_j$ is the sample mean of the measurement corresponding to the index j, which is an unbiased estimate of the population mean μ_j. We wish to have

$$Pr\left(|\bar{x}_j - \mu_j| \leq d_j,\ j=1,\ldots,4\right) = 1-\alpha \qquad \text{(C-2)}$$

where d_j is the chosen margin of error for the measurement corresponding to index j and α is a small probability. Eq. (C-2) can be rewritten as:

$$Pr\left(\frac{|\bar{x}_j - \mu_j|}{\sqrt{\frac{1}{N}\sigma_{jj}}} \leq \frac{d_j}{\sqrt{\frac{1}{N}\sigma_{jj}}},\ j=1,\ldots,4\right) = 1-\alpha \qquad \text{(C-3)}$$

where σ_{ij} is the covariance of x_i and x_j. Using the results of simultaneous confidence intervals (Appendix A), the required sample size N is determined to be:

$$N = \underset{i=1,\ldots,4}{\text{MAXIMUM}} \left[\frac{\sigma_{ii}\chi^2_{\alpha;4}}{d_i^2}\right] \qquad \text{(C-4)}$$

Before applying the above equations to our case, we note the following:

1) Eq. (C-2) is not the only expression that can be specified for the determination of the sample size in the multivariate case.

2) Sample size N obtained from Eq. (C-4) is a conservative number in the sense that $Pr(|\bar{x}_j - \mu_j| \leq d_j,\ j=1,\ldots,4) \geq 1-\alpha$.

3) Again, Eq. (C-4) is not the only way to obtain a conservative sample size. For example, for a particular set of $(\alpha_1, \ldots, \alpha_4)$ such that $\alpha_1 + \ldots + \alpha_4 = \alpha$, choose N to be

$$N = \underset{i=1,\ldots,4}{\text{MAXIMUM}} \left[\frac{\sigma_{ii} z^2_{1-\frac{\alpha_i}{2}}}{d_i^2}\right] \qquad \text{(C-5)}$$

4) In case the sample size determined from either Eqs. (C-4) or (C-5) is too large for practical consideration, the sample size N can be chosen as the smallest number satisfying the following equation:

$$P_r(|\bar{x}_j - \mu_j| \leq d_j,\ j = 1, \ldots, 4) \geq 1 - \alpha \qquad \text{(C-6)}$$

To do this, a computer program calculating multiple integrations with a truncated multinormal distribution function integrand is needed. In order to apply either Eqs. (C-4) or (C-5) to our case, we need the estimate of the covariance matrix of aggregate composition, based on non-composite samples. By considering the data in Table B-1 (Appendix B) composed of 20 groups, with five samples per group, and using Eq. (A-6) (Appendix A) we obtain the following matrix estimate:

$$\hat{\Sigma} = \left[\hat{\sigma}_{ij}\right] = \begin{bmatrix} 7.29 & 14.95 & 11.00 & 0.35 \\ & 57.26 & 47.69 & 1.78 \\ & & 47.15 & 2.17 \\ & & & 0.80 \end{bmatrix} \qquad \text{(C-7)}$$

The marginal error d_i in Eqs. (C-4) and (C-5) shall be chosen based on the specification for the composition of 22A aggregate which is specified in Table C-1.

TABLE C-1
THE CURRENT SPECIFICATION FOR
COMPOSITION OF 22A AGGREGATE

Sieve Size	Percent Passing		Half of the Band Width of the Specification
	Upper Limit	Lower Limit	
1-in.	100	100	0
3/4-in.	100	90	5
3/8-in.	85	65	10
No. 8	50	30	10
Loss-By-Washing	8	4	2

Based on the bandwidth of the specification in Table C-1, the following sets of marginal errors and α were chosen to determine the required sample size. The results are presented in Table C-2.

TABLE C-2
REQUIRED NUMBERS OF SCOOPS TO MAKE A COMPOSITE SAMPLE UNDER VARIOUS SETS OF MARGINAL ERRORS AND α

Marginal Errors				Risk Probability	Sample Size N	
d_1	d_2	d_3	d_4		Eq. (4)	Eq. (5)
5.00	10.500	10.0	2.0	0.05	5	4
				0.01	7	6
3.75	8.125	7.5	1.5	0.05	9	6
				0.01	12	10
2.50	5.750	5.0	1.0	0.05	18	12
				0.01	26	20

Note that each α_i used to determine sample size in Eq. (5) was set at $\alpha/4$.

As one can see from Table C-2, the sample size must be substantially increased to produce significant reduction in marginal error. For practical considerations, we decide to take $N = 10$.

It should be noted that the specification for 22A aggregate also requires that the percent crushed is at least 25. Since this measurement depends only on the percent passing the 3/8-in. sieve, a representative sample of a layer to determine the aggregate composition will also give a good estimate of the percent crushed of that layer. This was why the percent crushed was not included in the process of determining the optimal numbers of scoops to form a composite sample.

APPENDIX D

STATISTICAL EXPERIMENTS FOR THE COMPARISON OF THE CSM AND SRSM PLANS

The most straightforward experimental design for the purpose stated at the beginning of Chapter I-2 is the four-way completely randomized block design with two levels in each of the four factors:

Factor 1: Sampling method (CSM and SRSM)
Factor 2: Inspector (I_1 and I_2)
Factor 3: Sieve Set (S_1 and S_2)
Factor 4: Testing Location (L: Laboratory and F: Field)

There would be N blocks ($B_1, \ldots, B_N$), each representing a period of production time such as four-hours or one-day. The above design is presented in Table D-1.

According to this design, two inspectors would have to frequently travel between field and laboratory for each production day. This is not practical nor was it possible because of field limitations to schedule inspectors exactly as required by the experiment design. To be practical, we would have to complete the field test before the laboratory test. Since the sieving equipment might wear out after 4N tests we needed two sets of sieves, one for the laboratory test and the other for the field test. Consequently, the above design becomes a split-plot design as shown in Table D-2.

Unless we have additional information from the two sieves, the above design alone cannot tell the effects due to testing location. In this circumstance, it would be better to consider the above design as a two-way, completely randomized block design with 2N blocks; N blocks for laboratory tests using sieve set No. 1 and N blocks for field tests using sieve set No. 2, as shown in Table D-3.

Since our major concern is the estimation of effects due to sampling method, the design specified in Tables D-2 or D-3 satisfies both practical considerations and theoretical needs. Therefore, we shall take and test samples according to the design specified in Table D-2.

1) Data Preparation

According to the design specified in Table D-2, eight samples are needed to make a complete randomization within each block. It takes approximately 30 minutes to make a layer about 6 in. thick. Thus, each period of four to five production hours produced eight layers and was considered as one block. Eight combinations of three factors were randomly assigned to eight layers of each block. One sample was taken from each layer according to a predetermined combination. For example, the sample

TABLE D-1
2 BY 2 BY 2 BY 2 BY N CRB DESIGN

		CSM				SRSM			
		I_1		I_2		I_1		I_2	
		S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
B_1	L								
	F								
·	L								
	F								
·	L								
	F								
B_N	L								
	F								

TABLE D-2
SPLIT-PLOT DESIGN

		F - S_1		L - S_2	
		I_1	I_2	I_1	I_2
B_1	CSM				
	SRSM				
·	CSM				
	SRSM				
·	CSM				
	SRSM				
B_N	CSM				
	SRSM				

TABLE D-3
2 BY 2 BY 2N CRB DESIGN

		CSM	SRSM
B_1	I_1		
	I_2		
.	I_1		
	I_2		
.	I_1		
	I_2		
B_N	I_1		
	I_2		

from the first layer of the first production day might be taken according to SRSM and tested in the field by Inspector No. 1.

Samples for the field test were tested in the field to measure the percent passing the 1-in., 3/4-in., 3/8-in., and No. 8 sieves, the percent loss-by-washing and the percent crushed. Samples for the laboratory test were properly marked and stored in the laboratory. They were tested at the laboratory in the same order in which they were taken. Due to practical difficulties, we could not use field inspectors to perform the laboratory tests as originally planned. Thus, we assigned two other inspectors to perform the laboratory tests. This change makes the design specified in Tables D-2 and D-3 invalid because samples to be tested by laboratory inspectors were taken by field inspectors. In this situation, we would have to consider the original design as the following two, two-way completely randomized block designs (Tables D-4 and D-5).

For each sample tested in the laboratory, we measured the percent passing the 1-in., 3/4-in., 1/2-in., 3/8-in., No. 4, No. 8, No. 16, No. 30, No. 50, No. 100, and No. 200 sieves, and the percent loss-by-washing. The purpose of inserting extra sieves in the laboratory test is to obtain more accurate test results and to obtain other information such as aggregate size distribution.

Since our major interest is estimating the effects due to sampling method, we chose inspectors who had the same working experience and were similar in other respects. In this experiment 54 blocks were involved.

TABLE D-4
FIELD TEST - 2 BY 2 BY N CRB DESIGN

		I_1	I_2
B_1	CSM		
	SRSM		
•	CSM		
	SRSM		
•	CSM		
	SRSM		
B_N	CSM		
	SRSM		

TABLE D-5
LABORATORY TEST -
2 BY 2 BY N CRB DESIGN

		$I_1 - I_3$	$I_2 - I_4$
B_1	CSM		
	SRSM		
•	CSM		
	SRSM		
•	CSM		
	SRSM		
B_N	CSM		
	SRSM		

2) Data Analysis

Since the percent passing the 1-in. sieve was always 100, this measurement was included in the analysis. The reason for not including the percent crushed was explained in Appendix C.

Let x_{ijKL}, $L = 1, \ldots, 4$, be the percent passing the 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing, respectively, of the sample taken from the K^{th} block by using the i^{th} sampling method and tested by the j^{th} inspector, where $i = 1, 2$, $j = 1, 2$, and $K = 1, \ldots, 54$.

Then the linear model for the observation x_{ijKL} is

$$x_{ijKL} = \mu_L + \alpha_{iL} + \beta_{iL} + \rho_{KL} + (\alpha\beta)_{ij} + e_{ijKL} \qquad \text{(D-1)}$$

where α stands for sampling method effects, β stands for inspector effects, ρ stands for block effects and $(\alpha\beta)$ stands for sampling method-inspector interaction.

We are interested in testing the following three null hypotheses:

$H_0^{(1)}$: No inspector and sampling method interaction

$H_0^{(2)}$: Equal inspector effects

and $H_0^{(3)}$: Equal sampling methods effects

We shall test each hypothesis using field and laboratory tests.

A. Field Tests

The distribution parameters (Appendix A) for testing the three hypotheses are, $s = 1$, $m = 1$, and $n = 77$, and if we choose to test them at the 0.05 level, the critical value is $F_{0.05;4,\,156} = 2.432$. A multivariate analysis of variance computer program was used to obtain the following results (Table D-6):

The statistic used in testing $H_0^{(1)}$ was computed to be $F = \frac{156}{4} \frac{0.0499665}{1-0.0499665} = 2.05$ which is less than the observed statistic $F_{0.05;4,\,156}$. Thus, we do not reject the hypothesis of no inspector and sampling method interaction. With the possibility of an interaction dismissed, we are now ready to test the main effects.

TABLE D-6
MATRIX ELEMENTS FOR THE TWO-WAY
CRB A.O.V. - FIELD TEST

Responses		Treatment Effects			Error
		Inspector	Sampling Method	Interaction	
1	1	0.60400	1.5980	15.4500	436.70
1	2	-0.31670	7.1090	29.6300	458.20
1	3	-1.50200	3.9340	22.4400	279.10
1	4	-0.24160	0.4963	2.7940	25.86
2	2	0.16610	31.6300	56.8500	1,526.00
2	3	0.78790	17.5000	43.0600	996.90
2	4	0.12670	2.2080	5.3610	25.11
3	3	3.73700	9.6820	32.6100	430.80
3	4	0.60110	1.2220	4.0610	50.34
4	4	0.09669	0.1541	0.5056	49.59

The statistic used in testing $H_0^{(2)}$ was computed to be $F = \frac{156}{4} \frac{0.012488}{1-0.012488} = 0.49$ which is much less than $F_{0.05;4,\ 156}$. Thus, we do not reject the hypothesis of equal inspector effects. This is the expected conclusion since both inspectors have 15 years working experience as aggregate inspectors.

The statistic used in testing $H_0^{(3)}$ was computed to be $F = \frac{156}{4} \frac{0.025597}{1-0.025597} = 1.03$ which is less than the critical value $F_{0.05;4,\ 156}$. Thus, we do not reject the hypothesis of equal sampling method effects.

We present the 95 percent simultaneous confidence intervals (SCI) for the mean and mean difference of the two sampling methods on each type of measurement in Tables D-7 through D-9. Note that LCL and UCL in these tables stand for the lower and upper confidence limits, respectively.

TABLE D-7
95 PERCENT SCI OF AGGREGATE COMPOSITION
OF THE CSM - FIELD TEST

	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
Mean	100	94.55	71.88	45.69	4.95
LCL	100	93.96	70.78	44.83	4.75
UCL	100	95.14	72.98	46.55	5.15

TABLE D-8
95 PERCENT SCI OF AGGREGATE COMPOSITION
OF THE SRSM - FIELD TEST

	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
Mean	100	94.72	72.64	46.11	5.00
LCL	100	94.13	71.54	45.25	4.80
UCL	100	95.31	73.74	46.97	5.20

TABLE D-9
95 PERCENT SCI FOR THE DIFFERENCE BETWEEN SAMPLING
METHODS (CSM-SRSM) ON EACH MEASUREMENT

	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
Mean Difference	0	-0.17	-0.76	-0.42	-0.054
LCL	0	-0.88	-2.09	-1.46	-0.290
UCL	0	0.54	0.57	0.62	0.185

Note that every confidence interval in Table D-9 contains zero, which is consistent with the previous analysis. We also present the estimate of the covariance matrix of aggregate composition for each sampling method in Table D-10.

TABLE D-10
ESTIMATE OF THE COVARIANCE MATRIX OF AGGREGATE COMPOSITION - FIELD TEST

	Size Combination	Sampling Method	
		CSM	SRSM
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	2.970	2.870
	3/4-in. and 3/8-in.	2.670	3.650
	3/4-in. and No. 8	1.060	2.320
	3/4-in. and L.B.W.	0.042	0.088
	3/8-in. and 3/8-in.	12.250	13.490
	3/8-in. and No. 8	6.920	8.960
	3/8-in. and L.B.W.	-0.099	0.320
	No. 8 and No. 8	6.900	8.670
	No. 8 and L.B.W.	0.380	0.550
	L.B.W. and L.B.W.	0.440	0.300

B. Laboratory Tests

We delete one block in which one sample was missing. Thus, we have 53 blocks in this experiment. The distribution parameters (Appendix A) are $s = 1$, $m = 1$, and $n = 75.5$ and, thus, the critical value to test the hypotheses at the 0.05 level is $F_{0.05;4,\ 153} = 2.4335$. The following results were obtained by a multivariate analysis of variance computer program.

The statistic used in testing $H_0^{(I)}$ was computed to be $F = \frac{153}{4}\,\frac{0.024544}{1-0.024544} = 0.96$ which is less than $F_{0.05;4,\ 156}$. Thus, we do not reject the hypothesis of no inspector and sampling method interaction.

TABLE D-11
MATRIX ELEMENTS FOR THE TWO-WAY
CRB A. O. V. - LABORATORY TEST

Responses		Treatment Effect			Error
		Inspector	Sampling Method	Interaction	
1	1	11.8700	0.2042000	2.8730	675.20
1	2	5.0630	1.2350000	1.9090	698.20
1	3	4.4720	0.5071000	5.0940	409.50
1	4	1.7180	-0.0055880	0.8219	-1.65
2	2	2.1600	7.4640000	1.2690	1,643.0C
2	3	1.9080	3.0660000	3.3850	1,198.00
2	4	0.7328	-0.0337800	0.5462	45.86
3	3	1.6850	1.2590000	9.0330	1,210.00
3	4	0.6472	-0.0138700	1.4570	88.62
4	4	0.2486	0.0001528	0.2351	64.41

With the possibility of an interaction dismissed, we can now test the main effects. The statistic used in testing $H_0^{(2)}$ was computed to be $F = \frac{153}{4} \frac{0.02844}{1-0.02844} = 1.12$ which is less than $F_{0.05;4,\ 153}$. Thus, we do not reject the hypothesis of equal inspector effects. Again, this is the expected conclusion since the laboratory inspectors have the same number of years working experience.

The statistic used in testing $H_0^{(3)}$ was computed to be $F = \frac{153}{4} \frac{0.0094116}{1-0.0094116} = 0.3634$ which is less than $F_{0.05;4,\ 156}$. Thus, we do not reject the hypothesis of equal sampling method effects.

Other related results are presented in Tables D-12 through D-15.

This completes the analysis of the experiment designed to investigate the effects of sampling method.

Since both inspectors for field tests and laboratory testa are statistically comparable in measuring aggregate composition, we can pool the test results of all inspectors to obtain estimations of aggregate composition under the CSM and under the SRSM. That is, we now have a one-way completely randomized block design. The statistical analysis, of course, still shows the same conclusions as before; namely, that the two sampling methods are statistically the same.

TABLE D-12
95 PERCENT SCI OF AGGREGATE COMPOSITION
OF THE CSM - LABORATORY TEST

	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
Mean	100	94.30	72.50	46.59	4.584
LCL	100	93.50	71.33	45.59	4.350
UCL	100	95.05	73.67	47.60	4.820

TABLE D-13
95 PERCENT SCI OF AGGREGATE COMPOSITION
OF THE SRSM - LABORATORY TEST

	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
Mean	100	94.36	72.88	46.74	4.582
LCL	100	93.61	71.71	45.74	4.350
UCL	100	95.11	74.05	47.75	4.810

TABLE D-14
95 PERCENT SCI FOR THE DIFFERENCE OF THE SAMPLING
METHODS (CSM-SRSM) ON EACH MEASUREMENT

	Percent Passing Sieves				Percent Loss-By-Washing
	1-in.	3/4-in.	3/8-in.	No. 8	
Mean Difference	0	-0.06	-0.038	-0.150	0.002
LCL	0	-0.96	-1.442	-1.355	-0.276
UCL	0	0.84	1.366	1.055	0.280

TABLE D-15
ESTIMATE OF THE COVARIANCE MATRIX OF AGGREGATE COMPOSITION - LABORATORY TEST

	Size Combination	Sampling Method	
		CSM	SRSM
Covariance of Sieve Measurements	3/4-in. and 3/4-in.	4.19	5.12
	3/4-in. and 3/8-in.	4.92	6.21
	3/4-in. and No. 8	3.08	3.37
	3/4-in. and L.B.W.	0.20	-0.24
	3/8-in. and 3/8-in.	13.99	17.18
	3/8-in. and No. 8	10.78	12.17
	3/8-in. and L.B.W.	0.78	0.25
	No. 8 and No. 8	10.90	12.30
	No. 8 and L.B.W.	1.10	0.65
	L.B.W. and L.B.W.	0.49	0.70

APPENDIX E

EQUATIONS FOR ESTIMATING THE PRODUCT QUALITY AND FOR DETERMINING THE REQUIRED SAMPLE SIZE, LOWER AND UPPER ACCEPTANCE LIMITS OF A VARIABLES TYPE OF PLAN

1) Mathematical Expression for Estimating "Product Quality," P

If we denote (x_1, x_2, x_3, x_4) to be the aggregate composition, namely, the percent passing the 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing and x_5 to be the percent crushed aggregate, then $\mathbf{x} = (x_1, x_2, x_3, x_4, x_5)$ is approximately distributed according to multinormal law with mean vector $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_5)$ and covariance matrix Σ as was explained in Appendix B. Thus, we can express P in terms of the following integration function.

$$P = 1 - \int_{x_1=90}^{100} \int_{x_2=65}^{85} \int_{x_3=30}^{50} \int_{x_4=4}^{8} \int_{x_5=25}^{100} dF(\mathbf{x}; \boldsymbol{\mu}, \Sigma) \qquad \text{(E-1)}$$

where F is defined as

$$F(\mathbf{x}; \boldsymbol{\mu}, \Sigma) = \begin{cases} N(\mathbf{x}; \boldsymbol{\mu}, \Sigma), & 0 \leq x_4 \leq x_3 \leq x_2 \leq x_1 \leq 100 \\ & 0 \leq x_5 \leq 100,\ 0 < x_2 \\ 0 & \text{, otherwise} \end{cases}$$

and N is multinormal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix Σ. We note that the integration limits in Eq. (E-1) are the specification limits and the smaller value of P means the better quality product.

If the current sampling method (CSM) is used to sample stockpiled aggregate, then the covariance matrix Σ of aggregate composition is estimated to be

$$\Sigma = \begin{bmatrix} 3.82 & 3.78 & 2.06 & 0.12 & -0.60 \\ & 13.11 & 8.83 & 0.34 & -2.47 \\ & & 8.88 & 0.74 & -1.04 \\ & & & 0.47 & 0.56 \\ & & & & 18.57 \end{bmatrix} \qquad \text{(E-2)}$$

which was obtained in Chapter I-2 (see also Table 4). By substitution of various $\boldsymbol{\mu}$ and the above covariance matrix into Eq. (E-1), we obtain the following estimates of product quality. We note that each value of P in Table E-1 for a given mean vector was obtained by a simulation process based on 600 points which is a large enough data set to obtain reliable approximation to the integration in Eq. (E-1).

TABLE E-1
THE RELATIONSHIP BETWEEN
AGGREGATE COMPOSITION AND
PRODUCT QUALITY*

$(u_1, u_2, u_3, u_4, u_5)$	P
(100.0, 85, 50, 8, 46.57)	0.7111
(97.5, 80, 45, 7, 46.57)	0.1420
(95.0, 75, 40, 6, 46.57)	0.0180
(92.5, 70, 35, 5, 46.57)	0.2175
(90.0, 65, 30, 4, 46.57)	0.8175

* The percent crushed is set at 46.57 which was estimated from the Stillman pit.

2) Acceptance Probability of the Current Plan

For a given value of product quality P, the lot acceptance probability P_a of the current plan defined in Chapter I-3 is:

$$P_a = (1 - P) + P(1 - P) = 1 - P^2 \tag{E-3}$$

3) Equations for the Determination of the Required Sample Size, the Lower and/or Upper Acceptance Limits for Each Component of Aggregate Composition and Percent Crushed

If a sample of aggregate is taken from an aggregate lot according to the "Systematic Sampling Method with Random Start" specified in Chapter I-2, the covariance matrix of aggregate composition and percent crushed is estimated from Table 2, Chapter I-1 and matrix (E-2) to be

$$\Sigma = \begin{bmatrix} 0.8058 & 1.6762 & 1.2878 & 0.0422 & -0.200 \\ & 5.7918 & 4.7740 & 0.1108 & -1.160 \\ & & 4.7426 & 0.2160 & -0.540 \\ & & & 0.1450 & 0.220 \\ & & & & 9.375 \end{bmatrix} \tag{E-4}$$

By using the concept of simultaneous confidence intervals (see Appendix A) the required sample size N is:

$$N = \text{MAXIMUM} \left[\frac{\sigma_{ii}(\chi_{\alpha i5} + \chi_{2\beta;5})^2}{(c_i - b_i)^2}, \quad i = 1, \cdots, 4, \quad \text{and} \quad \frac{\sigma_{55}(\chi_{2\alpha i5} + \chi_{2\beta;5})^2}{(c_5 - b_5)^2} \right] \tag{E-5}$$

where σ_{ii} is the measurement variance corresponding to the index i, the $(ii)^{th}$ element of the covariance matrix. The values of a_i, b_i and c_i are specified in Table E-2.

TABLE E-2
TOLERANCE LIMITS OF AGGREGATE COMPOSITION

i	Upper Tolerance Limit a_i	Lower Tolerance Limit b_i	The Center of the Two Limits c_i
1	100	90	95
2	85	65	75
3	50	30	40
4	8	4	6
5	--	25	40

If we let U_i and L_i, $i = 1, \ldots, 4$, be the upper and lower acceptance limits for the percent passing the 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing, respectively, and L_5 be the lower acceptance limit for percent crushed aggregate, then, U_i and L_i can be obtained from the following equations.

1) β is held at the desired level

$$\begin{cases} U_i = a_i - \sqrt{\dfrac{\sigma_{ii}}{N}}\, \chi_{2\beta;5} \\ L_i = b_i + \sqrt{\dfrac{\sigma_{ii}}{N}}\, \chi_{2\beta;5} \end{cases}, \quad i = 1, \cdots, 4 \tag{E-6}$$

and $$L_5 = b_5 + \sqrt{\frac{\sigma_{55}}{N}}\ \chi_{2\beta;5}$$

2) α is held at the desired level

$$\left.\begin{aligned} U_i &= c_i + \sqrt{\frac{\sigma_{ii}}{N}}\ \chi_{\alpha;5} \\ L_i &= c_i - \sqrt{\frac{\sigma_{ii}}{N}}\ \chi_{\alpha;5} \end{aligned}\right\}, i = 1, \cdots, 4 \tag{E-7}$$

and $$L_5 = c_5 - \sqrt{\frac{\sigma_{ii}}{N}}\ \chi_{\alpha;5}$$

Once the desired sample size, N , and the lower and upper acceptance limits are determined, the lot acceptance probability, P_a , can be computed by the following integration:

$$P_a = \int_{x_1=L_1}^{U_1} \cdots\cdots \int_{x_4=L_4}^{U_4} \int_{x_5=L_5}^{100} dF(\mathbf{x}; \boldsymbol{\mu}, \Sigma/N) \tag{E-8}$$

where $\mathbf{x} = (x_1, x_2, \cdots, x_5)$, $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_5)$ and Σ is the covariance matrix.

APPENDIX F

A STATISTICAL EXPERIMENT TO STUDY THE EFFECTS OF THE HANDLING AND COMPACTION PROCESSES ON AGGREGATE COMPOSITION

Sampling Methods at the Construction Site

The construction site studied was 5.88 miles long. The gravel base of the road was 27 ft by 11 in. We defined "Section" as a segment of the road with dimensions of 13.5 ft (single lane, width) by 11 in. (thick) by 60 ft (length). Every 15 consecutive sections were defined as a "Group." Thus, we had 60 groups in total. For such a stratification, the total amount of aggregate in one group is approximately equal to one-day's production. For a section to be sampled, we stratified it into 10 strata as shown in Figure F-1.

That is, one scoop of aggregate was taken from each of the five basal strata and the five upper strata. Thus, we combine 10 scoops of aggregate from one section to form a composite sample.

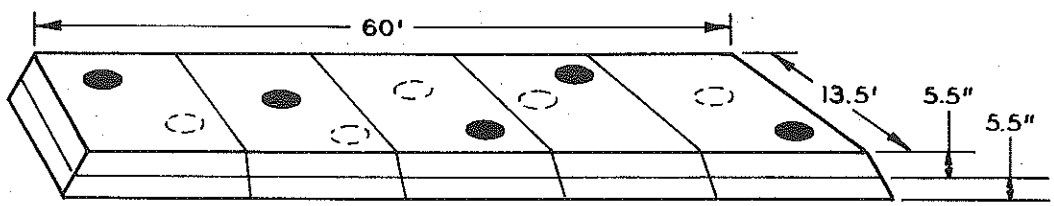

Figure F-1. The sampling layout of a section of construction road.

Data Preparation

To study the aggregate degradation due to compaction, we randomly chose 30 groups, then randomly chose two sections from each group. From each chosen section, one composite sample each was taken before and after compaction. Two samples from one section were randomly assigned to, randomly ordered, and tested by one inspector who used one set of sieves to measure the percent passing the 1-in., 1/4-in., 1/2-in., 3/8-in., No. 4, No. 8, No. 16, No. 30, No. 50, No. 100, and No. 200 sieves, and the percent loss-by-washing. Consequently, two samples from other sections of the same group were assigned to, and tested by another inspector who used another set of sieves. This experimentation constituted a one-way completely randomized block design with two treatments and 60 blocks. Each block is a combination of inspector and group. The reasons that we did not arrange the above experiment as a two-way completely randomized block design are as follows:

1) We knew from the previous experiment that the two inspectors produce about the same test results

2) Time efficiency

3) Information concerning these two particular sets of sieves is not important.

We note that two blocks involving missing samples were deleted. Thus, we have 58 blocks in total. We also note that the percent passing the 1-in. sieve was always 100, and therefore, was not included in the analysis.

Due to manpower limitations, those samples taken at the job before compaction for the aggregate degradation study were also used to study the effects of the handling process. Thus, at the stockpile, we randomly chose 30 blocks (one day's production) and randomly chose two layers from each block. One composite sample was taken from each layer according to the stratified random sampling method discussed in Chapter I-1. The only reasonable experiments that could be made to utilize those samples taken at the job before compaction for both studies (degradation due to compaction and handling process) are two, one-way, completely randomized designs; one design for each inspector with his own set of sieves. That is, those samples taken at the stockpile for this study were assigned and tested by the same two inspectors in the previous experiment such that samples taken before compaction, together with those taken after compaction, formed a one-way completely randomized block design as explained above. These, together with samples taken at the stockpile, formed two, one-way completely randomized designs. We also took one non-composite sample from each chosen section. That is, two non-composite samples were taken from each group. One of them was randomly assigned to and tested by one inspector and the other by the other inspector. These non-composite samples were collected to study the extent of aggregate uniformity of in-place aggregate.

Since liquid calcium chloride was spread over the construction site to help the compaction process obtain the required density, the sample weight of each of the in-place aggregate samples was increased by the amount of the content of calcium chloride which would be washed away by the washing process used in the current aggregate gradation test. In order to measure the true aggregate composition, the test results of each sample were adjusted by the amount of calcium chloride blended into that sample (which was estimated by a laboratory chemical test procedure).

Data Analysis

The mathematical model for the observations in the experiment to study the effect of compaction on aggregate composition is similar to the one presented in Chapter I-1, and, therefore, will not be repeated here. We would like to test the hypothesis that the mean aggregate composition before and after compaction is the same. The distribution parameters (Appendix A) are:

$$s = \min(2-1,\ 11) = 1$$

$$m = (\ 2-1-11\ \ -1)/2 = 9/2$$

and $$n = (2(58-1)-58-11)/2 = 45/2$$

The critical value of the test statistic to test the above hypothesis is $F_{0.05;11,\ 47}$ (=2.0113) at the 0.05 level and $F_{0.01;11,\ 47}$ (=2.0863) at the 0.01 level. The observed test statistic was computed to be

$$F = \frac{47}{11}\ \frac{0.621916961}{1-0.62191961} = 7.028$$

which is larger than the critical point at the 0.01 level. So, we reject the null hypothesis of equal treatment effects. That is, the compaction process significantly affects the aggregate composition. Since the hypothesis of equal treatment effects is rejected, we present the estimate of the mean aggregate composition and the 95 percent simultaneous confidence intervals of the difference of the mean aggregate composition before and after compaction in Tables 7 and 8, Chapter I-4. As we explained before, the loss-by-washing causes the rejection of the hypothesis of equal treatments.

We would also like to test the following hypothesis.

H_0 = The mean aggregate composition of the stockpile is the same as that at the construction site before compaction.

The test statistic for the above hypothesis is the Hotelling T^2 statistic and its critical value at the 0.05 level is 27.359 (based on N = 28). A computer program is used to compute the observed test statistics which are 56.65 for the experiment done by one inspector and 112.32 for the experiment done by the other inspector. Since both computed numbers are larger than the critical value, we reject the above hypothesis of equal mean aggregate composition. The simultaneous confidence intervals for the mean difference indicate that the mean aggregate composition of the aggregate material at the stockpile differs only with that of the aggregate at the job before compaction with respect to the fine aggregate.

Since the two inspectors are the same in performing aggregate testing, the tested results of the two are pooled to form one experiment. The above hypothesis is tested again using this revised data. The conclusion remains the same, as was expected.

APPENDIX G

STATISTICAL MODELS FOR ESTIMATING AGGREGATE COMPOSITION AND DEGRADATION RATES

In this appendix, we present statistical models for estimating the composition of in-place aggregate after compaction from either that before compaction or the stockpile aggregate composition. We also present statistical models for estimating the degradation rate of each aggregate size due to handling and compaction. The models' parameters are estimated based on the data obtained according to the statistical experiments described in Appendix F.

Models to Estimate Aggregate Composition After Compaction From Samples Taken Before Compaction.

For the purpose of this section, we introduce the following notation. For a sample taken from the roadbed before compaction, let X_i be the percentage retained on sieves corresponding to the index j, $j \leq i$, where the sieve size of index i, $i = 1, \ldots, 10$, is specified in Table G-1. We denote Y_i to be the measurement on a sample taken from the roadbed after compaction.

TABLE G-1
SIEVE SIZE OF EACH INDEX

Index i	1	2	3	4	5	6	7	8	9	10
Sieve Size	3/4-in.	1/2-in.	3/8-in.	No. 4	No. 8	No. 16	No. 30	No. 50	No. 100	No. 200

We also denote X_{11} and Y_{11} to be the percent loss-by-washing of the sample taken from the roadbed before and after compaction, respectively. Then, the relationship between X_i and Y_i can be expressed as:

$$Y_i = p_i X_i \quad , i = 1, \ldots, 11, \tag{G-1}$$

where the p_i 's are parameters to be estimated.

Let
$$d_i = 1 - p_i \quad , i = 1, \ldots, 10 \tag{G-2}$$

The d_i can be interpreted as the degradation rate of the aggregate, the grain size of which will retain on sieves corresponding to the index j, $j \leq i$. Once we know the p_i 's, the aggregate composition after compaction can be estimated from that before compaction. We present the following estimation procedures for the p_i 's.

Model 1: Ratio Estimate

Taking the expectation on both sides of Eq. (G-1), we have

$$E(Y_i) = P_i \, E(X_i) \quad , i = 1, \ldots, 11 \tag{G-3}$$

That is,

$$P_i = E(Y_i)/E(X_i) \quad , i = 1, \ldots, 11 \tag{G-4}$$

Thus, each P_i can be estimated by

$$\hat{P}_i = \overline{Y}_i / \overline{X}_i \quad , i = 1, \ldots, 11 \tag{G-5}$$

where $\overline{Y}_i$ and $\overline{X}_i$ are the sample means of samples taken from the roadbed before and after compaction, respectively. The results are presented in Table G-2.

TABLE G-2
ESTIMATES OF THE ADJUSTING PARAMETERS, p_i's, BY MEANS OF RATIO ESTIMATE

Index i	Sieve Size	Mean Percentage Retained Above Each Sieve		$\hat{p}_i$	$\hat{d}_i$	Standard Error of the Estimate
		Before Compaction	After Compaction			
1	3/4-in.	5.1378	4.7452	0.9236	0.0764	2.216
2	1/2-in.	15.3864	14.8221	0.9633	0.0367	3.412
3	3/8-in.	22.7616	22.3876	0.9836	0.0164	3.791
4	No. 4	37.9126	37.6759	0.9938	0.0062	3.715
5	No. 8	49.5303	49.3500	0.9964	0.0036	3.406
6	No. 16	60.7581	60.4991	0.9957	0.0043	3.111
7	No. 30	72.1976	71.7348	0.9936	0.0064	2.633
8	No. 50	85.6057	84.8121	0.9907	0.0093	1.725
9	No. 100	91.8266	91.0907	0.9920	0.0080	1.305
10	No. 200	93.8079	93.1931	0.9935	0.0065	1.170
11	L.B.W.	5.8193	6.5626	1.1277	--	0.929

We note the standard errors of the estimates are confounded with the sample splitting error (Gilson Sample Splitter). Thus, the actual standard errors of the estimates should be smaller than those presented in Table G-2.

Model 2: Fitting Eq. (G-1) by Using the Least Squares Criterion

Since the data of samples taken before and after compaction are in pairs (arranged by the experiment described in Appendix F), a non-linear curving fitting program (based on the least squares method) was used to fit Eq. (G-1) to obtain the following results in Table G-3.

TABLE G-3
ESTIMATES OF THE ADJUSTING PARAMETERS, p_i's, BASED ON THE LEAST SQUARES CRITERION

Index	Sieve Size	$\hat{p}_i$	$\hat{d}_i$	Standard Error of Estimate	Actual $\bar{y}_i$	Estimated $\bar{y}_i$
1	3/4-in.	0.8318	0.1682	2.156	4.7452	4.2734
2	1/2-in.	0.9315	0.0685	3.375	14.8221	14.3330
3	3/8-in.	0.9639	0.0361	3.763	22.3876	21.9409
4	No. 4	0.9863	0.0137	3.704	37.6759	37.3940
5	No. 8	0.9928	0.0072	3.401	49.3500	49.1748
6	No. 16	0.9938	0.0062	3.109	60.4990	60.3816
7	No. 30	0.9927	0.0073	2.632	71.7348	71.6699
8	No. 50	0.9905	0.0095	1.725	84.8121	84.7921
9	No. 100	0.9919	0.0081	1.305	91.0907	91.0799
10	No. 200	0.9934	0.0066	1.170	93.1931	93.1841
11	L.B.W.	1.1175	--	0.920	6.5626	6.5716

We note that the standard errors of the estimates of the above two methods are about the same. Since the correlation coefficient between the measurements obtained before and after compaction are very high ($r = 0.990649$) and the p_i , $i = 1, \ldots, 10$, are so close, it seems that we only need two adjusting parameters, one for percentage retained above each sieve and the other for percent loss-by-washing. This will allow a simple estimation of the aggregate composition after compaction from that before compaction. The estimating procedures are presented in Model 3.

Model 3:

The fitting equations in this model are:

$$\begin{cases} Y_i = pX_i \quad , i = 1, \ldots, 10 \\ Y_{11} = p_{11} X_{11} \end{cases} \qquad \text{(G-6)}$$

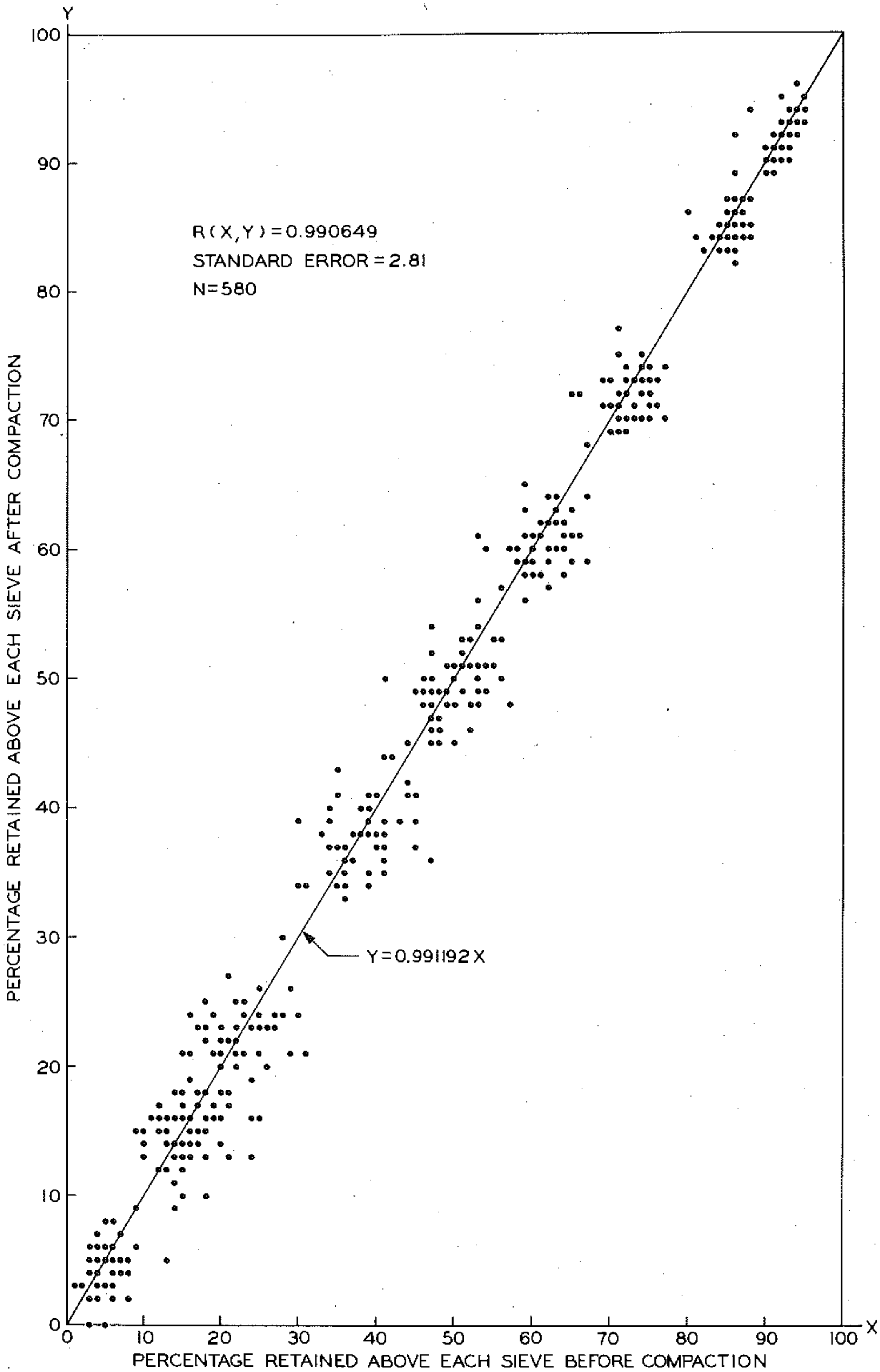

Figure G-1. Plot of percentage retained above each seive before and and after compaction.

The parameter p is estimated to be 0.9912 and the standard error of the estimate is 2.8122. The estimate of p_{11} was presented in Model 2. The actual versus estimated mean percentage retained above each sieve is presented in Table G-4. Figure G-1 is the plot of X_i versus Y_i, $i = 1, \ldots, 10$. Figure G-2 is the plot of X_{11} versus Y_{11}.

TABLE G-4
ACTUAL VERSUS ESTIMATED MEAN PERCENTAGE RETAINED ABOVE EACH SIEVE, $p = 0.9912$ and $p_{11} = 1.1175$

Index	Sieve Size	Actual $\overline{y}_i$	Estimated $\overline{y}_i$
1	3/4-in.	4.7452	5.0926
2	1/2-in.	14.8221	15.2509
3	3/8-in.	22.3876	22.5611
4	No. 4	37.6759	37.5787
5	No. 8	49.3500	49.0940
6	No. 16	60.4990	60.2229
7	No. 30	71.7348	71.5617
8	No. 50	84.8121	84.8517
9	No. 100	91.0907	91.0178
10	No. 200	93.1931	92.9814
11	L.B.W.	6.5626	6.5716

Comparing the standard errors and the actual versus the estimated values of the above three models, we conclude that they have the same precision in estimating aggregate composition after compaction from that before compaction. The choice between Model 1 (or Model 2) and Model 3 probably should depend on the type of aggregate.

Before turning to another subject, we would like to add some comments concerning estimation of the percent loss-by-washing. From the theoretical point of view, we know that

1) $Y_{11} \geq X_{11} \geq 0$

2) when $X_{11} = 0$, $Y_{11} \geq 0$

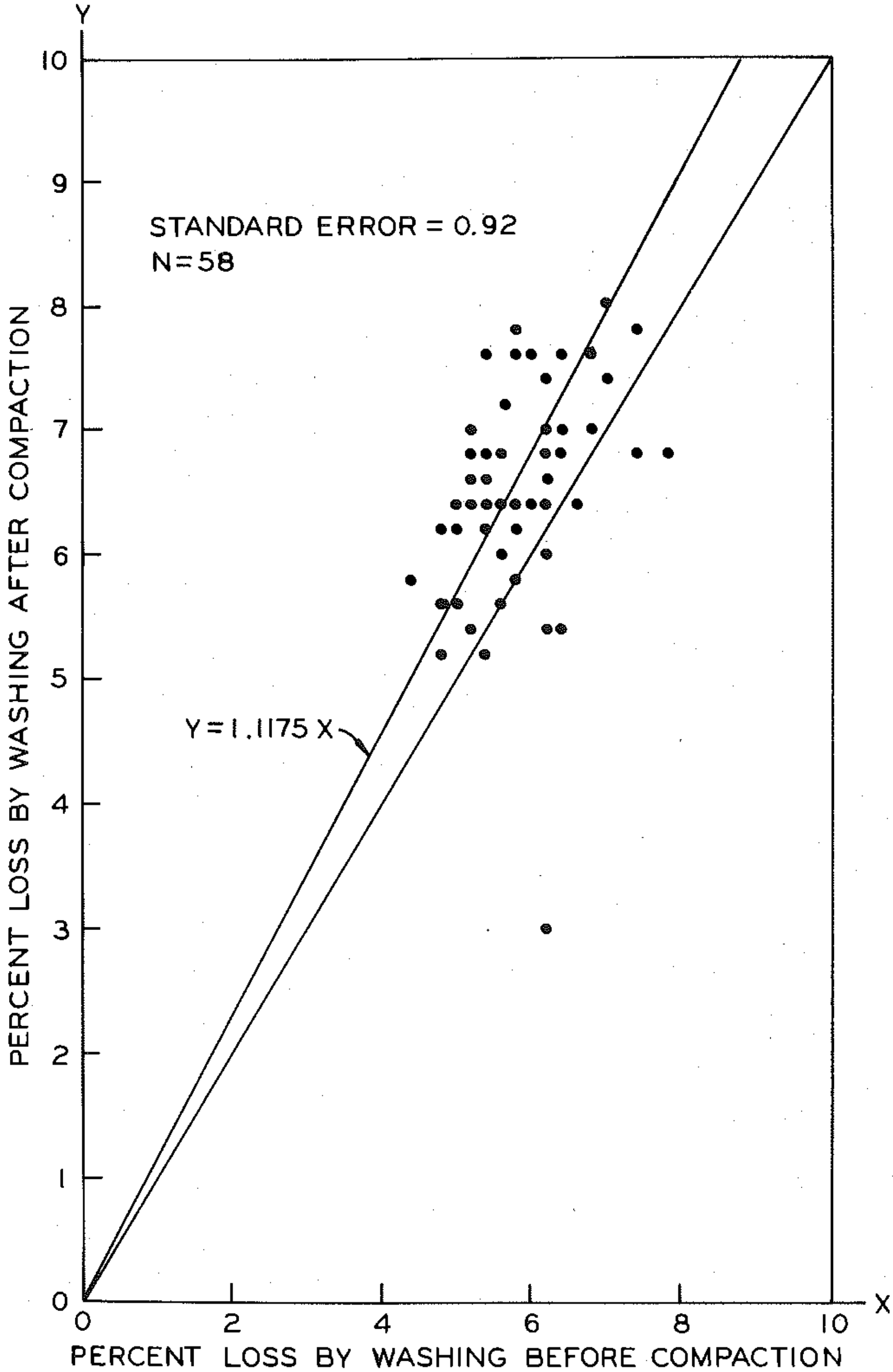

Figure G-2. Plot of the percent loss by washing before and after compaction.

Since the range of X_{11} is from 3 to 8 percent in our data, we could not determine the functional relation of Y_{11} and X_{11}, when X_{11} is near zero. Thus, the linear expression of Y_{11} as X_{11} in the above models might not be proper, especially, when X_{11} is near the zero. For this reason, we present the following two models for estimating the percent loss-by-washing:

Model 4:

The functional relation between X_{11} and Y_{11}, $Y_{11} = f(X_{11})$ is non-linear when X_{11} is near the zero, but almost linear when X_{11} is not zero, with $Y_{11} = a X_{11}$ as an asymptotic line of $Y_{11} = f(X_{11})$. Let $a = \tan \theta$. The above concept is graphically presented in the Figure G-3.

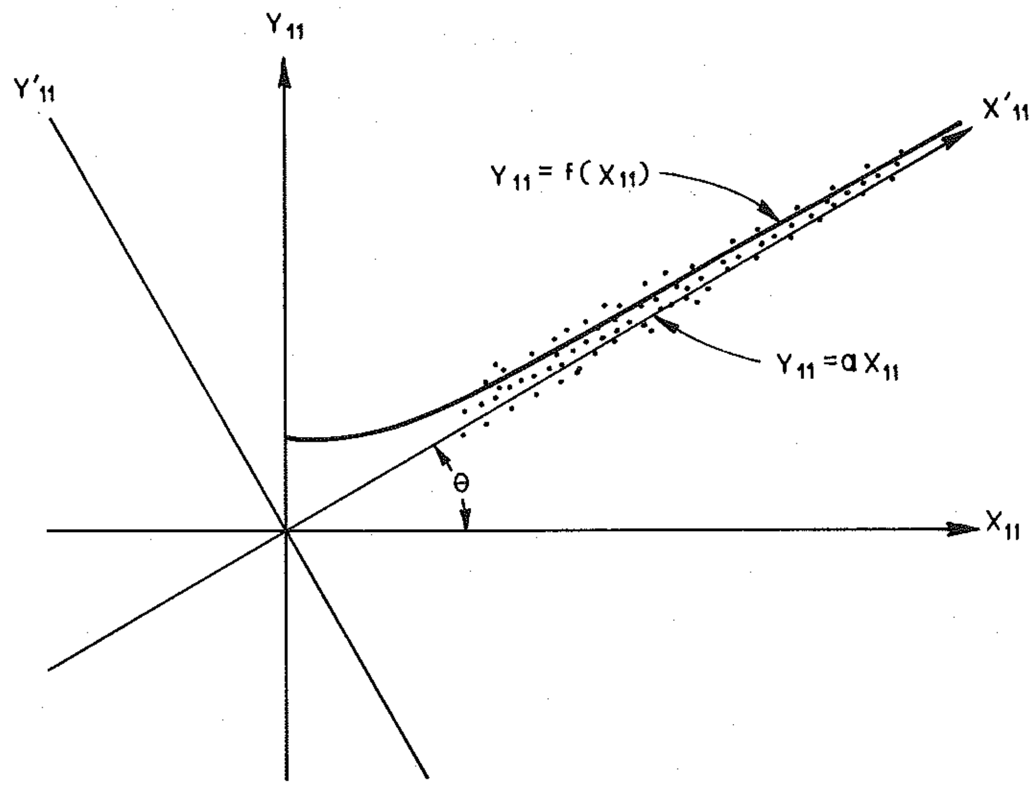

Figure G-3. The functional relation of X_{11} and Y_{11}.

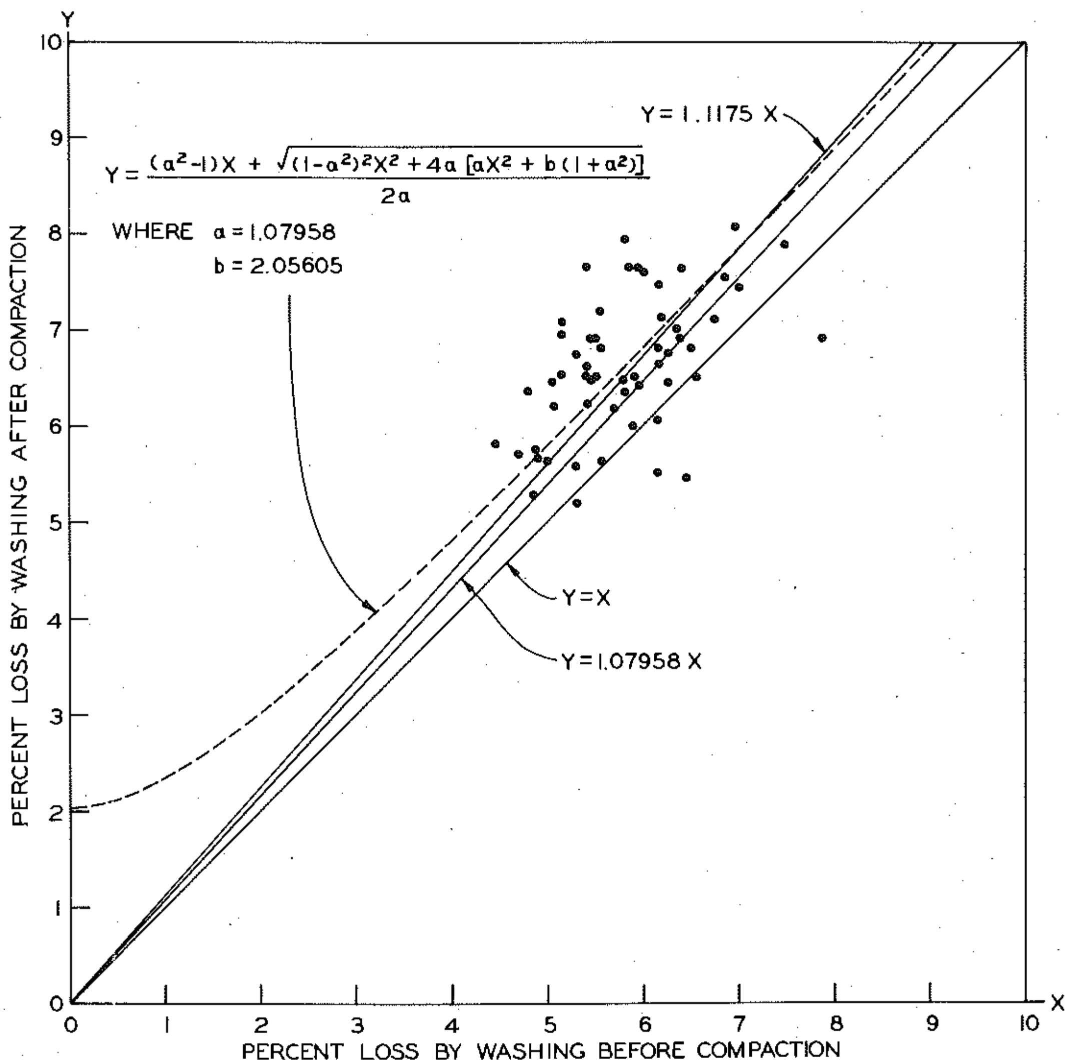

Figure G-4. The functional relationship of the percent loss by washing before and after compaction.

If we use the line $Y_{11} = aX_{11}$ as the new abscissa X'_{11} and rotate the original ordinate θ degrees, Y'_{11} (the functional relation between X_{11} and Y_{11} in terms of the new coordinate system) can be expressed as

$$Y'_{11} = \frac{b}{X'_{11}}, \quad Y'_{11} = be^{-c(X'_{11})^b}, \text{ or } \quad Y'_{11} = b(X'_{11})^{c}e^{-dX'_{11}}, \qquad \text{etc.}$$

It turns out that the first type is the best among those fitted to the data. A non-linear curve fitting the computer program is used to obtain the following estimates: a = 1.07958 and b = 7.05605.

By converting the (X'_{11} , Y'_{11}) coordinate system to the (X_{11} , Y_{11}) coordinate system, we obtain the following fitted equation.

$$Y_{11} = \frac{(a^2-1)X_{11}\sqrt{(1-a^2)X_{11}^2+4a[aX_{11}^2+b(1+a^2)]}}{2a} \qquad \text{(G-7)}$$

The standard error of the estimate obtained from the above equation is 0.734. The estimated mean percent loss-by-washing is 6.6256 compared to the actual value 6.5626. The functional curve of Eq. (G-7) is presented in Figure G-4.

Model 5:

Denote W to be

$$W = (100 - Y_{10}) - Y_{11} \qquad \text{(G-8)}$$

W stands for the difference of the percent passing the No. 200 sieve and the percent loss-by-washing. We have observed from this project that the variance of W is quite small. That is, the percent loss-by-washing after compaction can be estimated as:

$$\hat{Y}_{11} = (100 - P_{10}X_{10}) - \overline{W}$$

where $\overline{W}$ is the estimate of W.

It is computed from our data that $\overline{W}$ is 0.24431. Thus,

$$\hat{Y}_{11} = (100 - 93.1841) - 0.24431 = 6.57159$$

The above estimated value is very close to the actual mean percent loss-by-washing after compaction which is 6.56259.

The above models show the same precision in estimating the percent loss-by-washing after compaction when the range of the percent loss-by-washing before compaction is 4 to 8 percent. Without extra knowledge to support the functional relation between the percent loss-by-washing before and after compaction, we must conclude that Model 5 is the most desirable in estimating the percent loss-by-washing after compaction from that before compaction.

Model for Determining the Degradation Rate of Each Aggregate Size Due to Compaction

We have demonstrated in the previous section several models to estimate parameters, p_i, for adjusting the aggregate composition before compaction to that after compaction.

The $(1 - p_i)$, $i = 1, \ldots, 10$, are interpreted as the degradation rate of the aggregate the grain size of which will retain on sieves corresponding to the index j, $j \leq i$. However, there is no information on how much the aggregate retained on each sieve degrades. To answer this question, we consider the following model.

Due to computer limitation, and since the actual specification of 22A aggregate only involves the percent passing the 1-in., 3/4-in., 3/8-in., and No. 8 sieves, and the percent loss-by-washing, we present the following model dealing with three sieves, 3/4-in., 3/8-in., and No. 8.

Let B_i and A_i be the percentage retained on the sieve corresponding to the index i, before and after compaction, respectively. The relation of the index i and the sieve size is specified in the following table.

TABLE G-5
SIEVE SIZE OF EACH INDEX

Index i	1	2	3
Sieve Size	3/4-in.	3/8-in.	No. 8

Then, the relation between B_i and A_i can be expressed as

$$\begin{cases} A_1 = p_{11} B_1 \\ A_2 = p_{12} B_1 + p_{22} B_2 \\ A_3 = p_{13} B_1 + p_{23} B_2 + p_{33} B_3 \end{cases} \qquad \text{(G-9)}$$

where P_{ii}, $1 \leq i \leq 3$, is the proportion of aggregate retained on the sieve corresponding to index i after compaction, P_{ij}, $1 \leq i < j \leq 3$, is the proportion of aggregate retained on the sieve corresponding to the index i before compaction degraded in size to that retained on the sieve corresponding to index j after compaction and P_{i4}, $1 \leq i \leq 3$, is the proportion of aggregates retained on the sieve corresponding to the index i degraded in size to that passing the No. 8 sieve. Thus, P_{ij}, $j = i, \ldots, 4$, is the degradation distribution of the aggregates retained on the sieve corresponding to index i. It is obvious that

$$\sum_{j=i}^{4} P_{ij} = 1 \qquad \text{(G-10)}$$

The first expression in Eq. (G-9) is obvious. The second expression in Eq. (G-9) can be interpreted as follows:

The percentage retained on the 3/8-in. sieve after compaction = (the percent of 3/4-in. aggregates degraded due to compaction to that retained on the 3/8-in. sieve).

+ (the percent of 3/8-in. aggregates remaining on the 3/8-in. sieve after compaction).

The third expression in Eq. (G-9) is interpreted in a similar way.

Again, a non-linear curve fitting computer program is used to fit Eq. (G-9) to the actual data. The fitted results are:

$$\begin{cases} \hat{P}_{11} = 0.948877 \\ \hat{P}_{12} = 0.051123^{-} \\ \hat{P}_{13} = 0^{+} \\ \hat{P}_{14} = 0 \end{cases} \qquad \text{(G-11)}$$

$$\begin{cases} \hat{P}_{22} = 0.975332 \\ \hat{P}_{23} = 0.024668^{-} \\ \hat{P}_{24} = 0^{+} \end{cases} \qquad \text{(G-12)}$$

$$\begin{cases} \hat{P}_{33} = 0.97058 \\ \hat{P}_{34} = 0.02942 \end{cases} \qquad \text{(G-13)}$$

The standard error of the estimate is 2.46. The actual and the estimated mean percentage retained on each sieve are presented in the following table.

TABLE G-6
ACTUAL AND ESTIMATED
MEAN PERCENTAGE RETAINED
ON EACH SIEVE

Sieve Size	3/4-in.	3/8-in.	No. 8
Actual percentage	4.7452	17.6424	26.9624
Estimated percentage	4.8751	17.4517	26.4256

As one can see, the estimated values are very close to the actual ones. The major finding of the model is that aggregate retained on each sieve degrade only to the size of aggregate retained on the next size of sieve when degradation is due to compaction. The model estimates that about 5.1, 2.5, and 2.9 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade due to compaction.

Model to Estimate Aggregate Composition Before Compaction From That of the Stockpile

Since there is no way to pair the samples from the roadbed and the stockpile, Model 1 is the only way that can be used to estimate the composition of in-place aggregate before compaction from that of the stockpile. The results are shown in Table G-7.

Model for Determining the Degradation Rate of Each Aggregate Size Due to Handling

The model represented in Eq. (G-9) is no longer usable for this case because samples taken from the stockpile and construction site are not in pairs. However, based on the information obtained from the compaction process, it is reasonable to assume that each aggregate size degrades only to the next size. With this assumption, Eq. (G-9) can be modified to be:

$$\begin{cases} \hat{P}_{11} = 1 - \bar{A}_1 / \bar{B}_1 \\ \hat{P}_{22} = 1 - (\bar{A}_2 - \hat{P}_{11}\bar{B}_1) / \bar{B}_2 \\ P_{33} = 1 - (\bar{A}_3 - P_{22}\bar{B}_2) / \bar{B}_3 \end{cases} \qquad \text{(G-14)}$$

The $\bar{A}_i$ in Eq. (G-14) is the average percentage retained on the sieve corresponding to the index i, of samples taken from the roadbed before compaction. The $\bar{B}_i$ is for the measurement for samples taken from the stockpile. By using Eq. (G-14), we obtain the following estimates that 27.83, 20.51, and 14.42 percent of the total percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves, respectively, degrade due to handling.

TABLE G-7
ESTIMATES OF THE ADJUSTING PARAMETERS, p_i's, BY MEANS OF RATIO ESTIMATE

Index i	Sieve Size	Mean Percentage Retained Above Each Sieve		$\hat{p}_i$	$\hat{d}_i$
		Stockpile	Before Compaction		
1	3/4-in.	7.15	5.16	0.7217	0.2783
2	1/2-in.	18.81	15.41	0.8192	0.1808
3	3/8-in.	26.85	22.81	0.8495	0.1505
4	No. 4	42.01	33.98	0.8089	0.1911
5	No. 8	53.42	49.59	0.9283	0.0717
6	No. 16	64.33	60.81	0.9453	0.0547
7	No. 30	75.34	72.22	0.9586	0.0414
8	No. 50	88.08	85.61	0.9720	0.0280
9	No. 100	93.59	91.81	0.9810	0.0190
10	No. 200	95.30	93.78	0.9841	0.0159
11	L.B.W.	4.56	5.84	1.2800	--

APPENDIX H

EQUATIONS FOR DESIGNING SINGLE SAMPLING FRACTION DEFECTIVE SAMPLING PLAN AND WALD'S SEQUENTIAL PROBABILITY RATIO PLAN

In this appendix, we summarize the related equations for designing an aggregate inspection plan of so-called "acceptance sampling by attributing," such as the "single sampling fraction defective sampling plan" (SSFD) and the "Wald's truncated sequential probability ratio plan" (TSPR).

Basically, we desire a plan such that the lot acceptance probability is $1-\alpha$ for material of P_α quality and is β for material of P_β quality. The α and β are called the producer's and consumer's risk, respectively. The P_α and P_β are, respectively, called the acceptable and rejected product quality level. Details of each type of plan are discussed in the following.

Single Sampling Fraction Defective Sampling Plan (SSFD)

This type of plan specifies the sample size N that should be taken and the acceptance number, C , of the defective items that are allowed for lot acceptance. The desired N and C should satisfy the following equations:

$$\begin{cases} \sum_{i=0}^{C} \binom{N}{i} P_\alpha^i (1-P_\alpha)^{N-i} = 1-\alpha \\ \sum_{i=0}^{C} \binom{N}{i} P_\beta^i (1-P_\beta)^{N-i} = \beta \end{cases} \quad \text{(H-1)}$$

For a plan (N and C) determined by Eq. (H-1), the lot acceptance probability P_a for each given value of product quality P can be computed by the following equation.

$$P_a = \sum_{i=0}^{C} \binom{N}{i} P^i (1-P)^{N-i} \quad \text{(H-2)}$$

In the actual inspection procedure, the number of defective samples might be greater than C before all N samples are tested. Thus, the actual sample size needed to reach a decision (acceptance or rejection) is less than or equal to N . If we denote ASN to be the average sampling size, then

$$ASN = \sum_{i=C+1}^{N} i P_i \quad \text{(H-3)}$$

where

$$P_i = \binom{i-1}{C} P^{i+1} (1-P)^{i-C-1}, \quad i = C+1, \cdots\cdots, N-1,$$

and

$$P_N = 1 - \sum_{i=C+1}^{N-1} P_i$$

Wald's Sequential Probability Ratio Plan (SPR)

We describe this type of plan as follows. One sample is taken at a time. The cumulated sample results are plotted consecutively in a chart (Fig. H-1). For each point the abscissa N is the total number of samples drawn up to that time and the ordinate X is the total number of these samples that are defective. If the plotted points stay within a zone marked by parallel lines, the sampling is continued without a decision. As soon as a point falls outside that zone, we accept the lot if that point is on or below the lower line, and reject the lot if that point is on or above the upper line.

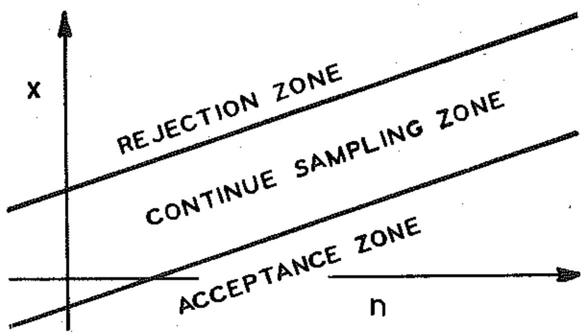

Figure H-1. Acceptance, rejection and continue sampling zone of a SPR plan.

Equations of the two parallel lines derived by Wald are presented in the following.

The lower line $X = sn - h_1$ (H-4)

The upper line $X = sn + h_2$ (H-5)

where

$$h_1 = \text{LOG}\left(\frac{1-\alpha}{\beta}\right) \Big/ \text{LOG}\left[\frac{P_\beta(1-P_\alpha)}{P_\alpha(1-P_\beta)}\right] \qquad \text{(H-6)}$$

$$h_2 = \text{LOG}\left(\frac{1-\beta}{\alpha}\right) \Big/ \text{LOG}\left[\frac{P_\beta(1-P_\alpha)}{P_\alpha(1-P_\beta)}\right] \qquad \text{(H-7)}$$

and

$$S = \text{LOG}\left(\frac{1-P_\alpha}{1-P_\beta}\right) \Big/ \text{LOG}\left[\frac{P_\beta(1-P_\alpha)}{P_\alpha(1-P_\beta)}\right] \qquad \text{(H-8)}$$

For a given value of θ , the following equations give the product quality P and the acceptance probability P_a .

$$P = \frac{1-\left(\frac{1-P_\beta}{1-P_\alpha}\right)^\theta}{\left(\frac{P_\beta}{P_\alpha}\right)^\theta - \left(\frac{1-P_\beta}{1-P_\alpha}\right)^\theta} \qquad \text{(H-9)}$$

and

$$P_a = \left[\left(\frac{1-\beta}{\alpha}\right)^\theta - 1\right] \Big/ \left[\left(\frac{1-\beta}{\alpha}\right)^\theta - \left(\frac{\beta}{1-\alpha}\right)^\theta\right] \qquad \text{(H-10)}$$

The sample size in this type of plan is not fixed and depends heavily on the product quality. However, the average sampling number ASN can be computed from the following equation derived by Wald.

$$\text{ASN} = \frac{P_a \text{ LOG} \frac{\beta}{1-\alpha} + (1-P_a) \text{ LOG} \frac{1-\beta}{\alpha}}{P \text{ LOG} \frac{P_\beta}{P_\alpha} + (1-P) \text{ LOG} \frac{1-P_\beta}{1-P_\alpha}} \qquad \text{(H-11)}$$

Wald's Truncated Sequential Probability Ratio Plan (TSPR)

This type of plan is the same as the SPR plan except that it includes a new rule for the acceptance or rejection at the N_0 th trial if the SPR procedures did not lead to a final decision for $N \leq N_0$. For a fixed N_0 , one method suggested by Wald is to accept the lot at the N_0 th trial if

$$\sum_{i=1}^{N_0} \text{LOG} \frac{f(X_i, P_\beta)}{f(X_i, P_\alpha)} \leq 0 \,, \qquad \text{(H-12)}$$

where

$$f(X_i, P) = P \text{ if } X = 1$$

$$= 1 - P \text{ if } X = 0$$

otherwise, the lot is rejected. That is, the lot will be rejected if the total number of defective items at the N_O th trial is less than or equal to C_O defined below.

$$C_O = \frac{N_O \log \frac{1-P_\beta}{1-P_\alpha}}{\log \frac{1-P_\beta}{1-P_\alpha} - \log \frac{P_\beta}{P_\alpha}} \tag{H-13}$$

Now, denote P_i to be the probability that a lot is accepted at the i^{th} trial and X_i to be the total number of defective items up to the i^{th} trial. Then, the acceptance probability P_a is

$$P_a = \sum_{i=1}^{N_O} P_i \quad , \tag{H-14}$$

where

$$P_1 = P(X_1 \leq S - h_1),$$

$$P_i = P(KS - h_1 < X_K < KS + h_2, K=1, \cdots, i-1, \text{ AND } X_i \leq iS - h_1), \; i = 2, \cdots, N_O - 1$$

and

$$P_{N_O} = P(KS - h_1 < X_K < KS + h_2, \; K = 1, \cdots, N_O - 1, \text{ AND } X_{N_O} \leq C_O)$$

Applying the above equations to the four TSPR plans defined in Chapter I-5, we obtain the following results.

For TSPR 1:

$$\begin{cases} P_i = 0, \; i = 1, \cdots, 23, 25, \cdots, 36, \\ P_{24} = (1-P)^{24} \\ P_{37} = 24(1-P)^{36} P \end{cases} \tag{H-15}$$

For TSPR 2:

$$\begin{cases} P_i = 0, \; i = 1, \cdots, 11, 13, \cdots, 17, \\ P_{12} = (1-P)^{12} \\ P_{18} = 12P(1-P)^{17} \end{cases} \tag{H-16}$$

For TSPR 3:

$$\begin{cases} P_i = 0, \ i = 1, \cdots, 6, 8, \cdots, 11, \\ P_7 = (1-P)^7 \\ P_{12} = 7(1-P)^{11} P \end{cases} \tag{H-17}$$

For TSPR 4:

$$\begin{cases} P_i = 0, \ i = 1, \cdots, 6, 8, 9, \\ P_7 = (1-P)^7 \\ P_{10} = 7(1-P)^9 P \end{cases} \tag{H-18}$$

In order to compute the average sampling size ASN, we first need to compute R_i, which is the probability of rejecting a lot at the i^{th} trial, $i = 1, \ldots, N_0$. Following the same pattern as before, it can be shown that

For TSPR 1:

$$R_i = \begin{cases} 0 & , i = 1, 20 \\ (i-1)(1-P)^{i-2} P^2 & , i = 2, \cdots, 19, \\ \left[19(i-20) + (i-20)(i-21)/2\right](1-P)^{i-3} P^3 & , i = 21, \cdots, 24 \\ \left[158 + 77(i-25)\right](1-P)^{i-3} P^3 & , i = 25, \cdots, 35 \\ 1005(1-P)^{33} P^3 + 1082(1-P)^{34} P^2 & , i = 36 \\ 24(1-P)^{35} P^2 & , i = 37 \end{cases} \tag{H-19}$$

For TSPR 2:

$$R_i = \begin{cases} 0 & , i = 1, 11 \\ (i-1)(1-P)^{i-2} P^2 & , i = 2, \cdots, 10 \\ \left[(K-13)I(K-13) + (K-12) + 10(K-11)\right](1-P)^{K-3} P^3 & , i = 12, \cdots, 16 \\ 69(1-P)^{14} P^3 + 81(1-P)^5 P^2 & , i = 17 \\ 12(1-P)^{16} P^2 & , i = 18, \end{cases} \tag{H-20}$$

where $I(x) = 0$ if $x \leq 0$, $= 1$ if $x > 0$.

For TSPR 3:

$$R_i = \begin{cases} 0 & , i = 1, 8 \\ (i-1)(1-P)^{i-2} P^2 & , i = 2, \cdots, 7 \\ 7(i-8)(1-P)^{i-3} P^3 & , i = 9, 10 \\ 21(1-P)^8 P^3 + 28(1-P)^9 P^2 & , i = 11 \\ 7(1-P)^{10} P^2 & , i = 12 \end{cases} \qquad \text{(H-21)}$$

For TSPR 4:

$$R_i = \begin{cases} 0 & , i = 1, 10 \\ (i=1)(1-P)^{i-2} P^2 & , i = 2, \cdots, 8 \\ R_8 & , i = 9 \end{cases} \qquad \text{(H-22)}$$

Once P_i and R_i are computed for each i, $i = 1, \ldots, N_O$, the average sampling size ASN can be easily computed by the following equation.

$$ASN = \sum_{i=1}^{N_O} i(P_i + R_i) \qquad \text{(H-23)}$$

APPENDIX I

STATISTICAL EVALUATION OF THE PROPOSED AGGREGATE TESTING METHOD

We proposed an aggregate testing method in Part III. In this appendix, we provide the statistical evaluation of that proposed testing method in estimating the aggregate composition.

A Simple Concept in Estimating Aggregate Composition

What could be the difference in aggregate composition measured by the current and the proposed testing methods? It is apparent that the percentage retained on each sieve measured by the proposed testing method is at least larger than that measured by the current testing method because fine aggregate which would be eliminated by the washing process could attach to aggregate retained on each sieve. Thus, to estimate the aggregate composition we must first estimate the amount of fine aggregate which ordinarily would be washed out and attach to aggregate retained on each sieve. Thus, the additional measurement needed is the percent loss-by-washing. This can be done with the help of a very fine sieve, say the No. 200 sieve, because the percent passing this sieve is very close to the percent loss-by-washing calculated by the current testing method. Moreover, the difference of these two measurements is nearly a constant. We present the above concepts in the following mathematical and statistical formulation.

For a sample of aggregates, let x_j, $j = 1, \ldots, 5$, be the percentage retained on the 1-in., 3/4-in., 3/8-in., No. 8, and No. 200 sieves, respectively; let x_6 be the percent passing the No. 200 sieve, and x_7 be the percent loss-by-washing measured by the current testing method. Define

$$A = x_6 - x_7 \qquad \text{(I-1)}$$

The random quantity A is the percent passing the No. 200 sieve, excluding the percent loss-by-washing. Thus, A is a non-negative random variable and, in general, has small variance. If we define Y_j as x_j, $j = 1, \ldots, 6$, measured by the proposed testing method, then we have the following relation,

$$Y_j - x_j = B_j + e_j, \; j = 1, \ldots, 5 \qquad \text{(I-2)}$$

where each B_j stands for the fine aggregate attached to aggregate retained on the sieve corresponding to the index j, which will be washed out if the current testing method is used, and the e_j's are random errors. By definition, B_j is a non-negative random variable, therefore the expected value, $E(B_j)$ of each B_j is non-negative.

In practice, each sample can be tested only by either one or the other of the two testing methods. Therefore, we use a Gilson Sample Splitter to split each sample into two sub-samples. We then randomly assign each sub-sample to one or the other of the testing methods. Further, if we denote W_{ij} as the true percentage retained on the sieve corresponding to the index j of sample i, $i = 1, 2$, then the e_j in Eq. (I-2) can be expressed as:

$$e_j = W_{1j} - W_{2j} \qquad \text{(I-3)}$$

Under these conditions, the e_j 's are random errors introduced by the Gilson Sample Splitter. If the above randomization takes place, it is expected that the expected value, $E(e_j)$, is zero. Therefore,

$$E(Y_j - X_j) = E(B_j) \geq 0 \qquad \text{(I-4)}$$

and

$$VAR(Y_j - X_j) = VAR(B_j) + VAR(e_j) \qquad \text{(I-5)}$$

The random error e_j can be reduced to a negligible amount with proper sample splitting technique. That is, Eq. (I-2) can be rewritten as:

$$Y_j - X_j = B_j \qquad \text{(I-6)}$$

If the random quantity B_j is "controllable" or "predictable" in the sense that the variance of B_j is very small or the B_j is proportional to X_j or Y_j , the aggregate composition measured by the proposed testing method can be accurately adjusted to that obtainable with the current testing method. One of the simplest estimation procedures suggested by Eq. (I-6) is to use the expected value of B_j which is estimated by the sample average of the quantity $X_j - Y_j$ as the adjusting parameter. That is, the X_j , $j = 1, \ldots, 5$, are estimated to be:

$$\hat{X}_j = Y_j - \bar{B}_j \qquad \text{(I-7)}$$

and X_7 , the percent loss-by-washing, is estimated to be:

$$\hat{X}_7 = \sum_{j=1}^{5} \bar{B}_j + Y_6 - \bar{A} \qquad \text{(I-8)}$$

The feasibility of the above estimating method for various types of aggregate is now discussed.

1) Sandy Aggregate - Since this type of aggregate is not sticky, the amount of fine aggregate attached to the coarse aggregate is negligible relative to sampling error, testing error, etc. Moreover, the variance of B_j for each j will be small. For this aggregate type, the estimated percentage is expected to be close to the actual percentage. Therefore, the proposed testing method is expected to be superior to the current method because of superior cost and testing efficiency.

2) Clayey Aggregate - For this type of aggregate, each B_j might be quite large, but should still have small variance. That is, the above estimating procedures are again good enough to be used to adjust the aggregate composition measured by the proposed testing method to that obtainable with the current testing method.

3) Mixture of Types (1) and (2) - Each B_j might have a sizable variance. If the variance is not large, the above crude estimation should still be satisfactory, and should be used because of its simplicity. Otherwise, other types of estimating procedures should be developed to fit the particular need.

Unless the aggregate type in a production pit varies greatly from sample to sample, the above method should give satisfactory estimates relative to other errors introduced by sampling methods, testing methods, sample splitter, etc.

We note that the variance of B_j could be large even for the sandy or clayey type of aggregate if the aggregate production process is not stable, i.e., the variance of X_j or Y_j is quite large. If this is the case, the above estimating method would still be good for estimating the mean aggregate composition, but not good enough for estimating aggregate composition of individual samples. For the latter case, other methods should be developed. For example, expressing X_j as:

$$X_j = P_j Y_j \tag{I-9}$$

where the P_j's are parameters to be estimated.

Each P_j in Eq. (I-9) can be estimated as:

$$\hat{P}_j = \overline{X}_j / \overline{Y}_j \tag{I-10}$$

or estimated by ordinary least squares techniques. Intuitively, this estimating method should be better than that presented in Eq. (I-7) and is therefore recommended. However, since the purpose of this study is to demonstrate the feasibility of the proposed test method in estimating aggregate composition, we shall use the simpler approach presented in Eqs. (I-7) and (I-8). If the simplest method works for the purpose, better methods would definitely work as well.

To support the above theoretical arguments concerning the feasibility of the proposed test method, a statistical experiment was conducted to acquire and test samples by both test methods as follows.

Data Preparation

Twenty-eight samples were taken from a stockpile in which aggregate can be considered as sandy (Type 1). In addition, 14 samples were taken from the construction site; this could be considered as clayey (Type 2) because it was blended with liquid calcium chloride. These 42 samples together could be considered as samples of Type 3. Each sample was divided in two sub-samples with a Gilson Sample Splitter. The first sub-sample was randomly assigned either the current or the proposed testing method. The other sub-sample was assigned the remaining method. Every sample was tested in the Laboratory by one inspector using one set of sieves to measure the percentage retained on the 1-in., 3/4-in., 3/8-in., No. 8, and No. 200 sieves; the percent passing the No. 200 sieve and the percent loss-by-washing (current testing method only). Since the percentage retained on the 1-in. sieve is always zero in this experiment, this measurement was not included in the analysis. The above experiment constitutes a one-way completely randomized block design.

Data Analysis

The statistical model for the above experiment is the same as that specified in Chapter I-2, and, therefore, will not be described here. We are interested in testing the hypothesis of equal testing method effects. That is, we are testing whether or not the aggregate composition measured by the two testing methods are the same. The analysis shows that the mean aggregate compositions measured by both methods are not the same. We present estimates of the aggregate composition measured by both testing methods in Table I-1. We also present the 95 percent simultaneous confidence intervals (SCI) for the mean differences between the two testing methods in Table I-2.

TABLE I-1
ESTIMATES OF AGGREGATE COMPOSITION MEASURED BY THE PROPOSED AND CURRENT TEST METHODS

Index	Sieve Size	Percentage Retained on Each Sieve		Difference
		Proposed Testing Method	Current Testing Method	
2	3/4-in.	5.88	5.65	0.22
3	3/8-in.	17.74	18.44	-0.40
4	No. 8	26.57	25.59	0.98
5	No. 200	47.90	45.20	2.70

TABLE I-2
95 PERCENT SCI FOR THE MEAN DIFFERENCES ON EACH MEASUREMENT BETWEEN TWO TESTING METHODS

Sieve Size	3/4-in.	3/8-in.	No. 8	No. 200
LCL	-1.12	-1.92	-0.47	0.83
UCL	1.57	1.11	2.42	4.58

Since the confidence interval for the percentage retained on the No. 200 sieve is the only one that does not cover zero, we conclude that the two testing methods disagree only in measuring the retained percentage on the fine sieves.

Before turning to other subjects, the following observations should be noted.

1) The differences between the test results of the two testing methods are not always positive, thus indicating that the random disturbance introduced by a Gilson Sample Splitter is much larger than the true difference between the two testing methods.

2) We see from Table I-1 that $\bar{B}_3$ is negative, and this contradicts Eq. (I-4). This situation can occur in any of the following circumstances:

a) The sample size is not large enough for complete randomization to take place.

b) The two samples obtained from a Gilson Sample Splitter are not randomly ordered and assigned to each testing method.

c) Recording error on some particular samples occurred.

At the present time we do not know the cause of the negative $\bar{B}_3$ in Table I-1. However, this is not a serious problem for the purpose of this representation.

Evaluation of the Estimation Procedures Based on the Conducted Experiment

We know that $\bar{B} = 0$. The $\bar{B}_j$ for $j = 2, \ldots, 5$, were presented in Table I-1. Thus, $\sum_{j=1}^{5} \bar{B}_j = 3.50$. The sample mean and standard deviation of A defined in Eq. (I-1) were computed as:

$$\bar{A} = 0.165$$

and

$$STDV(A) = 0.0241$$

Since the Gilson Sample Splitter is always used in the field operation to reduce a sample to test size, a random disturbance due to the splitter always exists. This fact, together with the conclusion of the previous analyses, suggests that there is no need to convert most of the test results of the proposed testing method to those obtainable with the current method. That is, we probably can consider the percentage retained on the 3/4-in., 3/8-in., and No. 8 sieves as the estimates of those obtainable with the current testing method. The only measurement requiring estimation is the percent loss-by-washing. This is accomplished by using Eq. (I-8) as follows:

The estimated percent loss-by-washing for each observation is

$$\hat{X}_7 = \sum_{j=1}^{5} \bar{B}_j + Y_6 - \bar{A} = Y_6 + 3.50 - 0.165 \qquad \text{(I-11)}$$

After this calculation a new data set is obtained in which the percentages retained on the 3/4-in., 3/8-in., and No. 8 sieves are obtained by using the proposed testing method, and the percent loss-by-washing is obtained by Eq. (I-11). This revised data set is compared with the one obtained by the current testing method. The statistical analysis shows that the aggregate composition of these two data sets is comparable. This indicates that the estimating procedures presented in Eqs. (I-7) and (I-8) are reasonably accurate.

Since the only measurement that was estimated is the percent loss-by-washing, let us look at this measurement more closely.

1) In-Place Samples ($N = 14$): The adjusting factors computed from the 14 pairs of samples taken from the construction site are:

$$\sum_{j=1}^{5} \bar{B}_j = 4.62 \quad \text{WITH} \quad \text{STDV}\left(\sum_{j=1}^{5} B_j\right) = 1.28$$

and

$$\bar{A} = 0.163571 \quad \text{WITH} \quad \text{STDV(A)} = 0.0247$$

Thus, the estimated percent loss-by-washing is:

$$\hat{X}_7 = Y_6 + 4.62 - 0.163571$$

for each observation Y_6 .

It is computed that the standard error of the estimating method for the percent loss-by-washing is 0.637. The actual standard error should be smaller than the above figure which is confounded with the error due to the sample splitter.

2) Stockpile Samples ($N = 28$): The adjusting factors computed from the 28 pairs of samples taken from the stockpile are:

$$\sum_{j=1}^{5} \bar{B}_j = 2.93 \quad \text{WITH} \quad \text{STDV}\left(\sum_{j=1}^{5} B_j\right) = 0.96$$

and

$$\bar{A} = 0.166 \quad \text{WITH} \quad \text{STDV(A)} = 0.0242$$

Thus, the estimated percent loss-by-washing is:

$$\hat{X}_7 = Y_6 + 2.93 - 0.166$$

for each observation Y_6.

The standard error in this case is computed to be 0.577.

From a practical point of view, 0.637 and 0.577 are not large relative to other errors caused by sampling, testing, and splitting methods. That is, the estimated percent loss-by-washing is accepted as a good estimate of the true measurement obtained by the current test method.

Based on above analyses we conclude that the proposed testing method, together with the estimation procedures, appear to be as good as the current testing method for measuring aggregate composition.