MEDIAN BARRIERS AND HIGHWAY SAFETY

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A LITERATURE SURVEY OF MEDIAN BARRIERS
AND HIGHWAY SAFETY

The question of guardrail and median barrier installation at a particular
location is complicated by the considerable doubts expressed in the literature
as to net safety benefits. In general, it is acknowledged that any barrier, suf-
ficiently strong to contain high velocity impact, is itself a hazard. Therefore,
engineers are cautioned as to the complex, 'trade-off' nature of decision making
in this area. There are numerous publications issued over the past 20 years
that point up the mixed benefit nature of guardrails and median barriers. Gen-
erally it is found that while fatal accidents remain the same or decrease slightly
after barrier installation, injury and property damage accidents substantially
increase.

Pennsylvania, for example, found that barrier installation in a narrow (4 to
10 ft) median dividing roadways of high volume (up to 130,000 ADT) seemed to
affect fatal accident expectations (four occurring rather than the expected six;
a decrease not considered statistically significant) but definitely affected injury
and property damage accidents:

> In 1964 the Pennsylvania Department of Highways published
> the technical report "Effects of Guard Rail in a Narrow Median
> Upon the Pennsylvania Driver." Part II of that report was
> concerned with a 'before' and 'after' accident study related to
> the installation of a back-to-back beam-type median barrier.
> The accident study was based on State Police and City of Phila-
> delphia Police accident reports. It was concluded using police
> data that in a one-year period before and after installation of
> the median barrier accident frequencies increased 73 percent
> and 38 percent in each of two sections studied with a 10 percent
> increase in volume. (1)

California, in a 1958 study, found that the fatality rate for traversable medians
was lower than that for non-traversable medians. In explaining this finding, they
say:

> On the other hand, the introduction of a physical barrier in
> a traversable or deterring median reduces the usable width of
> the median. If this usable width of the median is a factor in
> the over-all safety of a freeway, it would be a rational explan-
> ation of the noted increase in the accident rates with the instal-
> lation of a barrier. A driver's freedom to maneuver to avoid
> collision with other vehicles is reduced by a median barrier.
> There are undoubtedly vehicles which enter and in some cases
> cross the median and recover without a reportable accident
> when no barrier is present. More important, perhaps, is the
> fact that stalled vehicles are observed daily in median areas. (2)

Elsewhere in that study, the authors couch the barrier question in the form of a dilemma:

> In the basic study it was seen that if past experience is a guide, the installation of positive barriers in 'deterring-type' medians, when the volume is less than about 130,000 vehicles per day, would increase not only the total number of accidents, but the number of injuries and fatalities. On the other hand, the fact that, in three years, 19 percent of all fatalities on freeways were caused by cross-median collisions is extremely serious. The question is: would a reduction in the cross-median fatalities, accomplished by installing positive barriers, be accompanied by a rise in other types of fatalities that would more than offset the benefit? (2)

In conclusion, the authors state that barriers may be desireable even though some accident types will be increased by them:

> 1. The type of median influences the number of accidents on divided highways. On highways with traffic volume between 15,000 and 130,000 vehicles per day, the accident rate was 92 accidents per hundred-million vehicle-miles for earth and low curb medians, and 136 accidents per hundred-million vehicle-miles for the guardrail or concrete-wall-type median. Separate roadways had a rate of 139 in this volume range.

> 2. Traffic volume appears to be a factor in the relative safety of the various types of medians. Where traffic volumes were between 15,000 and 130,000 vehicles per day, the non-barrier-type median was superior. Where traffic volumes exceeded 130,000 vehicles per day, the advantage shifted to the non-traversable barrier-type median. (2)

In a later California study, designed to evaluate the effectiveness of barriers installed in the 1960's, Johnson writes that despite a drop in fatal accidents, narrow median roadways with either cable or beam barriers separating high opposing traffic volumes experience a definite rise in barrier-induced accidents:

> The effect of median barrier installation on accident rates is indicated by Table 1. Sections of highway where the beam barrier was installed had higher rates in both the before and after periods. Generally the beam barrier has been installed on freeways with narrower medians (less than 16 ft) which also tend to be the older freeways with higher volumes and lower geometric standards with an adverse effect on accident rates.
> The rise in accident rates can be attributed primarily to the median barrier installation. The accident rate on all urban freeways has increased slightly during the past few years. However, the accident rate on urban freeways with median barriers has increased more than the state-wide average for urban freeways. It is believed that the

primary reason for the increase in accident rates is that the
median barrier is a fixed object struck by out-of-control
vehicles that might have recovered without incident if the
barrier had not been installed. (3)

For these types of freeways, even cable barriers increase certain types of acci-

dents over the traversable median:

> Injury and fatal accidents combined increased after median
> barrier installation (Table 1). The beam barrier increases
> injury and fatal accidents approximately twice as much as
> does the cable barrier and it is believed that this is the rea-
> son for the increased severity.
> The ratio of the all accident rate to the injury and fatal
> accident rate is given in Table 1. The ratios in the before
> period are almost equal (2.2:1) and are normal for Cali-
> fornia freeways. In the after period, the ratio for the beam
> barrier is considerably lower than that for the cable, which
> is further evidence that the beam barrier increases the
> severity of accidents more than the cable barrier. (3)

TABLE 1

EFFECT OF MEDIAN BARRIER INSTALLATION ON ACCIDENTS

Barrier Type	Length (mi)	MVM	All Accidents				Injury and Fatal Accidents				Ratio[a]
			No.	Rate	Rate Change		No.	Rate	Rate Change		
					Abs.	Percent			Abs.	Percent	
(a) Before Installation											
Cable	26.6	1,195.6	1,586	1.33	—	—	713	0.60	—	—	2.22:1
Beam	27.6	1,633.8	2,690	1.65	—	—	1,204	0.74	—	—	2.23:1
Total	54.2	2,829.4	4,276	1.51	—	—	1,917	0.68	—	—	2.22:1
(b) After Installation											
Cable	26.6	1,277.8	2,231	1.75	+0.42	+32	904	0.71	+0.11	18	2.46:1
Beam	27.6	1,608.5	3,330	1.98	+0.33	+20	1,612	0.96	+0.22	30	2.06:1
Total	54.2	2,958.3	5,561	1.88	+0.37	+25	2,516	0.85	+0.17	25	2.21:1

[a] Of all accident rate to injury and fatal accident rate.

In an early California study, traversable (paved or hard earth median), deter-

ring (raised bar and berm, mountable double curb, and earth type), and non-travers-

able (physical obstruction) medians were compared. It was found that while tra-

versable median accidents tended to be more severe, non-traversable median acci-

dents were more frequent:

> When the sample was sorted on the basis of median width, the
> lowest accident and injury accident rates for deterring medians
> were definitely in the 4-to-6-ft range. Traversable medians
> showed lowest total accident rates in the 6-to-10-ft range and
> lowest injury accident rates in the 20-to-30-ft width group.

Widths of nontraversable medians were not significant in this study.

A breakdown on the basis of type of accident shows that approach type accidents are significant only for the undivided highway. Overtaking accidents increase slightly from traversable to nontraversable medians. The single-vehicle-accident rate for nontraversable medians is double that for other types.

On the basis of severity, deterring types of medians are lowest in casualty accidents per MVM, casualties per MVM, and casualties per 100 accidents. Nontraversable medians have markedly higher rates than all other types for casualty accidents and casualities per MVM, but the higher percentage of multiple-vehicle accidents occurring on traversable medians results in this group having the highest number of casualties per 100 accidents. (4)

The authors point out that from the perspective of total accidents, as well as injury accidents, traversable type medians appear superior:

In total accident rates, traversable types of median strips show a substantial advantage with a rate of 0.91 accidents per million vehicle-miles. Deterring type medians are second with a rate of 1.00 and the undivided highway at 1.18 exhibited a markedly lower accident rate than the nontraversable group at 1.35 accidents per million vehicle-miles. In injury accidents there was very little difference between deterring and traversable medians with rates of 0.56 and 0.58 per million vehicle-miles respectively. The undivided followed closely with a rate of 0.62 and nontraversable medians were again last with 0.78 injury accidents per million vehicle-miles. (4)

Finally, in a discussion closure, the authors question the wisdom of median barriers available at that time:

It is interesting to note in this connection that the nontraversable medians made a better showing in the higher traffic flows, although the sample was so small as to be merely indicative rather than conclusive.

Our observations over a period of years have tended to support the conclusion that nontraversable medians, or medians which have within their limits such obstacles as trees, power poles, etc., have a tendency toward a higher rate of reported accidents than those medians which are traversable and free of obstacles. It appears that with either the traversable or nontraversable median, substantially the same percentage of vehicles would enter the median for one reason or another. In the case of the nontraversable median, this inadvertent use of the median would result in a reportable accident, whereas, with a

traversable median, a substantial portion of those entering
the median would recover control and continue on their way.

The value of a positive barrier may, in some cases, off-
set the hazard it creates, but our belief is that it is a poor
substitute for usable space. (4)

For various median widths and types, a New York study found that, in general,
earth type medians appeared to be superior to other median types when using
injuries per 100 million vehicle miles (MVM) as a criterion. Fatalities per MVM
could not be readily compared because of small sample sizes:

The severity of accidents for the two types of medians is
given in Table 4. Using the number of injuries per 100 MVM
of travel as an index of severity, it is seen that both the
earth and miscellaneous features medians had the smallest
contribution to severity (47) in the deterring group. The
curbed median was next with a rate of 55. For the non-trav-
ersable type the index of severity ranged from 79 for the
double guide rail to 108 for the concrete posts. This con-
crete posts median index was more than twice that for the
deterring. It is also higher than the index for any of the
other median subgroups. (5)

TABLE 4

FATALITIES AND INJURY RATES BY TYPE OF MEDIAN FOR ACCIDENTS BETWEEN INTERSECTIONS

Type of Median	No. All Accidents		Accident Rates Per 100 MVM of Travel		
	Injury	Fatalities	Injury Accidents	Injuries	Fatalities
Deterring					
Earth	250	6	28	47	0.67[a]
Curbed	316	3	40	55	0.32[a]
Miscellaneous features	31	1	35	47	1.14[a]
Subtotal	597	10	34	51	0.57
Non-Traversable					
Double guide rail	20	0	16	79	0[a]
Concrete posts	93	2	70	108	1.51[a]
Single guide rail	5	0	64[a]	77[a]	b[a]
Guide rail and ditch	2	0	20[a]	61[a]	0[a]
Subtotal	120	2	43	92	0.72[a]
Total	717	12	35	56	0.59

[a]Number of accidents less than 10 for period of study.

Sacks, in a Pennsylvania study conducted in the mid-1960's, found that while
barriers on narrow medians eliminated head-on collisions, total accidents— including
injury accidents— increased considerably:

Overall accident resumes are presented in Tables 5 and 6.
Conventional classification based upon severity suffered by
individuals is used to define accident types.

The number of traffic accidents in Contract I increased from
50 before median barrier installation to 87 afterward. Based
on the 'before' period, this represents an increase of 74 per-
cent. If it is assumed that accident frequency is linearly in-
fluenced by amount of travel (vehicle mileage), then, for a
constant roadway length, it is also linearly affected by volume.

Thus, for a 10 percent volume increase approximately 55 accidents should have occurred. Therefore, 32 accidents represent a certain deviation from the 'expected norm,' a 64 percent 'abnormal' increase.

The accident frequency increase in Contract II was 112, representing a total percentage increase of 38 percent over the 'before' period. By similar reasoning to that presented above the 'abnormal' increase was 82 accidents or 28 percent. (6)

In conclusion, Sacks states that:

Although the median barrier does eliminate, for all intensive purposes, the accident severity associated with the cross-median fatality, the frequency of injury accidents was found to increase.

'Abnormal' accident frequency increase attributed to the median barrier is found normally distributed throughout all time periods.

Total property damage costs suffered, as well as costs of congestion arising from accidents occurring during peak periods, increased after median barrier construction. (6)

A later Pennsylvania study, covering a 4-ft median with a concrete barrier, did not show that this barrier installation was advantageous from either an injury or fatality point of view:

Accidents were analyzed for a one-year period May 1, 1965, to April 30, 1966, before the box-beam median barrier was installed, and police reports of accidents were analyzed for a one-year period, May 1, 1967, to April 30, 1968, after it was installed. The first year, 1965 - 1966, will be referred to as the 'before' period, and the second year, 1967 - 1968, as the 'after' period.

There were a total of 81 accidents reported by the police in the before period, and the ADT was 44,000. In the after period the police reported 93 accidents, and the ADT was 46,000. Volume increased 4.5 percent, whereas accidents increased 14 percent. The severity of these accidents is given in Table 3. The increase in the number of accidents and the reduction in the number of injury accidents are reflected in a 50 percent increase in property damage accidents. The number of persons injured increased 21 percent, though injury accidents were reduced 20 percent. (7)

TABLE 3

ACCIDENT SEVERITY DURING BEFORE AND
AFTER STUDY PERIODS

ADT and Severity	Before (1965-1966)	After (1967-1968)	Percent Difference
Average daily volume	44,000	46,000	+ 4.5
Total accidents	81	93	+14
Fatal accidents	2	2	0
Injury accidents	39	31	-20
Property damage accidents	40	60	+50
Total killed	2	2	0
Total injured	43	52	+21

Median Barrier Installation Practice

Even though the history of median barrier research has not provided clear policy direction, various departments and national highway research organizations have felt the necessity to provide direction in this uncertain, yet urgent, situation (8). Despite accident research in the 1950's and early 1960's, by the year 1967 no understanding of guardrail benefits had emerged:

> Operational experience with cable-chain link fence and double blocked-out beam median barriers reported by the State of California shows that although both types have been effective in reducing the frequency of cross-median accidents, the rate of accidents involving the median has increased at locations where barriers have been installed. An increase in accident frequency after median barrier installation was also revealed in before-and-after studies in Pennsylvania. The California studies revealed that, for the most part, both types of barriers were performing effectively, but that the cable-chain link fence median barriers were sometimes penetrated or vaulted in areas where it was installed on sawtooth-type medians. Another observed undesirable characteristic of the cable-chain link median barrier is that the impacting vehicles frequently undergo rather violent spinouts that can cause the occupants to be ejected and to thereby be exposed to greater danger. (9)

By 1968, median barrier "warrants" were considered in the NCHRP state-of-the-art literature. Apparently these warrants were based on California's policy, although no references or supporting research was cited. However, doubts concerning the wisdom of guardrail installation remained, as seen in NCHRP Report No. 54 (1968):

> Even properly designed guardrail and median barrier installations are formidable roadside hazards and provide errant vehicles with only a relative degree of protection. Although guardrail and median barrier installations should decrease accident severity, frequency of accident occurrence may increase with the added installations. This is because the guardrail/barrier system is usually a larger target and is located closer to the roadway than the roadside hazard itself. For this reason, guardrail and median barrier installations should be kept to a minimum, and highway designers should consider such installations only where they are clearly justified. Where guardrail and median barrier requirements are indicated, the designer should examine the roadway to determine the feasibility of adjusting site features so that guardrails will not be required (e.g., flattening an embankment slope or removing a tree). For borderline warranting cases, the action guide-

line is: <u>When in doubt, omit the guardrail or median barrier.</u> (<u>10</u>)

At about this time, the Organization for Economic Co-operation and Development reviewed world-wide guardrail practice and in a discussion of median warranting in the United States concluded:

> Where median width is the only factor considered it is doubtful if barriers are justified in medians wider than about 25 feet, since the possible hazard of a vehicle crossing a relatively wide median and colliding with a vehicle in the opposite roadway must be weighed against the increased hazard of reducing to less than half the manoeuvring space within the median. Traffic volume is also important, however, because the probability of both crossing the median and of striking a vehicle in the opposite roadway increases as the product of the two flows. (<u>11</u>)

By 1971, the situation had not changed, at least as far as the NCHRP literature reviews were concerned. Michie, Calcote, and Bronstad reiterate the apparent California warranting formula again with the same proviso:

> A basic aspect of the guardrail and median barrier technology is identification of locations along highways where protective installations are needed. Specific decision criteria to use a guardrail or median barrier in a given location are referred to as warrants. An ideal guardrail system— that is, one that safely redirects errant vehicles without endangering other traffic and without causing injuries or fatalities among the occupants— would improve safety at most highway sites, with the possible exception of those with flat embankments that are clear of obstacles. However, such ideal systems do not exist; guardrail and median barrier systems are intrinsic roadside hazards and provide the errant vehicles with only a <u>relative degree of protection.</u>
> Many existing installations are more hazardous than the roadside condition and may increase rather than reduce severity of ran-off-the-road accidents at a given site. For the period 1965-67, the California Highway Traffic Department has shown that in 33.8 percent of freeway fatal accidents involving single vehicles, the vehicles hit off-road fixed objects. Furthermore, 34.6 percent of these off-road fixed object fatal accidents involved a highway guardrail; therefore, it can be concluded that 11.7 percent* of single vehicle fatal accidents involved a barrier. From statistics compiled by Hosea on completed sections of the Interstate System for 1968, the percentage of single-vehicle fatal accidents involving guardrail and median divider— 364 and 71 accidents, respectively— is determined to be 23.6 per-

*Discrepancy between these figures (i.e., 11.7 versus 23.6) is attributed in large part to definition of single- and multi-vehicle accidents.

cent* (i.e., 435 out of 1,842 single-vehicle accidents). Although these accident statistics reflect performance of adequate as well as unsatisfactory barrier designs, the fact remains that highway barrier installations constitute a major roadside hazard. For this reason, highways should be designed with the specific intent of eliminating, or at least minimizing, the use of barrier systems, and at the same time upgrading the performance and functional capabilities of existing installations.

At some locations, guardrails and median barriers may decrease accident severity, but accident frequencies actually increase because these systems usually constitute larger targets and are located closer to traffic than a roadside hazard. This aspect adds to the basic concept that guard-rails and median barriers should be kept to a minimum. Accordingly, highway designers are well advised to examine the feasibility of adjusting site features (e.g., flattening an embankment slope or removing a tree) so that such installations will not be required. (12) See also (13).

*Ibid.

The Transportation Research Board recommendations of that time are presented in HRB Special Report 81 (14), but appear to reiterate the previously mentioned California practice. To this day, the research substantiating the California practice has not been published in any standard journal in the field. Thus, one cannot, through use of the standard literature, evaluate any California findings used to rationalize that state's median barrier policies.

Aside from the desireability of publishing barrier warrant research, it is essential that the safety criterion used as a warranting basis be widely understood. There are many criteria by which highway safety can be optimized. However, they may not all indicate the same response to a hazard. NCHRP Report No. 148 surveys several potential criteria:

As previously mentioned, any one of several different accident severity indices can be selected, depending on the objective of the roadside safety improvement program. The severity index is a numerical weighting scheme that ranks roadside obstacles by degree of accident consequence.

Generally, a safety improvement program is aimed at reducing total fatalities, injuries, and property damage. Therefore, any improvement program that assigns higher weights to the more severe accidents will tend to satisfy these aims. Basically, severity indices applied to obstacles could be based on any of the following measures:

1. Average property damage cost per accident.

2. Average direct cost per accident (includes property damage, hospitalization, insurance premiums, funeral expenses).

3. Average total cost per accident (in addition to direct cost, the total cost includes loss of future earnings and values for human suffering).

4. Average number of fatalities per accident.

5. Average number of fatal and nonfatal injuries per accident.

6. Proportion of fatal accidents.

7. Proportion of fatal and nonfatal injury accidents.

Severity indices are used solely for comparative purposes— for instance, comparison of the accident consequences of protective guardrail with those of bridge abutments. Because the severity indices for these obstacles are computed from historical accident data, the precision of the weighting scheme will depend on the accuracy and availability of accident records for each obstacle.

Fatal accidents are rare occurrences. They are rare enough that large volumes of accident data are needed to render the proportion of fatal accidents statistically reliable. As indicated in Appendix A, large volumes of accident data are not generally available for all types of roadside hazard situations. Therefore, even if a specific program objective is to reduce fatalities or fatal accidents, weighting scheme measures (Nos. 1,2,3,4, and 6) that give greater weight to fatal accidents than injury accidents will not necessarily be more successful in achieving that objective.

For a comprehensive program on roadside safety improvement, the weighting scheme that ranks the severity of obstacles on the basis of the proportion of fatal and nonfatal injury accidents associated with each obstacle is recommended. This measure is us ed in this report. (15)

Median Barrier Warranting Criteria

The research literature cited herein suggests that barriers installed in narrow medians of high traffic volume roadways may slightly decrease fatalities resulting from head-on collisions, but will also produce a substantial increase in property damage and injury accidents. If the safety criterion selected is fatality reduction, median barriers may or may not be justified for these particular highway configurations. On the other hand, if, as recommended in NCHRP Report No. 148, the combined injury – fatality rate is selected, then median barrier could not be justified even for the narrowest of medians. The problem is clear: what will be considered good median safety policy depends upon the accident statistic selected as the criterion.

While the national literature does not make it clear, California probably uses an accident index developed from an economic assessment of each accident

type for median barrier warranting ($\underline{16}$, $\underline{17}$, $\underline{18}$). As of the early 1960's, the relative cost, according to an Illinois study ($\underline{19}$), of fatal, injury, and property damage only accidents was 25, 6, and 1, respectively (figures adjusted to California's experience). This is an attempt to weight each type of accident—not by pain or other social consequences—but by relative dollar costs. Because these indices are based on value considerations, it should be abundantly clear that no highway department should adopt guardrail warrants based on California's or any other weighting scheme unless it is in full agreement with that scheme as a matter of public policy.

Considering the opposing traffic volumes on westbound I-94 and the eastbound service road near the Belleville exit ($25,000 + 1,000 = 26,000$), we find that no guardrail was warranted under the NCHRP/California specifications. However, the NCHRP/California warrants are for medians, not outer separations. The difference is not semantic. Median barrier warranting, especially in California, is based on the assumption (confirmed through traffic counts) that the median will separate two opposing roadways of nearly equal traffic volume. It is recognized that the probability of a vehicle crossing the median and colliding with another vehicle on the opposing roadway depends upon the density of the opposing traffic flow:

> It is frequently taken for granted that if a car crosses the median of a heavily-traveled freeway, it is bound to collide with a car proceeding in the opposite direction. This is not true. Even during daytime hours, there are many long spaces between vehicles, and during the hours from midnight to 5 A.M., when the fatal accident problem is the greatest, most of the spaces between vehicles are several hundred feet long ($\underline{2}$).

Moreover, it follows that the most probable collision situation occurs when the opposing flows are equal as with typical median conditions:

> Traffic volume is also important, however, because the probability of both crossing the median and of striking a vehicle in the opposite roadway increases as the product of the two flows. ($\underline{11}$)

Because the traffic flow of a service road is typically much less than the opposing flow on the paralleling main roadway, median warranting criteria cannot be used in this circumstance. Presumably for this reason, neither Michigan, the NCHRP/HRB

literature, nor California literature warrant barriers in the outer separation area. While there is strong intuitive feeling on this point, precise mathematical probability comparisons have not been worked out. In order to illuminate this situation vis-a-vis the I-94 - service road instance, as well as to advance understanding in general, this report will include a collision probability presentation for traversable medians separating dissimilar traffic density roadways.

COLLISION PROBABILITY MODEL

The review of research literature on the safety advantages of median barriers indicated that the installation of a barrier in any given case is not unambiguously supported by the accident statistics, even if the safety criterion was agreed upon. It is in this context that we feel a theoretical approach, modeling the I-94 - service road instance and the outer separation problem in general, is appropriate.

Fixed-object accident models have been prepared in the past in an attempt to rationalize guardrail warrants (15). However, no moving object collision model is currently available in the literature. Our development is similar in approach to the fixed-object model presented in NCHRP Report No. 148, but since moving objects raise additional analytical problems, we found it necessary to devise a more elaborate treatment.

Model Assumptions

1) In the model, the outer separation is variable (S). For the I-94 - service road instance, S = 30 ft.

2) Each roadway has two lanes.

3) All vehicles are traveling in the center of their respective lanes.

4) Vehicles on both I-94 and the service road cross any arbitrary milepost in accordance with the Poisson probability process (20) with rate,

$$\lambda = \frac{T}{24 \times 3600}$$

per second, with lane ADT of T.

5) Vehicles which encroach on the outer separation travel in a straight line.

6) Encroaching vehicles travel at a constant speed.

7) Once two vehicles are on a collision course, neither driver will take evasive action. In other words, a collision is not preventable by breaking or swerving.

8) A vehicle entering the opposing roadway will collide with another vehicle if that vehicle occupies the 'collision space' defined in the model.

9) The recovery point is defined to be the maximum lateral extent to which the encroaching vehicle has penetrated the outer separation.

10) No collision occurs at any time after the encroaching vehicle has reached the recovery point.

11) Under the section where guardrail is present, the guardrail is installed at the 'optimum' distance from the service road. Optimum means the location generating the least number of guardrail accidents.

12) If guardrail is present, it will fully contain the encroaching vehicle. This disallows vaulting or penetration into an opposing roadway.

13) Vehicles on I-94 are assumed to be traveling at 70 mph.

Findings

1) For 38 foot or greater roadway separations and equal opposing roadway traffic volumes, the optimal location for guardrail is the cross-sectional midpoint of the median.

2) For unequal opposing roadway traffic volumes, the optimal guardrail location is not the midpoint (Fig. 1). In the I-94 – service road instance, the optimal location— given that a guardrail was installed— would have been as close to the service road as possible, depending upon shoulder requirements. Even at a distance of 25 ft from I-94, however, the guardrail would generate about 160 guardrail accidents in five years for the 5.8 mile section considered.

3) The proportional split in opposing traffic volumes has a profound affect on the expected number of head-on collisions with no guardrail present (Figs. 2 and 3). The maximum number of collisions will occur when the split is 50-50. For an opposing ADT of 52,000 this results in about seven expected collisions over a five-year period. When the split is 96-4, as in the I-94 – service road instance, the expected number of collisions drops to between one and two for the same opposing ADT. Thus, it can be seen that guardrail warranting predicated on median experience with 50-50 splits of opposing traffic volumes is not applicable to the outer separation case, where the traffic is unequally distributed between the opposing roadways. This is the case for any total opposing ADT.

4) The expected number of I-94 – service road head-on collisions without guardrail increases almost linearly with I-94 ADT (Fig. 4). Doubling the service
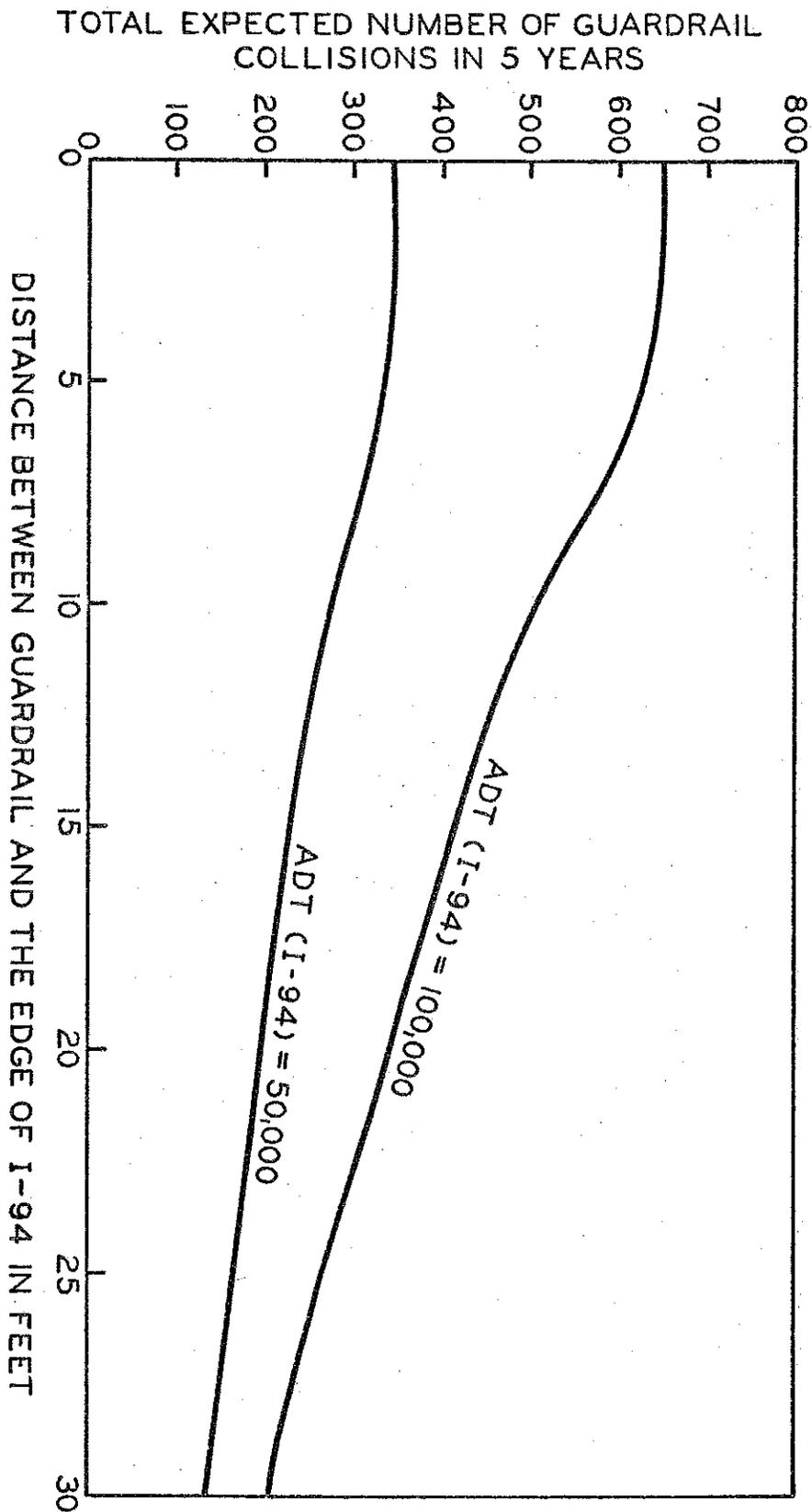
Figure 1. Total expected number of guardrail collisions in five years for various distances s* (the distance between guardrail and the edge of I 94) and various ADT of I 94 and Service Road.
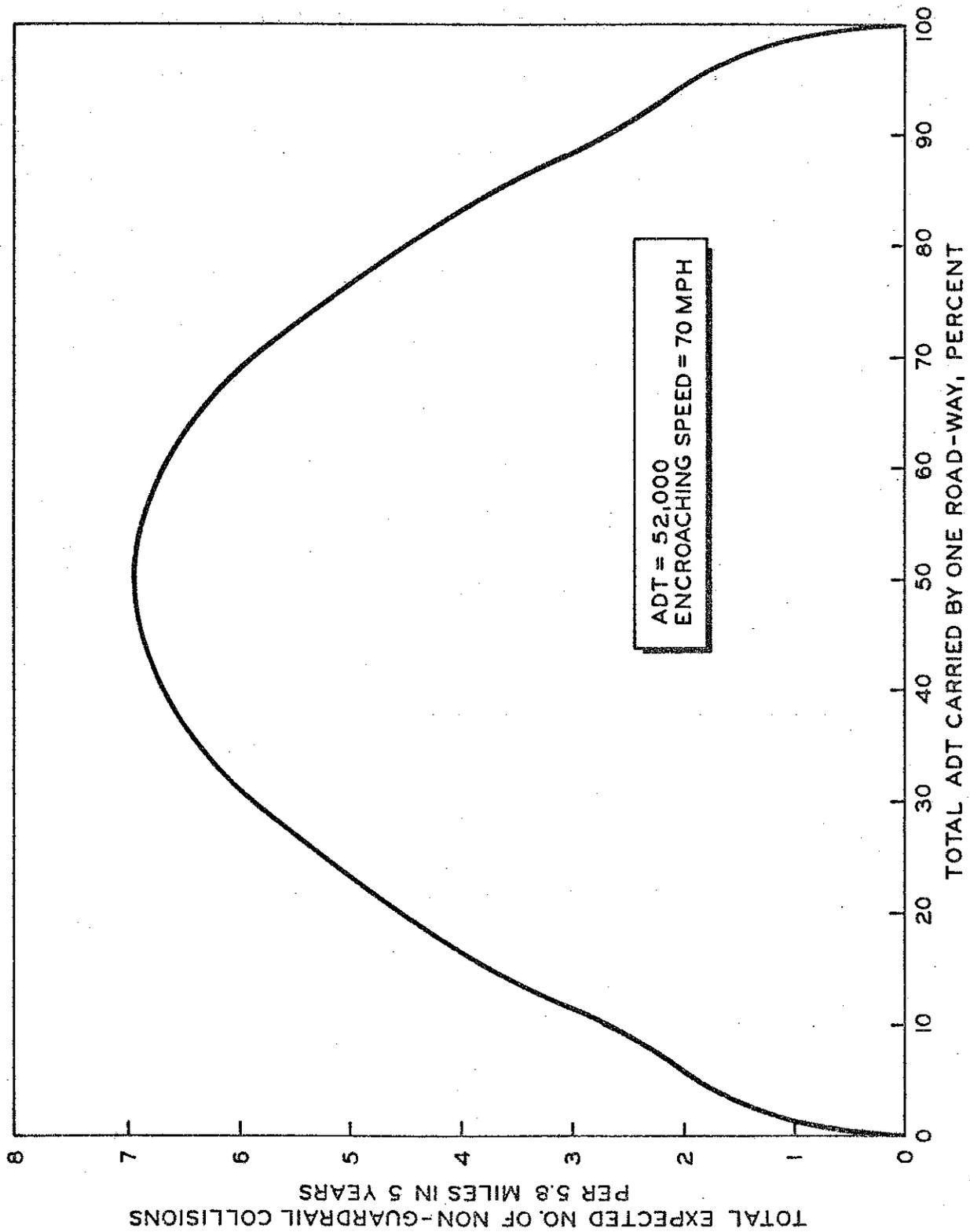
Figure 2. Effect of opposing traffic volume ratio on cross-median head-on collision expectancy.

TOTAL EXPECTED NO. OF NON-GUARDRAIL COLLISIONS
PER 5.8 MILES IN 5 YEARS

TOTAL ADT CARRIED BY ONE ROAD-WAY, PERCENT
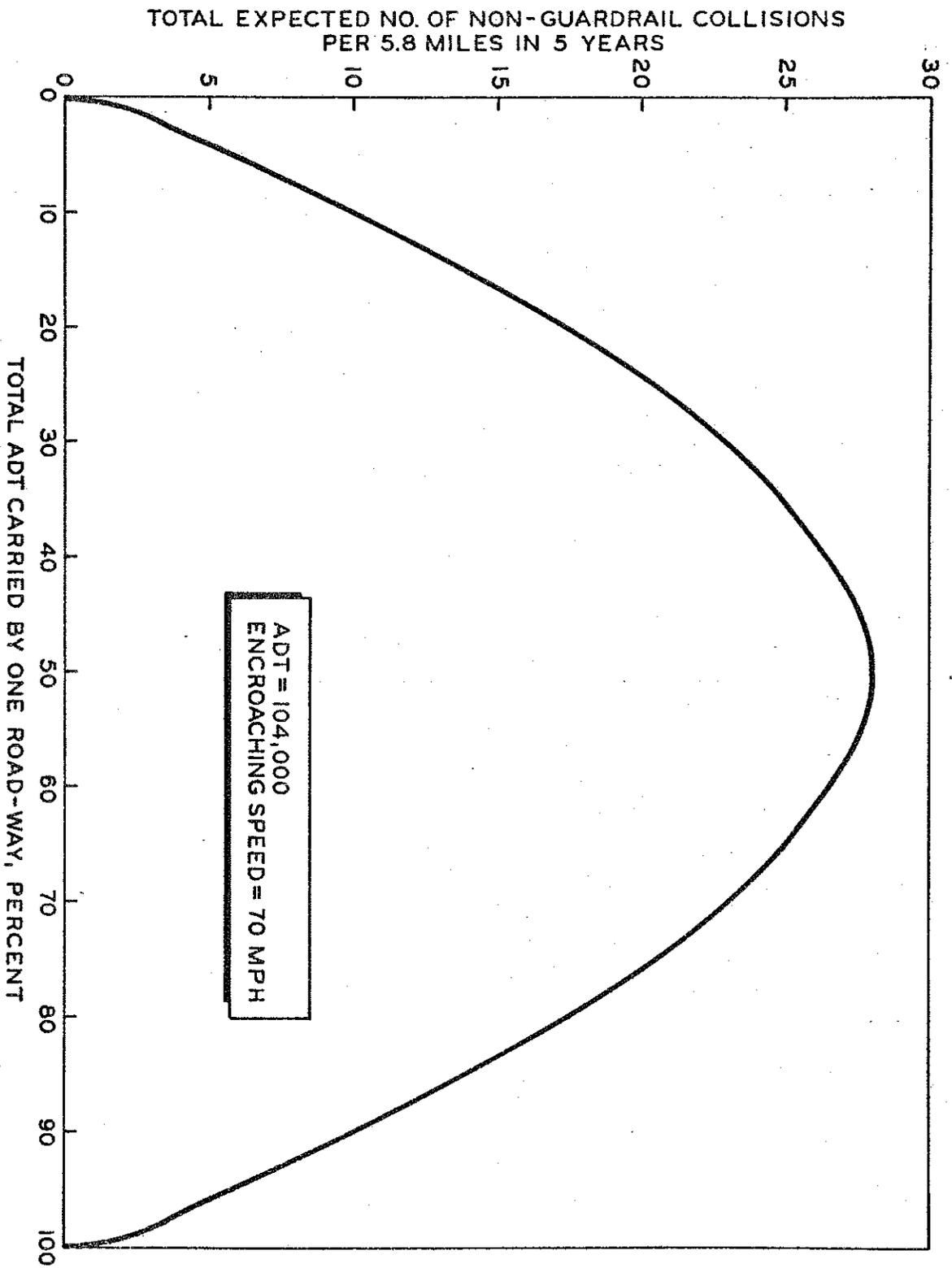
ADT = 104,000
ENCROACHING SPEED = 70 MPH

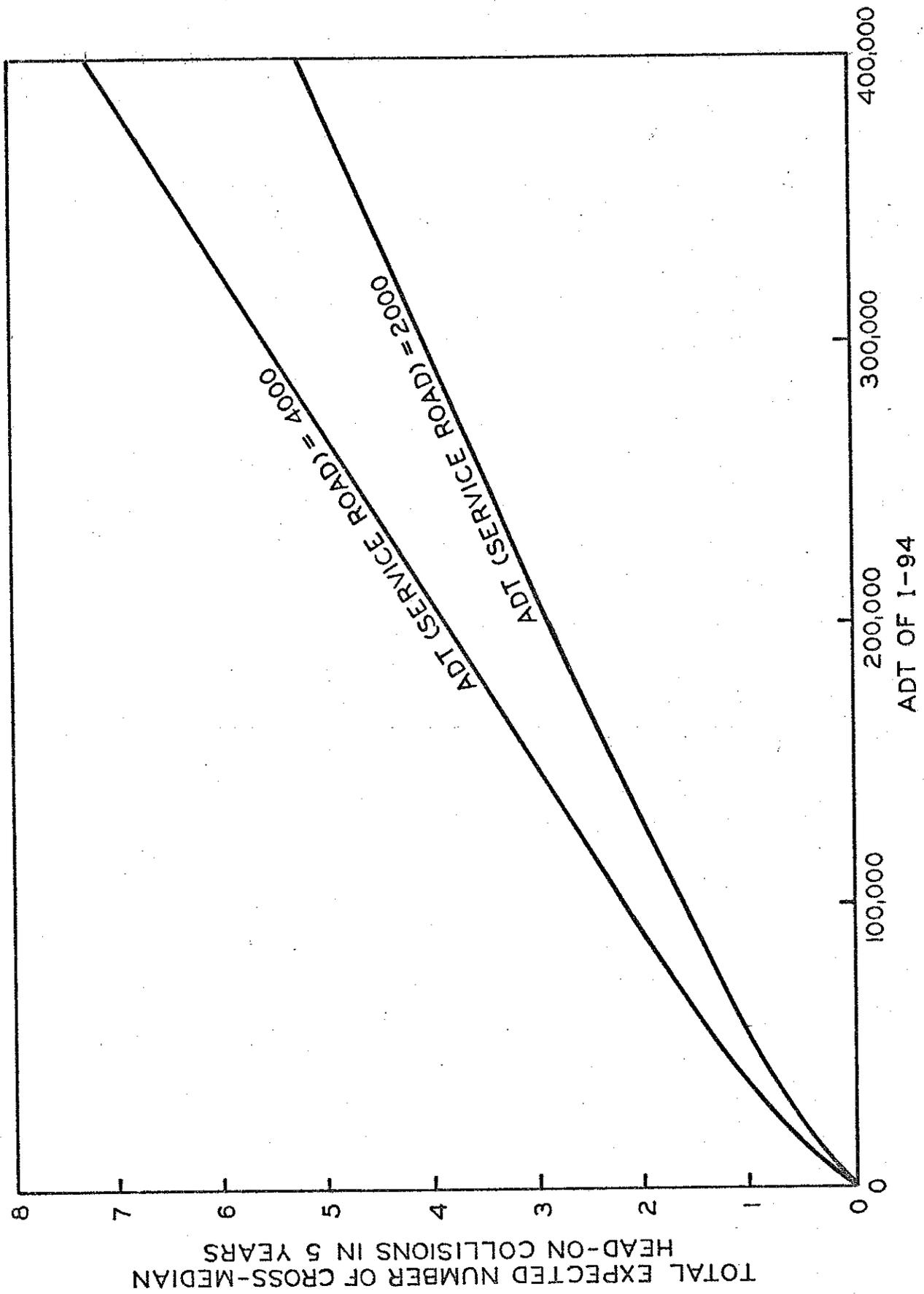Figure 3. Effect of opposing traffic volume ratio on cross-median head-on collision expectancy.

Figure 4. Cross-median head-on. Total expected number of collisions in five years for various ADT.

road ADT from 2,000 to 4,000 increases head-on collision probabilities, but not substantially. Figure 4 shows that if, for example, the ADTs of I-94 and the service road increased to 100,000 and 4,000, respectively, the expected number of cross-median collisions would be no more than two in five years.

5) As pointed out numerous times in the literature, the decision to install guardrail depends upon the relative hazard it will introduce. In the I-94 - service road instance, the accident model was used to estimate head-on collisions without guardrail and guardrail accidents with guardrail placed optimally at a distance of 25 ft from I-94. The ratio of guardrail accidents to head-on collisions without guardrail is shown for various I-94 and service road ADTs in Figure 5. It can be seen from Figure 5 that the best case for guardrail installation with a service road ADT of 2,000 is when the ADT of I-94 is about 70,000. At this point, the ratio of guardrail accidents to head-on collisions without guardrail is minimum. However, even at this ADT one would expect 120 times as many guardrail accidents as head-on collisions without guardrail. While guardrail accidents are generally not considered as severe as head-on collisions, they can still result in injuries and fatalities, as is amply documented (21,22). For example, NCHRP Report No. 148 rates the severity of guardrail accidents as about one-half that of tree accidents (i.e., one-half as many guardrail accidents are classified as injury or fatal as tree accidents) (15). Moreover, about 60 percent of the tree accidents result in injuries and/or fatalities. In the present case, even if all head-on collisions resulted in injury or death, one would not expect the far greater number of expected guardrail accidents to result in fewer injuries or fatalities.

6) When no guardrail is present, cross-median collision probability decreases sharply with increasing median width. However, the decline is much greater for opposing roadways of unequal traffic volumes (Figs. 6 and 7).

In conclusion, since optimally placed guardrail in the I 94 - service road instance would have resulted in at least 130 to 160 times as many accidents, including injury and fatal, as head-on collisions without it (Fig. 5), we believe, on the basis of probabilistic analysis, that the Department was correct in avoiding its installation.
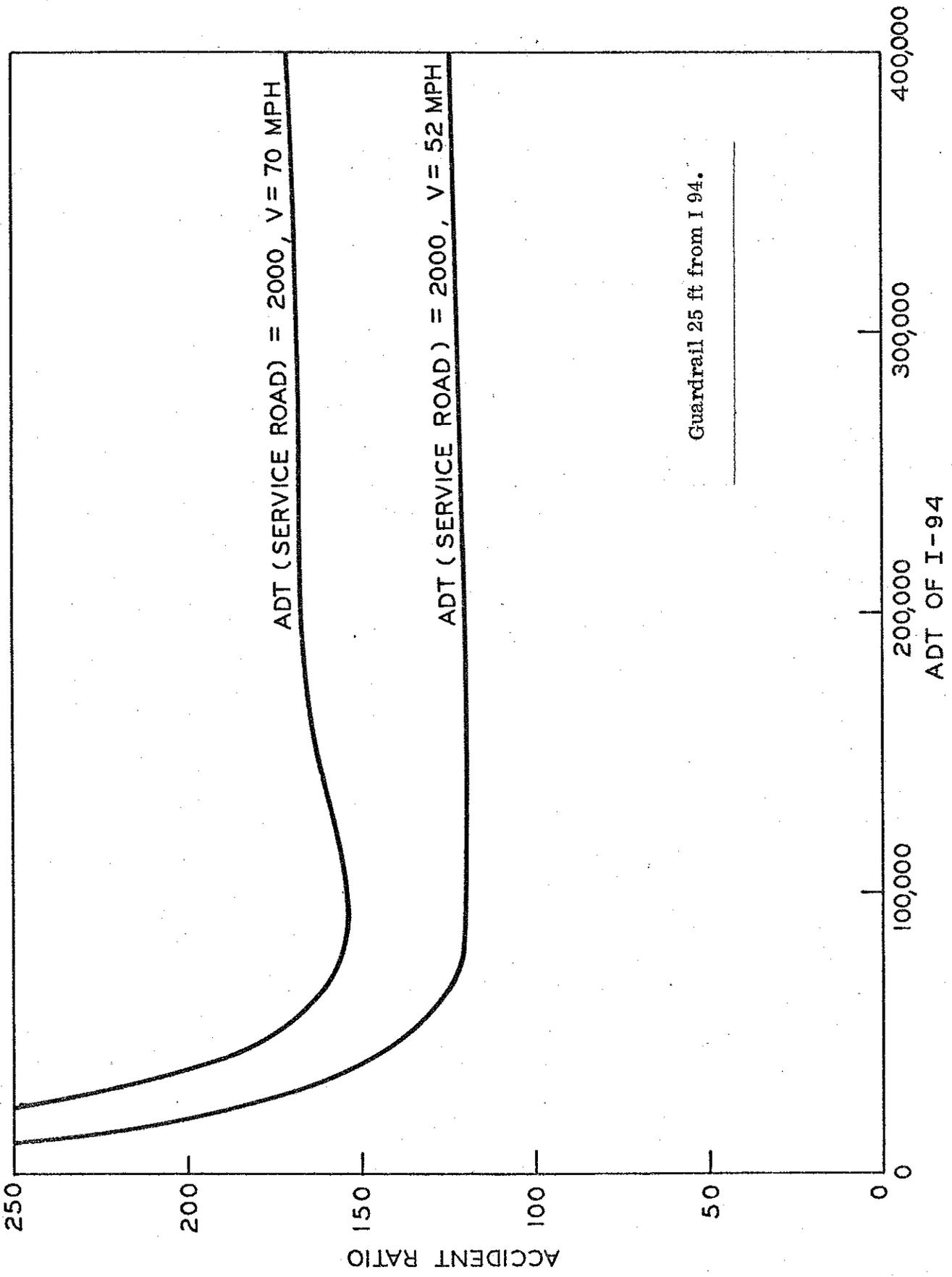
Figure 5. Ratio of the expected number of guardrail accidents to the expected number of cross–median collisions without guardrail.
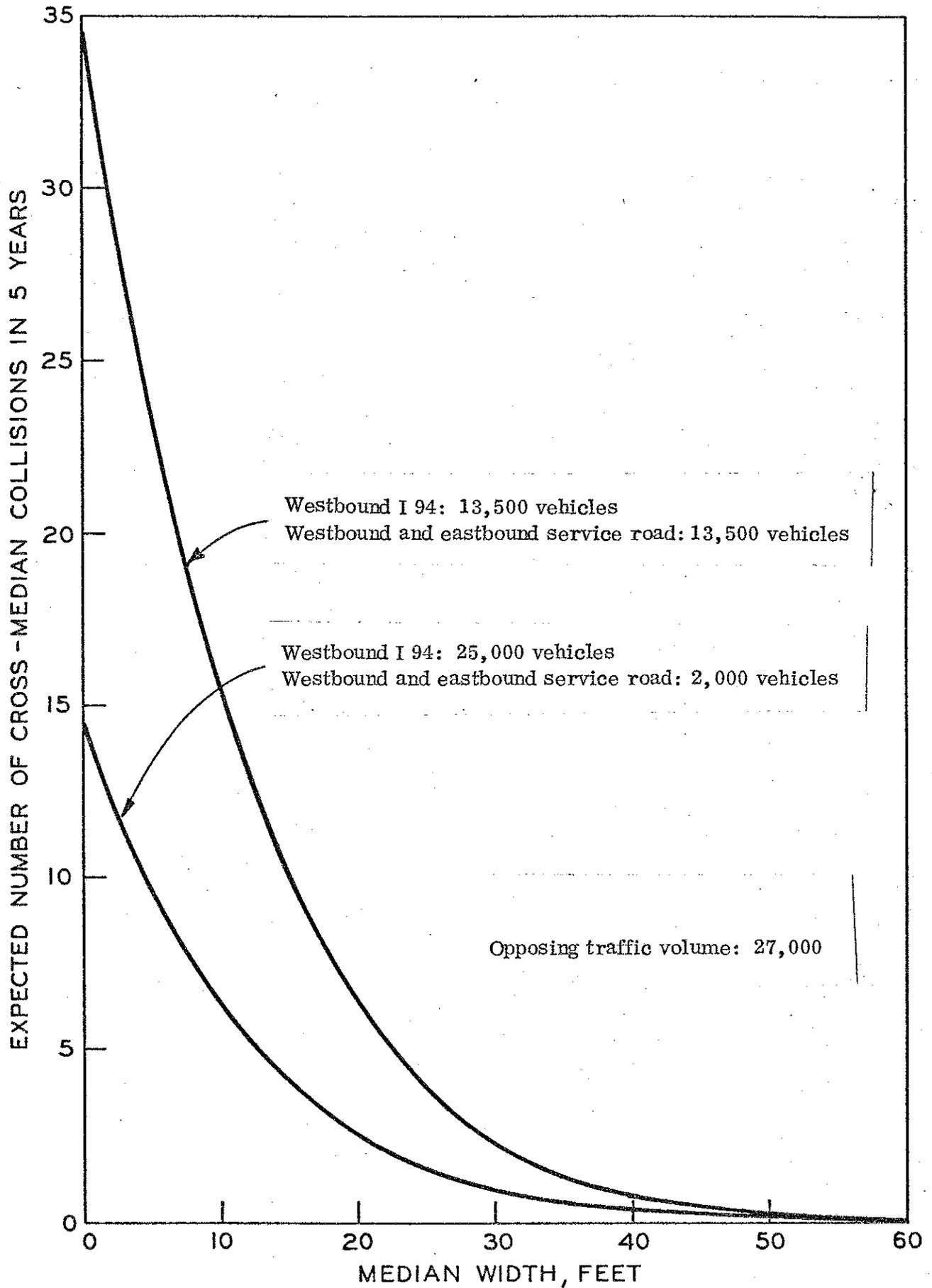
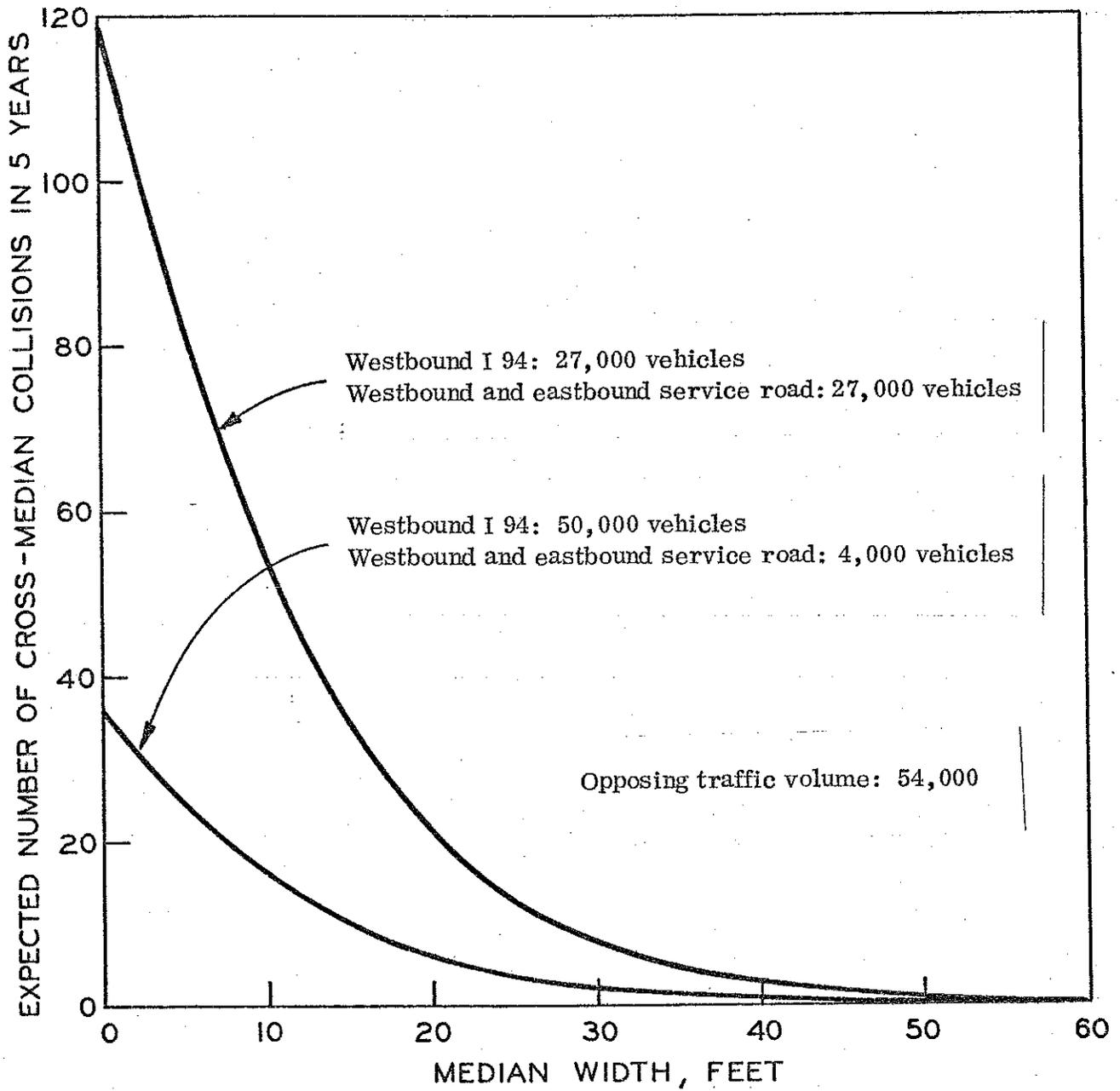Figure 6. Influence of median width on cross-median collisions without guardrail.

Figure 7. Influence of median width on cross-median collisions without guardrail.

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APPENDIX A

PROBABILITY MODEL FOR CROSS-MEDIAN
ACCIDENTS WITH AND WITHOUT GUARDRAIL

APPENDIX: THE PROPOSED MODEL

The purpose of this Appendix is to develop a probability model as a basis for answering the following questions:

1) Given that a barrier is installed between two roadways, and a median encroachment has occurred, what is the probability that this encroachment will result in an accident with the barrier?

2) If a barrier is installed between two roadways, where should it be located so that the probability of a collision with the barrier is minimal?

3) If a barrier is not installed between two roadways, what is the probability of an encroaching vehicle from one roadway entering, and colliding with a vehicle on the other roadway?

Notation

d : the width of a vehicle

ℓ : the length of a vehicle

θ : angle of encroachment

$G(\theta)$: probability distribution function of the angle of encroachment, $0 < \theta \leq \dfrac{\pi}{2}$

$(X,Y)_\theta$: the coordinate of the recovery point with respect to the encroaching point of an encroachment with angle θ

$F_\theta(y)$: the conditional probability distribution of the maximal lateral distance, y , given that the angle of encroachment is θ , $y > 0$

$H_\theta(x)$: the conditional probability distribution of the longitudinal distance, x , given that the angle of encroachment is θ , $x > 0$

$N(T)$: the expected number of encroachments per mile per
year if the average daily traffic volume of the road-
way is T

Collision Probability With The Barrier In Place

Suppose that two roadways are S ft apart from one another, and a barrier of length B ft is installed at s ft away from the edge of one roadway. In the practical situation, the thickness of a barrier such as guardrail, concrete barrier, etc., is relatively small with respect to the total installed length of the barrier. Therefore, the thickness of the barrier is assumed to be zero for the purposes of this representation. The above problem set-up is shown in Figure A1.

Given that an encroachment has occurred, we would like to compute the probability that this encroaching vehicle will collide with the barrier. This is demonstrated as follows:

For a given angle θ of encroachment, the probability that this encroachment will involve a collision with the barrier is, if the encroaching point (left corner of the vehicle in this case) is in the interval (A, B) with w ft to the left of the point B,

$$Q_\theta(s,w) = P_\theta\left(Y \geqslant s + w\sin\theta\cos\theta\right) = \int_{y=s+w\sin\theta\cos\theta}^{\infty} dF_\theta(y) \tag{1}$$

and, if the encroaching point is in the interval (B, C),

$$P_\theta(s) = P_\theta\left(Y \geqslant s\right) = \int_{y=s}^{\infty} dF_\theta(y) \tag{2}$$

Eqs. (1) and (2) are derived based on the assumption that the encroaching vehicle will travel a straight line along the angle of encroachment until it reaches the recovery point. The justification for the above assumption will be discussed later.

With the above two basic computational formulas, we are ready to answer the question stated above for the following two cases:

Case 1 - The term $Q_\theta(s, w)$ is negligible.

When the barrier is very long, so that the following conditional probability,

$$\int_{\theta=0}^{\frac{\pi}{2}} \frac{1}{C-A} \, dG(\theta) \int_{\omega=0}^{B-A} Q_\theta(A,\omega) \, d\omega = \int_{\theta=0}^{\frac{\pi}{2}} \frac{1}{B+d\csc\theta} \, dG(\theta) \int_{\omega=0}^{d\csc\theta} Q_\theta(A,\omega) \, d\omega \qquad (3)$$

is extremely small, the term $Q_\theta(s, w)$ is then negligible. On the other hand, when a barrier is required between two roadways such that the length of the barrier is the same as the length of the roadways, the term $Q_\theta(s, w)$ no longer exists under this situation. In general, the freeway system falls into this category. For the purpose of discussion, let us identify one of the two roadways as Roadway 1, and the other as Roadway 2. We further assume that the average daily traffic volume of Roadway 1 and 2 are T_1 and T_2, respectively. Thus, given that an encroachment has occurred and the barrier is installed at s ft away from the edge of Roadway 1, the probability, $P_B(s, S)$, that this encroachment will involve a collision with the barrier is,

$$P_B(s, S) = \frac{N(T_1)}{N(T_1)+N(T_2)} \int_{\theta=0}^{\frac{\pi}{2}} P_\theta(s) \, dG(\theta) + \frac{N(T_1)}{N(T_1)+N(T_2)} \int_{\theta=0}^{\frac{\pi}{2}} P_\theta(S-s) \, dG(\theta) \qquad (4)$$

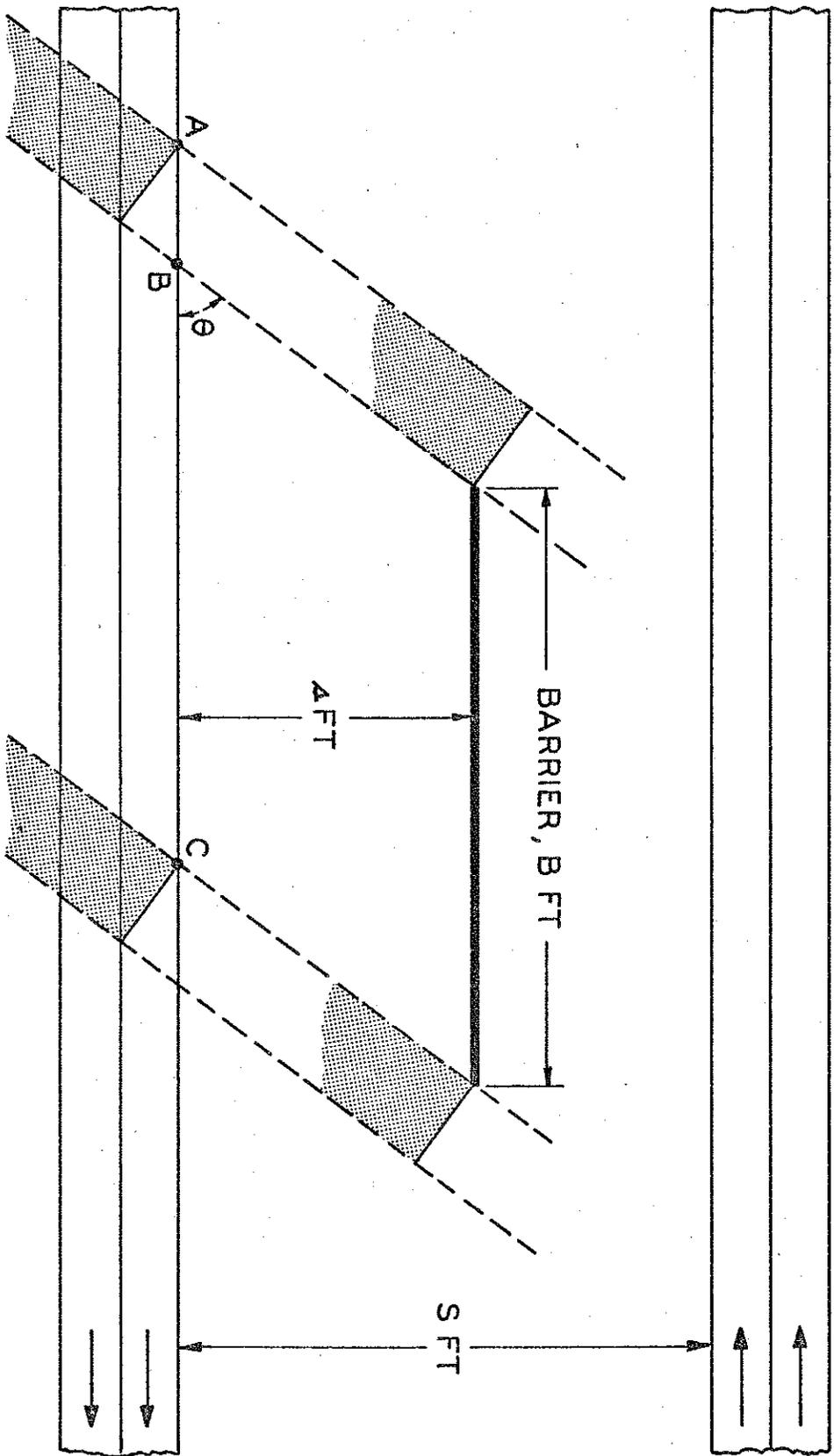The optimal distance from the edge of Roadway 1 that this barrier should be in-

Figure A1. Schematic illustration of a barrier and its relation to an encroaching vehicle.

stalled (in the sense that $P_B(s,S)$ in Eq. (4) is minimum) depends on the functional behavior of the integral $\int_{\theta=0}^{\frac{\pi}{2}} P_\theta(s) \, dG(\theta)$. For example, when $T_1 = T_2$ and $f_\theta(y)$ is the normal density function, $s^* = \dfrac{S}{2}$ is the optimal distance for the barrier installation provided that,

$$\int_{\theta=0}^{\frac{\pi}{2}} f_\theta'(\frac{S}{2}) \, dG(\theta) < 0$$

Otherwise, the optimal distance is toward the edge of either of the roadways as the distance between the two roadways becomes narrower.

The expected number of collisions with a barrier installed s ft away from the edge of Roadway 1 is $\left[N(T_1) + N(T_2) \right] P_B(s,S)$ per mile per year (one mile for each roadway).

Case 2 - The term $Q_\theta(s,w)$ cannot be ignored.

If the length of one roadway protected by a barrier is considered fixed, and the length of the other roadway is much longer, then the term $Q_\theta(s,w)$ in this case cannot be ignored as demonstrated below. A typical example of the above situation occurs if one roadway is a service road (fixed length) and the other roadway is one direction of a freeway. We shall limit our discussion to this typical example.

Assume that the length of the service road, identified as Roadway 2, is C ft. Let L^* be the length in ft determined by the following equation,

$$L^* = \underset{x}{\text{MAX}} \left\{ x = S \cot \theta + d \csc \theta \,\middle|\, \text{for those } \theta \text{ such that } P_\theta(S) > \alpha \right\} \quad (6)$$

where α is a very small predetermined probability. If the empirical frequency distribution of the recovery point of an encroachment is used to determine L*, the α can be set at zero. For example, if the data obtained by Hutchinson and Kennedy (23) are used, L* can be set as 828 ft, to be on the conservative side. Thus, the length of the section of Roadway 1 from which an encroachment could possibly reach Roadway 2 (the service road) is (C + 2 L*) ft. The required length of the barrier, B(s) is therefore a function of the distance s as shown in Figure A2.

It can easily be shown that,

$$B(s) = C + 2L^* (1 - s/S^*) \tag{7}$$

Thus, the length of the section of Roadway 1 from which an encroachment could possibly involve a collision with the barrier is,

$$L(s) = B(s) + K^* \tag{8}$$

where

$$K^* = \underset{x}{MAX} \left\{ x = s \cot \theta + d \csc \theta \,\middle|\, \text{for those } \theta \text{ such that } P_\theta(s) > \alpha \right\} \tag{9}$$

If we denote $\theta_1 = \tan^{-1} \left(\frac{s}{L(s)} \right)$ and $\theta_2 = \tan^{-1} \left(\frac{s}{L(s) - B(s)} \right)$, then, given that an encroachment has occurred on the roadway, the probability, $P_{\theta_1}(s, S)$, that this encroachment will involve a collision with the barrier is therefore,
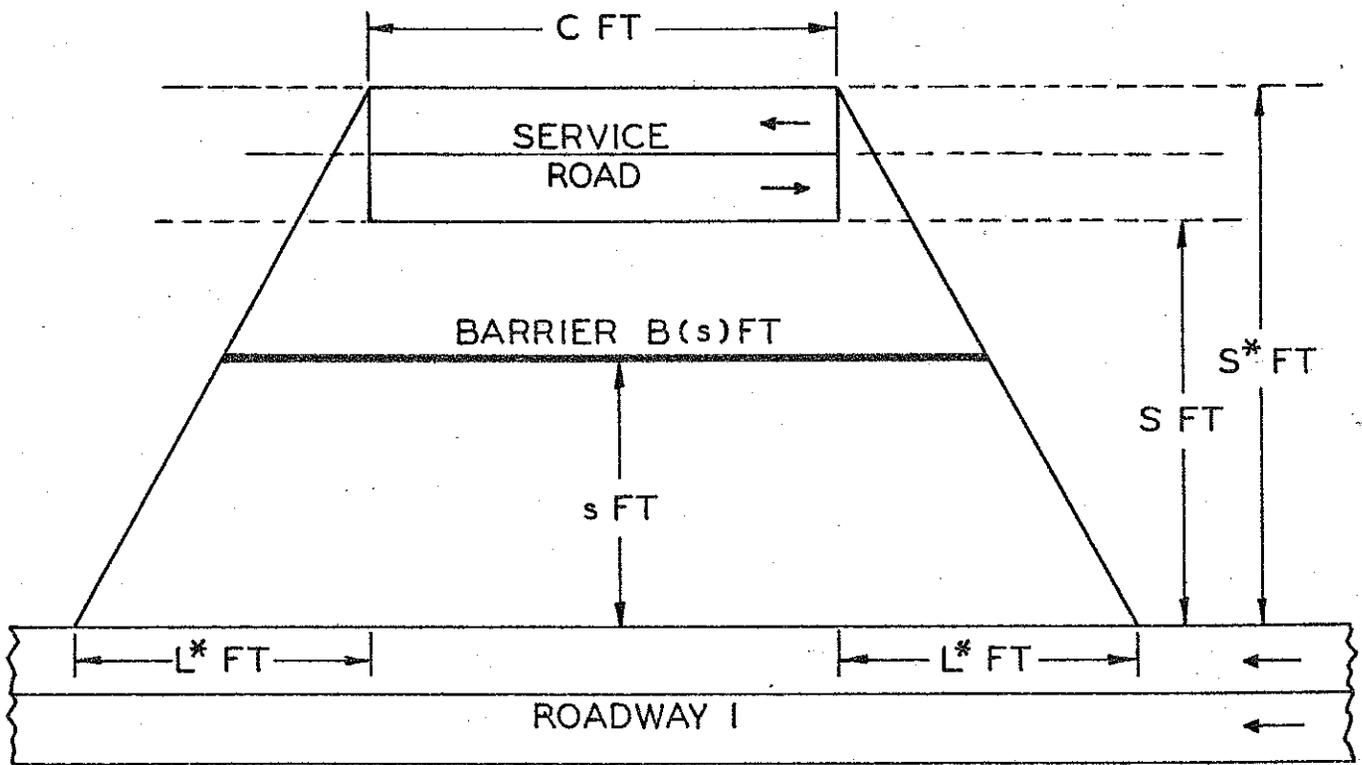
Figure A2. Schematic illustration of the required length of a barrier.

$$P_{s1}(s,S) = \int_{\theta=\theta_1}^{\frac{\pi}{2}} \frac{K(\theta)}{L(s)} P_\theta(s)\, dG(\theta)$$

$$+ \frac{1}{L(s)} \int_{\theta=\theta_1}^{\frac{\pi}{2}} dG(\theta) \int_{\omega=0}^{max(A(\theta),0)} Q_\theta(s,\omega)\, d\omega \qquad (10)$$

where $A(\theta) = Min\left(d\csc\theta,\ L(s)-s\cot\theta-B(s)\right)$ and $K(\theta)=Min\left(B(s),\ L(s)-s\cot\theta\right)$

If $L(s)$ is chosen such that,

$$s\cot\theta + B(s) + d\csc\theta \leq L(s) \qquad \text{for every}\quad \theta$$

such that $P_\theta(s) > 0$, then Eq. (10) becomes,

$$P_{\theta1}(s,S) = \frac{B(s)}{L(s)} \int_{\theta=0}^{\frac{\pi}{2}} P_\theta(s)\, dG(\theta) + \frac{1}{L(s)} \int_{\theta=0}^{\frac{\pi}{2}} dG(\theta) \int_{\omega=0}^{d\csc\theta} Q_\theta(s,\omega)\, d\omega \qquad (11)$$

If we denote $P_{B2}(s,S)$ to be the probability that an encroachment occurring on Roadway 2 will result in a collision with the barrier, and also denote,

$$B^*(s) = B(s) - L^*(1 - s/S^*),$$

$$\varphi_1 = \tan^{-1}\left(\frac{S-s}{B^*(s)}\right)$$

and

$$\varphi_2 = \tan^{-1}\left(\frac{S-s}{B^*(s)-C}\right)$$

then

$$P_{B2}(s,S) = \int_{\theta=\varphi_1}^{\varphi_2} \frac{B^*(s)-(S-s)\cot\theta}{C} P_\theta(S-s)\, dG(\theta)$$

$$+ \int_{\theta=\varphi_2}^{\frac{\pi}{2}} P_\theta(S-s)\, dG(\theta)$$

$$(12)$$

Since the number of encroachments per year occurring on Roadway 1 and 2 is

$$\frac{N(T_1)L(s)}{5280} \quad \text{and} \quad \frac{N(T_2)C}{5280} \quad \text{, respectively, we obtain,}$$

$$P_B(s,S) = \frac{N(T_1)L(s)}{N(T_1)L(s) + N(T_2)C} P_{B1}(s,S) + \frac{N(T_2)C}{N(T_1)L(s) + N(T_2)C} P_{B2}(s,S) \quad (13)$$

The value of s resulting in minimum barrier accident probability in Eq. (13) depends upon a number of functional relationships. The analytical solution will, therefore, not be provided here. However, the solution for an example related to this paper is shown in the main text (Figs. 2 and 3).

The expected number of collisions with the barrier in a year is equal to:

$$\frac{N(T_1)L(s) + N(T_2)C}{5280} P_B(s,S) \quad (14)$$

Collision Probability with No Barrier

We shall discuss this subject for the following two cases:

Case 1 — Both roadways are of equal length and are long enough to be considered as being of infinite length. Using Figure A3 we are ready to provide the basic formula for computing the probability that an encroaching vehicle (from one roadway) will enter and collide with a vehicle on the other roadway. This is demonstrated as follows:

Assume that the entering speed of the vehicle is V ft/second. Further assume that the Poisson input rates for Lane 1 and Lane 2 are λ_1 and λ_2 per second, respectively. Then, given that an encroachment has occurred with angle θ , the time $t_1(\theta)$ and $t_2(\theta)$ involving the potential collision with vehicles on Lane 1 and Lane 2, respectively, are,
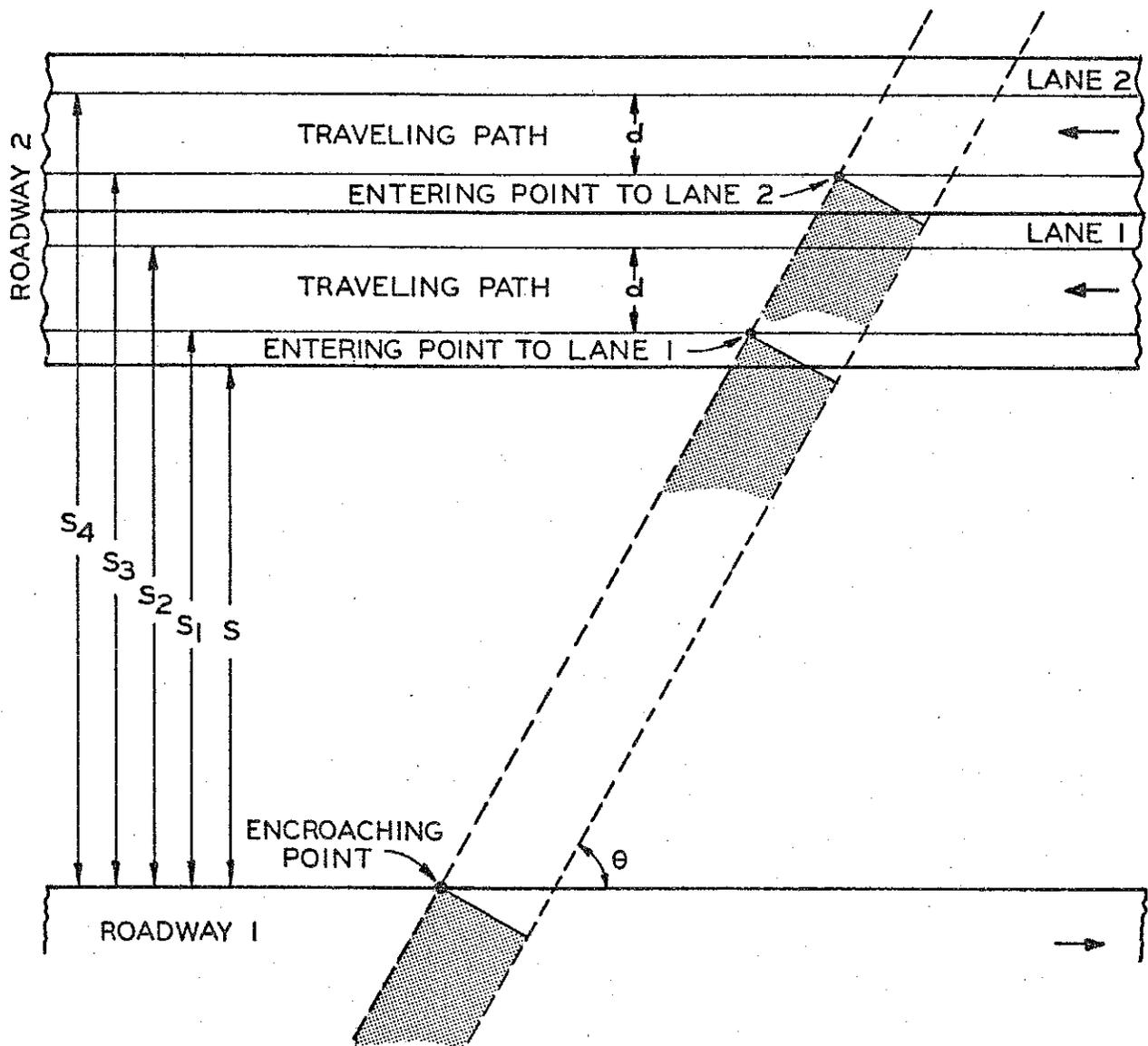
Figure A3. Schematic illustration of an encroaching vehicle entering the opposing roadway.

$$
t_1(\theta) = \begin{cases} \dfrac{(y-S_1)\csc\theta}{V} & \text{if } S_1 \le y < S_1 + (\ell + d\cot\theta + d\csc\theta)\sin\theta \\[2ex] \dfrac{\ell + d\cot\theta + d\csc\theta}{V} & \text{if } y \ge S_1 + (\ell + d\cot\theta + d\csc\theta)\sin\theta \end{cases} \tag{15}
$$

and

$$
t_2(\theta) = \begin{cases} \dfrac{(y-S_3)\csc\theta}{V} & \text{if } S_3 \le y < S_3 + (\ell + d\cot\theta + d\csc\theta)\sin\theta \\[2ex] \dfrac{\ell + d\cot\theta + d\csc\theta}{V} & \text{if } y \ge S_3 + (\ell + d\cot\theta + d\csc\theta)\sin\theta \end{cases} \tag{16}
$$

Let $Z(\theta) = \ell + d\cot\theta + d\csc\theta$. Then, given that an encroach-ment has occurred, the probability that the vehicle will enter and collide with vehicles on the other roadway, under the condition that $\ell = 3d$ ($S_1 + Z(\theta)\sin\theta \ge S_3$ for every $0 < \theta \le \dfrac{\pi}{2}$, is,

$$
P_H(S, V, \lambda_1, \lambda_2) = \int_{\theta=0}^{\frac{\pi}{2}} dG(\theta) \int_{y=S_1}^{S_3} \left(1 - e^{-\lambda_1 \frac{(y-S_1)\csc\theta}{V}}\right) dF_\theta(y)
$$

$$
+ \int_{\theta=0}^{\frac{\pi}{2}} dG(\theta) \int_{y=S_3}^{S_1+Z(\theta)\sin\theta} \left[\left(1 - e^{-\lambda_1 \frac{(y-S_1)\csc\theta}{V}}\right) + e^{-\lambda_1 \frac{(y-S_1)\csc\theta}{V}}\left(1 - e^{-\lambda_2 \frac{(y-S_3)\csc\theta}{V}}\right)\right] dF_\theta(y)
$$

$$
+ \int_{\theta=0}^{\frac{\pi}{2}} dG(\theta) \int_{y=S_1+Z(\theta)\sin\theta}^{S_3+Z(\theta)\sin\theta} \left[\left(1 - e^{-\lambda_1 \frac{Z(\theta)}{V}}\right) + e^{-\lambda_1 \frac{Z(\theta)}{V}}\left(1 - e^{-\lambda_2 \frac{(y-S_3)\csc\theta}{V}}\right)\right] dF_\theta(y) \tag{17}
$$

$$
+ \int_{\theta=0}^{\frac{\pi}{2}} dG(\theta) \int_{y=S_3+Z(\theta)\sin\theta}^{\infty} \left[\left(1 - e^{-\lambda_1 \frac{Z(\theta)}{V}}\right) + e^{-\lambda_1 \frac{Z(\theta)}{V}}\left(1 - e^{-\lambda_2 \frac{Z(\theta)}{V}}\right)\right] dF_\theta(y)
$$

where, if the width of each lane is w,

$$S_1 = S + \frac{w - d}{2}$$

$$S_2 = S_1 + d$$

$$S_3 = S_2 + d$$

and,

$$S_4 = S_3 + d.$$

Now, let us assume that the average daily traffic volume of the lane j of the roadway i is T_{ij}, i = 1,2, and j = 1,2. Then,

$$\lambda_{ij} = \frac{T_{ij}}{24 \times 3600}$$

Furthermore, assume that the entering speed into roadway i is V_i. The above assumptions are summarized in Figure A4.

Thus, given that an encroachment has occurred, the probability that there will be a collision is,

$$P_H(S) = \frac{N(T_{11} + T_{12})}{N(T_{11} + T_{12}) + N(T_{21} + T_{22})} P_H(S, V_2, \lambda_{21}, \lambda_{22})$$

$$+ \frac{N(T_{21} + T_{22})}{N(T_{11} + T_{12}) + N(T_{21} + T_{22})} P_H(S, V_1, \lambda_{11}, \lambda_{12})$$

(18)

Thus, the total expected number of collisions per mile per year will be,

$$P_H(S) \cdot \left[N(T_{11} + T_{12}) + N(T_{21} + T_{22}) \right]$$

Case 2 – The length of one roadway is fixed and the other roadway is much longer.

Again, we shall limit our discussion to the case of one roadway being the service road, C ft in length, and the other roadway is one direction of freeway. In this case, the time involving the potential collision with vehicles on Lane 1 and Lane 2 of Roadway 2 (service road) depends upon the angle of encroachment and the encroaching point on Roadway 1. Due to analytical complexity, we shall only demonstrate one typical angle θ, as shown in Figure A5.

For the angle θ, shown in Figure A5, the following equations are easily obtained:

$$A_1 = S_1 \cot \theta$$
$$A_2 = S_2 \cot \theta$$
$$A_3 = S_2 \cot \theta + (d) \csc \theta$$
$$A_1' = S_3 \cot \theta$$
$$A_2' = S_4 \cot \theta$$
$$A_3' = S_4 \cot \theta + (d) \csc \theta$$
$$A_4 = C + S_1 \cot \theta$$
$$A_5 = C + S_1 \cot \theta + (d) \csc \theta$$
$$A_6 = C + S_2 \cot \theta + (d) \csc \theta$$
$$A_4' = C + S_3 \cot \theta$$
$$A_5' = C + S_3 \cot \theta + (d) \csc \theta$$
$$A_6' = C + S_4 \cot \theta + (d) \csc \theta$$

For this angle of encroachment, the conditional probability that a vehicle will enter

and collide with vehicles on Roadway 2 is, if,

a) the encroaching point $w \in (A_1, A_2)$

$$P_\theta(\text{collision}|w) = \int_{y=S_1}^{S_1+h(w,A_1)\sin\theta} \left[1 - e^{-\lambda_{21}\frac{(y-S_1)\csc\theta}{V_2}}\right] dF_\theta(y)$$

$$+ \int_{y=S_1+h(w,A_1)}^{\infty} \left[1 - e^{-\lambda_{21}\frac{h(w,A_1)}{V_2}}\right] dF_\theta(y) \tag{19}$$

where,

$$h(w,A) = \left[\ell + (w-A)\cos\theta\right] \tag{20}$$

b) the encroaching point $w \in (A_2, A_1')$

$$P_\theta(\text{collision}|w) = \int_{y=S_1}^{S_1+h(w,A_2)\sin\theta} \left[1 - e^{-\lambda_{21}\frac{(y-S_1)\csc\theta}{V_2}}\right] dF_\theta(y)$$

$$+ \int_{y=S_1+h(w,A_2)\sin\theta}^{\infty} \left[1 - e^{-\lambda_{21}\frac{d\csc\theta+h(w,A_2)}{V_2}}\right] dF_\theta(y) \tag{21}$$

where, $h(w,A)$ was defined in Eq. (20).

c) the encroaching point $w \in (A_1', A_3)$

$$P_\theta(\text{collision}|w) = \int_{y=S_1}^{S_3} \left[1 - e^{-\lambda_{21}\frac{(y-S_1)\csc\theta}{V_2}} \right] dF_\theta(y)$$

$$+ \int_{y=S_3}^{S_1+K(w,A_2)\sin\theta} \left[\left(1 - e^{-\lambda_{21}\frac{(y-S_1)\csc\theta}{V_2}}\right) + e^{-\lambda_{21}\frac{(y-S_1)\csc\theta}{V_2}}\left(1 - e^{-\lambda_{22}\frac{(y-S_3)\csc\theta}{V_2}}\right) \right] dF_\theta(y)$$

$$+ \int_{y=S_1+K(w,A_2)\sin\theta}^{S_3+h(w,A_1')\sin\theta} \left[\left(1 - e^{-\lambda_{21}\frac{K(w,A_2)}{V_2}}\right) + e^{-\lambda_{21}\frac{K(w,A_2)}{V_2}}\left(1 - e^{-\lambda_{22}\frac{(y-S_3)\csc\theta}{V_2}}\right) \right] dF_\theta(y) \qquad (22)$$

$$+ \int_{y=S_3+h(w,A_1')\sin\theta}^{\infty} \left[\left(1 - e^{-\lambda_{21}\frac{K(w,A_2)}{V_2}}\right) + e^{-\lambda_{21}\frac{K(w,A_2)}{V_2}}\left(1 - e^{-\lambda_{22}\frac{h(w,A_1')}{V_2}}\right) \right] dF_\theta(y)$$

where,

$$K(w,A) = h(w,A) + d\csc\theta \qquad (23)$$

d) the encroaching point $w \in (A_3, A_2')$. If the $K(w,A_2)$ of P_θ (collision$|w$) in case (c) is replaced by $\ell + d\cot\theta + d\csc\theta$, we obtain the conditional probability for this case.

e) the encroaching point $w \in (A_2', A_3')$. If the $K(w,A_2)$ and $h(w,A_1')$ of P_θ (collision$|w$) in case (c) are replaced by $\ell + d\cot\theta + d\csc\theta$ and $K(w,A_2')$, respectively, we obtain the conditional probability for this case.

ROADWAY 2

λ_{22} ←

λ_{21} ←

V_2

V_1

ROADWAY 1
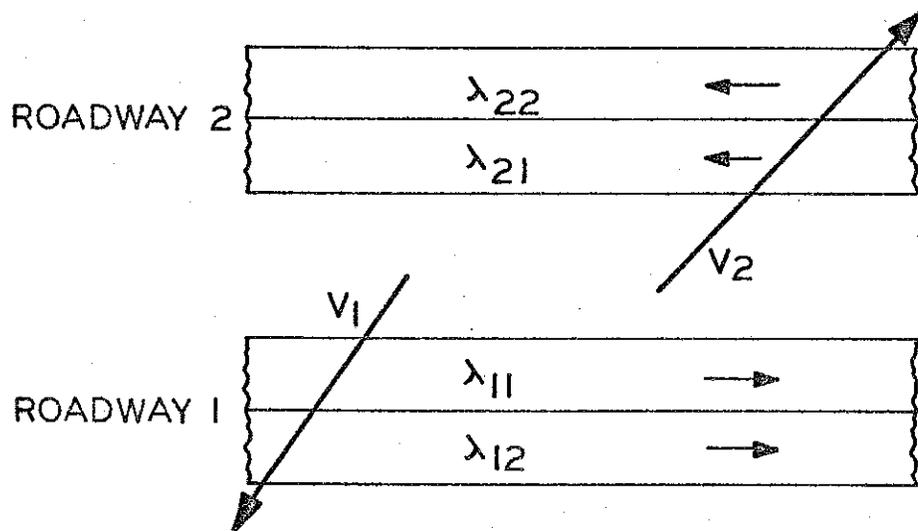
λ_{11} →

λ_{12} →

Figure A4. Schematic illustration of the entering vector in a given traffic density on each roadway.

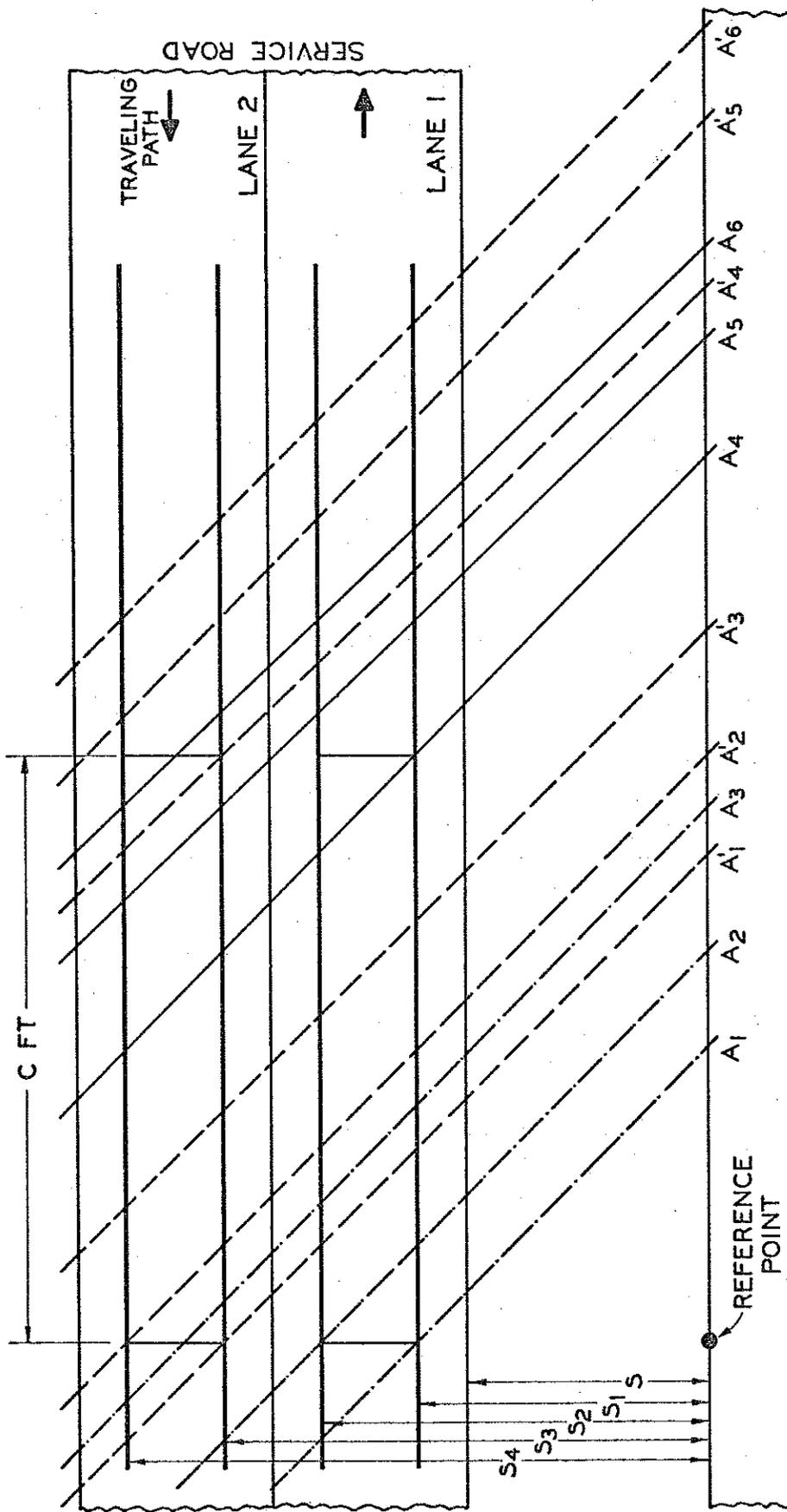Figure A5. Schematic illustration of the various encroachment zones.

f) the encroaching point $w \in$ (A'_3, A_4). The conditional probability P_θ (collision $\mid w$) for this case satisfies the following equation:

$$\int_{\theta=0}^{\frac{\pi}{2}} P_\theta (\text{collision} \mid w) \, d\, G(\theta) = P_H (S, V_2, \lambda_{21}, \lambda_{22}) \qquad (24)$$

where P_H was defined in Eq. (17).

The conditionaly probability P_θ (collision $\mid w$) for $w \in$ (A_4, A'_6) can be similarly derived as the cases (a) through (e) and is, therefore, omitted here. It is quite apparent that the conditional probability P_θ (collision $\mid w$) for $w \in (A'_3, A_4)$ is less than, or equal to, the conditional probability P_θ (collision $\mid w$) for $w \in (A'_3, A_4)$. If we denote θ^* to be the least angle such that $A'_3 < A_4$, and $P_{H1}(S)$ to be the probability that an encroaching vehicle from Roadway 1 will enter and collide with vehicles on Roadway 2 (service road), then,

$$\int_{\theta=\theta^*}^{\frac{\pi}{2}} \frac{A_4 \wedge L^* - A'_3}{L^*} P_\theta^* \, d\, G(\theta) \leq P_{H1}(s) \leq \int_{\theta=0}^{\frac{\pi}{2}} \frac{A'_6 \wedge L^* - A_1}{L^*} P_\theta^* \, d\, G(\theta) \qquad (25)$$

where $a \wedge b$ is defined to be the minimum of a and b , L^* was defined in Eq. (6), and P_θ^* is the conditional probability P_θ (collision $\mid w$) for any $w \in$ (A'_3, A_4).

Under the conditions that S is fairly large, or C/S is very large, the difference,

$$\int_{\theta=0}^{\frac{\pi}{2}} \frac{A'_6 \wedge L^* - A_1}{L^*} P_\theta^* \, d\, G(\theta) - \int_{\theta=\theta^*}^{\frac{\pi}{2}} \frac{A_4 \wedge L^* - A'_3}{L^*} P_\theta^* \, d\, G(\theta)$$

would be very small. Therefore, $P_{H1}(S)$ can be approximated by a quantity interpolated from the inequality Eq. (25).

The probability $P_{H2}(S)$ that an encroaching vehicle from Roadway 2 will enter and collide with vehicles on Roadway 1 is,

$$P_{H2}(S) = P_H (S, V_1, \lambda_{11}, \lambda_{12}) .$$

Thus, given that an encroachment has occurred, the probability that there will be a collision is,

$$P_H(S) = \frac{N(T_{11}+T_{12})L^*}{N(T_{11}+T_{12})\cdot L^* + N(T_{21}+T_{22})C}\, P_{H1}(S)$$

(26)

$$+ \frac{N(T_{21}+T_{22})C}{N(T_{11}+T_{12})L^* + N(T_{21}+T_{22})C}\, P_{H2}(S)$$

The expected number of collisions in a year is equal to,

$$P_H(S)\cdot\frac{N(T_{11}+T_{12})L^* + N(T_{21}+T_{22})C}{5280}$$

Data

For the purposes of this paper, we set d = 6 ft, ℓ = 18 ft, S = 30 ft, and C = 5.8 x 5280 ft. As mentioned before, L^* in Eq. (6) was set to be 828 ft.

We shall use the data (θ , X , Y) obtained by Hutchinson and Kennedy (23) to obtain the required distribution such that $G(\theta)$ and $F_\theta(y)$. The results are presented as follows:

The probability distribution function $G(\theta)$ of the angle of encroachment.

A non-linear curve fitting program was used to fit the empirical cumulative distribution of θ to obtain,

$$G(\theta) = \begin{cases} 1 - e^{-a\theta} & , 0 < \theta \leq \frac{\pi}{2} - \frac{1}{a} \\ 1 - (\frac{\pi}{2}-\theta)a\, e^{-a(\frac{\pi}{2}-\frac{1}{a})} & , \frac{\pi}{2} - \frac{1}{a} < \theta \leq \frac{\pi}{2} \end{cases}$$

(27)

where a = 0.0873978. The above fitting is acceptable in the sense that the Kolmogorov – Smirnov test does not reject the null hypothesis at the 0.025 significance level, that the data for the angle of encroachment was sampled from the population having the distribution $G(\theta)$ specified in Eq. (27).

The conditional probability distribution, $H_\theta(x)$, of the longitudinal distance x when y is the maximum, given that the angle of the encroachment is θ.

We first group θ-data into many intervals such that the number of x-points in each interval are at least 15. The x-data then determined by least squares a normal distribution. The fitted results are presented in Table A1.

TABLE A1
THE MEAN AND THE STANDARD DEVIATION OF THE FITTED X_θ-DISTRIBUTION

θ-Internal	Midpoint	$\mu(\theta)$	$\sigma(\theta)$	$C(\theta) = \dfrac{\sigma(\theta)}{\mu(\theta)}$
0.4-1.1	0.75	345.273	123.1990	0.357
0.4-2.3	1.35	333.350	136.9320	0.411
1.2-2.3	1.75	306.201	165.9700	0.542
2.4-2.9	2.65	284.925	66.2118	0.232
2.4-3.4	2.90	281.847	69.1359	0.245
3.0-3.4	3.20	272.265	68.5384	0.252
3.6-4.1	3.85	246.005	92.9195	0.378
4.4-4.8	4.60	246.169	87.1938	0.354
4.4-5.2	4.80	248.945	72.9535	0.293
5.2-5.7	5.45	197.252	88.8401	0.450
5.7-6.4	6.05	151.176	85.8139	0.568
6.4-7.1	6.75	142.461	96.9628	0.681
7.8-8.1	7.60	164.254	76.7960	0.468
7.1-9.5	8.30	166.663	81.8401	0.491
9.5	9.50	159.107	90.6593	0.570
11.3	11.30	138.420	74.5814	0.539
11.3-14.5	12.90	134.301	73.4639	0.547
14.0-14.5	14.25	122.899	73.3882	0.597
18.4-23.2	20.80	72.6085	34.6450	0.477
18.4-38.7	28.55	70.2708	35.0973	0.499
26.6-38.7	32.65	62.2810	30.1673	0.484
45	45.00	50.8907	26.5079	0.521

The normal fitting is good in the sense of the Kolmogorov-Smirnov test. The midpoint and $\mu(\theta)$ in Table A1 were further fitted by the following equation,

$$\mu(\theta) = \begin{cases} a + b\,e^{-c\theta} & \text{if} \quad 0 < \theta \leq d \\ \\ bc\,e^{-cd}\left(\frac{\pi}{2} - \theta\right) & \text{if} \quad d < \theta \leq \frac{\pi}{2} \end{cases}$$

The parameters obtained by a non-linear curve fitting program (Fig. A6) are as follows:

$$a = 62.3075, \quad b = 319.411, \quad c = 0.141815$$

and

$$d = 26.2131288766$$

Since the variance of the coefficient of variation in Table A1 is not large, it is reasonable to assume that

$$\sigma(\theta) = C_0 \mu(\theta)$$

where C_0 is the weighted average of $C(\theta)$ in Table A1. Thus, the distribution of X_θ is estimated to be for $0 < \theta \leq \dfrac{\pi}{2}$,

$$H_\theta(x) = \begin{cases} 0 & \text{if } \quad x \leq 0 \\[3mm] N\left(x; \mu(\theta), C_0\mu(\theta)\right) & \text{if } x > 0 \end{cases}$$

where $C_0 = 0.4525$ and N is the normal distribution function with mean $\mu(\theta)$ and standard deviation $C_0 \mu(\theta)$.

The conditional probability distribution of the maximum lateral distance $F_\theta(y)$, given that the angle of encroachment is θ.

For every angle of encroachment, define,

$$\psi = \tan^{-1}(Y/X)$$

The data obtained by Hutchinson and Kennedy [23] show that the correlation coefficient between θ and ψ is 0.831, and $\psi = 0.7807\theta$. The reason that the relation between ψ and θ is not a 45-degree line could be because there is a ditch between two roadways. If the area between two roadways is fairly flat, it is expected that $\psi = \theta$. In this situation, $Y_\theta = X_\theta \tan\theta$. Thus, the prob-
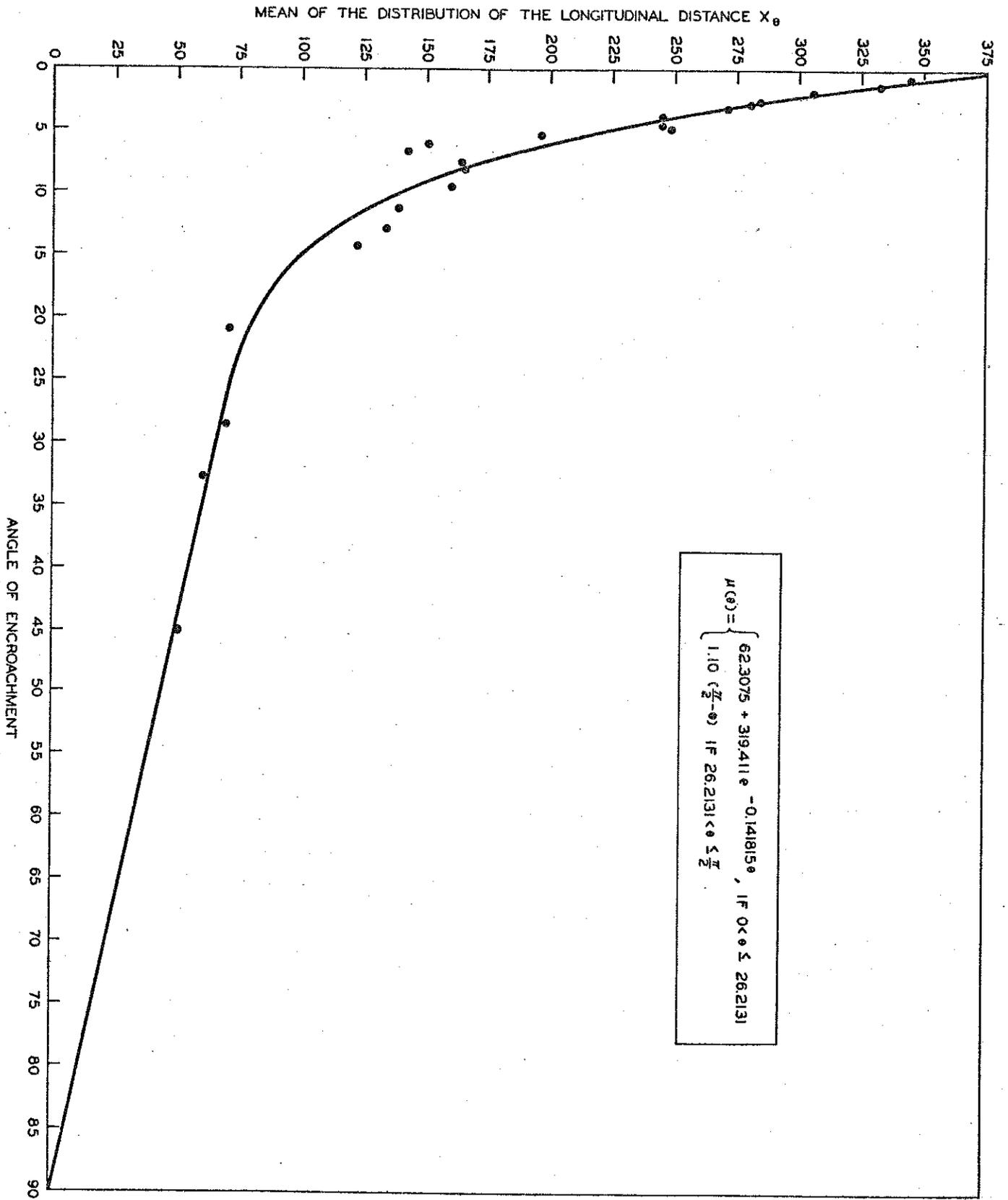
Figure A6. The actual and fitted mean of the distribution of the longitudinal distance for a given angle of encroachment.

MEAN OF THE DISTRIBUTION OF THE LONGITUDINAL DISTANCE X_θ

ANGLE OF ENCROACHMENT

$$\mu(\theta) = \begin{cases} 62.3075 + 319.411 e^{-0.141815\theta}, & \text{IF } 0 < \theta \leq 26.2131 \\ 1.10 \left(\frac{\pi}{2} - \theta\right) & \text{IF } 26.2131 < \theta \leq \frac{\pi}{2} \end{cases}$$

ability distribution of Y_θ is estimated to be for $0 < \theta \leq 89°$,

$$F_\theta(y) = \begin{cases} 0 & \text{if } y \leq 0 \\ \\ N_\theta(y; \; \mu(\theta)\tan(\theta), \; C_0\mu(\theta)\tan\theta) & \text{if } y > 0 \end{cases}$$

and $F_\theta(y) = F_{89°}(y)$ for $89° < \theta \leq \dfrac{\pi}{2}$.

APPENDIX B

COST EFFECTIVENESS OF MEDIAN GUARDRAIL
INSTALLATION FOR 30 FOOT SEPARATIONS

APPENDIX B

The presumption of median guardrail installation is that it will prevent more of the serious type accidents than it creates. It is acknowledged in the literature and verified in the theoretical investigation presented in this report that median guardrail does create more total accidents than it prevents. These findings notwithstanding, if median guardrail prevented the serious head-on collision while generating only the more numerous but less serious property damage accident, its installation could be favored. Michigan's experience as well as that of other states indicates that guardrail accidents can be quite serious and even result in fatalities. About one percent of reported guardrail accidents are fatal according to a Michigan study (21). Moreover, about 30 percent of reported guardrail accidents are either fatal or involve injury (15). However, if we are willing to disregard all accidents caused by guardrail, we still have the problem of justifying investment in this type of safety program as opposed to other competing programs of known value. The purpose of this appendix is to compare the estimated cost of preventing one fatality with guardrail optimally located for six miles between I 94 and its north service road and the cost of preventing one fatality with the Department's high accident intersection skid-proofing program.

Table B1 develops the cost-effectiveness comparison using 1973 data from California and Michigan. Naturally, some of the data will vary with sources. This is particularly true of costs. Accident data, on the other

hand, are quite stable since large samples are usually involved. Michigan intersection fatality rates, for example, vary only a few percent from year to year. What is attempted in this analysis is not a final, exact statement of the relative program merits, but a reasonable comparison based on available evidence. It is our belief that if other data sources were used, the cost comparison estimates would vary somewhat, but not enough to meaningfully alter the final conclusion.

We conclude that of the two programs compared, high accident intersection skid-proofing is, for the investment dollar, by far the most safety beneficial. Its return in lives saved is approximately 50 times as great as the median guardrail situation examined for the same cost. As mentioned before, even this is a conservative estimate since all guardrail accidents were excluded from consideration.

TABLE B1
COMPARISON OF SAFETY BENEFITS OBTAINABLE WITH GUARDRAIL INSTALLATION AND RESURFACING HIGH-ACCIDENT, LOW SKID RESISTANCE INTERSECTIONS

(Computed on the basis of six-miles of guardrail located between I 94 and its north service road at a distance of 25 ft from I 94. All figures based on 1973 costs.)

Item	Resurfacing High-Accident Intersections	Guardrail Installation in Outer Separation Between I 94 and Its North Service Road
Initial Cost Per Instance	$ 9,200 (a)	$ 498,326 (b)
Maintenance	-0-	$ 231,390 (c)
Total 25-Year Cost	$ 9,200	$ 729,716
Accidents Prevented	19.35 (d)	4.50 (e)
Fatal Accidents Prevented	0.13 (f)	0.14 (g)
Fatalities Prevented	0.16 (h)	0.22 (i)
Cost of Preventing One Fatality	$57,500 (j)	$3,316,891 (k)
Cost Ratio of Guardrail to Resurfacing	1.0	57.68 (l)

(a) Computed by assuming 1,000 lin ft of sand-asphalt resurfacing of an average of 2.3 lanes each 12-ft wide at $3.00/sq yd on a multiple project bid basis.

(b) Data taken from Tye, Edward J., "Median Barriers in California," Traffic Engineering, Se. 1975. Computations are:

Length of guardrail: 6 Mi x 5,280 = 31,680 ft
1973 Cost of Guardrail Installation per ft: $15.73
Total Cost of 6 Miles of Guardrail Installation: $498,326

(c) Data taken from: Tye, Edward J., "Median Barriers in California,"

Traffic Engineering, Se. 1975 and Michigan Report R-995. Compu-

tations are:

Beam Guardrail Accidents in 1973: 1,870
Number of Beam Guardrail Miles in 1973: 344
Beam Guardrail Accidents Per Mile: 5.436
Cost of Yearly Guardrail Repair Per Mile in 1973: $753
Cost of Yearly Guardrail Repair Per Accident in 1973: $138.52
Expected Number of Guardrail Accidents Per Year (from Fig. 5,
   Michigan Report R-995): 35
Total Yearly Repair Cost for 6 Mile Guardrail Section (35 x 138.52):
   $4,848.20
25 Year Service Life Repair Cost Assuming 5 Percent Yearly Inflation:
   $231,390.52

(d) It is estimated in Michigan Report R-998 that $10,000 invested in high-

accident, anti-skid resurfacing will, over 5 years, prevent an average

of 20 accidents. Since the 1973 per intersection cost averaged $9,200

the accidents prevented per intersection is 19.35.

(e) Data taken from Michigan Report R-995 (Fig. 4). With an I 94 ADT of

50,000, 0.9 cross-median collisions without guardrail are expected

to occur in five years for a service road ADT of 2,000. Therefore,

4.5 are expected in 25 years.

(f) Data taken from Michigan Accident Facts for years 1971-1974. Aver-

age proportion of rural, multi-vehicle intersection accidents which are

fatal is 0.68 percent; 0.68 percent of 19.35 is 0.13.

(g) Data from Michigan Accident Facts for years 1971-1974. Average

proportion of rural head-on collisions which are fatal is 4.63 percent.

However, since the service road is two-way only one-half of the collisions with vehicles from I 94 will be head-on. The other one-half will be at an angle. The average proportion of "angle" collisions which are fatal is 1.64 percent. The average of 4.63 and 1.64 is 3.12. 3.12 percent of 4.50 is 0.14.

(h) Since each fatal accident may result in more than one death, the proportion of fatalities per fatal accident is greater than unity. Data taken from Miehie, J. D., Calcote, L. R., and Broonstad, M. E., "Guardrail Performance and Design," NCHRP Report 115, 1971, Table 4 (Plaintiff's Exhibit No. 9, dated February 16, 1976). The proportion of fatalities per fatal broadside accident on the interstate system is 1.25. Thus, 1.25 x 0.13 = 0.16 fatalities prevented.

(i) Data also from Table 4 of reference (h). The proportion of fatalities per fatal head-on collision is 1.61. Thus, 1.61 x 0.14 = 0.22 fatalities. The proportion used agrees with California's experience of 1.68. See also HRB Bulletin No. 266 for New Jersey Turnpike experience.

(j) Since it costs $9,200 to prevent 0.16 fatalities, it will cost $9,200/0.16 = $57,500 to prevent one fatality.

(k) Since it costs $729,716 to prevent 0.22 fatalities, it will cost $729,716/ 0.22 = $3,316,891 to prevent one fatality.

(l) Since it costs $3,316,891 to prevent one fatality with six miles of guardrail and $57,500 to prevent one fatality with anti-skid resurfacing, the ratio is 3,316,891 to 57,500 or 57.68 to 1.0.