October 12, 2010
Memorandum

To: Rebecca Curtis
Load Rating Engineer
From: Sudhakar R. Kulkarni
Christal Larkins
Subject: TI-2126 Live Load Distribution Factors for Closely Spaced Prestressed Concrete Box Beam Superstructures with Reinforced Concrete Deck Slab

The objectives of the study are to complete a parametric study of Live Load Distribution Factors (LLDF) for moment and shear for bridge superstructures with concrete deck on prestressed concrete spread box beams. The parametric study of LLDF is completed using AASHTO-LRFD method and a Grillage Analysis using AASHTO-VIRTIS software. A comparison of results of these two methods is presented.

The LLDF represents a fraction of Design Live Load supported by a beam which is part of the bridge superstructure. The formula for LLDF given in the AASHTO LRFD Bridge Design Specifications has a few limitations listed, including a minimum spacing requirement of 6 ft . However, for a few bridges using PC Box beams, closer beam spacing may be necessary in order to provide the minimum under clearance and still meet design loads.

The scope of this investigation is limited to the moment and shear LLDFs of interior PC spread box beams of superstructures. The LLDFs determined by using the formulae in the current 2010 AASHTO LRFD Bridge Design Specifications are extrapolated to calculate results and compared to the computed LLDF from grillage analysis using AASHTOVIRTIS Bridge software. A comparison of results provided by these two methods is made.

The scope of this study on LLDF is limited as follows:
a) The LLDF is computed for interior beams only.
b) The three span lengths studied are $50 \mathrm{ft}, 80 \mathrm{ft}$ and 100 ft .
c) A typical bridge superstructure with PC Box beams was defined for the study. The details are as follows and shown in Figure 1: The bridge deck cross section is an out-to-out width of $47 \mathrm{ft}-3 \mathrm{in}$, providing two 12 ft lanes, two 10 ft shoulders, and two barrier railings, with a 9 in deck thickness.
d) For each span length the beam spacing varied from $10 \mathrm{ft}-0 \mathrm{in}$. to a minimum of 4 ft 6 in . The six feasible beam spacings are $10 \mathrm{ft} 0 \mathrm{in}, 8 \mathrm{ft} 0$ in, $6 \mathrm{ft} 8 \mathrm{in}, 5 \mathrm{ft} 8 \mathrm{in}, 5 \mathrm{ft} 0 \mathrm{in}$ and 4 ft 6 in .
e) PC Box beam depth is 33 in for 50 ft span, and 48 in for 80 ft and 100 ft spans.
f) Although it is current practice in the MDOT Design Division (Bridge Section) to use the HL-93 modified loading, the AASHTO formulas are based on the original HL-93 loading. Only the LRFD Design Load (HL93 ) is used for this study.

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Testing Box Beams - Case E - 8 Spaces at 5' 0"
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Figure 1 Typical cross-section for superstructure used in grillage analysis. Crosssection shown is for 50 ft span with 5 ft 0 in beam spacing.

Refer to AASHTO LRFD tables 4.6.2.2.2b-1 and 4.6.2.2.3a-1. The range of applicability for using the LLDF equations for PC spread box beams is as follows:
a) Beam Spacing is between 6.0 ft and 18.0 ft
b) Span Length is between 20 ft and 140 ft
c) Depth of PC box beam is between 18 in and 65 in, and
d) Number of Beams in the Superstructure is equal to or greater than three

Figure 2 shows LRFD Table 4.6.2.2.2b-1 for Distribution of Live Loads per Lane for Moment in Interior Beam and Table 4.6.2.2.3a-1 for Distribution of Live Load per Lane for Shear in Interior Beams. In this case the applicable cross-section of the deck is b, and c - Concrete Deck on Concrete Box Beams.

Table 4.6.2.2.2b-1 Distribution of Live Loads Per Lane for Moment in Interior Beams.

| Type of Superstructure | Applicable Cross-Section from Table 4.6.2.2.1-1 | Distribution Factors | Range of Applicability |
| :---: | :---: | :---: | :---: |
| Concrete Deck on Concrete Spread Box Beams | b, c | One Design Lane Loaded: $\left(\frac{S}{3.0}\right)^{0.35}\left(\frac{S d}{12.0 L^{2}}\right)^{0.25}$ <br> Two or More Design Lanes Loaded: $\left(\frac{S}{6.3}\right)^{0.6}\left(\frac{S d}{12.0 L^{2}}\right)^{0.125}$ | $\begin{gathered} 6.0 \leq S \leq 18.0 \\ 20 \leq L \leq 140 \\ 18 \leq d \leq 65 \\ N_{b} \geq 3 \end{gathered}$ |
|  |  | Use Lever Rule | $S>18.0$ |

Table 4.6.2.2.3a-1 Distribution of Live Load per Lane for Shear in Interior Beams.
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\begin{array}{|l|c|c|c|c|}\hline \hline \begin{array}{l}\text { Type of } \\
\text { Superstructure }\end{array} & \begin{array}{c}\text { Applicable } \\
\text { Cross-Section } \\
\text { from Table } \\
4.6 .2 .2 .1-1\end{array} & \begin{array}{c}\text { One Design Lane } \\
\text { Loaded }\end{array} & \text { Two or More Design Lanes } \\
\text { Loaded }\end{array}
$$ \quad \begin{array}{c}Range of \\

Applicability\end{array}\right]\)| $6.0 \leq S \leq 18.0$ |
| :--- |
| Concrete Deck on |
| Concrete Spread |
| Box Beams |

Figure 2 Tables from AASHTO LRFD Bridge Design Specifications, $4^{\text {th }}$ edition, pp. 4-38 \& 4-41

LLDF for moment and shear were computed using AASHTO LRFD formulas and Grillage Analysis using AASHTO- VIRTIS software. The LLDF given by AASHTO LRFD formulas are conservative, which contain the variables of span length(L), beam depth(d) and beam spacing(s) only. The contribution of sidewalks, bridge railings and possible fixity of supports is not accounted in this formulation. The formulas were extrapolated for computing LLDF on beam spacing less than 6.0 ft . The results are presented in tables 1 through 4 and in Figures 3 through 10. Side by Side box girder cases are provided for information only, and are not part of this study.

The percent difference between the grillage analysis and AASHTO formulas were calculated two ways to see which method would produce the largest difference when controlling. In table 3 grillage analysis controls and in table 4 the AASHTO formulas control. One reason that could account for the large difference in the shear LLDF for 50 ft span may be due to placement of truck load on the span, and beam depth to span ratio in the grillage analysis.

Table 1 Table listing the LLDFs computed from the AASHTO formulae in Table 4.6.2.2.2 b-1 for deck cross-section b and side-by-side cross section $f$

|  |  | Moment $\mathrm{gmi}_{\text {mi }}$ |  |  | Shear |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Beam Spacing | \# of Beams, $\mathrm{N}_{\mathrm{b}}$ | $\begin{aligned} & L=50^{\prime} \\ & d=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & L=80^{\prime} \\ & d=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & L=100^{\prime} \\ & d=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & L=50^{\prime} \\ & d=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & L=80^{\prime} \\ & d=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=100^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ |
| 10' - 0" | 5 | 0.751 | 0.700 | 0.662 | 0.952 | 0.943 | 0.922 |
| 8' - 0" | 6 | 0.639 | 0.595 | 0.563 | 0.796 | 0.789 | 0.771 |
| 6' - 8" | 7 | 0.560 | 0.521 | 0.493 | 0.688 | 0.682 | 0.667 |
| 5' - 8.6" | 8 | 0.500 | 0.466 | 0.441 | 0.608 | 0.603 | 0.589 |
| 5' - 0" | 9 | 0.454 | 0.423 | 0.400 | 0.547 | 0.542 | 0.530 |
| 4' - 5.3 | 10 | 0.417 | 0.389 | 0.368 | 0.498 | 0.493 | 0.482 |
| side by side |  |  |  |  |  |  |  |
| 4' - 1.5" | 11 | 0.282 | 0.262 | 0.251 | 0.469 | 0.464 | 0.454 |



Figure 3 Graph comparing LLDFs computed from the AASHTO formulae for 50 ft , 80 ft , and 100 ft spans. The data group at 4 ft beam spacing represents the side-by-side box beam configuration (for information only).

Table 2 Table listing the LLDFs computed from the Grillage Analysis using Virtis software

|  |  | Moment $\mathrm{gmi}^{\text {m }}$ |  |  | Shear |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | HL 93 (US) |  |  | HL 93 (US) |  |  |
| Beam Spacing | $\begin{gathered} \text { \# of Beams, } \\ N_{b} \end{gathered}$ | $\begin{aligned} & \mathrm{L}=50^{\prime} \\ & \mathrm{d}=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=80^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=100^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=50^{\prime} \\ & \mathrm{d}=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=80^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ | $\begin{gathered} \mathrm{L}=100^{\prime} \\ \mathrm{d}=48^{\prime \prime} \end{gathered}$ |
| 10' - 0" | 5 | 0.8 | 0.729 | 0.672 | 0.848 | 0.905 | 0.899 |
| 8' - 0" | 6 | 0.675 | 0.627 | 0.595 | 0.707 | 0.758 | 0.756 |
| 6' - 8' | 7 | 0.589 | 0.566 | 0.54 | 0.575 | 0.615 | 0.623 |
| 5' - 8.6" | 8 | 0.521 | 0.451 | 0.421 | 0.495 | 0.531 | 0.529 |
| 5' - 0 " | 9 | 0.458 | 0.409 | 0.384 | 0.430 | 0.480 | 0.479 |
| 4'-5.3 | 10 | 0.402 | 0.362 | 0.366 | 0.414 | 0.438 | 0.437 |
| side by side |  |  |  |  |  |  |  |
| 4' - 1.5" | 11 | 0.375 | 0.362 | 0.349 | 0.393 | 0.426 | 0.427 |



Figure 4 Graph comparing LLDFs computed from Grillage Analysis for $50 \mathrm{ft}, 80 \mathrm{ft}$, and 100 ft spans. The data group at 4 ft beam spacing represents the side-by-side box beam configuration (for information only).

Table 3 Table listing the percent difference between LLDFs from the AASHTO formulae and Grillage Analysis (GRILLAGE - AASHTO)/GRILLAGE

|  |  | Moment $\mathrm{gmi}_{\text {mi }}$ |  |  | Shear |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | HL 93 (US) |  |  | HL 93 (US) |  |  |
| Beam <br> Spacing | $\begin{gathered} \text { \# of Beams, } \\ \mathrm{N}_{\mathrm{b}} \end{gathered}$ | $\begin{aligned} & \mathrm{L}=50^{\prime} \\ & \mathrm{d}=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=80^{\prime} \\ & \mathrm{d}=48{ }^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=100^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=50^{\prime} \\ & \mathrm{d}=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=80^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=100^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ |
| 10' - 0" | 5 | 6\% | 4\% | 2\% | -12\% | -4\% | -3\% |
| 8' - 0" | 6 | 5\% | 5\% | 5\% | -13\% | -4\% | -2\% |
| 6' - 8" | 7 | 5\% | 8\% | 9\% | -20\% | -11\% | -7\% |
| 5'-8.6" | 8 | 4\% | -3\% | -5\% | -23\% | -13\% | -11\% |
| 5' - 0' | 9 | 1\% | -3\% | -4\% | -27\% | -13\% | -11\% |
| 4'-5.3 | 10 | -4\% | -7\% | 0\% | -20\% | -13\% | -10\% |
| side by side |  |  |  |  |  |  |  |
| 4' - 1.5" | 11 | 25\% | 28\% | 28\% | -19\% | -9\% | -6\% |

Table 4 Table listing the percent difference between LLDFs from the AASHTO formulae and Grillage Analysis (GRILLAGE - AASHTO)/AASHTO

|  |  | Moment $\mathrm{gmi}_{\text {mi }}$ |  |  | Shear |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | HL 93 (US) |  |  | HL 93 (US) |  |  |
| Beam Spacing | \# of Beams, $\mathrm{N}_{\mathrm{b}}$ | $\begin{aligned} & \mathrm{L}=50^{\prime} \\ & \mathrm{d}=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=80^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \end{aligned}$ | $\begin{aligned} & \mathrm{L}=100^{\prime} \\ & \mathrm{d}=48^{\prime \prime} \\ & \hline \end{aligned}$ | $\begin{aligned} & \mathrm{L}=50^{\prime} \\ & \mathrm{d}=33^{\prime \prime} \end{aligned}$ | $\begin{aligned} \mathrm{L} & =80^{\prime} \\ \mathrm{d} & =48^{\prime \prime} \end{aligned}$ | $\begin{gathered} L=100^{\prime} \\ d=48^{\prime \prime} \end{gathered}$ |
| 10' - 0' | 5 | 7\% | 4\% | 2\% | -11\% | -4\% | -3\% |
| 8' - 0' | 6 | 6\% | 5\% | 6\% | -11\% | -4\% | -2\% |
| 6' - 8' | 7 | 5\% | 9\% | 9\% | -16\% | -10\% | -7\% |
| 5'-8.6" | 8 | 4\% | -3\% | -5\% | -19\% | -12\% | -10\% |
| 5' - 0" | 9 | 1\% | -3\% | -4\% | -21\% | -11\% | -10\% |
| 4' - 5.3 | 10 | -4\% | -7\% | 0\% | -17\% | -11\% | -9\% |
| side by side |  |  |  |  |  |  |  |
| 4' - 1.5" | 11 | 33\% | 38\% | 39\% | -16\% | -8\% | -6\% |



Figure 5 Graph comparing moment LLDFs for 50 ft span computed from Grillage Analysis to the AASHTO Formula


Figure 6 Graph comparing shear LLDFs for 50 ft span computed from Grillage Analysis to the AASHTO Formula


Figure 7 Graph comparing moment LLDFs for 80 ft span computed from Grillage Analysis to the AASHTO Formula


Figure 8 Graph comparing shear LLDFs for 80 ft span computed from Grillage
Analysis to the AASHTO Formula


Figure 9 Graph comparing moment LLDFs for 100 ft span computed from Grillage Analysis to the AASHTO Formula


Figure 10 Graph comparing shear LLDFs for 100 ft span computed from Grillage Analysis to the AASHTO Formula

The following conclusions can be made:
a) The AASHTO LRFD formula for LLDF for moment can be used in lieu of grillage analysis for beam spacing of 6 ft 0 in or less with a span length between 50 ft and 100 ft . The results obtained by AASHTO-LRFD and grillage analysis in all cases had a percent difference less than 10 percent.
b) The AASHTO LRFD formula for LLDF for shear should be used for beam spacing closer than 6 ft 0 in for short span bridges with span length less than 80 ft . The AASHTO formula produced values that were 10 to 20 percent larger than the grillage analysis values. Using the formula may result in an overly conservative design but until it is clear why the grillage analysis has larger deviations at shorter spans, the AASHTO formula is the better choice.
c) For span lengths between 80 ft and 100 ft either the AASHTO LRFD formula for LLDF for shear or grillage analysis can be used as variations in the values computed by using the AASHTO LRFD formula are about 10 percent larger than the grillage analysis values.

It is recommended that further analysis be performed for span lengths ranging from 20 ft to 80 ft at 10 ft intervals. This investigation will expand upon the span length range that the AASHTO formulas can successfully be applied to for beam spacing less than 6 ft 0 in.

