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MICHIGAN
STATE HIGHWAY DEPARTMENT
Charles M. Ziegler
State Highway Commissioner

SLAB ACTION IN CONCRETE PAVEMENTS UNDER STATIC LOADS

by

L. D. Childs

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A STUDY OF SLAB ACTION IN CONCRETE PAVEMENTS

Early in 1944 an investigation was begun to determine the destructive effect of axle loadings upon concrete slabs. The Department of Engineering Research of the University of Michigan and the Michigan State Highway Department were the participants in this project. Preliminary tests were made upon a small model and these results have been published.(1)

Briefly, these results indicated that under static conditions the addition of wheels to an axle is not an expedient method of increasing the loading capacity; that two axles in tandem arrangement could carry a standing load more than twice that of a single axle if a proper axle spacing were used; and, that a three axle system could be spaced so as to support static loads three times the single axle values.

Although the model served very well to indicate the relative effects of various loading arrangements and locations upon the slab, it was necessary to repeat certain experiments upon a full scale slab in order to determine absolute values which could be used in slab and vehicle design. With this purpose in mind a 9 inch uniform slab 11 feet wide by 28 feet long was cast and tested in the Highway Research Laboratory at East Lansing.

Loads corresponding to full highway loads were applied through actual trailer axles with dual 10.00-20 tires at 70 p.s.i. air pressure. The loading positions were midway between the ends of the longitudinal free edge and also at the corner of the slab. Single axles, two axles spaced from 3-1/2 to 9 feet, and three axles spaced from 4 to 7 feet were loaded and the slab strains and deflections measured.

Note: Numbers in parentheses refer to bibliography.

Results of the tests on the full-size slab correlated quite well with those of the model. At a full 18,000 pound load per axle, greater stresses were produced in the slab by a single axle than by two or three axle combinations with four foot to eight foot axle spacings. At the usual four foot spacing between axles the three axle combination caused considerably less stress than either a two axle system or a single axle when the loads were applied at the corner of the slab.

This report includes a description of the materials and equipment used for this study, a discussion of the method of application of the loads and a graphical presentation of selected data. A comparison is made between the model study and the full scale investigation. The results of other investigations and theoretical computations are shown to corroborate certain data, and a bibliography of these sources is included.

DESCRIPTION OF EQUIPMENT AND TEST PROCEDURE

Materials and Equipment

The laboratory at East Lansing was chosen for the site of the test slab because there was sufficient space for the construction of a large slab and facilities for applying the loads. Other investigators had performed tests upon slabs cast out of doors, but the results were affected by warping due to changes in temperature and moisture. It was hoped that laboratory control would minimize this unfavorable condition.

A wooden form 15 feet by 32 feet by 2 feet was built upon the concrete floor of the laboratory. Tie bars were placed at the corners and at inner points to prevent spreading when the form was filled with subgrade material.

A system of perforated pipes was laid upon the floor and connected to the water supply for the purpose of controlling subgrade moisture, thereby giving some control over subgrade modulus. This stage of construction is shown in Figure 1.

Six inches of gravel were then placed in the bottom of the form and the remainder filled to within two inches of the top with a selected bank run sand. This sand was chosen because of its similarity to that used as a cushion under highway slabs and because of its bearing capacity. Table I is a summary of the properties of the sand.

TABLE I. CHARACTERISTICS OF SUBGRADE SAND

Sieve Analysis

Sieve No.	10	20	40	60	100	200
Percent Passing	99.43	97.57	85.30	43.15	7.80	1.55

Density Values

Percent Moisture	1	3	5	7	9	11	13
Density (p.c.f.)	98	104	107	104	102	101	92

In order to maintain a minimum temperature gradient in the slab some method of control had to be devised. Preliminary investigation showed that the difference in temperature between the top and bottom of the slab would be small, but that the lower surface of the slab would likely be cooler than the top. Of the various methods proposed for heating this lower surface, the one which appeared to affect the subgrade bearing capacity the least was an electrically heated wire grid. This was installed as shown in Figure 2 and it was covered with a two inch thickness of subgrade sand.

A wooden form 11 feet by 28 feet by 9 inches was built upon the prepared subgrade. It was carefully leveled and securely fixed. The subgrade was planed and auxiliary equipment incidental to the tests was installed.

Figure 1. Form for subgrade showing tie rods, water-proofing and moisture control pipe layout.

Figure 2. Wire heating grid placed in subgrade to control temperature differential in slab.

For the purpose of measuring strains on the bottom of the slab, SR-4 gages were attached to mortar blocks and those blocks were placed on the subgrade in such a way that the gages would be in the plane of the lower surface of the slab. Unfortunately these gages were not sufficiently insulated to give reliable results after a few weeks time.

For a separate study, incidental to the loading investigation, dowel bars of various sizes and lengths were installed at one foot intervals on all edges of the slab. These bars and the mortar blocks are seen in Figure 3.

The test slab was cast using a carefully designed transit mixed air entrained concrete. Table II gives the mix and strength data. At this time five installations of thermocouples and Bouyoucos moisture cells[2] were made for the purpose of keeping accurate record of temperature and moisture differential throughout the slab and to aid in their control. A diagram of the slab and the location of measuring equipment is given in Figure 4.

Curing was accomplished by applying a membrane curing compound to the slab four hours after pouring. The relative humidity of the room was maintained at about 70 percent and the temperature held at 75° for twenty-eight days. During this period moisture and temperature measurements were made and comparator readings were taken for length change and warping. Flat surfaces were ground on the slab surface according to the plan of Figure 4 and 1/2 inch by 1/16 circular brass discs were cemented to the slab for elevation and deflection measurements. SR-4 strain gages were applied and wired to junction boxes for facility in reading. The method of grinding smooth surfaces for these installations is shown in Figure 5.

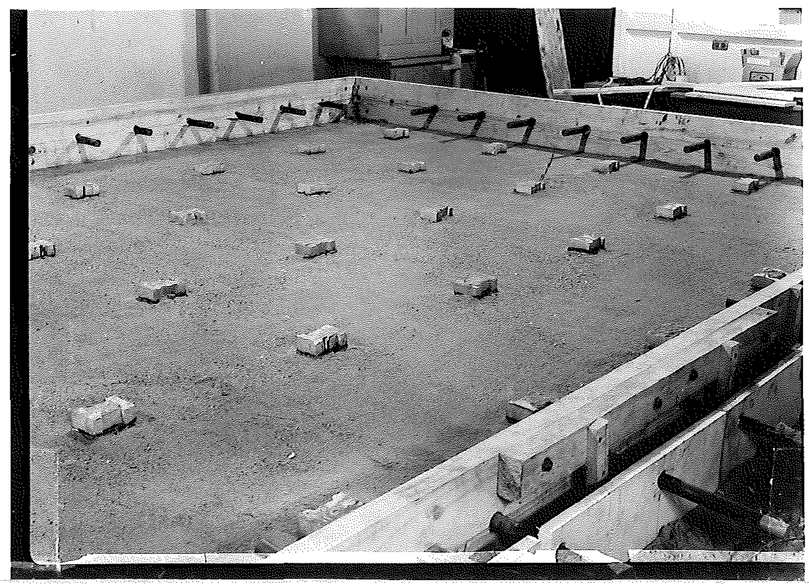


Figure 3. Form for test slab with mortar blocks and dowel bars in place.

TABLE II

MIX AND TEST DATA FOR CONCRETE SLAB

Material	Type	Weights per cu. yd. concrete
Cement	Peninsular V.R. (Raw)	517 lbs.
Fine Aggregate	Boichot 2NS	1182.5
Coarse Aggregate	American Aggregate Green Oak-10A	1892.0
Water		311.8

Fine Aggregate Gradation:

Sieve Size No.	4	8	16	30	50	100	200
Percent Passing	100	94	75	51	17	1	0

Coarse Aggregate:

Sieve Size, inches	1	3/4	1/2	3/8	No. 4
Percent Passing	100	93	59	34	1

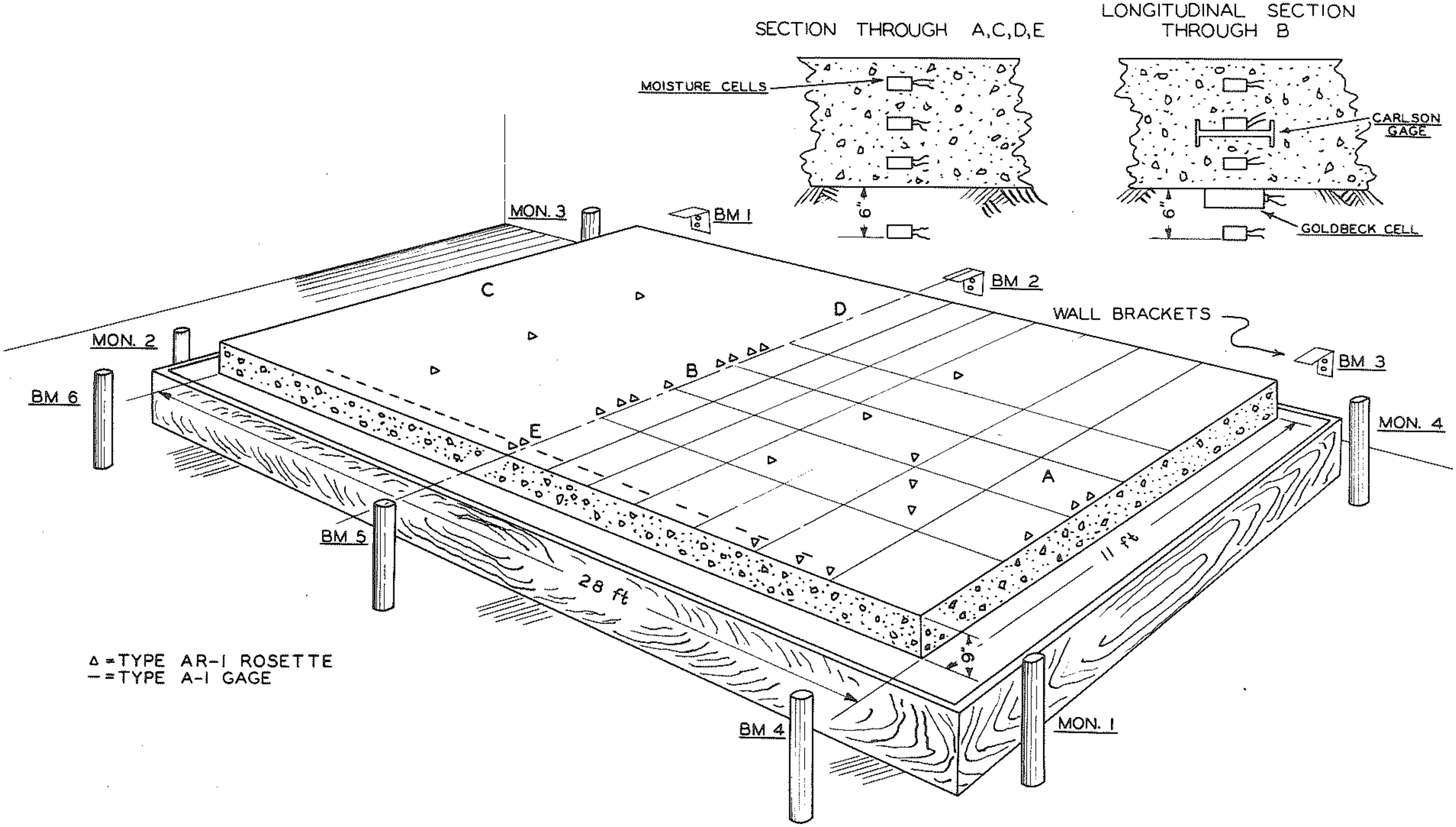
Average slump = 7-1/8 inches

Average air content = 7.5 percent

Average 28 day compressive strength 9-6"x12" test cylinders = 3660 p.s.i.

Average 28 day modulus of rupture 6-6"x8"x36" beams = 585 p.s.i.

Average 28 day modulus of elasticity of 6 cylinders at 400 p.s.i. = 5.25×10^6 p.s.i.



PLAN of TEST SLAB

Figure 4

Measuring Devices

The apparatus necessary for the measurement of moisture in the subgrade and concrete is thoroughly described in the technical bulletin to which reference was made. Temperatures were found by reading on a standard potentiometer the small e.m.f. generated by iron-constantan thermocouples. Strains were measured by resistance changes in bonded wire SR-4 type A-1 and AR-1 strain gages. These resistance changes were read directly as unit strain by a Baldwin Southwark SR-4 strain indicator. Federal one-thousandth dials at the slab edges and corners indicated deflections, while one-ten thousandth dials were used in calibrated rings to determine the load intensity. A special comparator was constructed to measure length change and warping. This is illustrated in Figure 6.

Application of Loads

All loads were applied by means of hydraulic jacks reacting against an "I" beam on the laboratory ceiling. Calibrated dynamometer rings served to indicate the load intensity. Although loads of any value up to 20,000 pounds could be applied, most of the tests were made at 10,000, 13,000, 16,000, and 18,000 pound axle loads.

MEASUREMENT OF DESTRUCTIVE EFFECT

Subgrade Modulus

In order to make a comparison of test results with theoretical values, it was necessary to determine the modulus of subgrade stiffness. This was done by two methods. First, before the slab was poured a number of loading

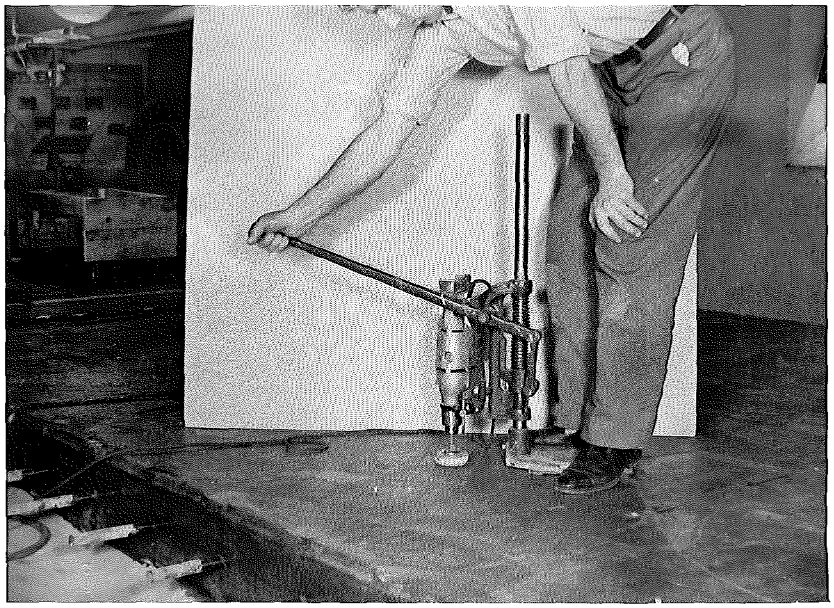


Figure 5. Method of grinding surface for application of strain gages and deflection targets.

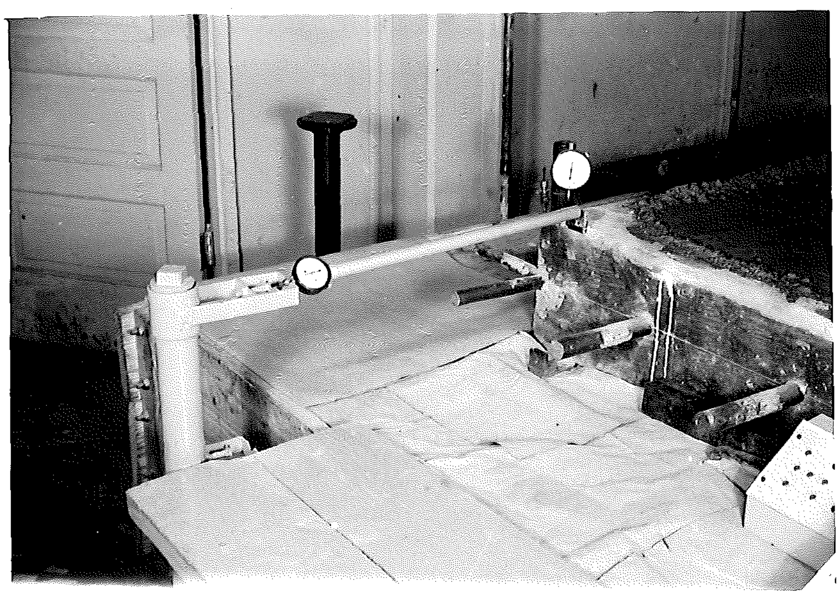
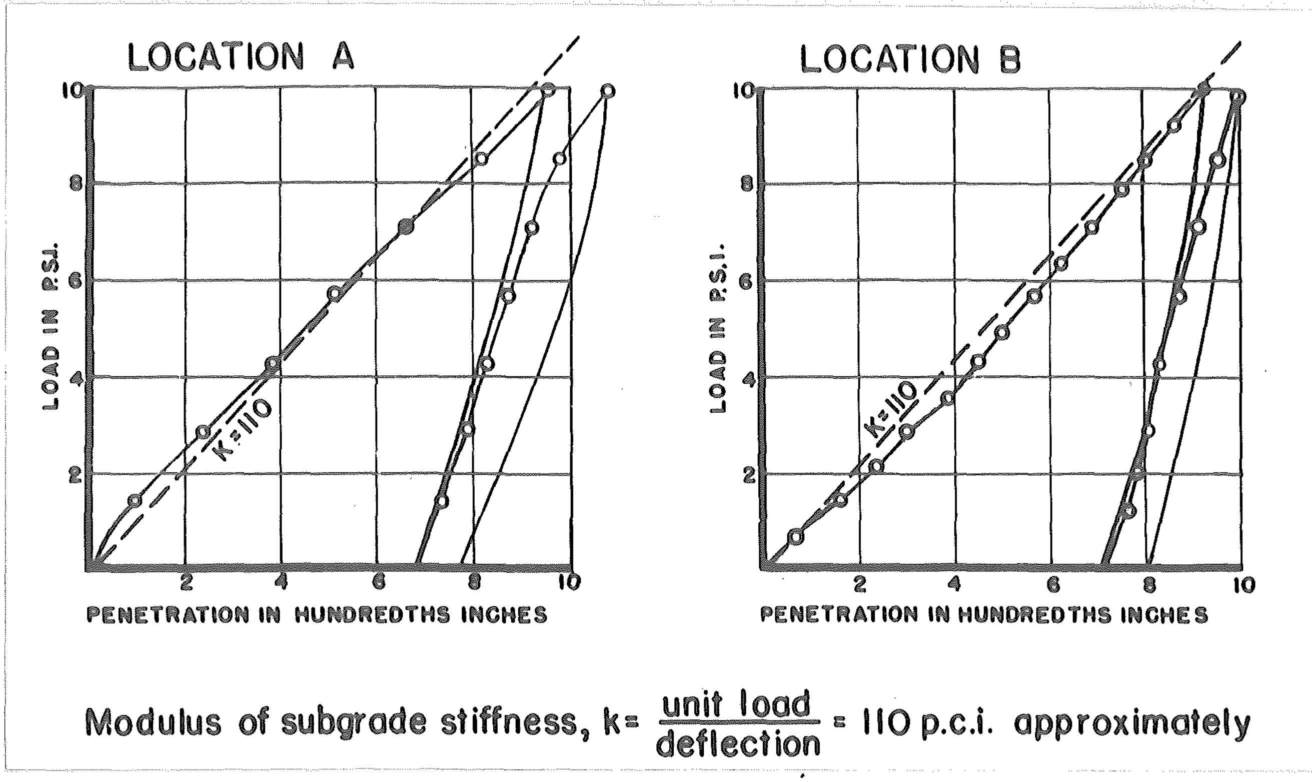


Figure 6. Length change and warping measurements being made by special comparator.

tests were made using a 30 inch plate. Figure 7 exhibits the apparatus, and data for two locations are shown in Graph 1. It is apparent that the modulus, k, is about 110 p.c.i. under the existing conditions.

At the end of a 28 day curing period further tests were made by loading the slab in four locations. Having found the modulus of elasticity of the concrete, the subgrade modulus was computed from both load-deflection data and load-stress data by formulas developed by Westergaard[3] and by the modified equations from the Arlington tests[4]. Figure 8 is an illustration of the apparatus used for these tests. The data are compiled and presented in Graph 2, and the accompanying table. There appears to be good correlation between these two methods of testing since the value 110 which was obtained by the bearing plate method also appears several times in the table. From these tests the subgrade modulus values for the two soil conditions which prevailed were chosen. For the first condition the value k=110 p.c.i. was used, and for the saturated condition k=60 p.c.i. seemed to be a fair value.

In spite of rigid control of temperature and humidity there was some upward warping of the concrete slab. A continuous record of comparator readings and dial readings at the slab corners showed that the slab corners had raised about two tenths inches. Since the temperature differential was small, the curling was attributed to moisture and fundamental differences in the concrete caused by the method of placement. In an attempt to rectify this condition the upper surface of the slab was flooded with water and left in that state until there was no further downward movement of the slab. The recovery was about fifty percent. A heavy coat of membrane curing compound was applied as soon as the water was removed.



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Graph 1. Thirty inch bearing plate tests for two locations on subgrade.

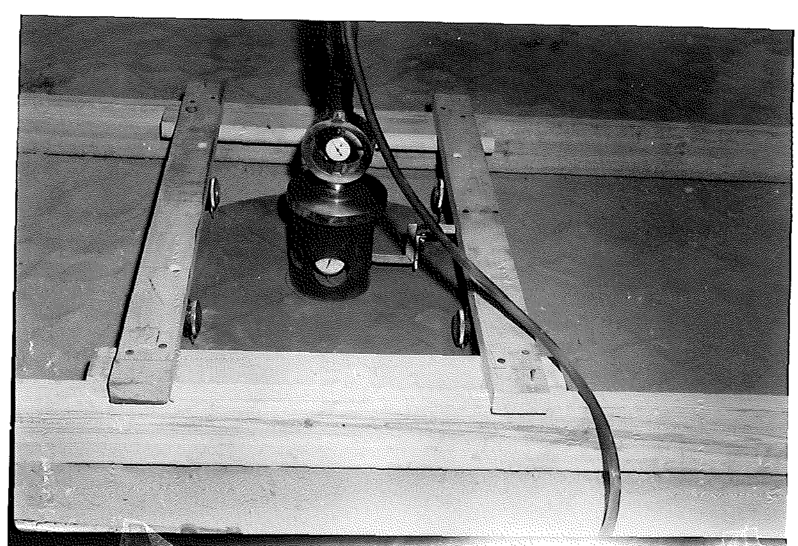


Figure 7. Arrangement of dials, calibrated ring and jack for tests on subgrade with 30 inch bearing plate.

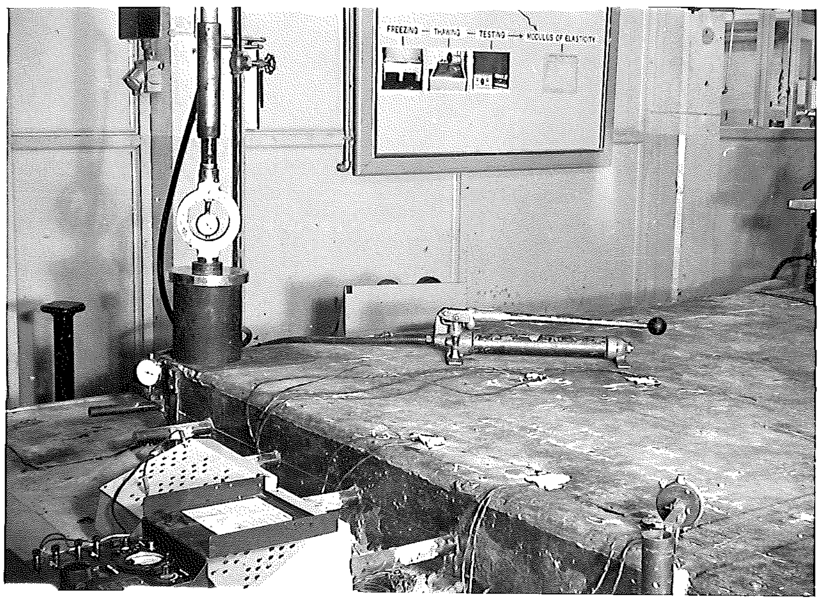
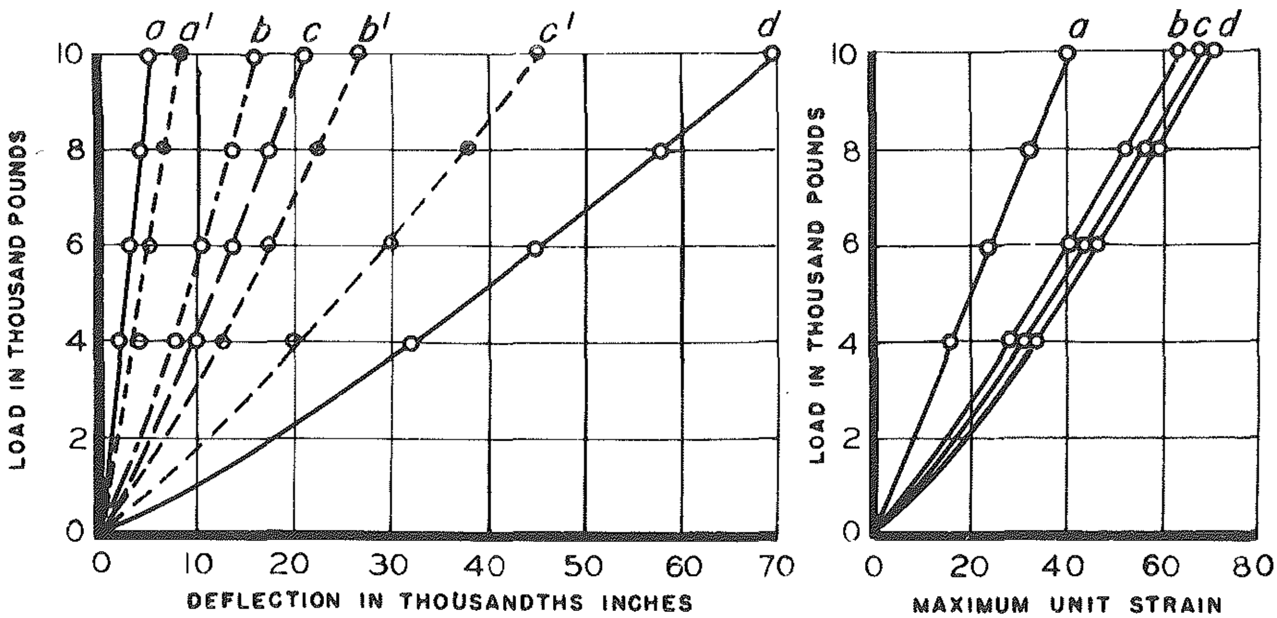


Figure 8. Method of slab loading for the determination of subgrade modulus by theoretical formulas.

DATA FROM LOADS ON 9″ SLAB *through* 6″ DIAMETER PLATE



IDENTIFICATION:

SAND NORMAL
- a = INTERIOR POINT
- b = FREE EDGE
- c = JOINT EDGE
- d = CORNER

SAND SATURATED
- a' = INTERIOR
- b' = FREE EDGE
- c' = JOINT EDGE

TABULATED VALUES OF SUBGRADE MODULUS K IN P.C.I.

LOCATION	1ST SUBGRADE CONDITION		2ND CONDITION	FORMULAS USED
	BY DEFLECTION	BY STRESSES	BY DEFLECTIONS	
INTERIOR	100	110	65	WESTERGAARD
FREE EDGE	110	110	65	WESTERGAARD
JOINT EDGE	80	95	40	B.P.R.
CORNER	55	60	—	B.P.R.

Graph 2

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The Loading Program

As soon as a series of slab elevation readings had been completed the loading program was begun. A preliminary series of tests were made at symmetric points with a metal plate for bearing area to determine the local differences in the slab and to attempt to attain good bearing between slab and subgrade.

A single 10.00-20 tire at 70 p.s.i. inflation pressure was now located at several points on the slab and deflection and strain readings were taken. Figure 9 is an example of this test. The data curves are shown in Graph 3. A comparison of these curves with those of Graph 2 shows that the strains and deflections under the wheel are comparable to those under the metal plate. Apparently the greater contact area under the tire and consequent reduced unit pressure upon the slab does not cause any appreciable decrease in slab stresses below those produced under the metal plate.

This study was followed by similar tests on a single axle equipped with dual tires. Due to the small strain magnitudes and the difficulty in obtaining reliable deflection measurements at interior points of the slab, tests at these locations were discontinued, and the only data presented for these and subsequent tests are those for the edge and corner locations.

Next, two axles were placed with outer wheels on the free edge of the slab. One series of tests was run with the axles symmetrically placed about the middle point of the edge, and another series was made with one axle at the slab corner. A variety of axle spacings was used in each group of tests.

Finally three axles were loaded in the same test pattern as was used for two axles. The maximum axle spacing was necessarily limited because of the length of the test slab. For large spacings, the tests at the center

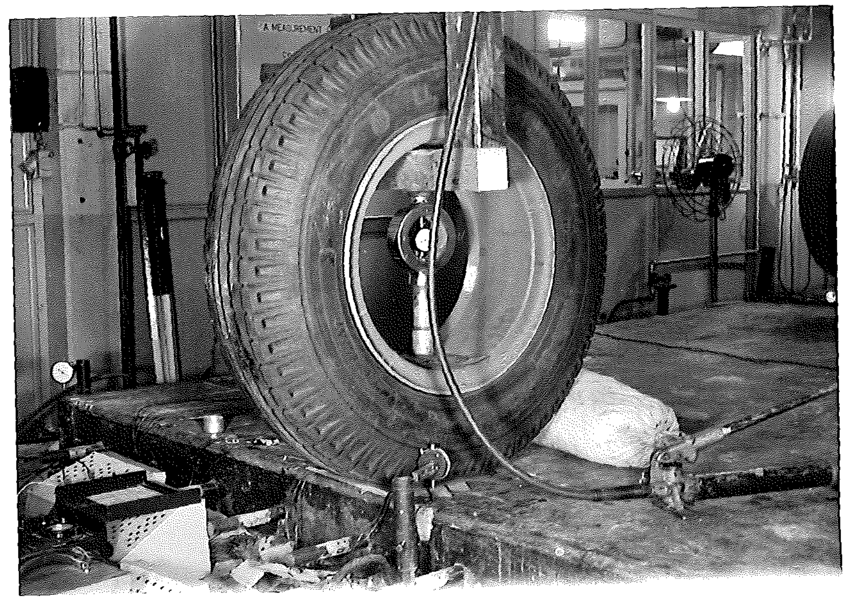
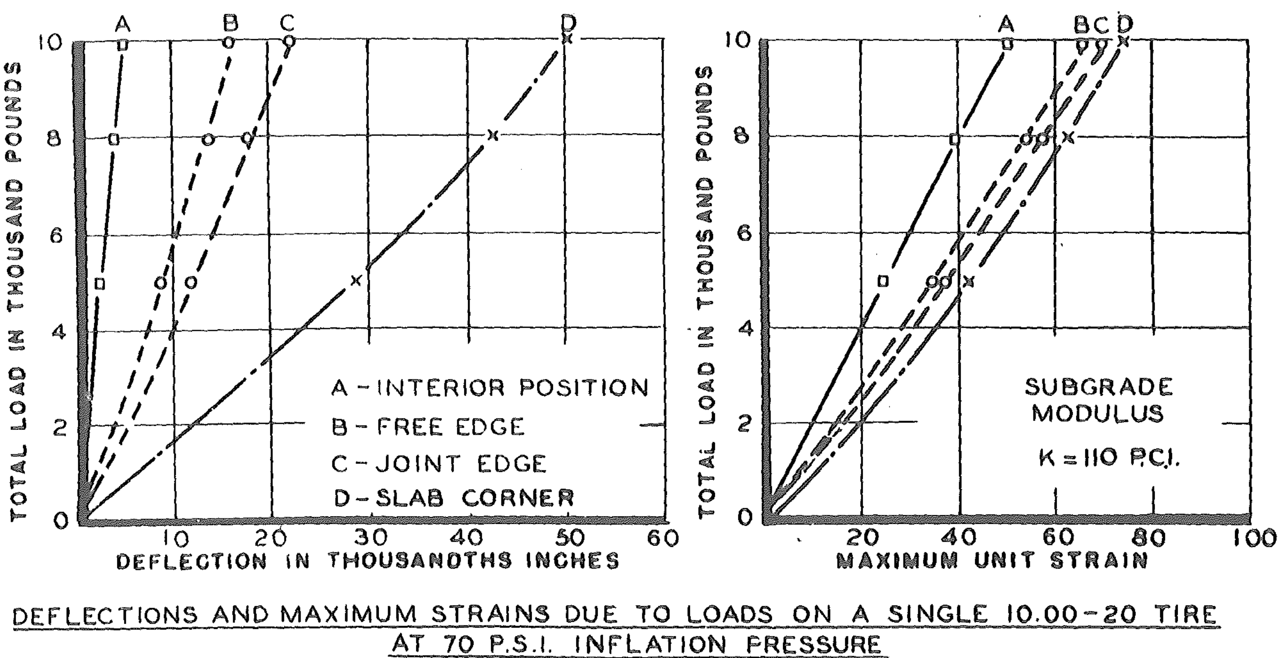


Figure 9. Load being applied through a single tire.



Graph 3. Single Wheel Loading Data

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were affected by the ends, and the tests at the end were influenced by the center. However, a fair comparison may be made between two and three axle systems for small axle spacings.

Free Edge Loading

Single Axle: An arrangement for measuring strains and deflections at the edge of the slab due to a load on one axle may be seen in Figure 10. A total of fifteen tests were made at edge locations for each of two subgrade conditions. In order to avoid eccentric results due to local conditions the axle was shifted to positions both sides of the lateral center line of the slab and all of these results were averaged for the presentation in Graph 4.

The strains from which the stresses were computed were measured longitudinally in a line on the top of the slab parallel to the edge and nine inches inward from the edge. This location was chosen because this line fell midway between the dual tires when the wheels were at the edge of the pavement. Although this is not the line of maximum strain it is sufficiently close for the purposes of these tests. It is also true that the longitudinal strains are not necessarily maximum, but calculations from 45° rosette readings gave values within 10 percent of the longitudinal magnitudes and within a few degrees of the longitudinal direction.

Two Axles: A second axle was placed in tandem with the first and strain and deflection readings were noted when these axles were loaded simultaneously. The distance between the axles was varied from the mechanical minimum of 3-1/2 feet to a maximum of 9 feet. Figure 11 pictures one of these arrangements.

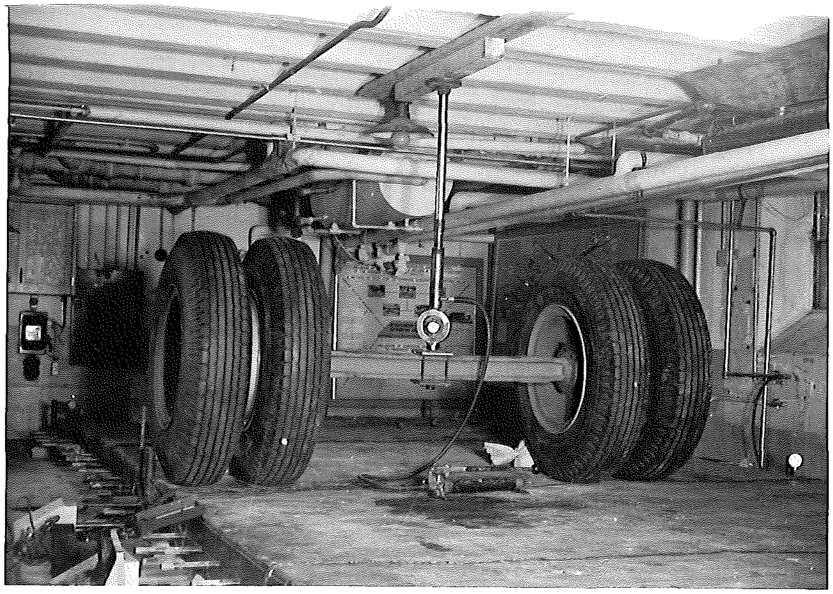


Figure 10. One axle at free edge of slab.

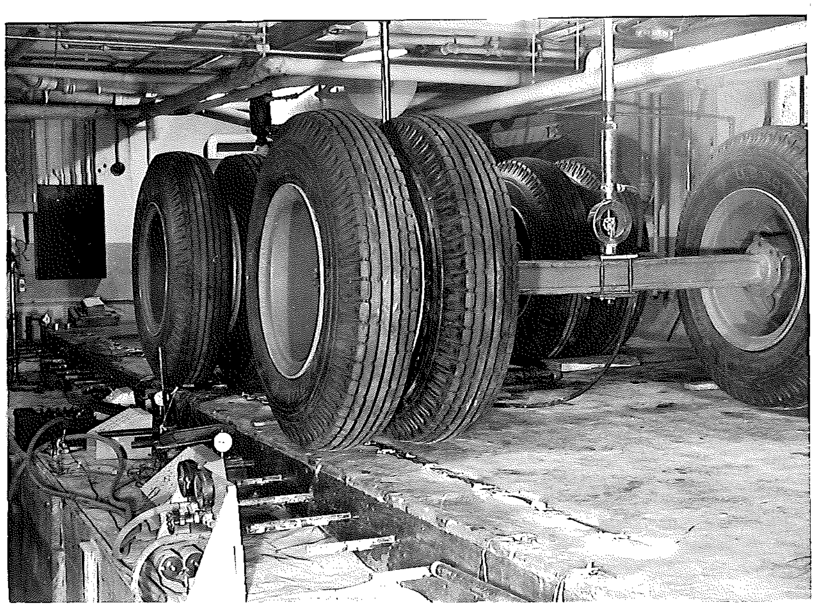
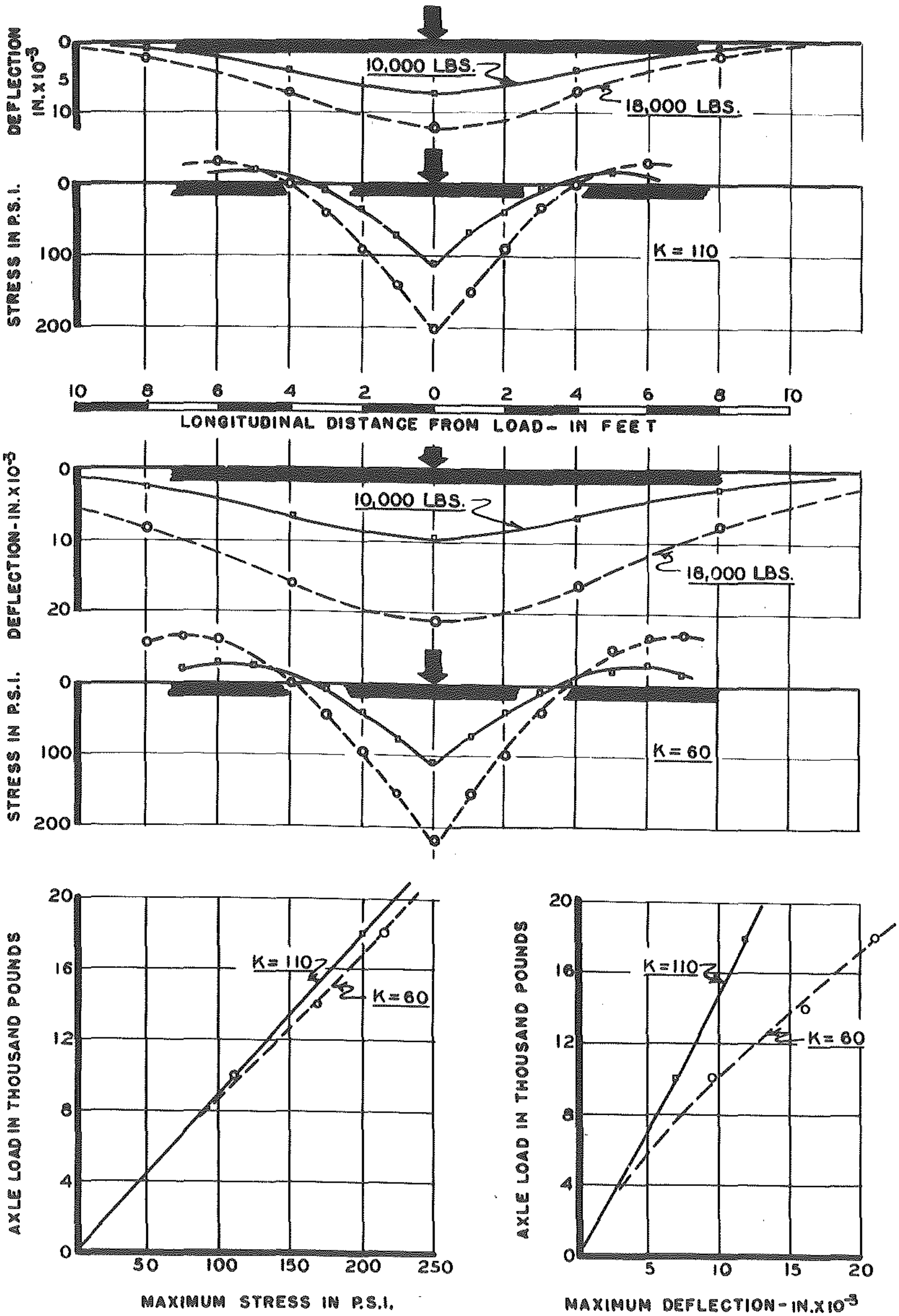


Figure 11. Two axles placed on slab and loaded preparatory to measuring strains and deflections.

STRESS AND DEFLECTION CURVES FOR ONE AXLE AT FREE EDGE OF SLAB



Graph 4

Since four feet is a standard spacing for axles on a heavy trailer, a number of tests were made at this spacing and the averages of these results are shown in Graph 5. The maximum stresses for this arrangement do not differ significantly from those due to the single axle. However, the deflections are greater under the two axle system than under one axle.

Three Axles: A third axle was added to the group and the loading tests were repeated for this system. The spacings for this group were from four to seven feet. Again for comparative purposes the four foot spacing was emphasized and averages of these tests are given in Graph 6.

The deflections increased over those of one axle and the two axle systems. The stresses, however, were only slightly less than the values under the single axle. The differences are not significant.

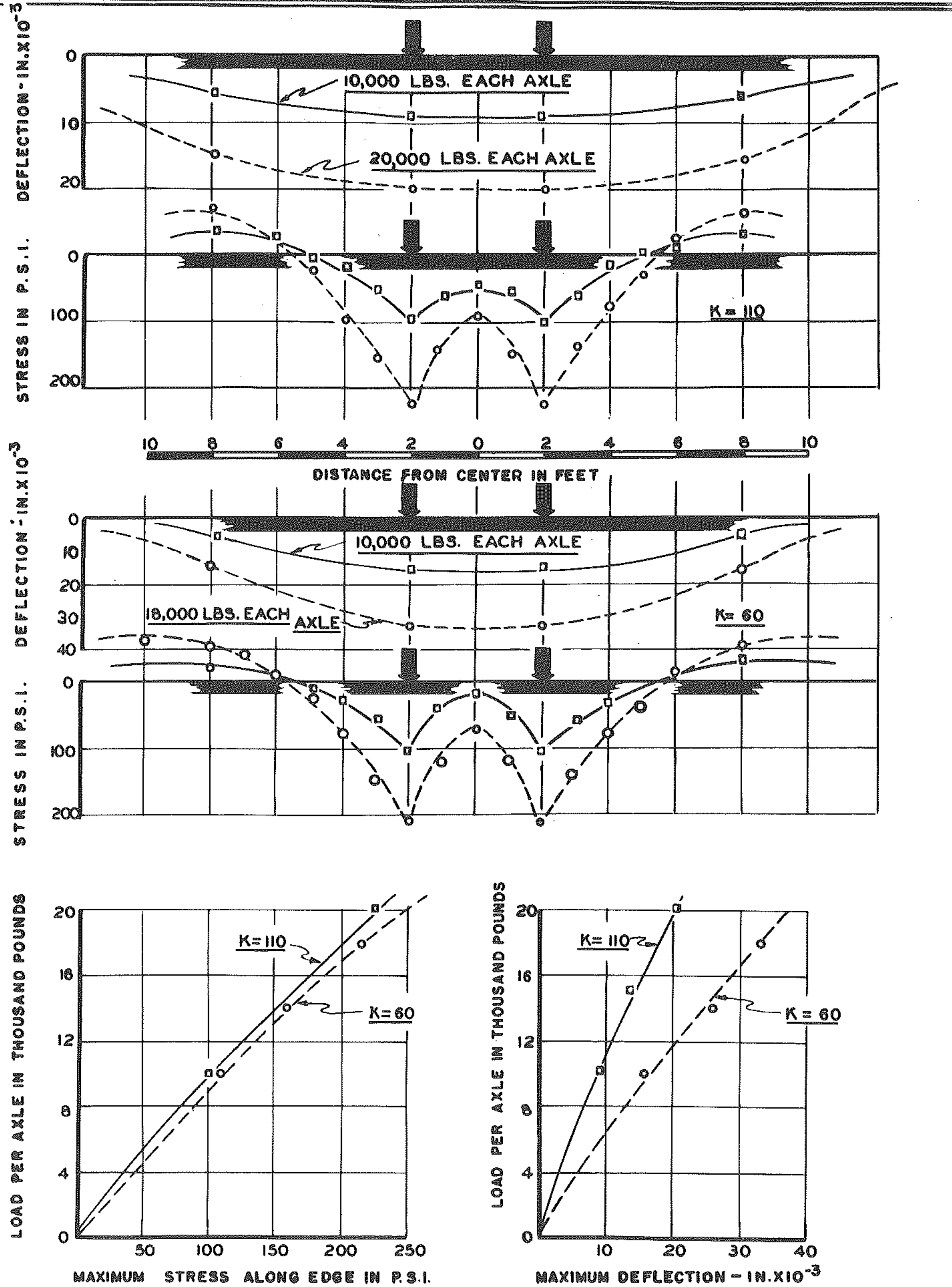
Corner Loading

Single Axle: Strains and deflections made by an axle at a corner of the pavement slab were measured at two corners at extreme ends of the slab. As in the case of edge loading, Graph 7 is a portrayal of average values.

An inspection of these data and a comparison with Graph 4 reveals that the corner deflections are much greater than the deflections at the edge at both high and low loads. For the softer subgrade, the maximum stresses also are greater at the corner than at the edge for corresponding loads. However, very little difference is noted for maximum stresses at the two locations when the subgrade modulus was 110 p.c.i.

Two Axles: Loads were applied to a two axle system with one axle remaining at the corner and the second axle being inward from the first at distances from four to nine feet. Repeated tests were made at two corners and averages of these data for the four foot spacing are given in Graph 8.

STRESS AND DEFLECTION CURVES FOR TWO AXLES ON SLAB EDGE–4FT. SPACING



Graph 5

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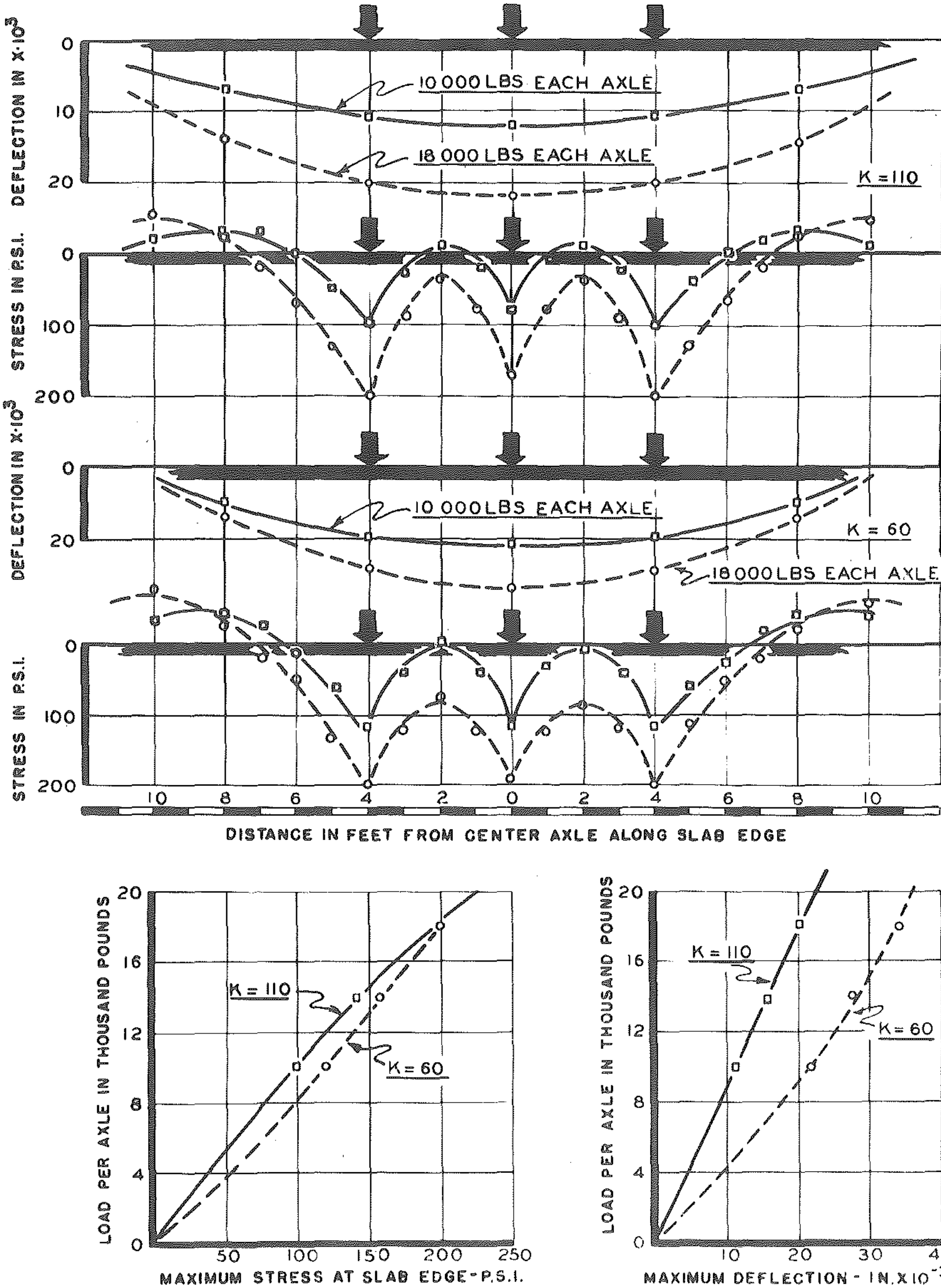
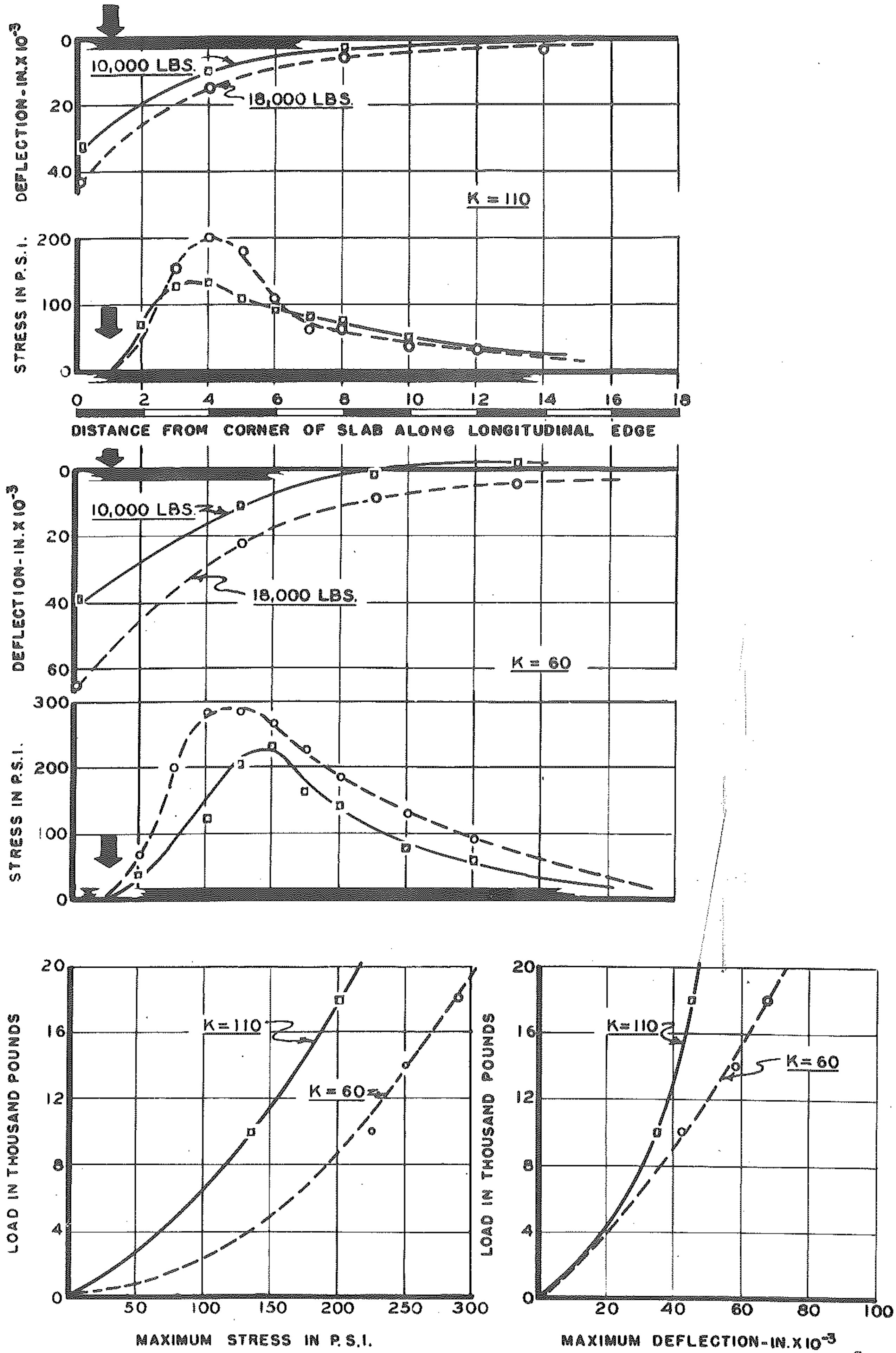
STRESSES AND DEFLECTIONS CAUSED BY LOADS ON 3 AXLES - SLAB EDGE



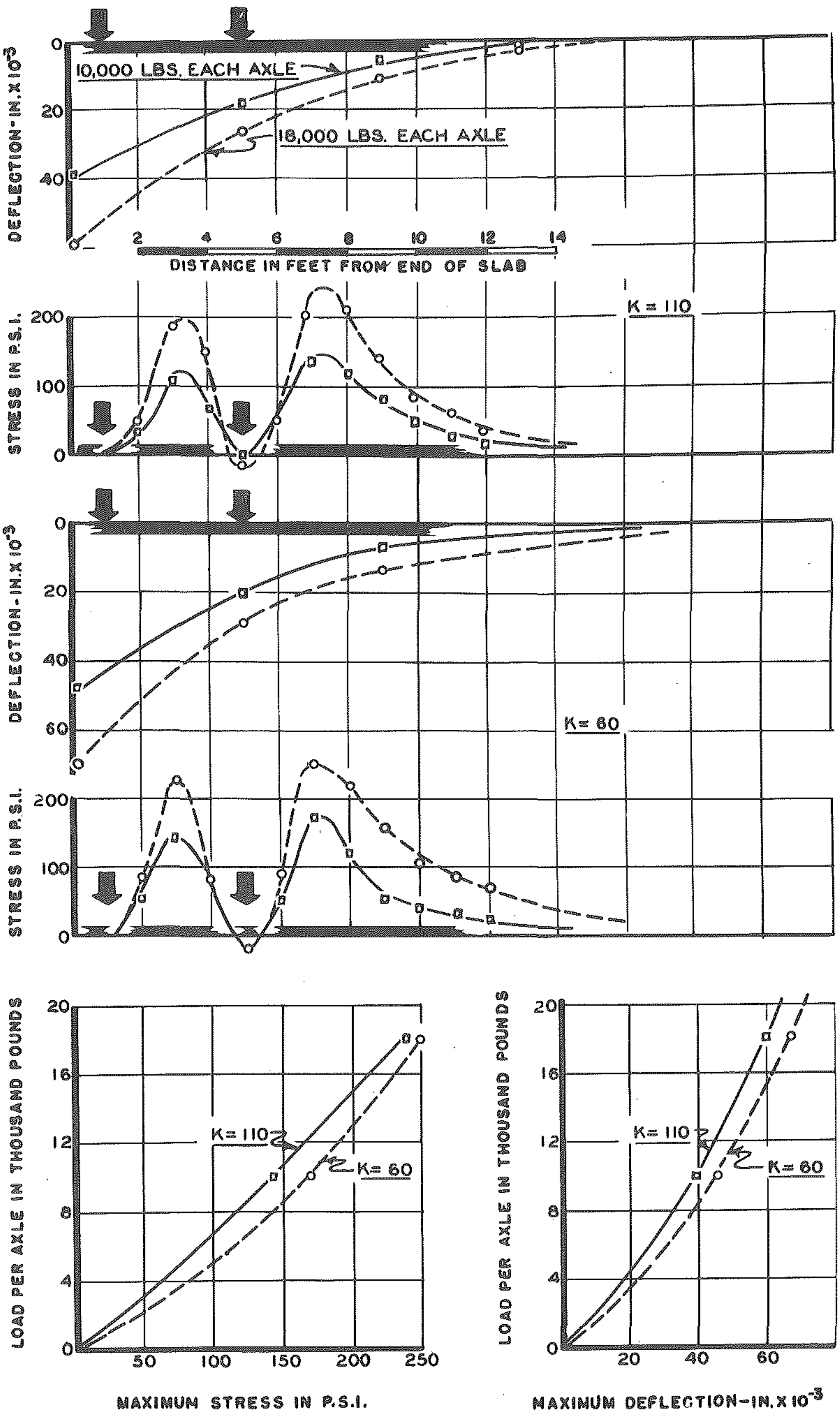
Graph 6

STRESS AND DEFLECTION CURVES FOR ONE AXLE AT SLAB CORNER



Graph 7

STRESS AND DEFLECTION CURVES FOR TWO AXLES AT SLAB CORNER-4 FT. SPACING



Graph 8

Deflections for this case were larger than for the single axle. Although the maximum stresses were greater than those caused by one axle when the slab was supported by the stiffer subgrade, the stresses were considerably less than those for one axle when the test was made upon the soft subgrade.

<u>Three Axles</u>: Finally, three axles were so placed that the first was on a corner and the others were equally spaced inwardly at distances of four, five and six feet. The arrangement may be clearly seen in Figure 12. Average data from loading tests at the four foot spacing are shown in Graph 9.

Although the deflections for the three axle system are greater than the corresponding deflections for the single axle and two axle arrangements, the stresses are less. Apparently the deflection curve is flattened to such an extent that larger subgrade displacement is obtained with a smaller slab curvature.

<u>Comparative Tests</u>

<u>Effect of Multiple Axles</u>: Although the previously described tests provided average values for stresses and deflections for the arrangements specified, it was noted that the differences in maximum stresses as produced by the three systems at the edge of the slab were not significant. The several tests made in each group gave maximum strain values which differed substantially from a mean value. However, repeated tests upon a system whose position on the slab was not disturbed usually produced results in close agreement. This fact led to the conclusion that some local condition in or below the slab, such as grouping or size of aggregate or perhaps subgrade bearing beneath the slab, influenced the strain readings.

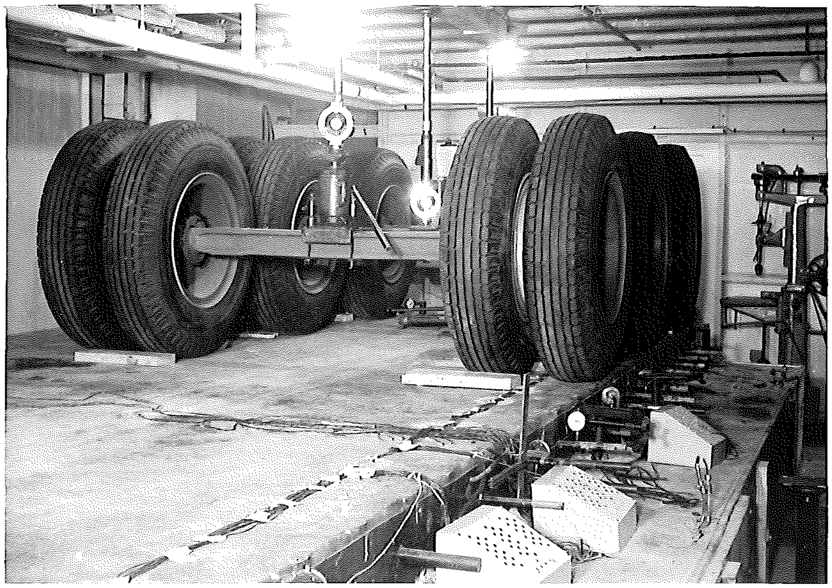
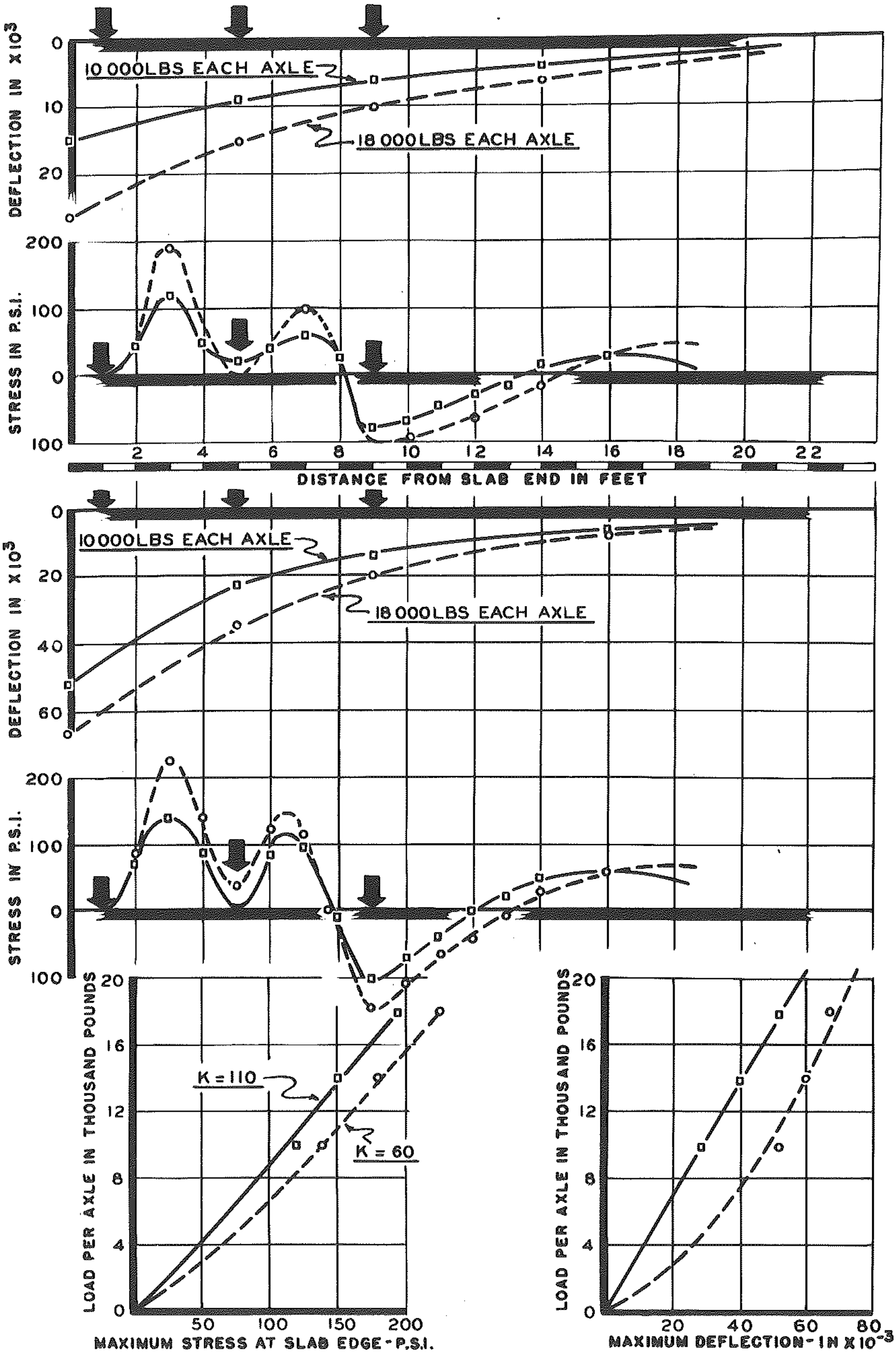


Figure 12. Three axles in the corner region of the slab.

STRESSES AND DEFLECTIONS CAUSED BY LOADS ON 3 AXLES - SLAB CORNER



Graph 9

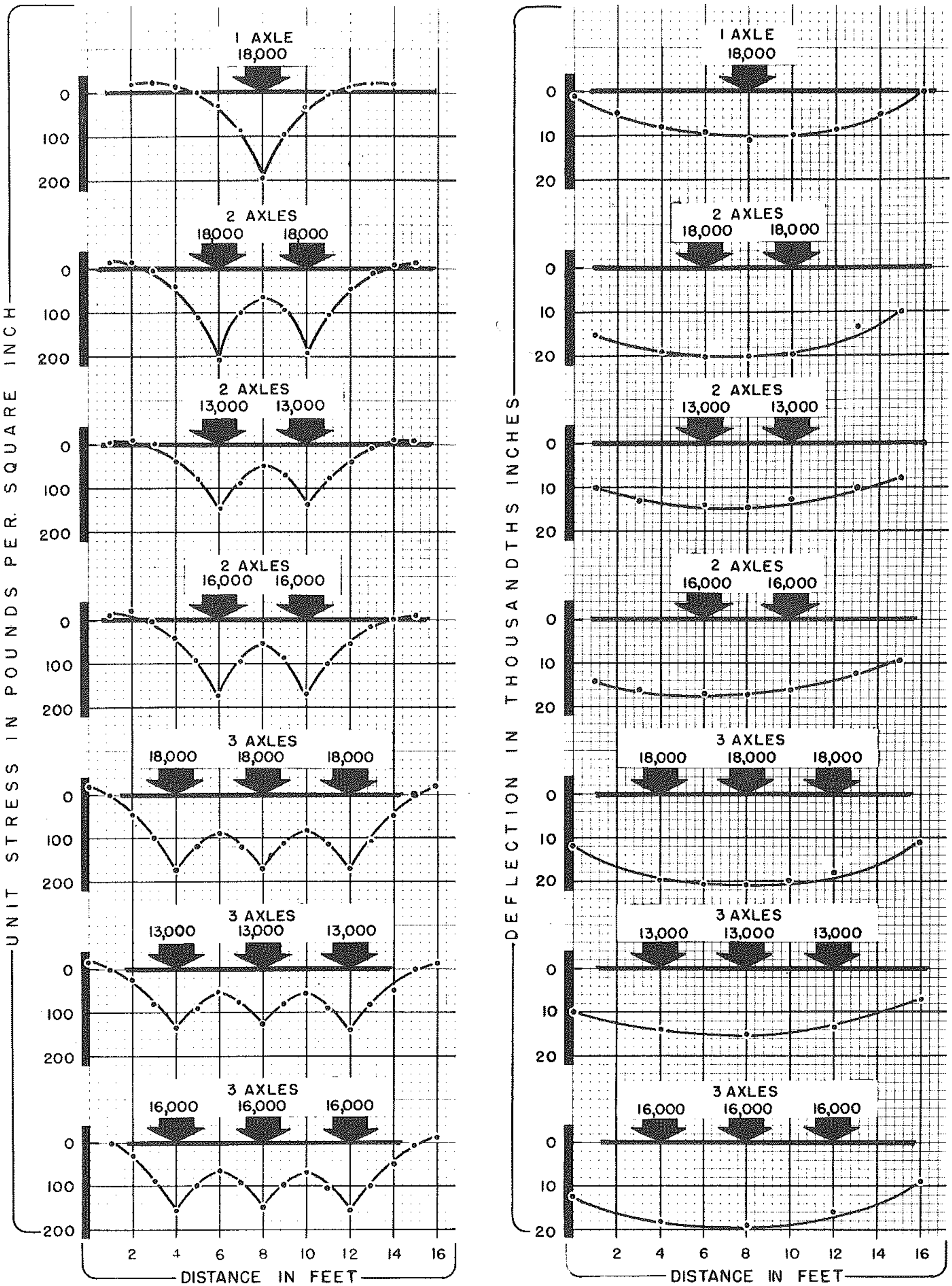
With this thought in mind, and the object a direct comparison of the maximum stresses under the one, two, and three axle arrangements, the systems were tested in such an order that an axle once placed was not disturbed. The results of this method applied to the axles located at the slab edge are given in Graph 10.

A similar set of comparative tests was made at one corner of the slab. The curves are shown in Graph 11. It may be seen that the stresses are quite high for these tests. This fact may be explained by the warped condition of the slab at this time. The irregularity of the stress curve for one axle is evidence that warping affected the results.

Loads of 13,000 pounds, 16,000 pounds and 18,000 pounds were used in this latter series of tests in order to make comparisons among the legal loading values. These data bring out the fact that from the standpoint of slab stresses, the single axle 13,000 pound load is more severe than any other loading system tested. The detrimental effect of large deflections under multiple axle loads has not been determined.

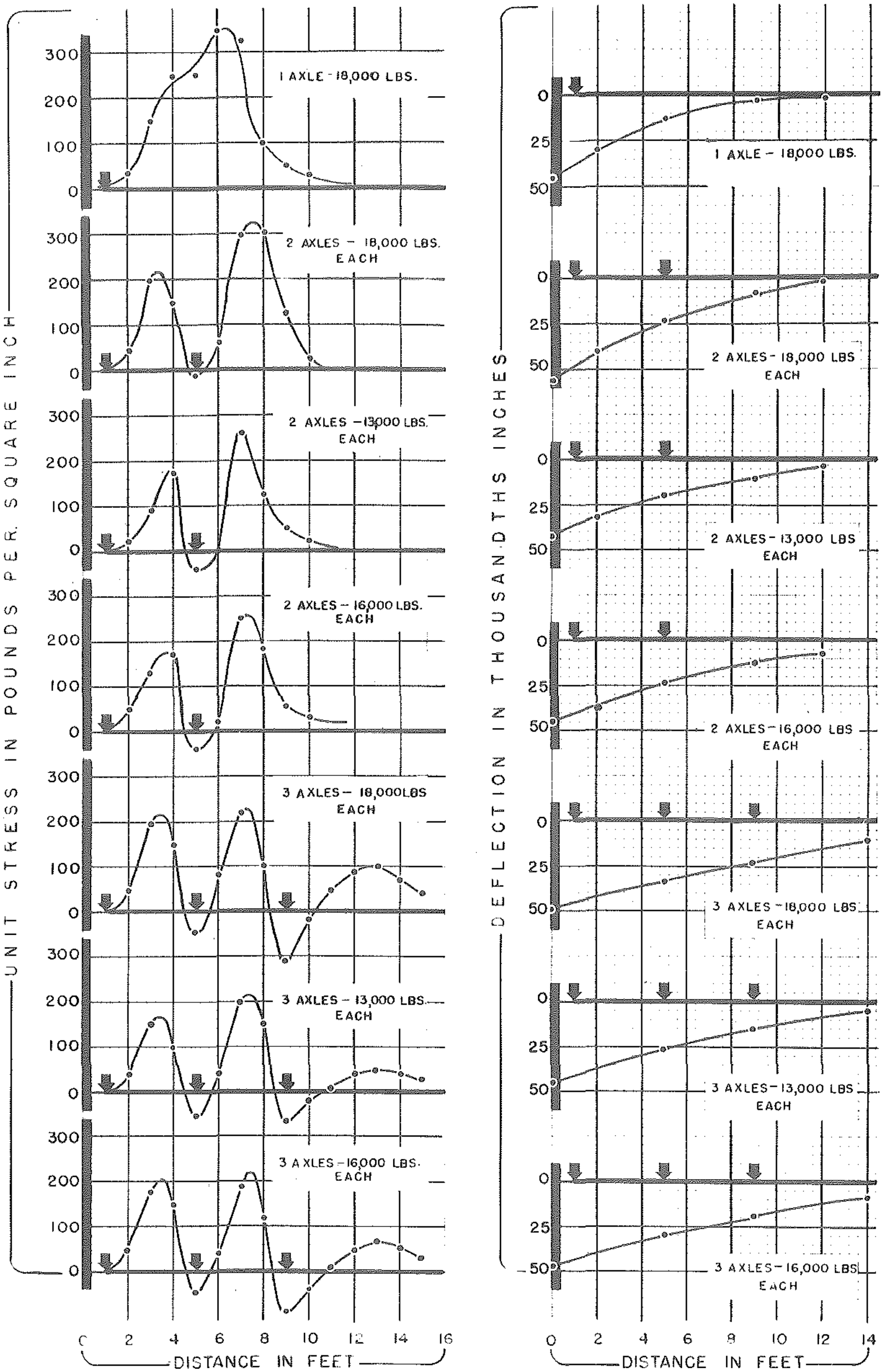
Effect of Axle Spacing

Although the stress and deflection curves for two axles shown in this report are drawn from data obtained when the axles were spaced four feet apart, other spacings were used in an effort to find the distances at which the slab strains would be the least. When the axles were located along the slab edge the minimum strain was found to occur at a six foot spacing, while a four foot distance between axles produced the least strain in the corner region. These figures may be easily verified by examination of Graph 12.



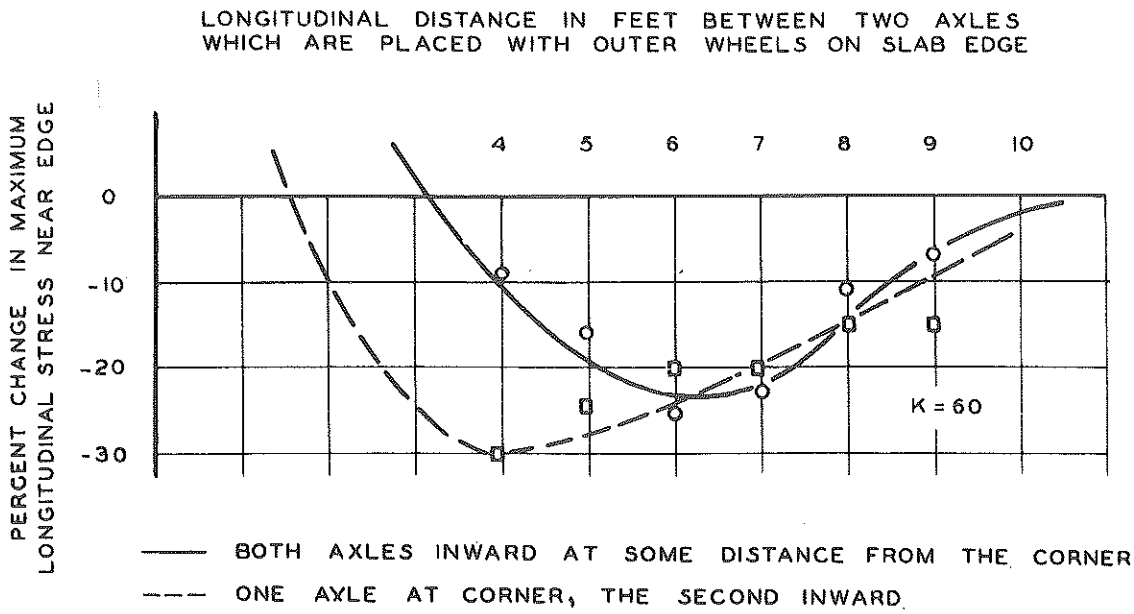
STRESS *and* DEFLECTION *at the* EDGE

Graph 10



STRESS *and* DEFLECTION *at the* CORNER

Graph 11



EFFECT of AXLE SPACING on MAXIMUM STRESS NEAR SLAB EDGE

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A COMPARISON OF THE MODEL INVESTIGATION WITH RESULTS OF THIS STUDY

A group of curves representing data from the model study are repeated here as Graph 13. Comparisons between these curves and the corresponding curves for similar loading on the full-size slab show a marked similarity. The strain curves for the model have several times the amplitude of the curves for the large slab. This indicates that the model was considerably overloaded. These excessive loads magnified the differences in deflections, however, and brought out fluctuations that are not apparent in the full-size slab study.

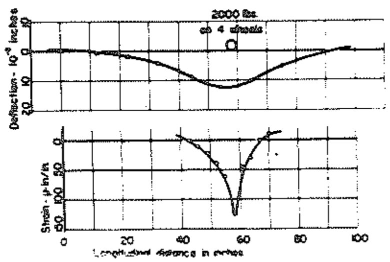
No conflicts are seen between data from the large slab and data from the model. The prototype study was necessary for the determination of working values for slab design, but the relative effects of different loading arrangements are brought out clearly in the model study and are corroborated by this later investigation.

RESULTS OF OTHER INVESTIGATIONS

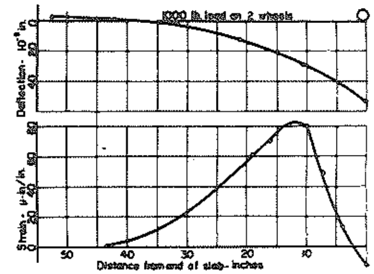
Static load tests somewhat similar to those made in this study have been conducted by other investigators. A six wheel truck study was made by Teller(5) in 1925. A six inch plain concrete slab was tested, and deflection and strain curves for one and two axles were found. These were similar to those of the present study. In 1931 the Illinois Division of Highways(6) made tests on the edge of a 9-6-9 pavement when it was subjected to loading by four and six wheel trucks. Again the one axle and two axle data compare well with the results of the present investigation. A thorough study of stresses in the corner region of concrete pavements was made by Spangler(7) in 1942. These results were used as a guide for gage placement in the present test.

DATA *from the* MODEL STUDY

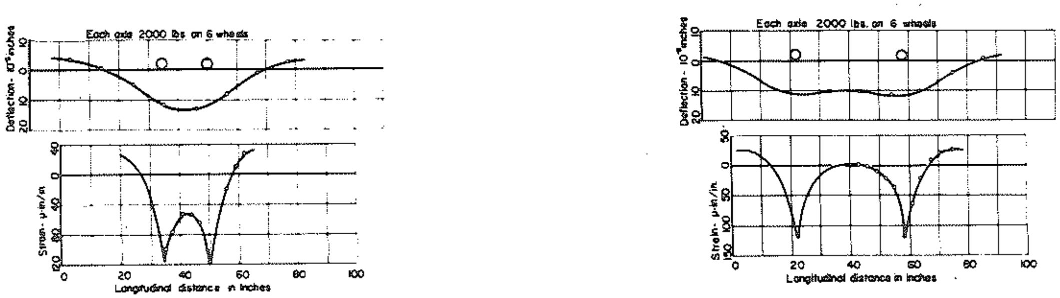
TYPICAL CURVES FROM THE LOADING STUDY
ON 33IN BY 15FT BY 2IN MODEL SLAB



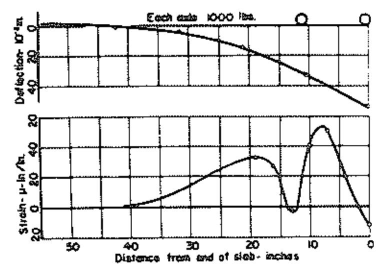
ONE AXLE AT SLAB EDGE



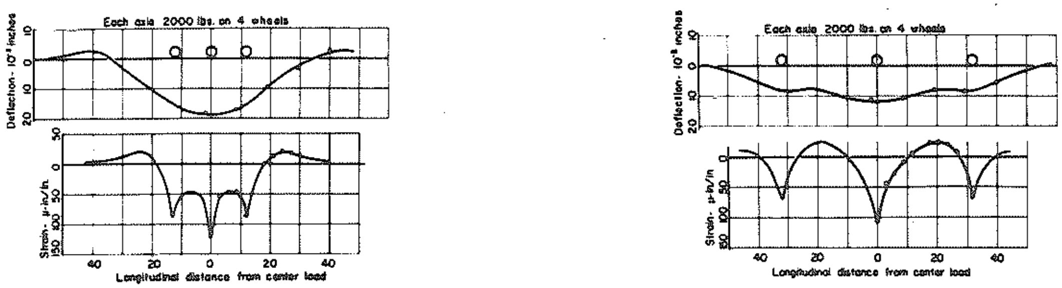
ONE AXLE AT SLAB CORNER



TWO AXLES AT SLAB EDGE

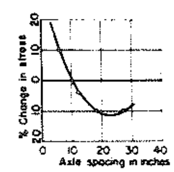


TWO AXLES AT SLAB CORNER

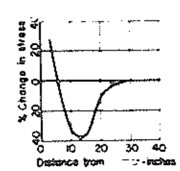


THREE AXLES AT SLAB EDGE

CHANGE IN STRESS OVER THE SINGLE AXLE VALUE
CAUSED BY LOAD ON A SECOND AXLE



TWO AXLES AT SLAB EDGE



EFFECT OF SECOND AXLE WHEN FIRST IS AT SLAB CORNER

Graph 13

Numerous other studies have been made where the investigators used actual vehicles and also loading plates to apply loads to the concrete slabs. The strains have been measured by mechanical gages of standard and self recording types. Investigations by O. Graf[10] and G. Weil[11] are particularly thorough.

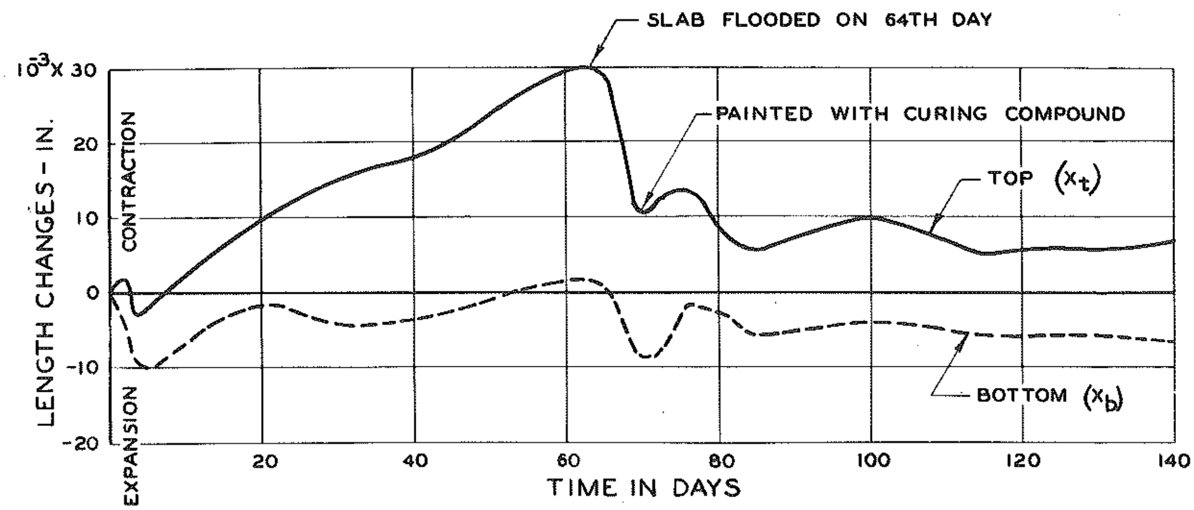
LIMITATIONS OF THIS STUDY

Although the concrete slab under investigation was constructed in the laboratory, test conditions did not prove to be as ideal as anticipated. Lack of room precluded the possibility of making any study under moving loads, and a curling phenomenon which materially affected the stress values was encountered.

The extensometer shown in Figure 6 gave a record of slab warping at each corner. Graph 14 provides a typical curve. This upward movement of the corners and edges was verified by a precise level. The cause of this slab distortion cannot be attributed to a temperature differential, for continuous records showed little or no differences in temperature readings between the top and bottom of the slab. Further experimentation and analysis is necessary before a satisfactory solution to the causes of such behavior can be presented.

RESULTS OF THEORETICAL ANALYSIS

The foregoing section suggests that the strains measured in this experiment were not total strains. The upward warping of the slab no doubt produced stresses causing tensile strains on the upper surface. These



VERTICAL MOVEMENT AT CORNER, $y = \frac{L}{h}(x_t - x_b)$

WHERE L = DISTANCE IN INCHES FROM CORNER TO START OF CURL.

h = SLAB THICKNESS IN INCHES.

x_t AND x_b ARE DISPLACEMENTS AT TOP AND BOTTOM OF SLAB.

EXTENSOMETER RECORD *at* SLAB CORNER

values should be added to those measured under corner loading and subtracted from the edge loading values. Such warping strains were not measured because of the failure of the gages imbedded in the concrete.

For the purpose of this study of the relative effects of static loads it was not necessary to know the total stresses. However, their magnitudes are of interest and a theoretical examination of the slab stresses is presented.

The Westergaard formulas(3) provide a theoretical check on some of the results of this experimental study. It must be remembered that these formulas were developed under the assumption that the slab was of infinite extent and that the subgrade pressure was proportional to the deflection. The finite dimensions of the test slab and the curling of the edges necessarily modified these results.

Stresses at the slab corner, interior, transverse edge, and longitudinal edge produced by a single loading area were computed by the following formulas:

$$\text{Corner stress} = \frac{3P}{h^2}\left(1 - \frac{a\sqrt{2}}{t}\right)^{0.6}$$

$$\text{Interior stress} = 0.31625\,\frac{P}{h^2}\left(4\log\frac{t}{b} + 1.0693\right)$$

$$\text{Edge stress} = 0.57185\,\frac{P}{h^2}\left(4\log\frac{t}{b} + 0.3593\right)$$

where P = load, h = slab thickness, a and b are functions of the loading area and

$$t = \sqrt[4]{\frac{Eh^3}{12(1-u^2)k}}.$$

For these computations P = 10,000 pounds; h = 9 inches; a and b are average values from Bradbury[13]; the modulus of elasticity of concrete, $E = 5.25 \times 10^6$ p.s.i.; Poisson's ratio, u = 0.15, and the subgrade modulus, k = 110 p.c.i. The tables presented by Kelley[12] were used to facilitate the computation.

The same data were used in the modified versions of these formulas which evolved from the Arlington tests[4]. These equations are listed herewith:

$$\text{Corner stress} = \frac{3P}{h^2}\left(1 - \frac{a\sqrt{2}}{t}\right)^{1.2}$$

$$\text{Interior stress} = 0.31625\,\frac{P}{h^2}\left(4\log\frac{t}{b} + 0.1788\right)$$

$$\text{Edge stress} = 0.57185\,\frac{P}{h^2}\left(4\log\frac{t}{b} + \log b\right)$$

The results of these computations together with the results of experiment are presented in the following table:

TABLE III

Stresses Under a Single Wheel with 10,000 lb. Load

	Westergaard formulas	Arlington formulas	Test Slab (Experimental)
Corner	222 psi.	311 psi.	350 psi.
Interior	166	142	250
Longitudinal Edge	194	238	330
Transverse Edge	234	271	350

It is readily seen that the measured stresses exceeded the values computed by both methods, although the Arlington formulas give a closer approach to the experimental values than the Westergaard results.

Loads on Two Wheels

When the slab was loaded through two wheels on a single axle, these stresses were somewhat modified. Computations are shown herewith for principal stresses due to a 20,000 pound axle load at the interior, and at longitudinal and lateral edges of the slab.

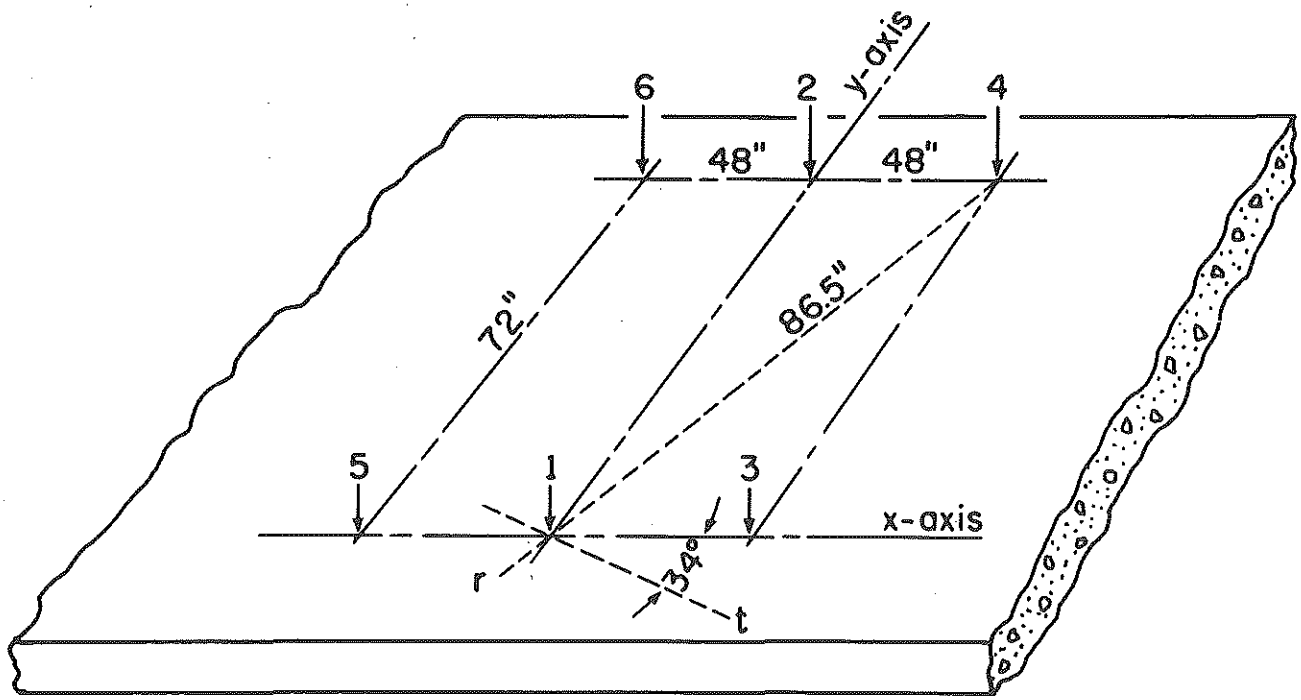
Interior Location: Consider the stresses at point 1 (Figure 13) due to 10,000 pound loads at positions 1 and 2 only. The stresses due to the load at 1 were found previously and are equal in all directions. Let us call this value S_1. Stresses at 1 due to load 2 which is 72 inches in the y-direction from 1 are found with the help of Westergaard's[3] moment curves shown in Graph 15.

For this slab the radius of relative stiffness t = 41.5 inches. The distance from 1 to 2 is 72 inches = 1.74t. Hence, $M_r = M_y = -.021$ and $M_t = M_x = +.020$. The section modulus for the rectangular section, $\frac{h^2}{6} = 13.5$ in.^{2}. Then $S_y = \frac{-.021 \times 10,000}{13.5} = -16$ p.s.i. Similarly $S_x = +15$ p.s.i., consequently the principal stresses at 1 expressed as functions of S_1 are,

Longitudinally: $S_1 + S_x = S_1 + 15$

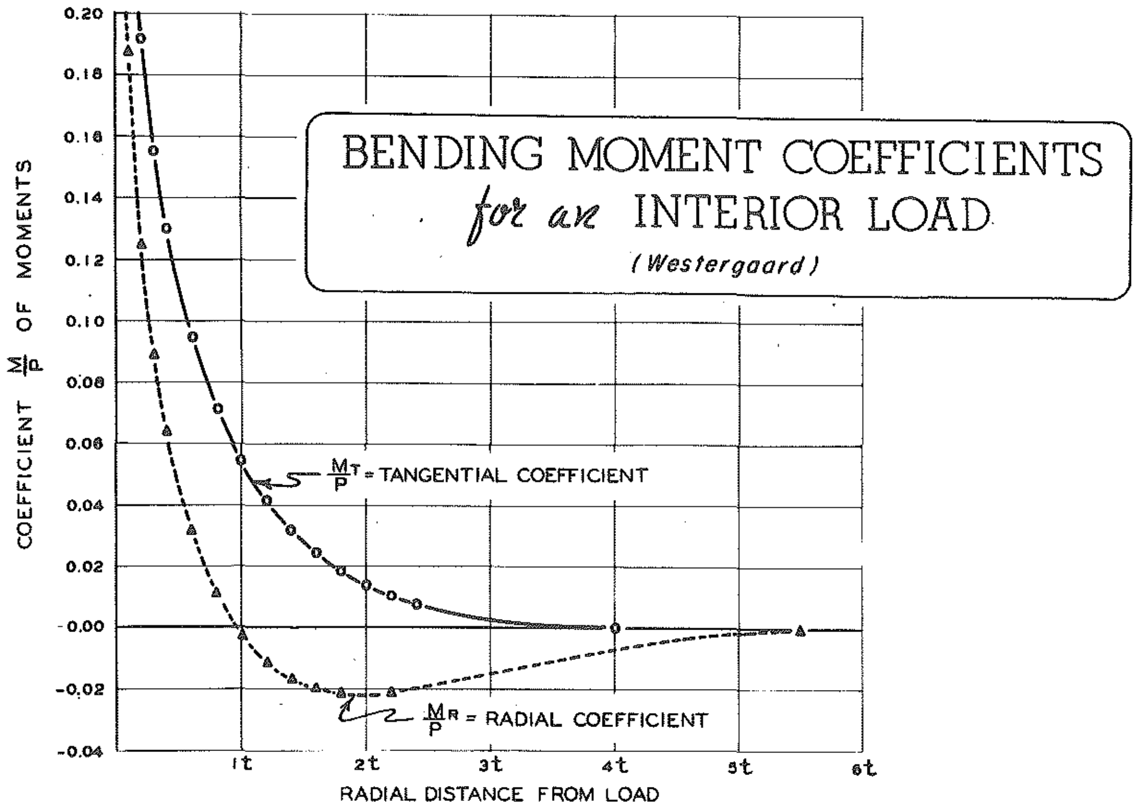
Laterally: $S_1 + S_y = S_1 - 16$

Longitudinal Edge: The longitudinal stress S_e under load 1 due to the load at 2 when point 1 is on the longitudinal edge of the slab has been

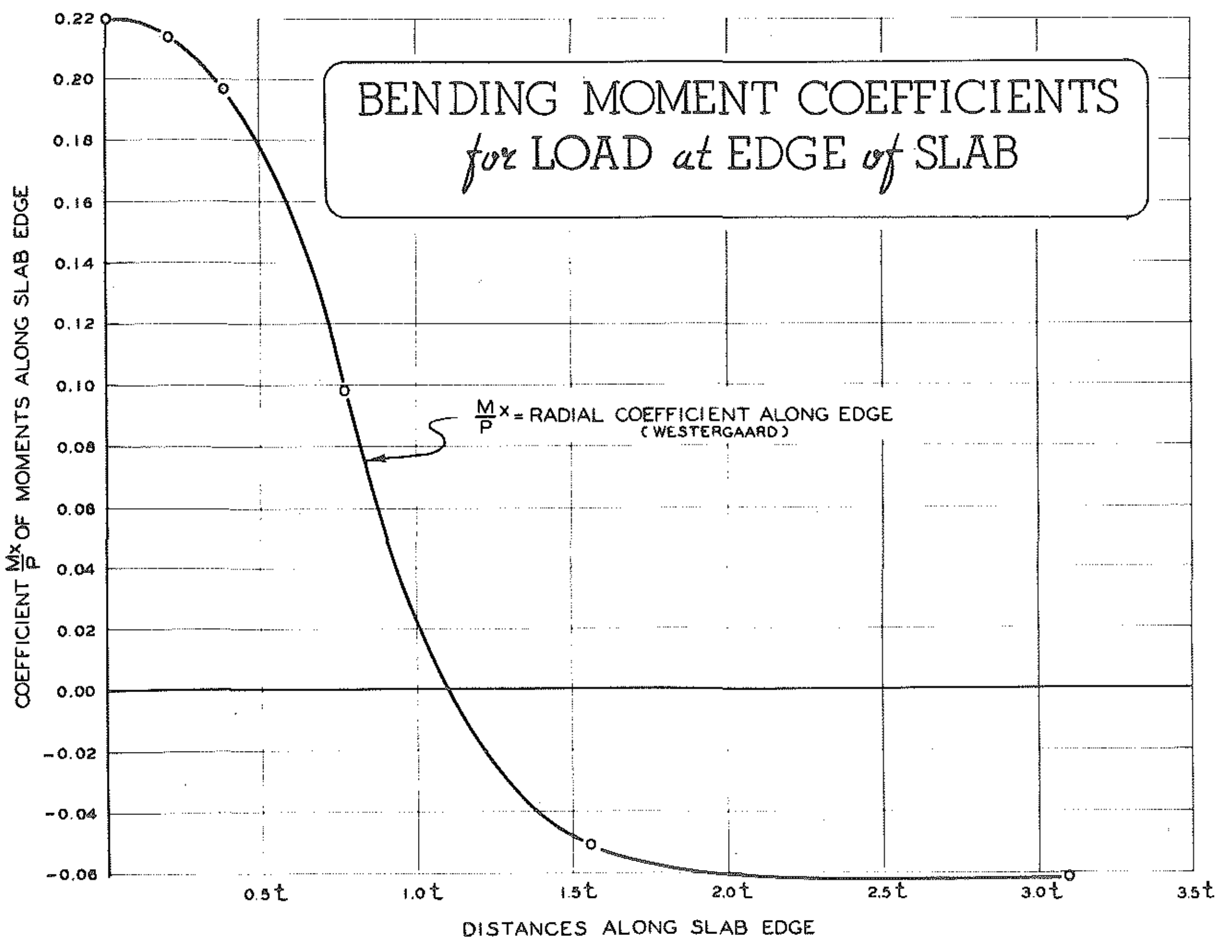


LOAD SPACING for MULTIPLE AXLES

Figure 13



Graph 15



Graph 16

previously computed. The effect of load 2 at position 1 is found in a manner similar to that for an interior location. Wheel 2 is at an interior point, and its effect on 1 will approximate the value found in the interior case, namely 15 p.s.i. The longitudinal stress will then be $S_c + S_x = S_e + 15$.

Transverse Edge: When loads 1 and 2 are on the transverse edge, the load at 1 produces stress S_e. The influence of load 2 is found by use of Graph 16, which is drawn by interpolation from Westergaard's graphs in order to obtain a direct reading curve for u = 0.15. At distance 1.74t, the coefficient $\frac{M}{P} = -0.057$, hence $S_x = \frac{-0.057 \times 10,000}{13.5} = -42$ p.s.i. Then the total transverse stress at 1 is $S_e -42$ p.s.i.

Loads on Four Wheels

Interior Location: Consider loads at interior positions 1, 2, 3 and 4. The accumulation of stresses due to loads at 1 and 2 have been found. We now find the effects of loads at 3 and 4.

Figure 13 shows load 3 to be 48 inches from 1. Since 48 inches = 1.156t, we find from Graph 15 that $\frac{M_y}{P} = \frac{M_t}{P} = 0.044$ and $\frac{M_x}{P} = \frac{M_r}{P} = -0.01$. Hence $S_y = \frac{0.044 \times 10^4}{13.5} = 33$ p.s.i. and $S_x = \frac{-0.01 \times 10^4}{13.5} = -7.4$ p.s.i.

The effect of the load at 4 upon position 1 is slightly more complex. In this case the tangential and radial stresses are not in the x and y directions but make angles of 34° with those axes. Position 4 is 86.5 inches = 2.08t from 1 so by Graph 14 again $\frac{M_t}{P} = 0.012$ and $\frac{M_r}{P} = -0.022$ whence $S_t = +9$ and $S_r = -16$. Since the principal stresses from positions 2 and 3 are on the x and y directions, the effects of S_t and S_r in these directions must be computed before the longitudinal and lateral stresses can be accumulated. The stresses in directions x and y due to load 4 are:

$$X = 9 \cos^2 34° - 16 \sin^2 34° = 1$$

$$Y = 9 \sin^2 34° - 16 \cos^2 34° = -7$$

and shear $T = 1/2\ (-16\ -9) \cos^2 34° = -9$

The accumulated stresses in the x and y directions are:

$$S_x = S_1 + 15 - 7 + 1 = S_1 + 9$$

$$S_y = S_1 - 16 + 33 - 7 = S_1 + 10$$

and $T = 9$

The principal stresses are found by the formula

$$S_{max} = \frac{S_x + S_y}{2} + \quad \frac{S_x - S_y}{2}^2 + T^2$$

$$S_{min} = \frac{S_x + S_y}{2} - \quad \frac{S_x - S_y}{2}^2 + T^2$$

Whence $S_{max} = S_1 + 19$ and $S_{min} = S_1 + 0.5$

<u>Longitudinal Edge</u>: The longitudinal stress is necessarily maximum at the edge of the slab. The load at 2 adds 15 p.s.i. as was shown in the two wheel case. Load 3 is at distance 1.156t from 1, hence from Graph 16, $\frac{M_r}{P} = -0.012$ and $S_x = \frac{-0.012 \times 10^4}{13.5} = -8.9$ p.s.i. The effect of load 4 is not determined, but from the computations above, for the interior, it is

apparent that it is small. An approximate value for stress at the edge is then

$$S_x = S_e + 15 - 9 = S_e + 6$$

Loads on Six Wheels

Interior: When the load is distributed through six wheels, the greatest stresses are at positions 1 and 2 (Figure 13). Consider the stresses at point 1. Computations for stresses under four wheel loading may be used for this calculation. The stresses at 1 due to load 1 are S_1 in all directions. Load 2 contributes +15 p.s.i. longitudinally and -16 p.s.i. laterally. Loads 3 and 5 each affect 1 by longitudinal stresses of -7 and lateral stresses of +33. Loads at 4 and 6 produce tangential and radial stresses of +9 and -16 respectively, and the t and r axes make angles of -34° and +34° respectively with x. These stresses, accumulated, are equivalent to $S_x = 2$, $S_y = -14$, and $T = -18$. Finally, the sum of all the stresses at 1 is:

$$S_x = S_1 + 15 - 14 + 2 = S_1 + 3$$

$$S_y = S_1 = 16 + 66 - 28 = S_1 + 22$$

$$T = 18$$

The principal stresses are:

$$S_{max} = S_1 + 31 \qquad S_{min} = S_1 - 6$$

Longitudinal Edge: For three axles at the longitudinal edge it is sufficient to compute the longitudinal stress under 1. The stress due to load 1 is S_e. By Graph 15 that due to 2 is +15. Loads at 3 and 5 each cause $\frac{M_x}{P}$ to be -0.012 from Graph 16, whence each $S_x = \frac{-0.012 \times 10,000}{13.5} =$ -8.9. The effects of 4 and 6 must be approximated by the method used for interior loads. From above the effect of these two loads in the longitudinal direction was only 2 p.s.i. Hence the total longitudinal stress due to all loads is:

$$S_x = S_e + 15 - 18 + 2 = S_e - 1$$

Tabulation of Computed Stresses

The foregoing computations were made in terms of a variable S_1 or S_e in order that we might make a comparison between the formulas for stress computation. A tabulation of these results is given below:

TABLE IV

Stresses Under Multiple Wheels with 10,000 lb. Load Per Wheel

Load Type	Position	Maximum Experimental Stress	Maximum Computed Stress Westergaard	Maximum Computed Stress Arlington
1 axle	Interior	----	182	157
1 axle	long. edge	220	209	253
1 axle	trans. edge	----	192	229
2 axles	Interior	----	185	161
2 axles	long. edge	230	200	244
3 axles	Interior	----	198	173
3 axles	long. edge	220	193	237

It is readily seen that the Public Roads formula yields greater stresses than Westergaard's except for the interior location, and in all cases of multiple wheel loading our experimental values lie between the values computed by these formulas.

OBSERVATIONS AND CONCLUSIONS

The greater part of this report has been limited to the effects of one axle, two axles at an axle spacing of four feet, and three axles at a four foot spacing. Except where otherwise stated, all observations and conclusions given here are restricted to a discussion of results found under these limitations. The outstanding results are as follows:

1. A comparison of the curves presenting average values shows that the maximum stresses for a constant axle load are produced by a single axle on the corner of the slab.
2. Stresses due to corner loading by all three systems were considerably greater when the slab rested on a subgrade with modulus k = 60 p.c.i. than they were when k = 110 p.c.i.
3. At k = 110 p.c.i. stresses due to corner loading did not greatly exceed those due to edge loading for any of the three systems tested, but at k = 60 p.c.i., corner loading produced greater stresses than edge loading.
4. The special comparative tests, the data for which were given in Graph 10, indicate the following relationships for edge loading:
   (a) The maximum stress for the two axle system under loads of 18,000 pounds per axle did not exceed that for the single axle at 18,000 pounds.
   (b) The three axle system at 18,000 pounds per axle caused less stress than either of the other two systems.

(c) The maximum deflection under the two axle system was twice that under the single axle, but the three axle system did not cause a corresponding increase in deflection.

5. The corner loading tests which furnished the data for Graph 11 showed the following:

   (a) The single axle produced a greater maximum stress than either other system.

   (b) Three axles at 18,000 pounds each produced less than 70 percent as great a maximum stress value as the single axle, and two axles at 18,000 pounds each caused a maximum stress which was more than 90 percent of the single axle value.

   (c) The deflections under two axles were twenty percent greater than under one axle, but the three axle deflections were only ten percent greater than those for a single axle.

6. Upward warping of the slab at the ends affected the total stresses, but measured values were found to lie between those computed by the Public Roads and Westergaard formulas.

7. The results of this study are so nearly those that might have been predicted by the model investigation that model studies are recommended for further investigation in slab stresses.

8. Strains as measured by the SR-4 type A-1 and AR-1 gages vary somewhat due to local conditions. Strain differences as high as ten

micro-inches per inch were found, and when the strains due to loading were small, these local differences caused decided irregularities in the curves.

9. The intensity of maximum stress caused by the two-axle systems was influenced by the axle spacing. The optimum distance between axles is about five feet.

ACKNOWLEDGMENT

Throughout this study the research staff has been guided by the suggestions of W. O. Fremont, and his help in outlining procedures and criticism of results is gratefully acknowledged.

Special equipment for this investigation was constructed by F. C. Filter and A. G. Davis. These men participated in the entire testing program and their aid in the simplification of technique and in the compilation of data is appreciated.

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