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THE EXPERIMENTAL DETERMINATION OF THE STRESS DISTRIBUTION ALONG A DOWEL AT A TRANSVERSE JOINT IN A CONCRETE PAVEMENT

by

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THE EXPERIMENTAL DETERMINATION OF THE STRESS DISTRIBUTION ALONG A DOWEL AT A TRANSVERSE JOINT IN A CONCRETE PAVEMENT

The problem of determining the stresses in and around dowels through transverse joints in concrete pavements has been solved analytically by means of the theory of an elastic structure in a yielding mass[1]. This solution is based on the assumption that the dowel will behave similarly to the elastic structure and the concrete around the dowel will simulate the yielding mass. Certain additional assumptions are necessary in order to translate the actual conditions to the analytical solution such as assuming a value for the modulus of support of the concrete around the dowel.

This study was undertaken for the purpose of checking experimentally the stresses in and around dowels and comparing the experimental values with analytical values. The immediate objective was to measure the bending moment variation throughout the length of a steel dowel embedded in concrete at a joint which was subjected to a truck wheel load. This is a part of the Michigan State Highway Department's research program on concrete pavement design. The Department's research laboratory has conducted extensive laboratory research on load-transfer devices but this is the first successful program for obtaining the distribution of the stresses in load-transfer devices in their natural environment.

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[1] "Design of Dowels in Transverse Joints of Concrete Pavements", Bengt F. Friberg, Transactions, American Society of Civil Engineers, Vol. 105, 1940, pp. 1076 -1095.

PROCEDURE

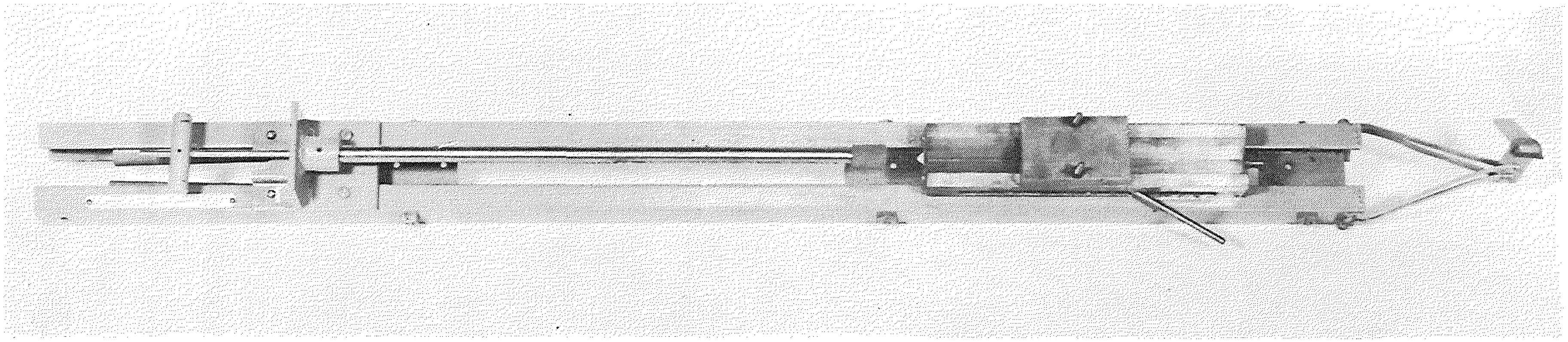
Three instrumented dowels 1-1/4 inches in diameter and 20 inches long were used in this investigation. Since strain gages mounted on the outer surface along the embedded portion of a dowel would influence the bearing pressure between the dowel and the concrete surrounding it, hollow dowels were used so that strain gages could be mounted on the inner surface. These dowels were cut from steel tubing having a 7/8-inch inside diameter.

The moment of inertia of this 1-1/4 inch diameter tube is 0.0911 inch^4, which means that the removal of the 7/8-inch cylinder at the center decreases the moment of inertia, I, by only 24 per cent and gives an I equivalent to that of a 1.165-inch dowel.

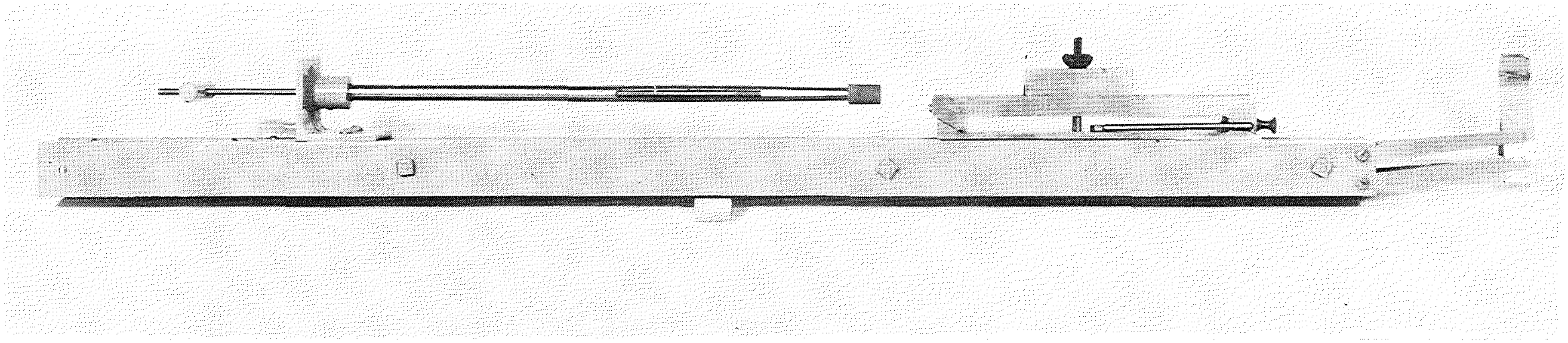
SR-4, Type AD-7 strain gages were mounted in each dowel by means of the special tool shown in Figure 1 and Figure 2. With this tool it was possible to mount the gage within ± 0.02 inch of any desired location. The positioning of the strain gages in each dowel is illustrated in Figure 3.

By mounting the gages on the top and bottom of dowel No. 2, it was possible to space the strain gages at 1/2-inch intervals. A compensating gage was placed inside dowels No. 1 and No. 3. Each of these gages was mounted on a small piece of steel and placed in a small rubber casket, which in turn was suspended in one of the two dowels.

The leads to the strain gages were wrapped in a bundle extending from one end of each dowel. A rubber tube (Figure 6) was slipped over each bundle of leads and clamped around the end of the dowel to protect the leads from moisture. The dowels were filled with petrosene wax to prevent moisture from reaching the gages.



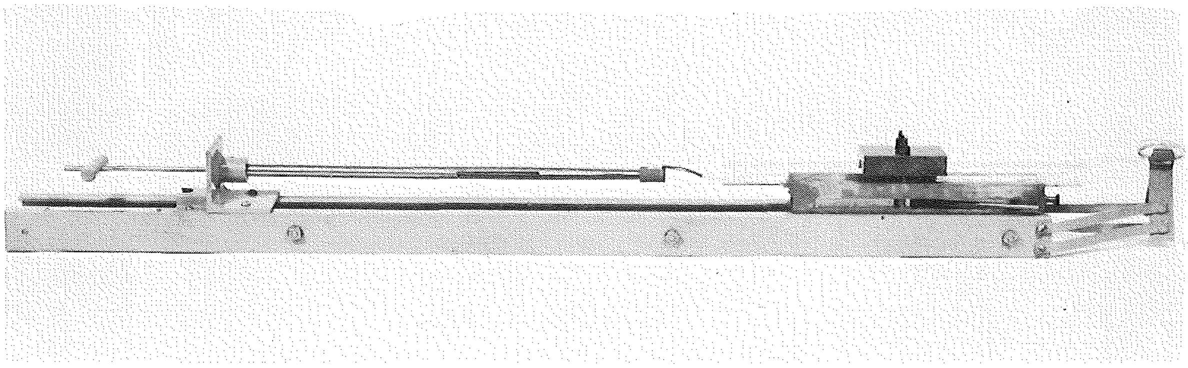
TOP



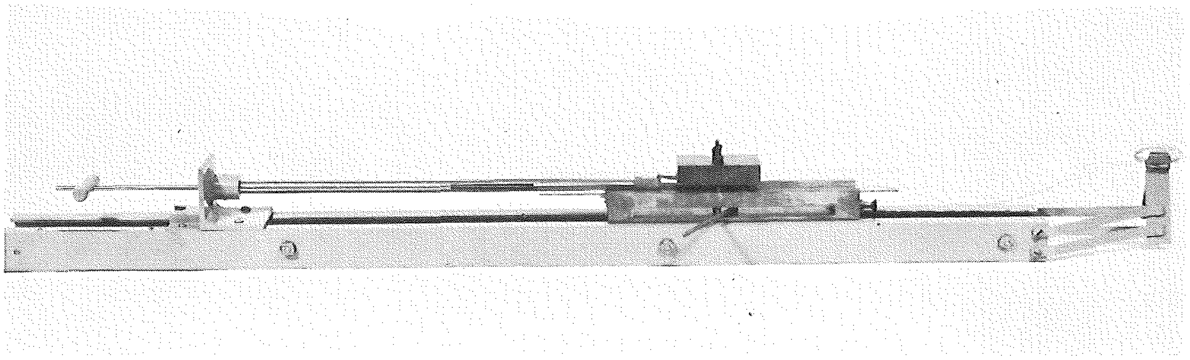
SIDE

FIGURE I

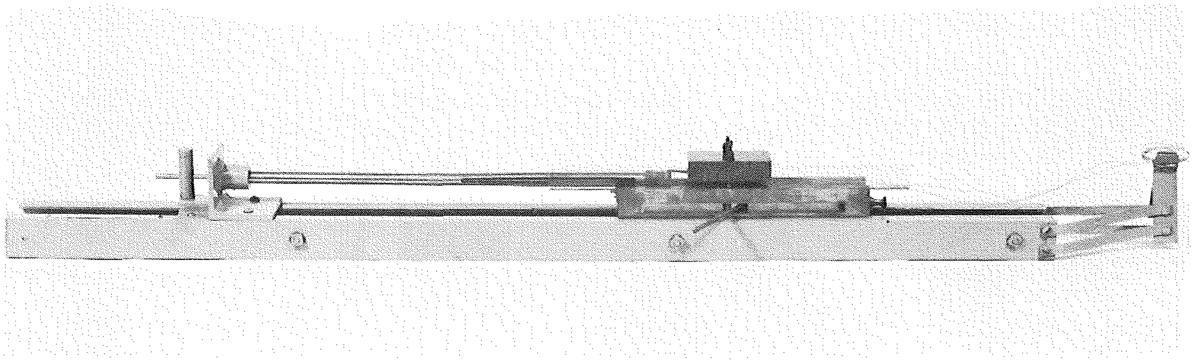
TOP AND SIDE VIEWS OF THE SPECIAL TOOL
FOR MOUNTING STRAIN GAGES IN DOWELS.



A. GAGE POSITIONED ON RUBBER TIP READY FOR INSERTION INTO DOWEL.



B. GAGE INSERTED INTO DOWEL - SLIDING "V" BLOCK LOCKED PREPARATORY TO APPLICATION OF GAGE PRESSURE.

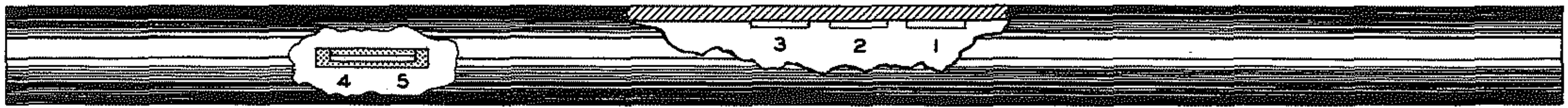


C. GAGE PRESSURE APPLIED AND MAINTAINED UNTIL GAGE CEMENT IS CURED.

NOTE:
THE PLEXIGLAS DOWEL OF THESE PHOTOGRAPHS WAS USED ONLY TO FACILITATE DEMONSTRATION OF THE GAGE MOUNTING DEVICE.

FIGURE 2. APPLICATION OF STRAIN GAGES TO INTERIOR OF HOLLOW DOWEL

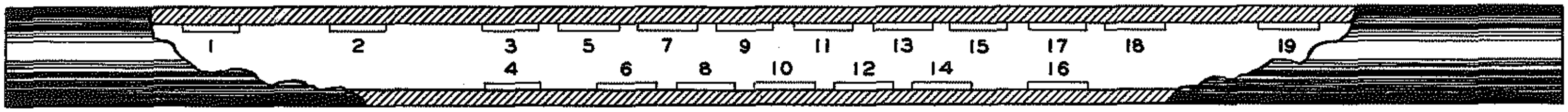
DOWELS NO. 1 AND 3



GAGE NO.	1	2	3
DISTANCE FROM CENTER-INCHES	2	1	0

NOTE:
GAGES NO. 4 AND 5 ARE COMPENSATING GAGES, BOTH OF WHICH ARE CONTAINED IN SMALL RUBBER CASKETS PLACED IN DOWELS NO. 1 AND 3 RESPECTIVELY.

DOWEL NO. 2



GAGE NO.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
DISTANCE FROM CENTER - INCHES	7 1/2	5 1/2	3 1/2	3 1/2	2 1/2	2	1 1/2	1	1/2	0	1/2	1	1 1/2	2	2 1/2	3 1/2	3 1/2	4 1/2	6 1/2

ALL THREE DOWELS:
1 1/4 INCH OUTSIDE DIAMETER - 7/8 INCH INSIDE DIAMETER - 20 INCHES IN LENGTH.
ALL GAGES ARE SR - 4 TYPE AD - 7.

FIGURE 3. LOCATION OF STRAIN GAGES IN INSTRUMENTED DOWELS

The dowels were then supported as cantilevers and loaded to determine the relationship between strain and bending moment. The results are shown graphically in Figures 4 and 5, where the strains were plotted without regard to sign. The strains along the upper surface of dowel No. 2, were larger than those along the lower surface, indicating that the two surfaces were not equidistant from the neutral axis. The strain recorded for gage No. 3 in dowel No. 1, was small. From this data it appears that this gage may be defective.

Two concrete slabs 9 inches thick, 3 feet wide, and 15 feet long were poured end to end on 10 inches of consolidated sand fill, and separated by an expansion joint filler strip 1 inch thick. During the pouring operation the three dowels were held in place across the joint by a small dowel basket cut from a standard expansion joint assembly. (See Figures 6 and 7). The major test dowel, No. 2, was placed in the center of the slab and the minor dowels, No. 1 and 3, were placed on each side of it and 12 inches away as shown in Figure 6. A metal cap was slipped over the free end of each dowel and that half of the dowel was covered with a mixture of cutback asphalt (RC-1) to provide for longitudinal movement of one slab with respect to the other. The electrical leads were brought out through the wooden forms.

Three test cylinders and three sonic beams were made from the batch of concrete used in pouring the slabs. These were allowed to cure for one month before testing. From cylinders, the average ultimate 28-day compressive stress of the concrete was found to be 4200 psi. and from the beams, the tangent modulus of elasticity was found to be 5.9×10^6 psi.

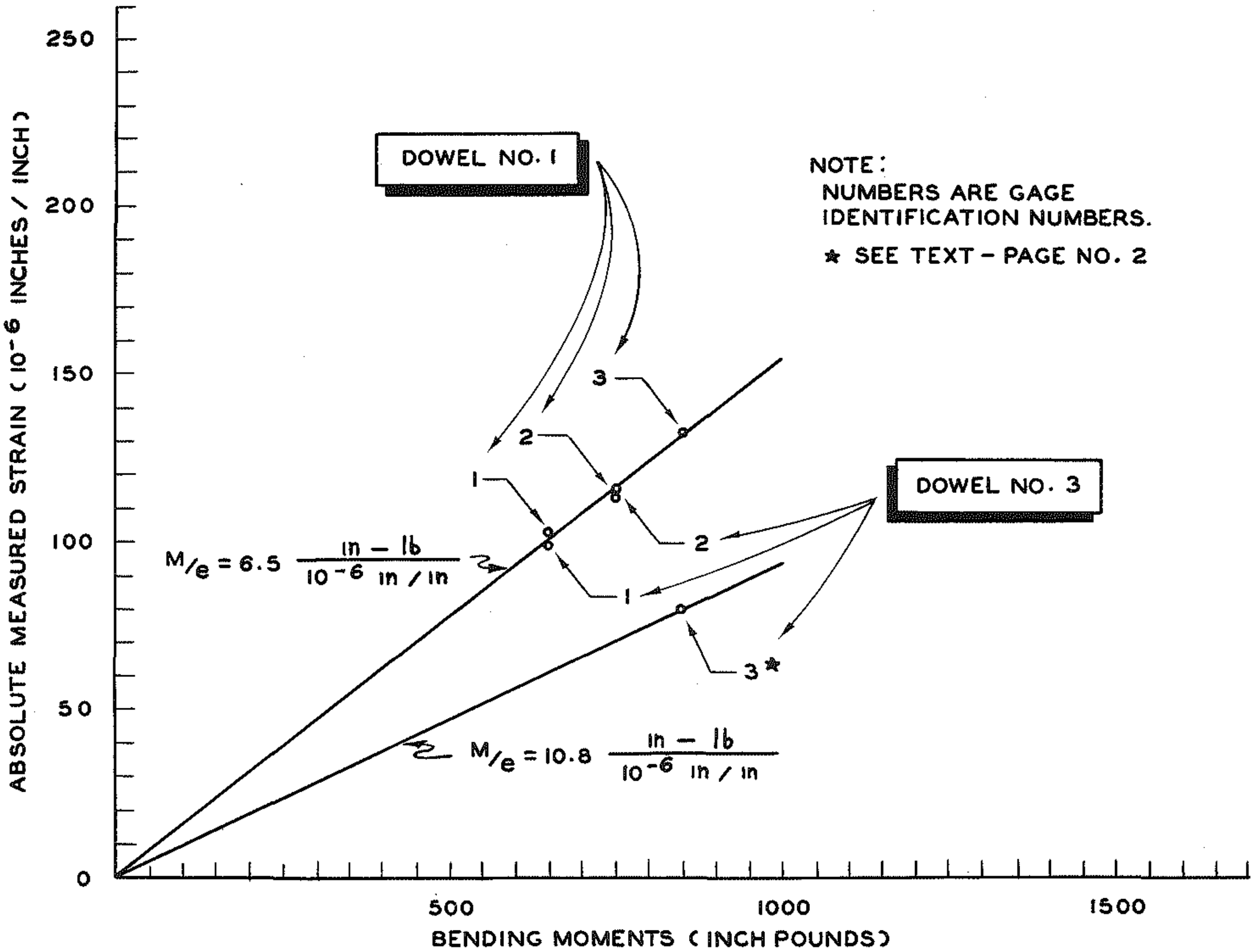


FIGURE 4. CALIBRATION FOR INSTRUMENTED DOWELS NO. 1 AND 3

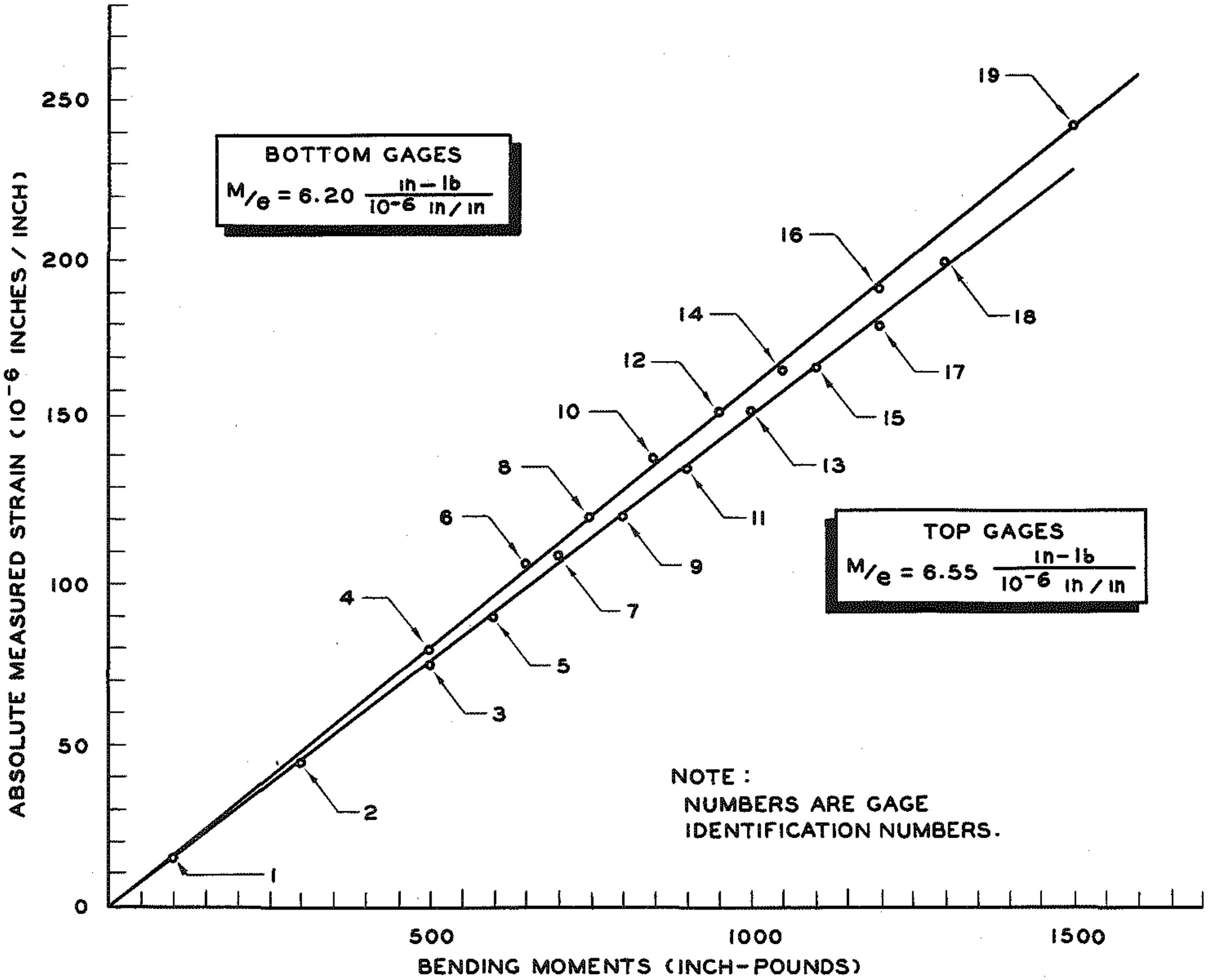
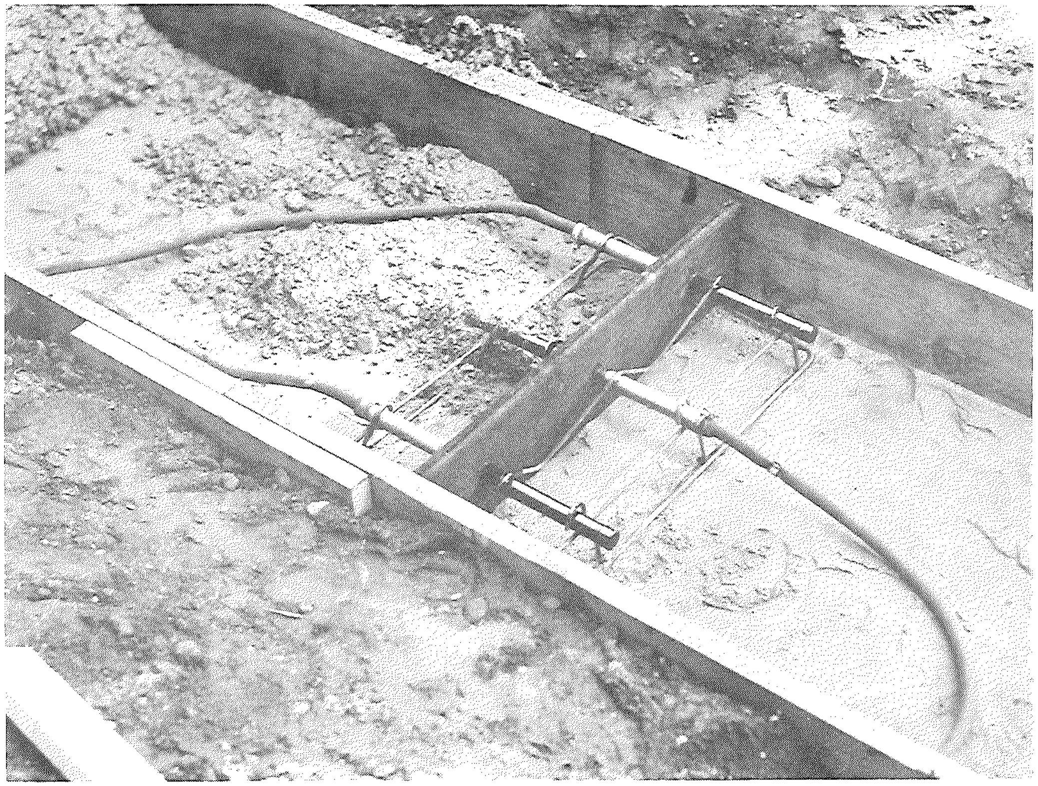


FIGURE 5. CALIBRATION FOR INSTRUMENTED DOWEL NO. 2



▲ FIGURE 6. INSTRUMENTED DOWELS AND DOWEL ASSEMBLY PRIOR TO THE POURING OF CONCRETE.

FIGURE 7. POURING OF CONCRETE SLABS CONTAINING INSTRUMENTED DOWELS. ▶

Loading of the slabs was accomplished by means of the MSHD Special Weigh Truck. It was backed into position so that the right rear dual wheels were centered on the slab at each of the three locations shown in Figure 8. Wheel loads of 7000 and 9700 pounds were applied.

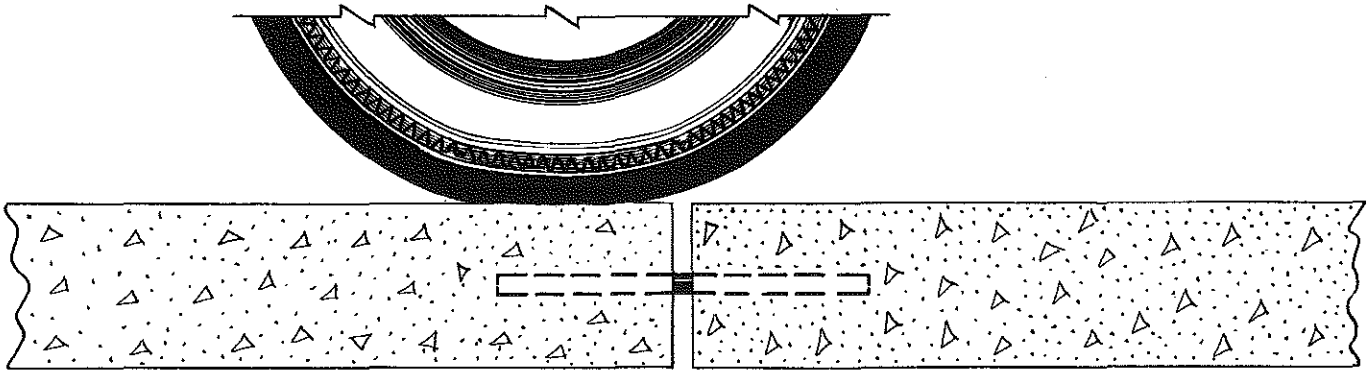
With the aid of an Anderson switching device, the strains were measured rapidly with an SR-4 strain meter. This made it possible to run three trials with the load at each of the three positions in a period of less than 4 hours. Tests were made between 1:00 and 4:00 p.m., which is the period during which the slab receives the most support from the subgrade.

The deflection of the slabs at each of the four corners adjacent to the joint, was measured with 0.001 inch dials, when the wheel load was applied. From these deflections the percent of load being transferred across the joint was obtained.

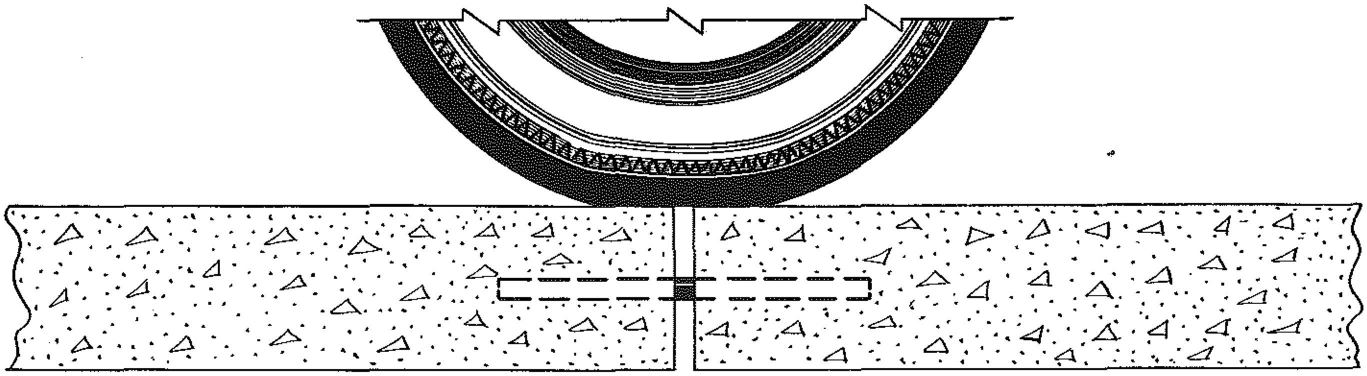
TEST RESULTS

The variation in flexural strain along dowel No. 2 with the load on the approach slab, directly over the joint, and on the leaving slab, is presented graphically in Figure 9. Each point represents the average strain recorded for three successive trials. Also, the points shown represent two separate sets of tests. The first being made in November of 1954 while the subbase was in a normal non-frozen condition, and the second set in March of 1955 while the subbase was frozen. Because of the good agreement between these two sets of readings only one curve is drawn for each loading condition.

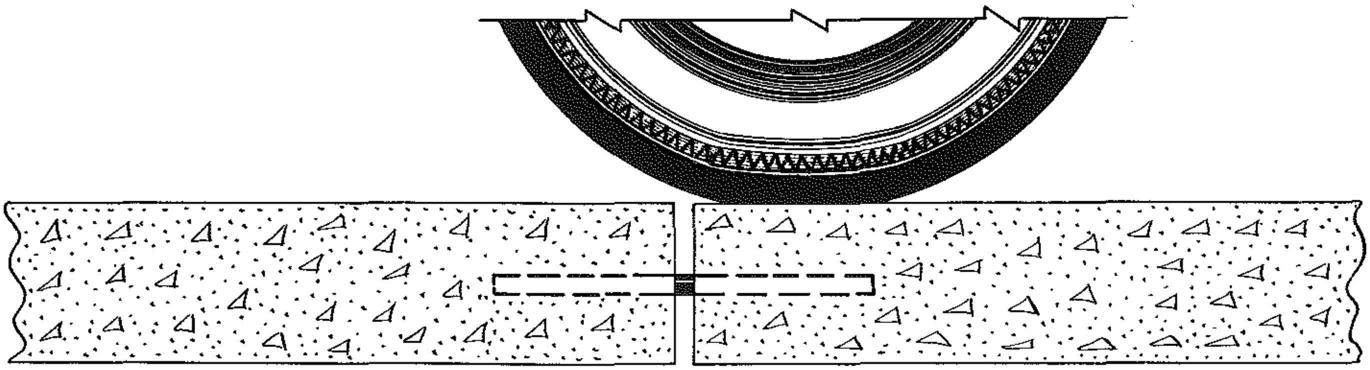
Furthermore, to facilitate the plotting of curves and because of the fact that the top and bottom gages were on opposite sides of the neutral axis, while the dowel



LOAD WEST



LOAD CENTER



LOAD EAST

FIGURE 8. THE THREE LOADING POSITIONS OF THE REAR WHEEL WITH RESPECT TO THE JOINT.

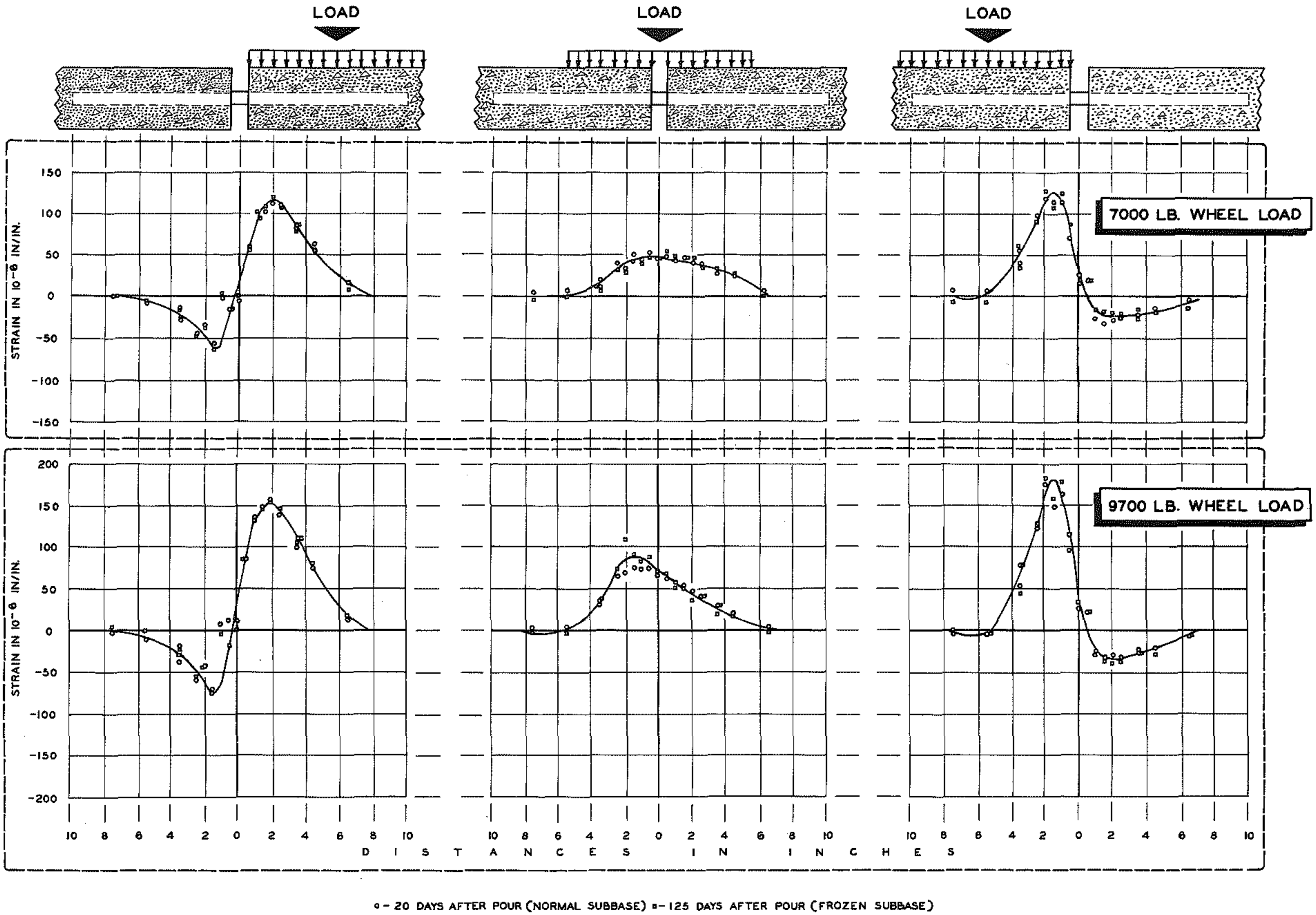
was in bending, the signs of the strains associated with the "top" gages were changed to agree with those of the bottom gages.

The poor agreement which exists between the strains recorded for the top and bottom gages under wheel load tests (Figures 9 and 10) cannot be attributed entirely to the variation in the calibration shown in Figure 5. It appears that the flexural strains were accompanied by compressive strains when the load was applied at the joint and by tensile strains when the load was applied at the other two locations. This phenomena was more pronounced in the portion of the test dowel in the approach slab; the half not covered with cutback asphalt.

When the 9700 pound wheel load was centered over the joint the maximum strain occurred one to one and one-half inches from the center of the joint. This may be due to environmental conditions associated with the test dowel installation, for example, the bond-reduction coating, on one side.

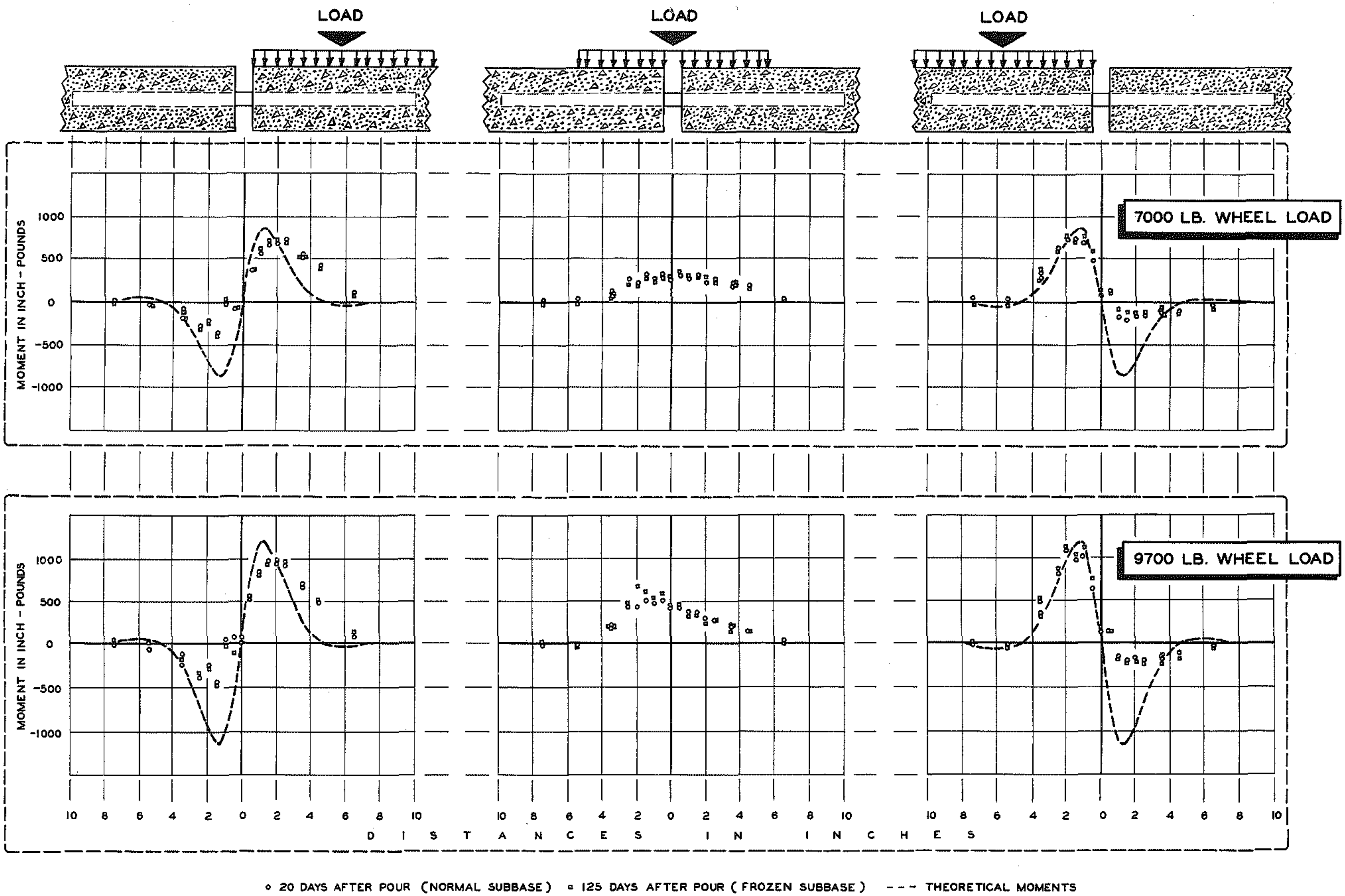
If the measured dowel strains are assumed to be flexural in nature, then the moments in the dowel may be computed by making use of the relationship between strain and moment shown in Figure 5. In Figure 10 the bending moment values were plotted for dowel No. 2 and smooth curves were drawn through the points. In some cases, the difference between the values obtained from "top" gages as compared to the values obtained from "bottom" gages were decreased and in other cases the differences were amplified by the difference in the strain-moment relationships. In general, the curves have by necessity the same shape as the strain curves.

The fact that the maximum moment in the dowel under the load is much larger than the maximum moment in that portion of the dowel embedded in the "unloaded" slab



STRAIN DISTRIBUTION IN LOAD TRANSFER DOWEL

FIGURE 9



MOMENT DISTRIBUTION IN LOAD TRANSFER DOWEL

FIGURE 10

is of considerable importance. In effect, the present method for the design of dowels is based on the stresses in and around that portion of the dowel embedded in the "unloaded" slab. Also, in the present theory it has been assumed that stresses equal to these in magnitude and opposite in sign, exist in and around the portion of the dowel embedded in the "loaded" slab.

In order to compare these curves with theoretical curves, it is necessary that we know what percent of the load is being transferred across the joint and what percent of that is being transferred by the center dowel. If we consider the moment in the dowels resulting from the application of the 9700 lb. wheel load, we find as shown in Table I the percent of load transferred across the joint to be approximately 45 percent.

TABLE I

Deflection of Slab in 10^{-3} in. and Percent Load Transfer

	Approach Slab			Leaving Slab			
	Corner 1	Corner 2	Ave.	Corner 1	Corner 2	Ave.	Load Transfer*
Load on Approach Slab	18.1	15.1	16.6	14.9	12.7	13.8	45.4%
Load on Leaving Slab	14.3	16.5	15.4	16.8	18.9	17.8	46.3%

$$\text{*Load Transfer (\%)} = \frac{100\ (\text{deflection of unloaded slab})}{\text{Deflection of unloaded slab + defl. of loaded slab}}$$

If we assume that each dowel is transferring the same percent of the load, then the center dowel is transferring 15 percent of 9700 pounds or 1450 pounds. According to the accepted equations controlling the design:[2]

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[2] ibid. pp. 1078-1080.

$$M = -\frac{e^{-\beta x}}{\beta}\left[P \sin \beta x - \beta M_o (\sin \beta x + \cos \beta x)\right]$$

and

$$M \max = -\frac{P e^{-\beta x_m}}{2 \beta} \sqrt[4]{1 + (1 + \beta a)^2}$$

where:

$$\beta = \sqrt{\frac{d G}{4 E_s I}}$$

M = the bending moment at any point in the dowel in inch-pounds

M_o = M at the face of the joint in inch-pounds

e = the base of Naperian logarithms

d = dowel diameter in inches. (1.25 inches)

G = Modulus of support of concrete in p.c.i. (2.5×10^6)

E_s = Modulus of elasticity of steel in psi (29×10^6)

I = Moment of inertia of dowel in inch^4 (0.0911)

P = the concentrated load in pounds acting on the dowel at the face of the slab

a = the width of the joint in inches (1.0)

x = the distance along the dowel from the slab face at the joint

$x_m = \frac{1}{\beta}$ arc tan. $\frac{1}{1 + \beta a}$ = the distance from the face of the slab to the point of maximum moment in the dowel.

These equations are applied to the problem assuming that the moment at the face of the slab is equal in magnitude to $\frac{-Pa}{2}$. If G, the modulus of support of concrete,[3] is taken equal to 2.5×10^6 pci and the general equation is solved for values of x from

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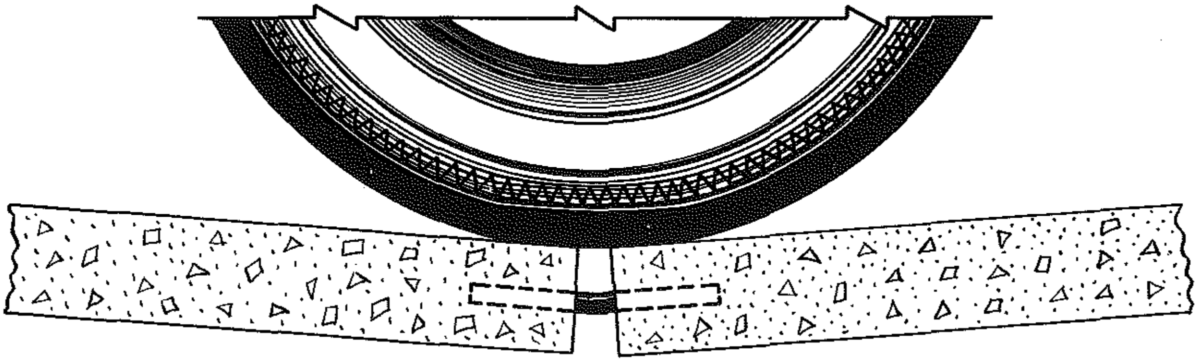
[3] Journal of the American Concrete Institute, July 1956.

0 to 9-1/2 inches, the moment distributions are those represented by the dotted line curves of Figure 10 (assuming the loaded side to have the same distribution as the unloaded). From these curves the theoretical maximum bending moment M is found to be -1170 inch-pound which compares favorably with the maximum moment on the loaded side for the 9700 pound wheel load in Figure 10. However, when compared with the two "unloaded side" M max values, the theoretical is found to be approximately two to four times as great.

Due to the fact that the load transfer at the test joint was found to be approximately 45 percent, it was assumed that the experimental moment distribution would be of the same general shape as the theoretical distribution. However, as mentioned above and observed in Figures 9 and 10, this was quite obviously not the case.

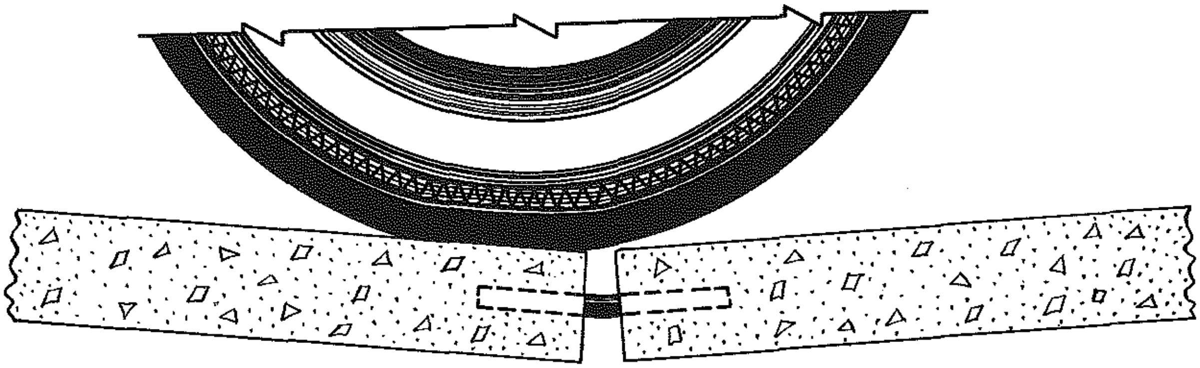
In attempting to determine the cause of the lateral non-symmetry (ignoring sign) of the experimental curves, it was concluded that at least part of the answer must lie in the elimination, in the design theory, of the moments caused by the hinge action of the joint when a load is applied near the joint. As the stiffness ratio of slab to joint is of the order of 2500 to 1, it seems proper to assume that for a 9-inch slab loaded 5-1/2 inches from the joint, that a deflection will occur at the joint which will induce a symmetrical moment distribution proportional to the distribution obtained when the load is applied over the joint, but reduced in magnitude. (See Figure 11).

If this assumption is correct, then it must be assumed that the experimental load transfer moment distributions obtained in this study are the result of the combination of the moments induced in the dowel by the load transfer action and those induced by the hinge action of the joint.



CENTER LOAD

MAXIMUM MOMENT = M_c



SIDE LOAD

MAXIMUM MOMENT = M_s

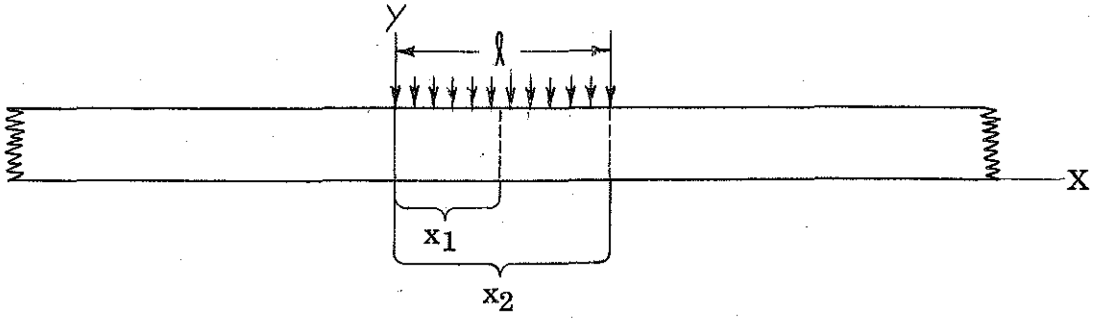
$$\frac{M_s}{M_c} = \frac{2e^{-\beta X_s}\ \sin\beta X_s - e^{-\beta(\ell - X_s)}\cos\beta\ell}{2e^{-\beta X_c}\ \sin\beta X_c - e^{-\beta(\ell - X_c)}\cos\beta\ell} = 0.77$$

(FOR THE CONDITIONS OF THIS STUDY)

FIGURE 11. RELATIONSHIP BETWEEN BENDING MOMENTS RESULTING FROM LOAD APPLICATION OVER THE JOINT AND THOSE RESULTING FROM LOAD APPLICATION AT ONE SIDE OF THE JOINT.

To test the above hypothesis it is necessary to determine the ratio of the moments in the dowel when the load is applied over the joint, to the induced moments of like distribution but reduced magnitude when the load is applied at one side of the joint.

This ratio was determined by again using the theory of beams on elastic foundations.[4]



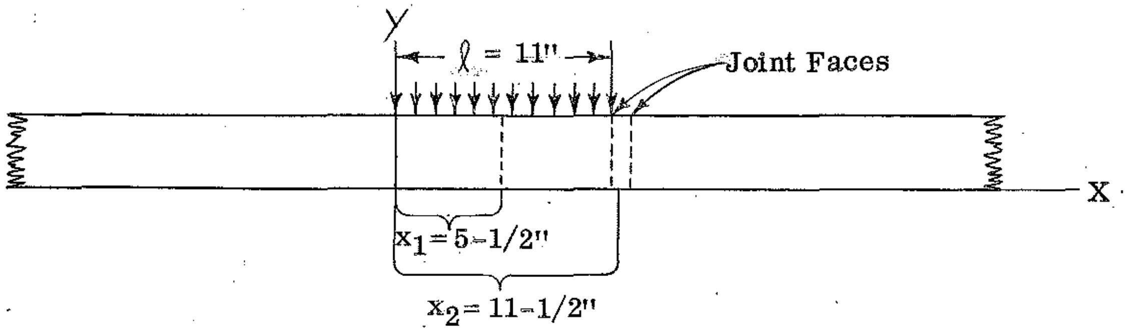
$$M = \frac{W}{8\beta^2}\left(2e^{-\beta x}\sin\beta x - e^{-\beta(\ell - x)}\cos\beta\ell\right)$$

where

$$\beta = \sqrt{\frac{K}{4EI}}$$

$$W = \text{load in pounds per linear inch} = \frac{P}{\ell}$$

Now, if this method is applied to the actual case, an approximation of the ratio of the maximum moment at x_1 to the lesser moment at x_2 may be obtained:



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[4] "Beams on Elastic Foundation; R. J. Roark, Formulas for Stress and Strain, McGraw-Hill, 1938, p. 128.

$$\frac{M_2}{M_1} = \frac{\frac{W}{8\beta^2}\left(2e^{-\beta x_2}\sin \beta x_2 - e^{-\beta(\ell - x_2)}\cos \beta \ell\right)}{\frac{W}{8\beta^2}\left(2e^{-\beta x_1}\sin \beta x_1 - e^{-\beta(\ell - x_1)}\cos \beta \ell\right)}$$

$$= 0.77$$

Therefore, assuming our hypothesis to be valid, it would follow that each experimental moment distribution of Figure 10 is the resultant of a moment distribution similar to the theoretical, plus a moment distribution similar to the experimental distribution obtained when the load was applied over the joint, but reduced in magnitude by the factor 0.77.

The moment curves of the 7000 pound wheel load were selected to test the above hypothesis because of the non-symmetry of the obtained curves when the 9700 pound wheel load was applied over the joint.

In Figure 12, the theoretical moment distribution is combined with the moment distribution resulting from joint hinge action, to result in a "corrected" theoretical load transfer moment distribution curve. If the latter curve is compared with the experimentally determined load transfer moments of Figure 10 it becomes obvious that the experimental data is a much better approximation of the "corrected" theory than of the uncorrected. In each of the experimental curves the points of inflection are shifted towards the unloaded side, and the magnitude of the maximum moment of the loaded side is considerably greater than that of the unloaded side. Both of these characteristics are in accord with the corrected theoretical curve.

However, there does exist a significant difference in the maximum moments (both plus and minus) of the experimental distribution and those of the corrected

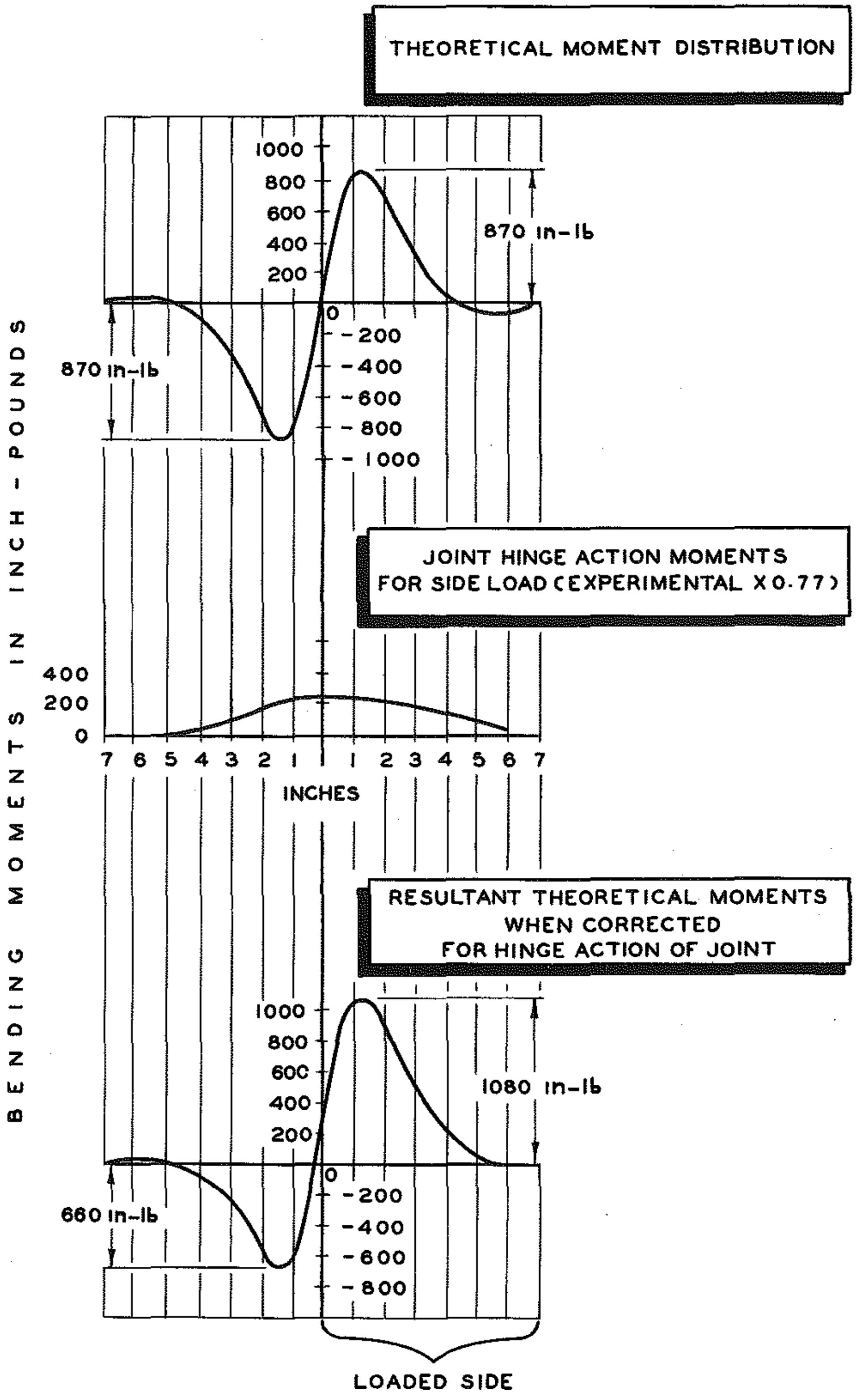

FIGURE 12. PROPOSED CORRECTION OF THEORETICAL LOAD TRANSFER MOMENT DISTRIBUTION – BY COMBINATION WITH MOMENTS RESULTING FROM JOINT HINGE ACTION TO RESULT IN A CORRECTED THEORETICAL DISTRIBUTION

theoretical distribution. Because of the limited amount of data available for this report no attempt will be made at this time to explain these differences. There are a number of possible explanations, some of which may be indicated by further experimentation.

A cursory examination of Figures 9 and 10 will indicate to the reader, one of the more apparent observations which may be made from the data of this experiment. That observation being that the 20-inch dowel length could be reduced (for the dowel size and wheel loads used here) by approximately four inches without significantly affecting the moment distributions.

As a matter of interest it was decided that the strains in the dowel resulting from conditions other than the application of load should be measured. Readings were taken for each gage, approximately two and one-half hours after the concrete slabs were poured and used as a reference. The strains in the dowel on each of the three later dates are shown in Figure 13. The strains for 14 and 20 days after pour are probably due to a combination of factors such as warpage and shrinkage of the slabs with age variations in moisture and temperature. These readings indicate that small flexural strains are superimposed on large tensile strains. These strains are two to four times as large as the maximum strains developed under the loads.

The strains recorded for 125 days after pour are of such a magnitude that they quite likely indicate a change in the reference value of the compensating gage.

The last readings on these gages were taken in July, 1956 and at that time the resistance to ground of all of the gages was reduced to the point where they were no longer usable.

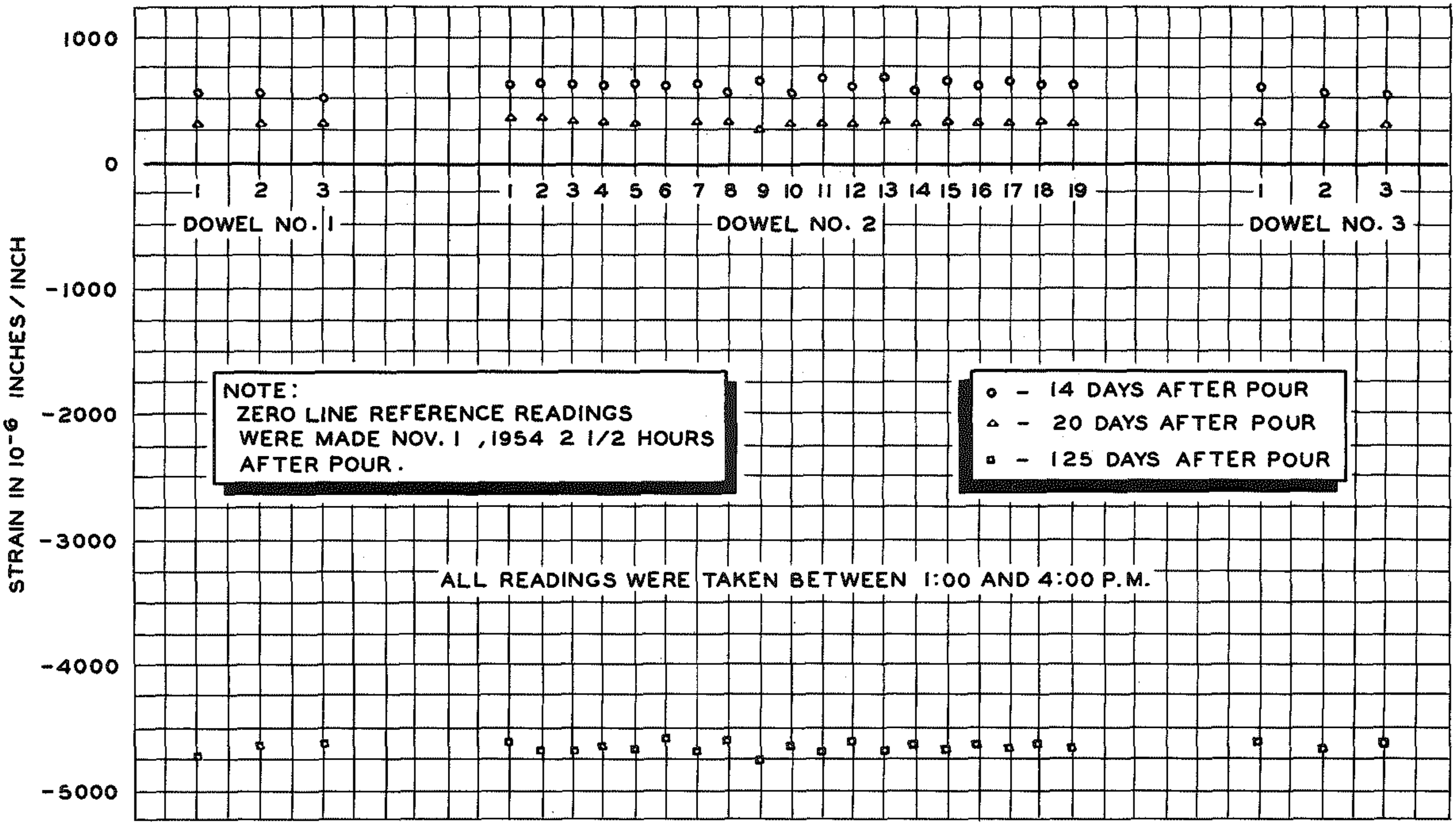

FIGURE 13. DOWEL STRAINS RESULTING FROM CONCRETE SHRINKAGE AND ENVIRONMENTAL FACTORS

CONCLUSION

With full cognizance to the limitations of this study, it is possible to make some interesting observations from the experimental data:

1. The fact that the moments in the loaded portion of the dowel, while agreeing quite well with the theoretical, are from two to four times as great as the moments of the unloaded or design portion. This would seem to indicate that the theoretical data has more application to the loaded portion of a dowel than to the unloaded portion.

2. The possibility that the differences in the distribution of the experimentally determined moments and those determined theoretically, results from the non-consideration in the theory, of the moments occurring as a result of the hinge action of the joint. When the moments caused by the joint action are combined with the theoretical load transfer moment distribution, the resulting moment distribution curve is a very good approximation of the load transfer distributions determined experimentally in this study. With, however, the exception of the case where the load is applied to that side of the joint which contains the unlubricated dowel end. When this is the case the experimental moments of the unloaded side are considerably less than the corrected theoretical.

3. That the maximum strains observed appear to be approximately proportional to the applied loads. This is demonstrated by the fact that the maximum strain under the 7000 pound wheel load was 68 to 70 percent of that under the 9700 pound wheel load, and that 7000 is 72 percent of 9700.

4. That the dowel length could be reduced substantially without seriously affecting the stress distribution.

A significant accomplishment of this study to date is the development of a device for the mounting of SR-4 strain gages along the inside wall of hollow dowel bars. Utilizing this device, it will be possible to undertake a much more comprehensive and conclusive study.