R-370

A SYMPOSIUM ON THE MICHIGAN NUCLEAR GAGE FOR SOILS

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ABSTRACT

The work described in the papers comprising this symposium supports several major conclusions.

The first paper, dealing with known characteristics of radioactivity, shows that the random nature of the radioactive decay process does not impose a barrier to attainment of desired precision with the nuclear gage. In fact, it permits any degree of precision depending on the time the operator is willing to expend on each test. The two-minute counting period normally used with the gage appears to be satisfactory.

In the second paper, field results which were published in Research Report No. R-316, "The Michigan Nuclear Combination Density-Moisture Surface Gage," were analyzed statistically in an attempt to discover whether the accuracy of nuclear density determinations is affected by the moisture content of the soil. It was found that the 124 measurements available for analysis did not tend to substantiate such an effect. Either there is no such effect, or the data were 1) insufficient or 2) tended to mask the effect. The problem requires further study.

The third paper is a report covering work done to test the influence of temperature and humidity on the operation of the nuclear gage. A commercial environmental chamber was made available for this study and performance tests were conducted at temperatures from -10 to 130 F and at relative humidities from 31 to 94 percent. It was found that the gage operated properly at all normal conditions of temperature and humidity, although some difficulties were experienced under grossly abnormal conditions.

The fourth paper describes a series of experiments designed to check the stabilities of the electronic components of the gage under controlled laboratory conditions. A total of 12 experiments comprising 2600 individual tests were conducted on various portions of the scaler-gage system in order to determine the stabilities of the radioactive source, the detector tubes, the preamplifier, and the portable scaler; to compare different radioactive sources; to check the effect of warm-up time; and to discover whether any other instability exists within the system. A total of 400,000 radioactive disintegrations were recorded in each experiment. In general it was found that all components are stable under controlled conditions, strongly suggesting that any "instabilities" reported from the field are caused by factors external to the gage. One such factor could be nonuniform compaction of the soil; another could be appreciable sensitivity to soil type. Such possible effects require further study.

The final paper deals with a statistical analysis of results obtained from a field experiment designed to compare the relative precision of the nuclear instrumentation with that of conventional equipment. Moisture and density determinations were made by both methods of six sections each of sand, clay, and gravel. It was found that the relative precision of nuclear and conventional density determinations is equal for sand and clay soils. In the case of gravel, the data were inconclusive. The conventional method is relatively less precise than the nuclear method for soil moisture measurements.

CONTENTS

PREFACE

In view of widespread interest in the accuracy and stability of nuclear gages for soil density and moisture determinations under field conditions, the Research Laboratory Division undertook a series of experiments specifically designed to shed light on these questions. The papers contained in this symposium constitute a report on the success of the experiments.

Scalers used in the study included one Nuclear-Chicago Ultrascaler, Model 192, operating on a-c current; one Nuclear-Chicago scaler, Model 183, also operating only on a-c current; and five Nuclear-Chicago portable scalers, Model 2800, each operating on both a-c and battery power. The five portable scalers are designated by their company serial numbers: 156, 157, 169, 170, and 257.

The nuclear gages used were designed and fabricated by the Michigan State Highway Department. These combination moisture-density gages are designated as Nos. 1, 2, 3, and 4. Each gage normally used 5 mc of $Ra^{226}Be$, a source of multi-ergic gamma rays and fast neutrons; in one series of density experiments a 5-mc Cs^{137} single-energy gamma source was substituted. Detectors used in conjunction with the gages were Amperex 90NB gamma detectors for density determinations, and N. Wood Laboratories $B^{10}F_3$ slow neutron detectors for moisture.

THE LIMITING VALUE OF ACCURACY OBTAINABLE IN MEASUREMENTS USING RADIOACTIVITY

Synopsis

The random nature of radioactive processes establishes a limit to the precision with which measurements depending on these processes can be made. This limit becomes broader as the number of disintegrations used in the measurement increases. By using a sufficiently large number of disintegrations, it is possible to reduce the statistical uncertainty to an insignificant level.

Energy from a radioactive source is produced by atoms within the source breaking down spontaneously and converting some of their mass into kinetic energy in accordance with Einstein's equation, $E = mc^2$. Because several million atoms are contained in even a very small source, there is no assurance that any particular atom will break down at a given time. For this reason we say that radioactivity is a random process, just as the number of raindrops falling on one's house during a spring shower is a random process. We can't say, for example, that exactly 1,248,765 drops will fall on a roof in one minute, but if we knew the size of the drops and the total amount of water falling on the roof, and the duration of the shower, we might come up with this figure.

Suppose we made this calculation and our result indicated mathematically that 1,248,765 drops actually fell on the roof in one minute. Would this be an exact number? Would it be the same for every minute? I am sure we would have to say that this would be a sort of statistical average, and that even if we could count fast enough, the chances of selecting any minute at random during the shower and counting exactly 1,248,765 drops in that minute would be practically nil.

This does not mean, however, that such a figure is useless. By knowing it, we could probably figure how many seconds a small scientific instrument could stay on the roof before it became completely wet, if we needed to have this knowledge.

In the case of radioactivity, we might count the number of gamma rays picked up by a Geiger tube in an hour and find that the result is 5940. If we divide this by the number of minutes in an hour, we get 99. Does this mean, then, that if we count gamma rays for exactly one minute we will get 99? We might, and we might not. Most likely not. Even if we counted for another hour we would probably not get exactly 5940. But we would be closer to 5940 than we would be to 99 for individual minutes.

Let us see how this works out in actual practice. The following table is taken from Friedlander and Kennedy's Introduction to Radiochemistry (John Wiley & Sons, 1949):

Minute	Counts	Difference from Mean	Difference^2
1	89	-10	100
2	120	21	441
3	94	- 5	25
4	110	11	121
5	105	6	36
6	108	9	81
7	85	-14	196
8	83	-16	256
9	101	2	4
10	95	- 4	16
Totals	990	0	1276
Mean	99		

This table tells us that we don't get 99 for any particular minute, although the average happens to be 99 counts per minute. For the counts actually made, we see that we got as low as 83 and as high as 120. Is that bad? It certainly doesn't look very good, since we have a pretty wide range, and 120 is almost 45 percent greater than 83. If we depended on these figures for critical results we could be 45 percent off. This would not be very likely, however, since we would probably use the mean as a true value, in which case we could only be off by half that amount. The table tells us, moreover, that we have only one chance in ten of being off by as much as 22 percent, and we have just as good a chance of being right within 2 percent.

Does all this mean, then, that if we use radioactivity as a tool to give us accurate information about certain properties of materials, we are taking chances--playing a sort of guessing game with nature? The absolutely correct answer to this question is yes, but let us see just how much of a chance we are taking. After all, we are all familiar with tolerances. We can hardly expect to measure anything with complete accuracy.

Let us first of all compare the above table with a similar one taken from Picker X-Ray Corporation's Scintillator (Vol. 4, No. 2, Dec. 29, 1959):

Minute	Counts	Difference from Mean	Difference^2
1	3210	89	7921
2	2982	-139	19321
3	3105	- 16	256
4	3217	96	9216
5	3086	- 35	1225
6	3142	21	441
7	3074	- 47	2209
8	3101	- 20	400
9	3204	83	6889
10	3089	- 32	1024
Totals	31210	0	48902
Mean	3121		

This table tells us that in this case we have one chance in ten of being off by as much as 4.5 percent, with just as good a chance of being right within 0.5 percent. This looks much better than the data in the first table, and it is only natural to inquire why.

The only difference between the two sets of data is that the number of counts obtained in a minute in the second set is about 31 times the number obtained in the first set. The fact that the count rate is also 31 times as great turns out to have nothing to do with it; it is the actual number of counts obtained, regardless of the length of time required to obtain them, that is all-important. This is only another illustration of a universal principle of nature, that whenever you are going to use a sampling procedure you will never get anywhere unless your sample is

representative. All applications of radioactivity involving measurement are really nothing but sophisticated sampling procedures, and to get the right results we have to pay attention to the rules of sampling.

These rules have been worked out in great detail by many persons and are published throughout the literature dealing with radioactivity. They are generally applicable to all experiments involving statistics. One of their most important implications is the fact that we can attain any desired degree of accuracy in measurements employing radioactivity simply by taking large enough counts. This follows from the well-known principle that the larger the sample, everything else being equal, the more representative the sample will be of the whole.

From a practical standpoint it becomes necessary to weigh the required accuracy against the time needed to obtain a large enough count to attain that accuracy. Also, of course, there is no justification for insisting upon a higher accuracy from the standpoint of radioactivity statistics than one can expect from the instrumentation employed or from the method of its employment. Total counts obtained with the nuclear gage within the counting times specified in the instructions were calculated to provide acceptable accuracy within reasonable time.

And now a word about statistics. Webster defines statistics as the "systematic compilation of instances for the inference of general truths." As far as we are concerned, the "instances" referred to by Webster are the breakdowns of radioactive atoms. His "general truths" are the answers that we want, within the degree of accuracy that we want. If we are systematic about it, we can have confidence in the answers.

If we take the number 1,276 from the first table and divide it by 10 (the total number of individual one-minute counts), then take the square root of the result, we obtain the number 11.3, known as the standard deviation. This is a measure of the closeness of the individual one-minute counts to the mean. Since the mean was 99, we have to compare 11.3 with 99. If we do the same thing with the number 48,902 from the second table, we end up comparing the standard deviation for that table with the mean of 3,121. This is a much more favorable comparison, 69.93 being a much lower percentage of 3,121 than 11.3 is of 99 (the actual percentages are 2.24 and 11.41 percent respectively). It is obvious that there is much less deviation in the second table. This is simply a statistical way of comparing things.

Or, just to be different, we might divide the number 48,902 by the mean of 3,121, and arrive at the number 15.669. This is known as Chi Square for the second table, and is a measure of how closely the data fit the Poisson distribution curve. The following table, also from the Picker X-Ray Corporation, tells us that the value 15.669 falls within the standard confidence range, and therefore the equipment used to obtain the data can be considered to be functioning properly.

CHI SQUARE VALUES

No. of Measurements in Series Being Analyzed	Ranges of Chi Square Values		
	High Confidence (80% Probability)	Standard Confidence (90% Probability)	Basic Confidence (98% Probability)
3	0.211- 4.605	0.103- 5.991	0.020- 9.210
4	0.584- 6.251	0.352- 7.815	0.115-11.345
5	1.064- 7.779	0.711- 9.488	0.297-13.277
6	1.610- 9.236	1.145-11.070	0.554-15.086
7	2.204-10.645	1.635-12.592	0.872-16.812
8	2.833-12.017	2.167-14.067	1.239-18.475
9	3.490-13.362	2.733-15.507	1.646-20.090
10	4.168-14.684	3.325-16.919	2.088-21.666
11	4.865-15.987	3.940-18.307	2.558-23.209
12	5.578-17.275	4.575-19.675	3.053-24.725
13	6.304-18.549	5.226-21.026	3.571-26.217
14	7.042-19.812	5.892-22.362	4.107-27.688
15	7.790-21.064	6.571-23.685	4.660-29.141
16	8.547-22.307	7.261-24.996	5.229-30.578
17	9.312-23.542	7.962-26.296	5.812-32.000
18	10.085-24.769	8.672-27.587	6.408-33.409
19	10.865-25.989	9.390-28.869	7.015-34.805
20	11.651-27.204	10.117-30.144	7.633-36.191
21	12.443-28.412	10.851-31.410	8.260-37.566
22	13.240-29.615	11.591-32.671	8.897-38.932
23	14.041-30.813	12.338-33.924	9.452-40.289
24	14.848-32.007	13.091-35.172	10.196-41.638
25	15.659-33.196	13.848-36.415	10.856-42.980
26	16.473-34.382	14.611-37.382	11.524-44.314
27	17.292-35.563	15.379-38.885	12.198-45.642
28	18.114-36.741	16.151-40.113	12.879-46.963
29	18.939-37.916	16.928-41.337	13.565-48.278
30	19.768-39.087	17.708-42.557	14.256-49.588

Actual applications of all of the principles above are explained and illustrated in detail in the papers by Zapata and Schwartje. It suffices here to point out that there is an inherent statistical variability in radioactivity itself, and that this variability sets an upper limit to the degree of accuracy possible with an instrumental setup. Radioactive variability,

however, may be treated statistically to yield any degree of accuracy desired short of 100 percent. This may sound like a paradox, but it isn't.

It will be noted, for example, that the mean count rate in the second table is 3,121 counts per minute, and we have seen that for any minute we can have confidence that we will be within at least 4.5 percent of that number--probably much closer. However, to say that we will get a correct answer is not a very satisfactory statement, even though we know exactly what the odds are. It is much more satisfying to use the standard deviation, which in this case turned out to be 69.93, and to say that for any one-minute count we will get 3,121 $\pm$ 70. The next thing to do is to make this tolerance of 70 small enough to be insignificant.

To make things still easier, Friedlander and Kennedy point out that for reasonably large numbers the standard deviation is very close to the square root of the mean. So if we count 100 in one minute we could be justified in saying that the count rate is 100 $\pm$ 10 counts per minute. But we might count for a longer time and get 1000 counts in ten minutes. Now we would be able to state that the counting rate is 100 $\pm$ 3.2 counts per minute. (The square root of 1000 is 32, and one-tenth of that is 3.2.)

Calibration curves for density and moisture, originally published in Research Laboratory Report No. 316*, are given here in Fig. 1. The density curve indicates that the count rate corresponding to a density of 120 pcf is 10,000 counts per minute. The square root of 10,000 is 100, exactly 1 percent of 10,000. This means that the true count rate is 10,000 $\pm$ 100 counts per minute, and therefore if the density were exactly 120 pcf and if one counted for only one minute, he might count 9,900 or 10,100. Conversely, if he counted for only one minute and counted exactly 10,000 counts on an unknown soil, he would know that the density of that soil must be between 119.5 and 120.6 pcf according to the curve. This degree of accuracy would be acceptable for most purposes. However, if one counted for two minutes and counted exactly 20,000, his observed counting rate would be 10,000 $\pm$ 71 counts per minute, and he would report the density as being between 119.7 and 120.4 pcf. The ultimate limit of accuracy has been improved this much by counting 20,000 instead of only 10,000 total counts, although the time

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* Pocock, B. W., Smith, L. W., Schwartje, W. H., and Hanna, R. E. "The Michigan Nuclear Combination Density-Moisture Surface Gage." Michigan State Highway Department Research Report No. 316 (1959).

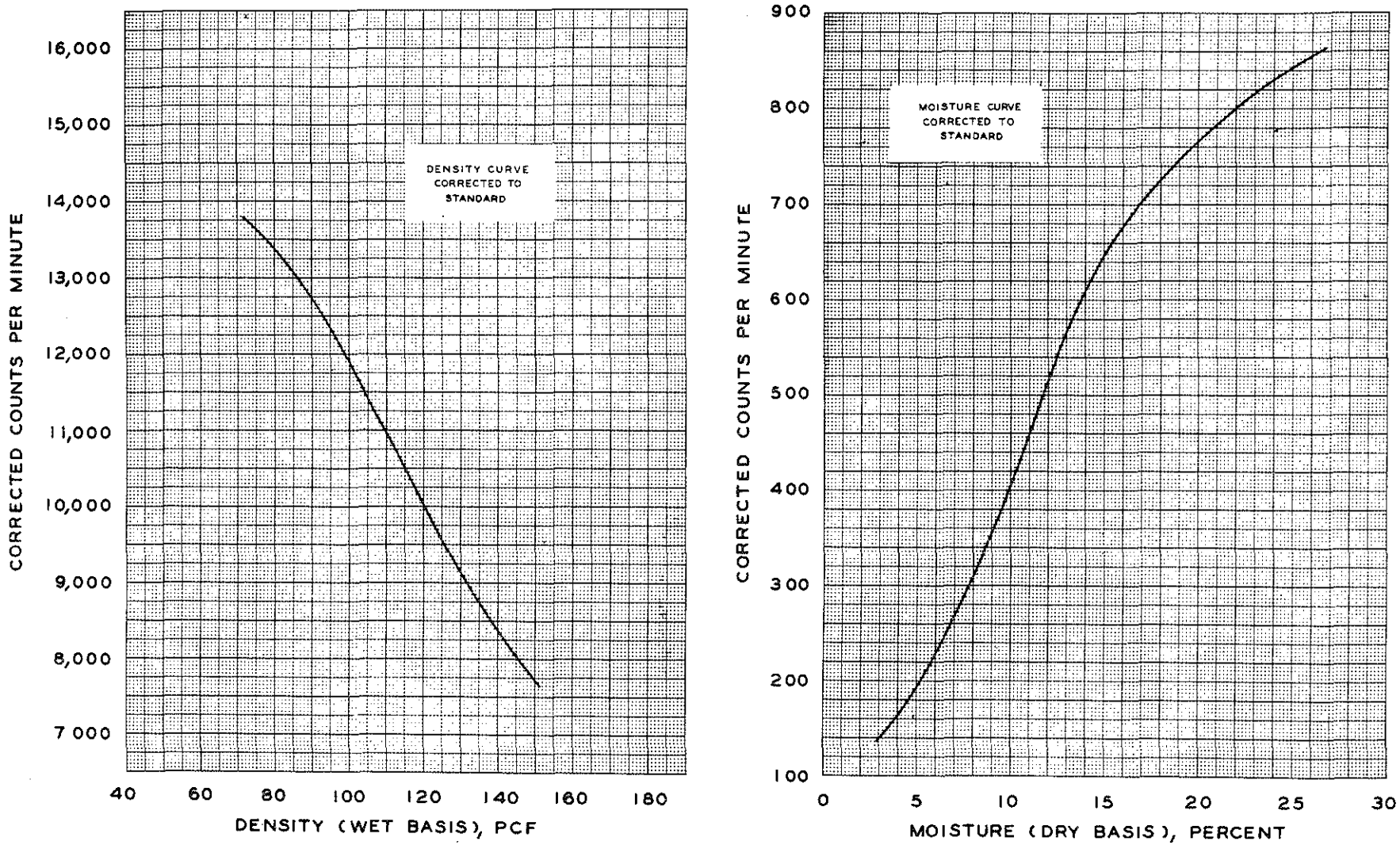

Figure 1. Calibration curves for density and moisture for the Michigan combination nuclear density-moisture gage.

involved has been doubled. It is easy to see that if we counted all day and counted several million counts, we would always have an uncertainty which would never quite be zero. But it would become infinitesimal.

The calibration curve for moisture in Report 316 shows a count rate of 404 counts per minute for a moisture content of 10 percent. Since the square root of 404 is 20.1, this count rate could be written as 404 $\pm$ 20.1 counts per minute, and this would correspond to a range of moisture content from 9.6 to 10.4 percent according to the curve. Counting for two minutes would increase the accuracy of the count rate to 404 $\pm$ 14.2 counts per minute, so that the moisture content could be reported as between 9.7 and 10.3 percent. Here again the ultimate limit of accuracy can be increased indefinitely by taking large enough counts, but there is no point in spending valuable time for this purpose if two-minute counts are sufficiently accurate, as they are, or if factors other than the source contribute to greater inaccuracies, such as human errors in operating a stopwatch, the presence of nonuniformity of soil density or moisture content, erratic behavior of an electronic component, an environmental effect, or some other variable which is not being controlled. Such factors as these form the subjects of the papers which follow.

THE EFFECT OF MOISTURE CONTENT ON CORRELATION BETWEEN NUCLEAR AND CONVENTIONAL DENSITY DETERMINATIONS

Synopsis

Limited statistical data presented do not indicate that soil moisture content per se has any influence on the accuracy of nuclear density determinations, or that nuclear density calibration curves for specific ranges of moisture content will be required.

Statement of the Problem

In the course of several field and laboratory studies with the nuclear gage, certain results obtained seemed to indicate a possible effect of moisture content on the accuracy of density determinations. It appeared on the basis of meager data that the density portion of the gage might be more accurate at some soil moisture contents than at others, or that density calibration curves might have to be established for each of several ranges of moisture content.

Since insufficient data were available from the above-mentioned studies to prove or disprove this hypothesis, the writers undertook to analyze all of the 1959 field results published in Research Report No. 316* for which complete data were recorded, to confirm or refute the alleged moisture-dependency of the accuracy in density determinations.

Method of Analysis

It was found that complete data were available for a total of 124 individual 1959 field determinations of moisture and density. On the basis of conventional measurements, the moisture contents ranged from 4 to 13 percent by weight, dry basis. At the suggestion of L. T. Oehler,

*Pocock, B. W., Smith, L. W., Schwartje, W. H., and Hanna, R. E. "The Michigan Nuclear Combination Density-Moisture Surface Gage." Michigan State Highway Department Research Report No. 316 (1959).

head of physical research, these determinations were arranged in groups as shown in Table 1. The moisture content within each group is "constant," the variation having been held to $\pm$ 0.5 percent of the nearest whole number. As can be seen from the table, however, the range of densities within each group was quite high, and susceptible to statistical determination of the correlation coefficient, r, between nuclear and conventional density determinations within the group.

Obviously, to the extent that statistical prerequisites were met, if the correlation coefficients of all groups were approximately equal, there would have been no effect of moisture content on the accuracy of density determinations. If they were not approximately equal, the accuracy of density determinations would be greatest within the moisture content ranges of the groups having the highest correlation coefficients.

Unfortunately, not all statistical prerequisites were met. The total numbers of cases within each group were too low, with different numbers of cases in different groups. In spite of this, however, it was felt that if any trend existed, it should at least become apparent in the results.

It was decided to base the separation of the 124 cases into groups arbitrarily by their moisture contents as determined by conventional methods rather than by the nuclear method, without assuming that either method is more accurate.

Results of the Analysis

The results of the analysis are shown in Table 2. All correlation coefficients are approximately equal with the exception of one for 13-percent moisture content. Therefore, the question of whether the departure of this correlation coefficient is significant was important for this analysis.

It will be recalled that the groupings were based upon their average moisture contents as determined by conventional means. When, however, as is shown in Table 3 and in the note appended to Table 2, one reestablished the 13-percent moisture content group on the basis of moisture as determined by the nuclear method, the correlation coefficient of nuclear density versus conventional density determinations becomes approximately equal to all the other coefficients.

TABLE 1
1959 TESTS ARRANGED BY MOISTURE CONTENT AS DETERMINED BY CONVENTIONAL METHODS

	Moisture, dry basis, percent by weight		Density, wet basis, pcf		Point Number	Soil Type
	Conventional	Nuclear	Conventional	Nuclear		
4.5 ± 0.5	4.7	4.2	110.8	111.8	200	Sand subbase
	4.3	8.3	117.1	116.9	164	Sand
	4.1	6.0	117.9	116.2	165	Sand
	4.6	4.2	120.8	121.5	201	Sand subbase
	4.9	6.5	127.2	125.4	167	Sand
	4.3	9.0	133.5	132.2	189	Unknown
			r = 0.9904			
6.0 ± 0.5	6.1	6.7	106.9	113.4	103	Sand
	6.4	10.7	107.4	112.1	188	Sand
	5.9	7.4	111.2	113.9	191	Sand subbase
	5.9	5.9	111.5	111.9	57	Sand
	5.9	5.9	113.9	117.0	36	Sand-gravel
	5.6	5.2	114.1	113.9	185	Sand
	6.5	6.5	115.8	116.5	122	Sand
	6.3	6.9	117.3	116.7	206	Sand subbase
	5.8	4.9	117.6	114.5	1	Sand-clay
	5.6	5.8	122.2	129.7	105	Sand
	5.7	8.2	122.7	119.5	11	Sand-clay
	5.9	11.7	123.3	123.5	161	Unknown
	6.5	4.9	127.1	126.1	168	Sand-stone
	6.0	6.7	127.8	127.1	158	Sand-stone
	6.5	7.6	130.6	128.0	37	Sand
	5.7	8.3	136.2	134.8	19	Sand
			r = 0.9249			
7.0 ± 0.5	6.9	6.9	104.1	113.0	61	Sand
	6.5	6.5	115.8	116.5	122	Sand
	7.1	8.1	116.3	118.3	33	Sand-gravel
	7.5	7.9	118.1	121.5	64	Sand
	6.5	6.5	118.3	122.6	66	Sand
	6.9	6.9	119.7	121.3	126	Sand
	6.7	9.9	121.3	121.6	187	Unknown
	6.7	9.9	121.3	121.6	202	Unknown
	7.2	3.6	121.9	122.7	169	Unknown
	6.8	7.4	123.7	118.8	9	Sand-clay
	7.2	7.7	124.1	125.9	209	Sand subbase
	7.3	8.2	124.1	119.0	21	Sand
	7.4	7.4	126.2	127.9	101	Sand
	7.3	9.5	126.5	127.4	193	Sand subbase
	7.2	8.2	127.0	128.3	130	Sand
	6.5	4.9	127.1	126.1	168	Stone-sand
	7.5	7.9	127.9	127.0	171	Sand
	6.8	7.9	128.3	130.0	211	Sand subbase
	6.5	7.6	130.6	128.0	37	Sand
	7.0	8.2	134.8	133.6	108	Sand
			r = 0.8604			
8.0 ± 0.5	8.3	7.4	103.4	99.8	7	Sand-clay
	8.0	10.7	114.2	116.7	157	Clay-stone
	7.6	7.6	115.8	120.2	5	Sand-clay
	7.6	9.4	118.0	119.9	132	Sand
	8.2	8.3	121.9	122.0	142	Sand
	7.7	7.7	122.2	122.6	48	Sand
	7.8	5.7	122.9	124.7	186	Sand-stone
	8.2	6.7	123.5	123.7	204	Sand subbase
	8.0	8.3	123.7	120.2	29	Sand
	7.6	7.2	125.5	126.3	134	Sand
	8.1	8.1	125.5	129.2	49	Sand
	8.1	8.9	125.9	126.0	203	Sand subbase
	8.2	8.2	126.0	126.1	135	Sand
	7.5	7.9	127.9	127.0	171	Sand
	8.4	7.4	128.1	125.9	192	Sand subbase
	8.5	8.4	129.4	127.6	199	Sand subbase
	8.0	9.0	131.7	131.2	174	Unknown
	8.1	7.3	131.7	132.9	208	Sand subbase
	8.3	6.5	131.8	122.2	18	Sand
	8.0	11.0	131.8	129.1	195	Sand subbase
			r = 0.9292			

TABLE 1 (Cont.)
1959 TESTS ARRANGED BY MOISTURE CONTENT AS DETERMINED BY CONVENTIONAL METHODS

	Moisture, dry basis, percent by weight		Density, wet basis, pcf		Point Number	Soil Type
	Conventional	Nuclear	Conventional	Nuclear		
9.0 ± 0.5	9.2	9.2	115.7	119.8	36	Sand-gravel
	8.7	8.6	117.4	121.8	184	Unknown
	9.0	9.9	118.0	122.0	136	Sand
	8.8	9.3	120.6	114.8	95	Sand
	8.8	9.7	124.6	126.0	205	Sand subbase
	8.9	10.0	125.9	126.4	190	Unknown
	9.3	9.9	126.2	126.9	41	Sand
	8.7	9.5	126.4	125.4	159	Sand-clay-gravel
	8.9	9.9	127.9	127.0	76	Sand-gravel
	8.9	7.2	128.8	128.5	207	Sand subbase
	8.5	8.4	129.4	127.6	199	Sand subbase
	9.2	9.9	130.5	128.0	117	Sand
	8.9	9.7	131.2	129.4	98	Sand-gravel
	9.1	10.3	132.2	121.4	86	Sand-gravel
	8.9	9.7	133.1	131.8	129	Sand-clay-silt
	8.8	11.8	138.3	138.2	68	Sand
	8.6	9.1	138.9	133.1	23	Clay-sand
	8.6	12.4	140.5	139.7	149	Sand-silt
			r = 0.8448			
10.0 ± 0.5	10.3	9.9	106.8	111.3	146	Sand
	10.2	10.0	115.5	118.2	138	Sand
	9.8	10.0	118.3	119.0	210	Sand subbase
	9.6	9.6	120.0	118.7	87	Sand
	10.0	10.6	122.5	121.8	17	Sand-clay
	10.1	8.9	125.3	127.7	170	Sand
	10.0	10.8	127.5	126.0	194	Sand subbase
	9.7	10.7	127.7	128.9	125	Sand
	10.3	10.3	129.5	132.1	83	Sand-gravel
	9.9	9.9	131.6	132.2	172	Unknown
			r = 0.9596			
11.0 ± 0.5	11.2	9.1	115.9	119.2	154	Sand
	10.6	10.8	121.1	121.1	52	Sand
	11.3	10.1	122.3	124.7	94	Sand-clay
	10.7	12.5	122.8	118.6	24	Clay-sand
	11.4	10.7	127.6	128.7	198	Sand subbase
	11.5	11.5	129.8	128.0	112	Sand-clay
	10.7	9.9	132.7	129.7	46	Sand-clay
	10.9	10.7	134.6	135.0	156	Unknown
	10.9	12.7	134.9	136.8	140	Clay
	11.1	15.4	139.5	140.2	69	Sand-clay
	11.2	12.1	141.2	141.9	162	Sand gravel
			r = 0.9679			
12.0 ± 0.5	12.4	12.4	121.0	120.9	3	Sand-clay
	11.9	11.8	129.4	130.6	82	Clay
	11.5	11.5	129.8	128.0	112	Sand-clay
	11.9	13.3	133.5	131.9	67	Sand
	12.5	11.3	134.0	139.0	13	Sand-clay
	12.1	12.1	134.0	135.4	78	Sand-gravel
	11.7	13.3	134.3	134.9	113	Sand
	12.5	12.3	134.5	134.8	127	Sand-gravel
	12.5	13.8	134.8	135.9	88	Sand-clay
	11.7	11.5	134.9	135.6	163	Sand-clay
	12.4	12.4	135.1	133.6	131	Sand-clay
	12.0	14.4	137.8	136.7	109	Sand-clay
	11.7	17.7	140.3	139.7	70	Sand-clay
			r = 0.9491			
13.0 ± 0.5	12.6	15.1	123.9	123.9	196	Sand subbase
	13.5	17.7	127.3	135.9	152	Sand
	13.5	13.5	129.4	126.5	27	Sand
	13.4	13.4	130.5	138.8	20	Sand
	12.5	11.3	134.0	139.0	13	Sand-clay
	12.5	12.3	134.5	134.8	127	Sand-gravel
	13.0	13.0	134.8	133.8	45	Sand
	12.9	16.8	134.8	133.9	166	Clay
	13.0	13.0	136.6	137.4	72	Sand-clay
	12.6	12.4	138.9	139.1	155	Sand-clay
			r = 0.6845			

TABLE 2
CONVENTIONAL VERSUS NUCLEAR CORRELATION COEFFICIENTS
ARRANGED BY CONVENTIONAL METHODS
1959 Data

Number of Cases	Moisture, percent	Density Correlation Coefficient, r
6	4-5	0.9904
16	6	0.9249
20	7	0.8604
20	8	0.9292
18	9	0.8448
10	10	0.9596
11	11	0.9679
13	12	0.9491
10	13	0.6845
124 (total)	4-13	0.8965
Note: 1959 data selected at random:		
40 (random)	4.1-15.9	0.9496
Arranged according to nuclear moisture:		
8	13	0.8385

TABLE 3
13-PERCENT CONVENTIONAL MOISTURE CONTENT GROUP
ARRANGED BY NUCLEAR MOISTURE CONTENT

Moisture, dry basis, percent by weight		Density, wet basis, pcf		Point Number	Soil Type
Nuclear	Conventional	Nuclear	Conventional		
12.5	10.7	118.6	122.8	24	Clay-sand
12.7	10.9	136.8	134.9	140	Clay
13.0	13.0	133.8	134.8	45	Sand
13.0	13.0	137.4	136.6	72	Sand-clay
13.3	11.7	134.9	134.3	113	Sand
13.3	11.9	131.9	133.5	67	Sand
13.4	13.4	138.8	130.5	20	Sand
13.5	13.5	126.5	129.4	27	Sand
		r = 0.8385			

Conclusion

The rather sparse statistical data which form the basis for analysis do not support the contention that soil moisture content alone has any noticeable influence on the accuracy of nuclear density determinations when conventional density determinations are used as a control measure. Nuclear density calibration curves for specific ranges of moisture content are not justified by this analysis.

ENVIRONMENTAL EFFECTS ON THE MICHIGAN NUCLEAR GAGE FOR SOILS

Synopsis

The Michigan State Highway Department nuclear gage for soils has been found to be stable under all normal conditions of temperature and humidity.

Although the Michigan State Highway Department's nuclear gage for soils has undergone considerable testing, both in the laboratory and the field, the question has frequently been raised as to just what effects if any on the accuracy of the gage are produced by changes in temperature and humidity. As a corollary to this question, it is desirable to know the limits of temperature and humidity variation within which there is no significant effect, either on the gage or on the scaler.

A contract was therefore entered into with the Abrams Instrument Company of Lansing, providing the use of that company's environmental chamber for a period of six consecutive days. An extra day was provided without cost, to partially compensate for unforeseen technical difficulties with the chamber controls. All testing was conducted by Research Laboratory Division personnel.

Procedure

A testing procedure was worked out in advance calling for density and moisture readings to be taken at 20-deg increments from -10 F to 130 F, at 30, 60, and 90 percent relative humidities. It was felt that two days would be required to obtain readings under all these environments, so the first two days were set aside for readings with both gage and scaler in the chamber. The tests were to be repeated on the third and fourth days with the gage in the chamber but the scaler outside, and the fifth and sixth days were reserved for duplicate gages and scalers under environments shown to have the greatest effect during the first four days.

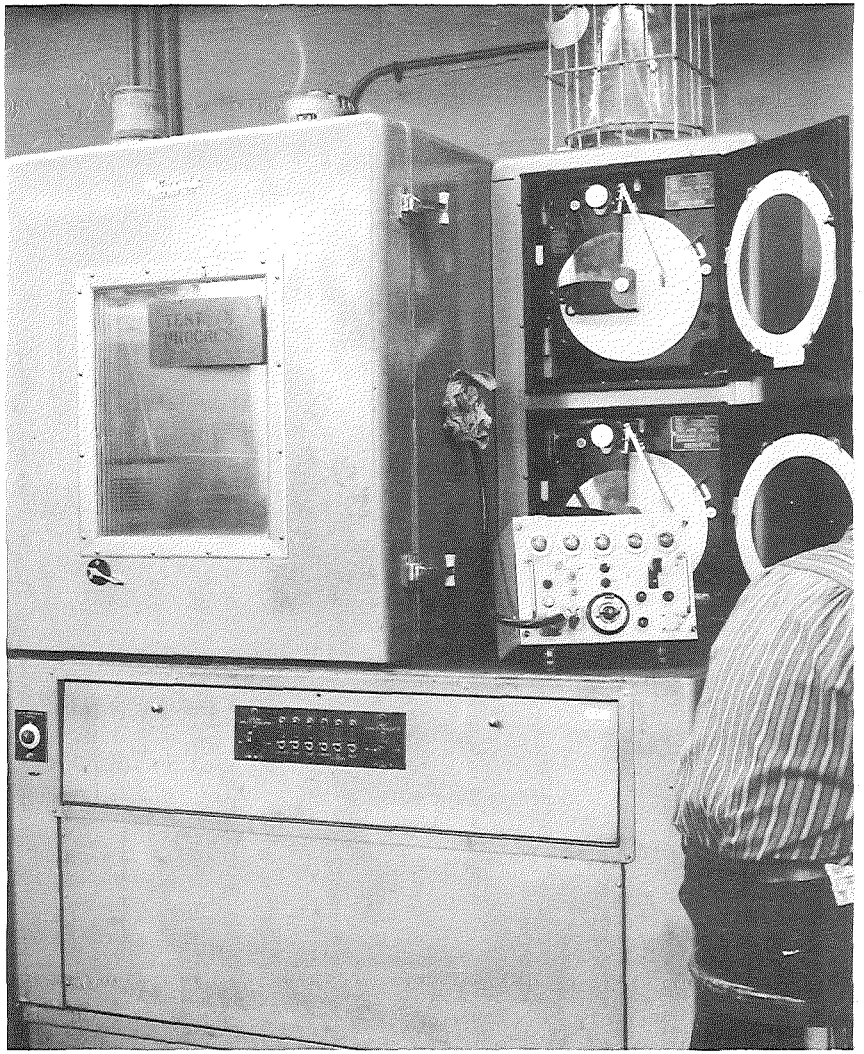

Figure 1. Environmental chamber and controls.

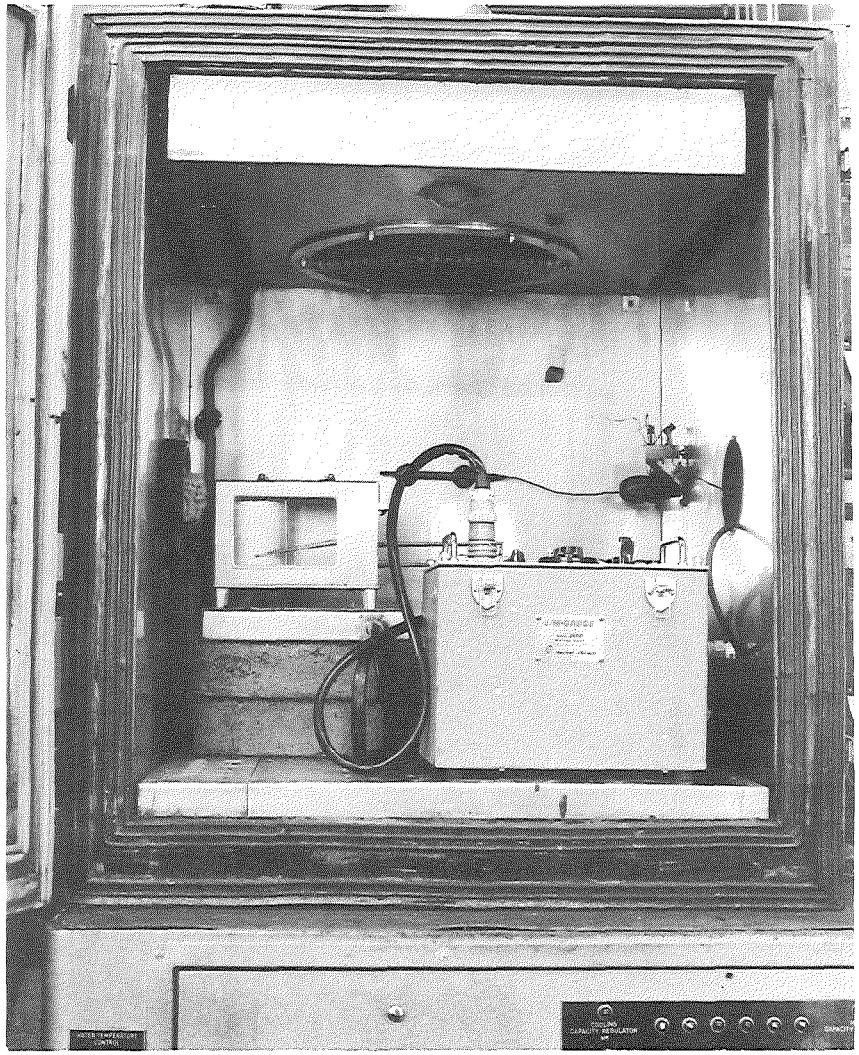

Figure 2. Interior of environmental chamber showing gage resting on concrete block.

Actually, technical difficulties with the chamber controls slowed down the experimental procedure considerably, and the desired temperatures and humidities were not obtained in all instances. However, enough variation in the environment was obtained to justify drawing some conclusions.

All readings were taken using five Portable Scalers Models 2800 (Serial Nos. 156, 157, 169, 170, 257), with the gage resting on a concrete block sealed with epoxy resin. Geometry was maintained as constant as possible throughout the experiment. Figs. 1 and 2 show the experimental setup used.

Results

As shown in Fig. 3, Scaler 170 and Gage 3 gave essentially identical density count rates at 30 and 50 F at all relative humidities attained from 36 to 93 percent, with both gage and scaler in the chamber. When Scaler 170 was removed from the chamber, the count rate dropped because of the change in geometry, but remained statistically constant at 90 F and again at 130 F, from 36 to 90 percent relative humidity at both temperatures. The graph also shows the poor response of Scaler 156, which was traced to defective components in its power supply. Two condensers and two rectifiers of the semiconductor diode type had to be replaced in this scaler. These electronic components should normally withstand the environments used, and are assumed to have been defective in manufacture.

Fig. 3 also shows moisture count rates at various temperatures plotted against relative humidity. It will be noted that Scaler 170 used in the chamber with Gage 3 at 30 F reflects the increase in relative humidity from 31 to 90 percent. The fact that this is not the case at 50 F, or with the scaler used outside the chamber at 90 F, is not significant, however, since all three curves are within acceptable statistical limits at these low count rates. It can be said, therefore, that no environmental effect is indicated by these curves. In this figure also, the relatively poor response of Scaler 156 is shown by the moisture curves obtained with this scaler used with Gage 3.

Fig. 4 shows varying relationships of moisture count rate, relative humidity, and temperature with scalers and gages inside and outside the environmental chamber. "Outside" in these tests meant resting an instrument on the concrete floor just outside the chamber.

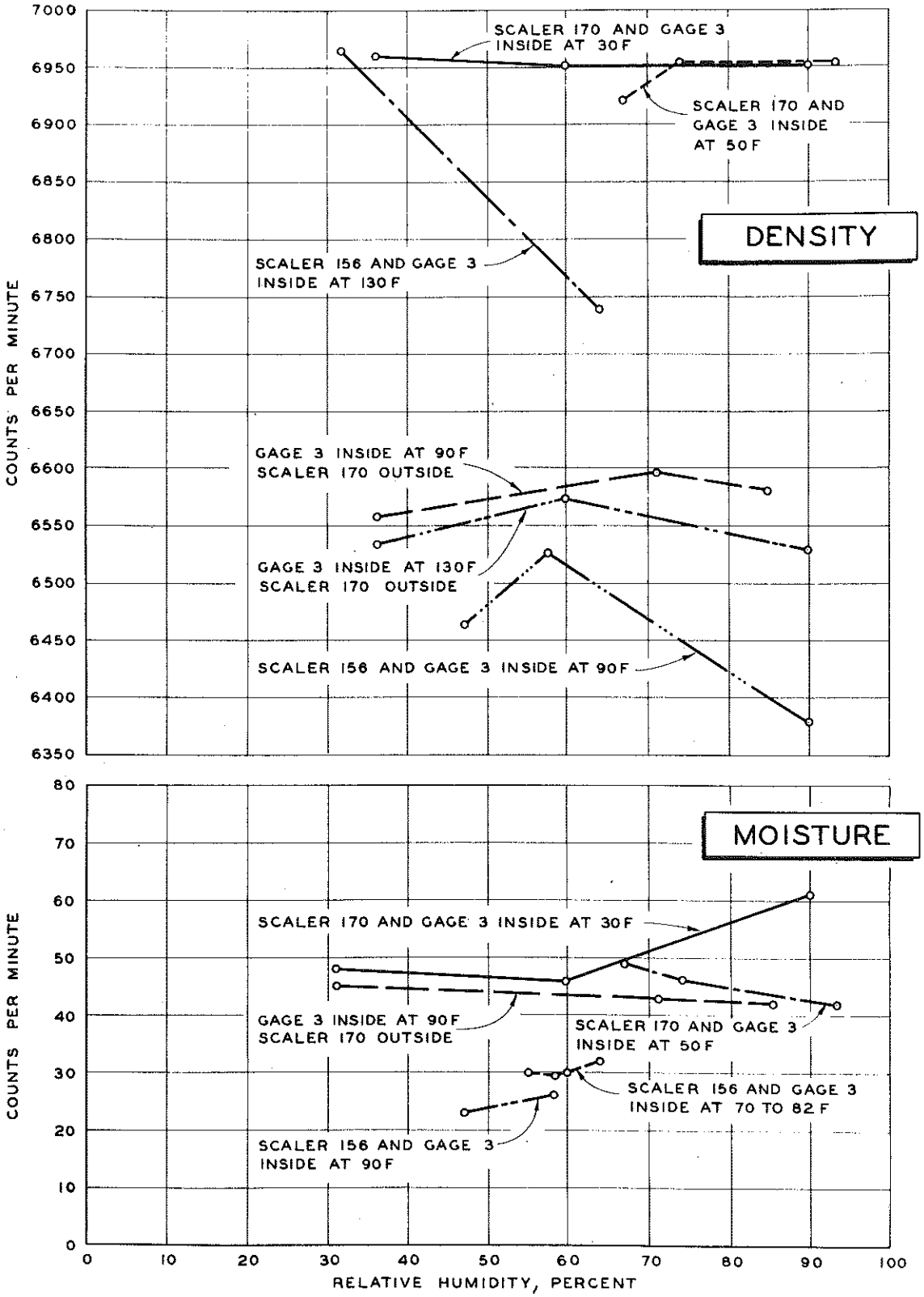

Figure 3. Relative humidity versus count rate at various temperatures, with instruments inside and outside environmental chamber.

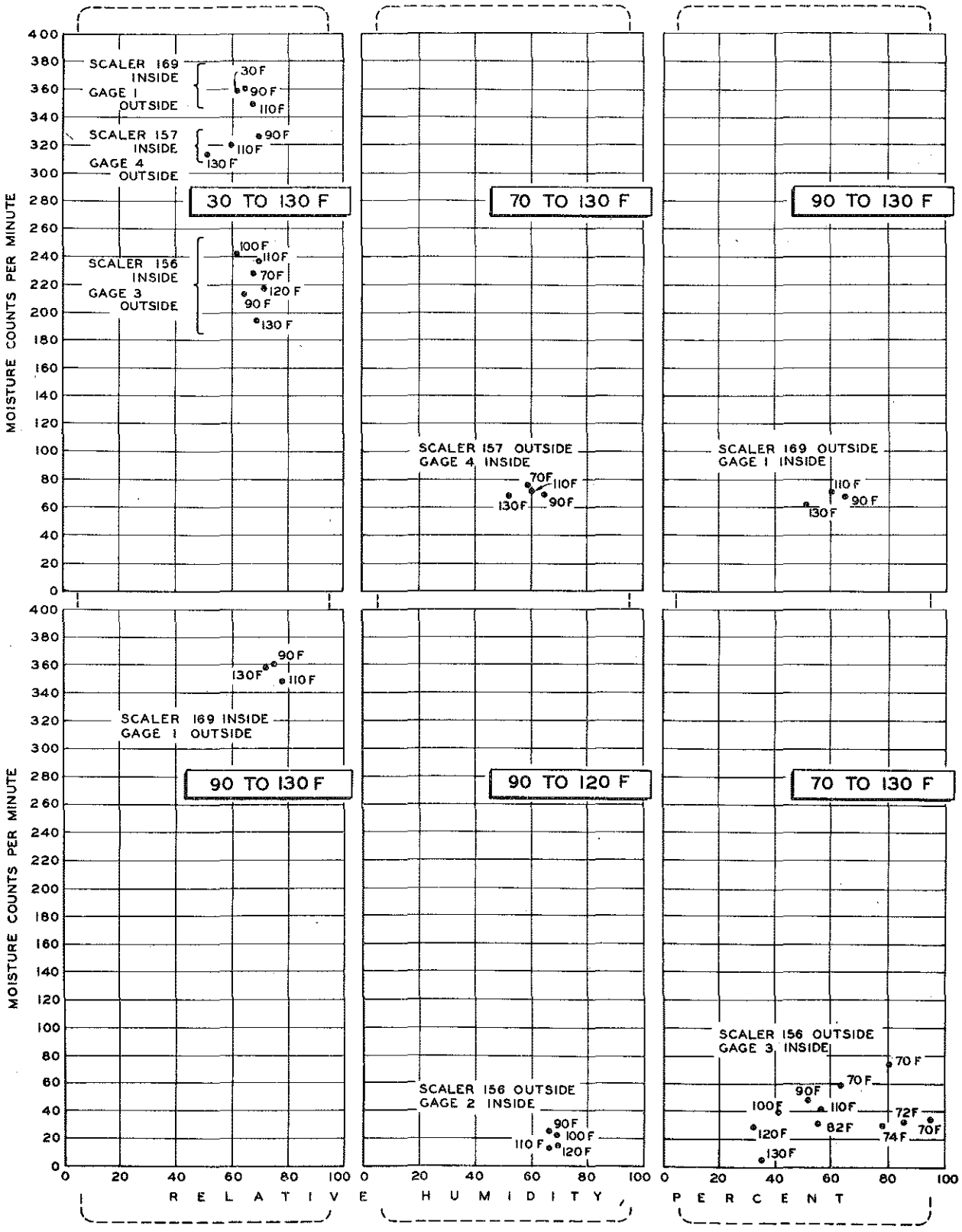

Figure 4. Varying relationships of moisture count rate, relative humidity, and temperature, with instruments inside and outside environmental chamber.

In the first graph of Fig. 4, showing those tests with scalers inside and corresponding gages outside, the results for Scaler 156 are outside statistical limits, indicating some scaler malfunction. The results for Scaler 169 with Gage 1, and for Scaler 157 with Gage 4, are within acceptable statistical limits and indicate no effect on these scalers for temperatures of from 90 through 130 F, with relative humidity from 52 to 68 percent.

The next graph shows the moisture count rate vs relative humidity plot for Scaler 157 outside the chamber and Gage 4 inside. It is noted that these results are well within statistics, and there is no effect of environment on the moisture response of Gage 4 from 70 to 130 F, between 52 and 65 percent relative humidity.

TABLE 1
PERFORMANCE OF SCALERS AND GAGES*

	No.	Density	Moisture
SCALER	156	High voltage power supply became defective: glow tubes "froze" at -10 F	High voltage power supply became defective; glow tubes "froze" at -10 F
	157	OK at room temperature	OK 90 to 130 F, 52 to 68 percent RH
	169	OK at room temperature; glow tubes "froze" between 0 and 10 F	OK 90 to 130 F, 52 to 68 percent RH; glow tubes "froze" between 0 and 10 F
	170	OK 30 to 50 F, 36 to 93 percent RH; glow tubes "froze" between 0 and 10 F	OK 30 to 50 F, 31 to 90 percent RH; glow tubes "froze" between 0 and 10 F
	257	OK at room temperature	OK at room temperature
GAGE	1	OK at room temperature	OK 90 to 130 F, 52 to 65 percent RH
	2	OK at room temperature	OK at room temperature and 70 to 130 F
	3	OK 30 to 50 F, 90 to 130 F, and 36 to 93 percent RH	OK 70 to 130 F, 31 to 93 percent RH; became defective at 130 F, at 36, 60, and 90 percent RH
	4	OK at room temperature	OK 70 to 130 F, 52 to 65 percent RH

* Gage 5 not included.

The third graph for Scaler 169 outside the chamber and Gage 1 inside, shows essentially the same results as the second graph. No effect of environment is noted on the moisture response of Gage 1 from 90 to 130 F between 52 and 65 percent relative humidity.

No effect of environment is noted in the fourth graph on the moisture response of Scaler 169 from 90 to 130 F between 62 and 68 percent relative humidity.

The last two graphs show that Scaler 156 was just as defective in moisture response as it was in density response.

All the experimental results are listed in Table 1, for all five scalers and four gages. This table indicates that all of the three scalers taken to low temperatures became defective near zero F in that their glow tubes "froze" and they ceased counting, both for density and for moisture. This was entirely temporary, however, and with the exception of Scaler 156 they operated normally when the temperature was raised again. As mentioned above, Scaler 156 was discovered to have a defective high-voltage power supply, and this was repaired later.

Tests included in the experiment and presented in summary form in Table 1 show that at least two scalers operated efficiently up to 130 F within normal ranges of relative humidity, and a third scaler was satisfactory between 30 and 50 F and between 31 and 93 percent humidity.

The single gage tested under all conditions that time permitted operated normally between 30 and 130 F and from 36 to 93 percent relative humidity for density. It also operated normally for moisture from 30 to 90 F and from 31 to 93 percent humidity, but was abnormal at 130 F from 36 to 90 percent humidity. This is interpreted as being caused by a temperature-sensitive transistor used in the preamplifier circuit.

Three other gages operated perfectly for moisture counting up to 130 F and 65 percent humidity.

Conclusions

Although the experimental data were not complete for all five gages or for all five scalers, actually representing a random sampling of the ten units involved, the following conclusions appear justified from the evidence cited:

1. The nuclear gages designed by the Michigan State Highway Department are reliable between 30 to 130 F and from 30 to 90 percent relative humidity, except for the possibility that an occasional temperature-sensitive transistor may give trouble at 130 F in its moisture section. Any such trouble is immediately apparent.

2. The scalers used are not reliable at low temperatures (zero and below), because their glow tubes "freeze" and the scalers cease to count. This condition is transitory and appears and disappears suddenly. The scaler either works or it doesn't work.

3. No environmental effect was noted either on the gage or on the scaler which was progressive in nature. Response was constant in all environments.

4. The Michigan State Highway Department nuclear gage may be expected to be stable under all normal operating conditions of temperature and humidity.

VARIABILITY OF TEST RESULTS WITH THE NUCLEAR GAGE UNDER CONSTANT LABORATORY CONDITIONS

Synopsis

Gage instabilities reported from the field are caused by factors external to the gage. Variations in positioning the gage and nonuniformity of soil density are suspected as prime factors. The exact effect of soil type (chemical identity and physical state) requires further study. Such an effect could probably be reduced through addition of discriminator circuitry, the use of carefully prepared standards, etc.

As part of an overall evaluation of the nuclear gage, a study was undertaken to investigate the stability of the gage under controlled laboratory conditions, in order to pinpoint any discernible abnormal variation in test results as being caused by electronic or mechanical instabilities within the gage or scaler, by the type of source employed, or by some factor external to the scaler-gage system.

Twelve different experiments were conducted on scaler-gage systems. Each experiment was designed to check operating characteristics of one section of the system independently of other sections. The twelve experiments were then compiled into six groups for statistical analysis (Table 1).

Experiments 1 and 6 each consisted of 300 tests. The remaining experiments consisted of 200 tests each. In all, the results of 2600 tests were analyzed.

Description of Experiments

Details of the following experiments are given in the Appendix:

Group 1 (Experiments 1 and 6). The purpose of these two experiments was to determine stability of the source and detector used in the density section of the gage under normal operating conditions.

TABLE 1
EVALUATION STUDY OF SIX NUCLEAR MEASURING SYSTEMS

	Group No.	Experiments To Be Compared	Source	Scaler No.	Gage No.
Density Determinations With Tube 90NB	1*	1	Ra^{226}-Be	192	
		6	Cs^{137}	192	
	2	2	Ra^{226}-Be	2800 (169)	2
		3	Ra^{226}-Be	183	2
		4	Cs^{137}	2800 (169)	2
		5	Cs^{137}	2800 (169)	2
	3	2	Ra^{226}-Be	2800 (169)	2
		11	Ra^{226}-Be	2800 (169)	3***
Moisture Determinations With Tube $B^{10}F_3$	4	7	Ra^{226}-Be	192	
		10	Ra^{226}-Be	192**	
	5	8	Ra^{226}-Be	2800 (169)	3
		9	Ra^{226}-Be	183	3
	6	8	Ra^{226}-Be	2800 (169)	3
		12	Ra^{226}-Be	2800 (169)	3***

* In statistical analysis, collected data from Experiment 1 were more consistent than those from Experiment 6. In all other groups, collected data were equally reliable for all experiments.

** Plus transistor preamplifier.

*** Intermittent recording. All others continuous recording.

The tests were made with the Amperex 90NB tube and source placed inside a lead counting chamber to insure minimum background variation. Ultrascaler 192 was used.

Experiment 1 was made using Ra^{226}-Be as the gamma source; this is a multi-energy source. Experiment 6 was conducted with the single-energy gamma source Cs^{137}. A total of 400,000 counts was recorded in each experiment.

Group 2 (Experiments 2, 3, 4, and 5). The purpose of these experiments was to compare the stability of laboratory Scaler 183 with several portable scalers (Model 2800), when counting gamma rays. The tests were made using Ra^{226}-Be and Cs^{137} as gamma sources, separately. These tests were also intended to determine whether the density section of the gage was more stable when counting gamma backscatter from the multi-energy or single-energy sources.

The experiments were made with Gage 2 on a wood standard. The test combinations were as follows:

Experiment No.	Source in Gage 2	Scaler Model
2	Ra^{226}-Be	2800 (169)
3	Ra^{226}-Be	183
4	Cs^{137}	183
5	Cs^{137}	2800 (169)

Group 3 (Experiments 2 and 11). The purpose of these experiments was to determine the minimum warm-up time required by the system and the effect of turning the scaler off at the end of each field density test, on the system's stability. The comparison was made between Experiment 2, where the tests were run continuously with the portable scaler on, and Experiment 11 where the portable scaler was turned off for a few seconds after each test.

Group 4 (Experiments 7 and 10). The purpose of these experiments was to check the stability of the three-stage transistor preamplifier used in conjunction with the $B^{10}F_3$ slow neutron detectors in the gage. The tests were made with $B^{10}F_3$ detectors and a Ra^{226}-Be source on a paraffin block. In Experiment 7 the detector was fed directly to the preamplifier input of Ultrascaler 192; input sensitivity was 2 mv. In Experiment 10 the detector was connected to the transistorized preamplifier whose output was fed to Ultrascaler 192; input sensitivity of the scaler was 100 mv.

Group 5 (Experiments 8 and 9). The purpose of these experiments was to compare the stability of the portable scaler to that of Scaler 183, when used for moisture determinations. Tests were made with Gage 3 on a standard.

Group 6 (Experiments 8 and 12). The purpose here was the same as for Group 3, to determine warm-up time. Group 6 differed from Group 3 in being designed for moisture tests.

Discussion of Results

To check statistically for consistency within the experimental data, the series of measurements for each experiment were divided into 10 subgroups. The sample average ($\bar{x}$), the standard deviation (s), and the number of observations (n) are presented in the appendix. Also, the sample variance ratio $\frac{s^2}{\bar{x}}$ and the 99-percent probability limits (control limits) within which the measurements are supposed to fall, are given for each experiment.

The control limits can be given a physical meaning in terms of density and moisture units for each gage, using as an example the calibration curves (Figs. 1 and 2) of Research Report No. 316. The slope of the density curve is 280 counts per 2 min per pcf. The slope of the moisture curve is 100 counts per 2 min per 1-percent moisture.

If one now considers the upper and lower control limits of the individual counts in each experiment in terms of density or moisture unit, and the percentage of the total number of counts in each experiment falling outside the control limits, the results are as shown in Table 2.

The control charts for each experiment indicate that the Chi-Square ratio test $\frac{s^2}{\bar{x}}$ shows all groups were within the Chi-Square limits. This indicates that all the results were obtained under well-controlled laboratory conditions. Control of these conditions would have been perfect if less than 1-percent "defective" counting had occurred in all 12 experiments.

TABLE 2
DEPARTURES FROM CONTROL LIMITS
FOR TWELVE EXPERIMENTS

Experiment Number	Average ($\bar{x}$), counts per 2 min	Standard Deviation (s), counts per 2 min	Coefficient of Variation (V) $=\frac{s}{\bar{x}}$ percent
1	21,664	124.4	0.57
2	21,856	148.0	0.68
3	22,431	151.2	0.67
4	24,956	156.9	0.63
5	24,276	157.4	0.65
6	22,229	171.6	0.77
7	426	15.4	3.62
8	1,538	47.5	3.09
9	1,187	35.6	3.00
10	551	18.6	3.38
11	23,307	146.3	0.63
12	1,581	52.2	3.30

In the present analysis, a knowledge of the relative variation of the nuclear measuring system is informative and valuable, especially in the presence of the average, ($\bar{x}$), and the standard deviation, (s). Therefore, the relative variability or coefficient of variation (V) was computed from the equation $V = \frac{100s}{\bar{x}}$ and is presented in Table 3. The tables and statistical treatments of the data from individual experiments are given in the Appendix.

TABLE 3
AVERAGE STANDARD DEVIATION
AND COEFFICIENT OF VARIATION
FOR TWELVE EXPERIMENTS

Experiment No.	Control Limit Deviation from Average, pcf	Individual Counts Outside Control Limits	Total Defective Counting, Percent
1	±1.15	10	3.3
2	±1.36	8	4.0
3	±1.38	0	0.0
4	±1.44	0	0.0
5	±1.43	0	0.0
6	±1.37	8	2.0
7	±0.40	6	3.0
8	±1.01	4	2.0
9	±0.89	0	0.0
10	±0.48	0	0.0
11	±1.35	0	0.0
12	±1.35	6	3.0

Conclusions

1. The results of the statistical analysis of Group 1 indicated that the Amperex 90NB detector, used in the density section of the gage, showed slightly greater stability when counting gamma rays from the Ra^{226}-Be source than when counting gamma rays from the Cs^{137} source.

2. The results of Group 2 showed no significant difference in stability among the four possible combinations. This would indicate that the portable scaler is as stable as the laboratory scaler. As far as gamma backscatter from a constant uniform geometry is concerned, the detector has essentially the same efficiency for both the Cs^{137} and the Ra^{226}-Be sources. This means that some factor which had been assumed constant, such as background, changed during the tests of Group 1. On the basis of these results, it would be preferable to use the Ra^{226}-Be source since it has a much longer half-life than Cs^{137}.

3. Group 4 indicated no significant difference in moisture testing between laboratory and portable scalers.

4. The results of Groups 3 and 6 indicated that operating the scaler intermittently--turning it off and on after each observation--produced no measurable effect on stability as compared with operating the scaler continuously.

5. The results of Group 4 indicate that the transistorized preamplifier in the moisture section of the gage is as stable as the sensitive preamplifier in the laboratory Ultrascaler 192.

6. In general, the results of the various experiments indicate that any instabilities caused by electronic or mechanical components within the scaler-gage system are negligible under laboratory conditions. The greatest variations for any combination of components were $\pm$ 1.36 pcf for density and $\pm$ 1.35 percent for moisture. These variations represent the limits within which the gage should function 99 percent of the time on materials of a given type possessing uniform density and moisture content, as shown in the calibration curves published previously in Research Report No. 316.

7. The results strongly indicate that larger variations reported from the field are due to something external to the gage. Two of the most logical explanations are 1) variations in positioning of the gage, and 2) greater gage sensitivity to material types than had been antici-

pated. The first possibility involves nonuniformity in density and/or moisture content of the materials tested. This nonuniformity might be a) abrupt and occurring within radii of a few inches, in which case it could be detected by rotation of the gage, or b) gradual and occurring within radii of a few feet, in which case it would be detected by moving the gage laterally.

The second explanation suggests the possible need for more than a single density calibration curve, perhaps two or three depending on the type of material. By "type" is meant both the chemical identity and the physical state. A direct dependency exists between chemical identity (atomic number) and density, although this is not a linear relationship, and this situation operates in favor of the gage, which is sensitive to the atomic numbers of materials which in turn help to establish their densities. Physical state includes particle size, plus size and content of the spaces between particles; and it includes the nature of the water which is present--whether bound as water of crystallization or free to be evaporated by conventional means. The gage detects and reports all water, regardless of physical state, and it is obvious that all matter including water contributes to density.

It remains to be pointed out that the Michigan nuclear gage appears to be a very sensitive and rapid device for detecting and measuring the extent of nonuniformity of soil density and moisture content. Such knowledge could go a long way toward eliminating one possible cause of pavement failure.

APPENDIX

Notation

s = standard deviation

$\bar{x}$ = sample average

$\frac{s^2}{\bar{x}}$ = sample-variance ratio (or Chi-Square ratio)

$\frac{s}{\bar{x}}$ = coefficient of variation or relative variability

n = total readings per subgroup

$\bar{\bar{X}}$ = Grand Mean

UCL = upper control limit

LCL = lower control limit

Experiment No. 1
Density Determinations from Subgroups 1 through 10

$n = \frac{300}{10} = 30$ readings per subgroup.

Ultrascaler 192: 100 x 400 counts, 900 v operating voltage.
Lead counting shield contained 5 mc Ra^{226}-Be source and
Amperex 90 NB detector.
Readings recorded in minutes, then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
1	21,788.37	84.88	0.33*
2	21,689.53	112.40	0.58
3	21,665.90	120.32	0.67
4	21,712.63	128.64	0.76
5	21,648.70	93.51	0.40*
6	21,544.23	101.81	0.48
7	21,716.43	124.67	0.72
8	21,598.63	93.02	0.40*
9	21,597.80	84.23	0.33*
10	21,684.87	97.38	0.44*

Grand Mean $\bar{\bar{X}} = 21,664$
1-percent statistical error = $\pm$ 379

* 2 individual readings fell outside control limits.

Control limits at 99-percent probability level
a. For individual readings: UCL = 22,043; LCL = 21,285
b. For subgroups (n = 30): 21,664 $\pm$ 69
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.71; LCL = 0.49

Experiment No. 2
Density Determinations from Subgroups 11 through 20

$n = \frac{200}{10} = 20$ readings per subgroup.

Portable Scaler 2800: 2-min time count, 900 v operating voltage.

Gage 2 (placed on standard for gamma counting): 5 mc Ra^{226}-Be source and Amperex 90 NB detector.

Readings recorded in counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
11	21,802.45	108.04	0.54
12	21,837.20	151.71	1.04
13	21,853.90	163.75	1.22
14	21,859.50	220.84	2.23*
15	21,855.10	212.11	2.06*
16	21,863.75	107.12	0.52
17	21,938.75	134.40	0.82
18	21,840.55	118.12	0.64
19	21,860.65	135.32	0.84
20	21,846.55	129.19	0.76

Grand Mean $\bar{\bar{X}} = 21,856$

1-percent statistical error = $\pm$ 381

* 4 individual readings fell outside control limits.

Control limits at 99-percent probability level

a. For individual readings: UCL = 22,237; LCL = 21,475
b. For subgroups (n = 20) = 21,856 $\pm$ 85
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90; LCL = 0.40

Experiment No. 3
Density Determinations from Subgroups 21 through 30

$n = \frac{200}{10} = 20$ readings per subgroup.

Scaler 183: 100 x 256 counts precount, 900 v operating voltage.

Gage 2 (placed on standard for gamma counting): 5 mc Ra^{226}-Be source and Amperex 90 NB detector.

Readings recorded in minutes and then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
21	22,499.45	149.54	0.99
22	22,507,60	122.84	0.67
23	22,498.30	139.84	0.87
24	22,498.90	143.88	0.92
25	22,345.55	137.60	0.85
26	22,380.50	114.04	0.58
27	22,345.45	125.53	0.71
28	22,443.45	128.80	0.74
29	22,413.25	181.52	1.47
30	22,379.60	118.92	0.63

Grand Mean $\bar{\bar{X}} = 22,431$

1-percent statistical error = $\pm$ 386

Control limits at 99-percent probability level

a. For individual readings: UCL = 22,817; LCL = 22,045
b. For subgroups (n = 20): 22,431 $\pm$ 86
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90; LCL = 0.40

Experiment No. 4
Density Determinations from Subgroups 31 through 40

$n = \frac{200}{10} = 20$ readings per subgroup.

Scaler 183: 100 x 256 counts precount, 900 v operating voltage.

Gage 2 (placed on standard for gamma counting): 5 mc Cs^{137} source and Amperex 90 NB detector.

Readings recorded in minutes and then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
31	24,975.15	112.15	0.50
32	24,915.95	179.49	1.29
33	24,951.45	169.33	1.15
34	24,954.65	141.82	0.81
35	24,040.20	150.98	0.91
36	24,939.40	131.00	0.69
37	24,939.65	137.56	0.76
38	24,908.80	202.49	1.64
39	24,950.45	153.05	0.94
40	24,983.85	125.68	0.63

Grand Mean $\bar{X} = 24,956$

1-percent statistical error = $\pm$ 404

Control limits at 99-percent probability level

a. For individual readings: UCL = 25,360; LCL = 24,502
b. For subgroups (n = 20): 24,956 $\pm$ 90
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90, LCL = 0.40

Experiment No. 5
Density Determinations from Subgroups 41 through 50

$n = \frac{200}{10} = 20$ readings per subgroup.

Portable Scaler 2800: 2-min time count, 900 v operating voltage.
Gage 2 (placed on standard for gamma counting): 5 mc Cs^{137} source and Amperex 90 NB detector.
Readings recorded in minutes and then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
41	24,303.00	164.61	1.11
42	24,255.80	164.35	1.11
43	24,369.60	206.51	1.75
44	24,345.15	146.99	0.89
45	24,307.40	117.44	0.57
46	24,272.20	133.58	0.74
47	24,260.30	129.12	0.69
48	24,229.05	129.93	0.70
49	24,209.40	151.01	0.94
50	24,208.65	120.24	0.60

Grand Mean $\bar{\bar{X}} = 24,276$
1-percent statistical error = $\pm$ 401

Control limits at 99-percent probability level
a. For individual readings: UCL = 24,677; LCL = 23,875
b. For subgroups (n = 20): 24,276 $\pm$ 90
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90, LCL = 0.40

Experiment No. 6
Density Determinations from Subgroups 51 through 60

$n = \frac{300}{10} = 30$ readings per subgroup.

Ultrascaler 192: 100 x 400 counts precount, 900 v operating voltage.
Lead counting shield contained 5 mc Cs^{137} source and Amperex 90 NB detector.
Reading recorded in minutes and then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
51	22,221.40	142.26	0.91
52	22,197.43	94.66	0.40*
53	22,263.70	92.07	0.38*
54	22,286.10	130.48	0.76
55	22,263.90	90.16	0.37*
56	22,180.13	131.21	0.78
57	22,156.60	121.98	0.67
58	22,289.10	120.61	0.65
59	22,195.86	89.75	0.36*
60	22,297.30	149.64	1.00

Grand Mean $\bar{\bar{X}} = 22,229$
1-percent statistical error = $\pm$ 384

* Two individual readings were outside the control limits.

Control limits at 99-percent probability level
a. For individual readings: UCL = 22,613; LCL = 21,845
b. For subgroups (n = 30): 22,227 $\pm$ 70
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.71, LCL = 0.49

Experiment No. 7
Moisture Determinations from Subgroups 61 through 70

$n = \frac{200}{10} = 20$ readings per subgroup.

Ultrascaler 192 using preamplifier stage: 10 x 100 counts precount, 1350 v operating voltage, 2 mv sensitivity.

Gage 3 (placed on standard for neutron counting): 5 mc Ra^{226}-Be source and $B^{10}F_3$ slow neutron detector.

Readings recorded in minutes and then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
61	417.45	17.76	0.76
62	422.20	12.94	0.40
63	431.00	11.18	0.29*
64	416.70	17.10	0.70
65	426.94	12.33	0.36*
66	430.26	15.17	0.53
67	428.71	12.95	0.39
68	424.57	15.15	0.54
69	436.92	10.43	0.25*
70	426.27	15.58	0.57

Grand Mean $\bar{\bar{X}} = 426$

1-percent statistical error = $\pm$ 53

* 2 readings fell outside the control limits

Control limits at 99-percent probability level .

a. For individual readings: UCL = 479, LCL = 373
b. For subgroups (n = 20): 426 $\pm$ 12
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90, LCL = 0.40

Experiment No. 8
Moisture Determinations from Subgroups 71 through 80

$n = \frac{200}{10} = 20$ readings per subgroup.

Portable Scaler 2800: 2-min time count, 1350 v operating voltage
Gage 3 (placed on standard for neutron counting), 5 mc Ra^{226}-Be source and $B^{10}F_3$ slow neutron detector.
Reading recorded in counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
71	1491.70	39.81	1.06
72	1517.55	43.51	1.25
73	1521.55	27.45	0.49
74	1532.55	28.44	0.53
75	1574.05	53.18	1.80
76	1549.35	60.14	2.33*
77	1532.85	33.77	0.74
78	1552.80	25.83	0.43
79	1549.95	38.87	0.97
80	1560.05	50.53	1.64

Grand Mean $\bar{\bar{X}} = 1538$
1-percent statistical error = $\pm$ 101

* 4 readings fell outside the control limits.

Control limits at 99-percent probability level
a. For individual readings: UCL = 1639, LCL = 1437
b. For subgroups (n = 20): 1538 $\pm$ 23
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90, LCL = 0.40

Experiment No. 9
Moisture Determinations from Subgroups 81 through 90

$n = \frac{200}{10} = 20$ readings per subgroup.

Scaler 183: 10 x 128 counts precount, 1400 v operating voltage.

Gage 3 (placed on standard for neutron counting): 5 mc Ra^{226}-Be source and $B^{10}F_3$ slow neutron detector.

Readings recorded in minutes and then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
81	1203.45	37.90	1.19
82	1210.70	27.31	0.62
83	1194.35	29.74	0.74
84	1187.60	34.00	0.97
85	1218.25	24.06	0.48
86	1185.55	24.41	0.50
87	1179.95	26.38	0.59
88	1167.40	26.85	0.62
89	1157.05	37.22	1.20
90	1166.00	28.80	0.70

Grand Mean $\bar{\bar{X}} = 1187$

1-percent statistical error = $\pm$ 89

Control limits at 99-percent probability level

a. For individual readings: UCL = 1276, LCL = 1098

b. For subgroups (n = 20): 1187 $\pm$ 20

c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90, LCL = 0.40

Experiment No. 10
Moisture Determinations from Subgroups 91 through 100

$n = \frac{200}{10} = 20$ readings per subgroup.

Ultrascaler 192: 10 x 100 counts precount, 1450 v operating voltage, 100 mv sensitivity with gage pre-amplifier.

Gage 3 (placed on standard for neutron counting): 5 mc Ra^{226}-Be source and $B^{10}F_3$ slow neutron detector.

Readings recorded in minutes and then converted to counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
91	549.70	15.16	0.42
92	559.35	18.93	0.64
93	550.05	18.72	0.64
94	555.35	18.50	0.62
95	545.10	16.66	0.51
96	557.85	19.34	0.67
97	552.00	17.92	0.58
98	550.55	19.73	0.71
99	547.75	18.13	0.60
100	544.65	16.39	0.49

Grand Mean $\bar{\bar{X}} = 551$

1-percent statistical error = $\pm$ 60

Control limits at 99-percent probability level

a. For individual readings: UCL = 611, LCL = 491
b. For subgroups (n = 20) = 551 $\pm$ 13
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90, LCL = 0.40.

Experiment No. 11
Density Determinations from Subgroups 101 through 110

$n = \frac{200}{10} = 20$ readings per subgroup.

Portable scaler 2800: 2-min time count, 900 v operating voltage, battery-operated.

Gage 3 (placed on standard for gamma counting): 5 mc Ra^{226}-Be source and Amperex 90 NB detector.

Readings recorded in counts per 2 min (scaler turned off a few seconds after each test).

Subgroup	Sample Average, $\overline{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\overline{x}}$
101	23,198.20	123.73	0.66
102	23,245.45	143.72	0.89
103	23,383.15	105.28	0.47
104	23,302.50	167.26	1.20
105	23,333.95	133.47	0.76
106	23,353.60	176.03	1.33
107	23,335.40	115.80	0.57
108	23,305.55	151.39	0.98
109	23,343.35	108.26	0.50
110	23,271.55	120.88	0.63

Grand Mean $\overline{X} = 23,307$
1-percent statistical error = $\pm$ 393

Control limits at 99-percent probability level

a. For individual readings: UCL = 23,700; LCL = 22,914
b. For subgroups (n = 20): 23,307 $\pm$ 88
c. For $\frac{s^2}{\overline{x}}$ chart: UCL = 1.90, LCL = 0.40

Experiment No. 12
Moisture Determinations from Subgroups 101 through 110

$n = \frac{200}{10} = 20$ readings per subgroup.

Portable Scaler 2800: 2-min time count, 1400 v operating voltage, battery-operated

Gage 3 (placed on standard for neutron counting): 5 mc Ra^{226}-Be source and $B^{10}F_3$ slow neutron detector.

Readings recorded in counts per 2 min.

Subgroup	Sample Average, $\bar{x}$, counts per 2 min	Standard Deviation, s, counts per 2 min	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
111	1,467.05	34.40	0.81
112	1,520.40	29.57	0.58
113	1,541.85	61.50	2.45*
114	1,586.10	44.79	1.26
115	1,570.05	49.44	1.55
116	1,604.75	37.46	0.87
117	1,602.50	39.64	0.98
118	1,607.80	39.35	0.96
119	1,586.05	96.14	5.82
120	1,587.60	37.77	0.90

Grand Mean $\bar{X} = 1581$

1-percent statistical error = $\pm$ 102

* 3 readings fell outside the control limits.

Control limits at 99-percent probability level

a. For individual readings: UCL = 1683; LCL = 1479
b. For subgroups (n = 20): 1581 $\pm$ 23
c. For $\frac{s^2}{\bar{x}}$ chart: UCL = 1.90, LCL = 0.40

NUCLEAR VS. CONVENTIONAL DENSITY-MOISTURE MEASUREMENTS: STATISTICAL ANALYSIS OF RESULTS OF A FIELD EXPERIMENT

Synopsis

The relative precision of nuclear and conventional density determinations is equal for sand and clay soils, although in the case of gravel the data were inconclusive. The conventional method is relatively less precise than the nuclear method for soil moisture measurements.

During the period from May 18 to July 11, 1960, an experimentally-designed series of field tests was conducted by the Research Laboratory Division to determine: 1) the relative precision of the MSHD nuclear gage in measuring density and moisture contents of soils, 2) the relative precision of the conventional balloon testing method in making these measurements, 3) whether relative precision of the two methods differed significantly, 4) the degree of correlation between balloon and nuclear values, and 5) the statistical significance of such a correlation.

Data used in the study were secured from measurements made by MSHD personnel of moisture content and density on six sections each of sand, clay, and gravel. The sections were selected from construction sites on US 27 and M 46 in the area north and west of St. Louis. Each section measured 10 by 50 ft and was divided into ten numbered sub-sections of approximately equal area. Five sub-sections per section were chosen at random for test. The conventional balloon device was a standard 6-in. unit. The nuclear device consisted of MSHD Gage 4 and Portable Scaler 157. A single field operator made the measurements by using both devices.

PROCEDURE

Experimental Design

The staff of the Isotopes Section was asked to design a sampling procedure for comparative field study by using the nuclear method and the conventional equipment in such a fashion that experimental errors could be reduced to a minimum. This was done, taking into account the limited time, equipment, and material available to carry out the tests. The following work outline was presented to the field operator:

1. The uniformity of the selected area should be checked before deciding on a suitable sampling procedure.

2. If soil lacks uniformity, divide the surface into ten or more equal sub-sections.

3. Divide each sub-section into four equal parts and select two parts at random from each sub-section to determine density and moisture by both balloon and nuclear methods.

4. In the first sub-section chosen, make four nuclear and eight balloon tests (two balloon tests will be taken at same spot as nuclear test to check any deviation in test method or soil characteristics).

5. At each spot the nuclear gage should be rotated through four positions about 90^{o} apart and four counts should be recorded. The counting time should be two minutes for each count or eight minutes for every four observations at each spot.

6. Make two nuclear and two balloon tests in the other four selected sub-sections. The total tests for a typical test area should be 16 balloon and 12 nuclear.

7. During field evaluation of the nuclear gage, readings should be periodically taken on the wood standard reference block to check reliability of the overall system.

Effective use of this experimental design depends on knowing how to break the selected section into sub-sections so that the variations within

such sub-sections are relatively small. It depends also on how the testing spots are selected. Randomization, if applied, reduces the effects of systematic bias in any part of any testing procedure.

Analysis of Test Data

The test data were analyzed with computations performed by a digital computer. The data were treated according to the normal distribution law and by statistical methods used in the measurement of radioactivity (1, 2, 3).

In addition to the expected variation in the counting rate due to the random nature of the radioactive process, there were other extraneous factors which tended to increase the actual error found in the counting data.

The presence of these extraneous factors was established at the 99-percent probability level, so that the disturbing effects of these factors would be expected to occur once in 100 similar determinations. Possible factors which might contribute to the overall variability of the results may be represented by a linear equation such as

$$S^2 = S_i^2 + S_o^2 + S_s^2 + S_m^2 \qquad (1)$$

in which S^2 = total variance of individual readings,
S_i^2 = instrument variance,
S_o^2 = operator variance,
S_s^2 = soil variance, and
S_m^2 = variance from miscellaneous sources.

For practical reasons, the present work was not designed to separate these possible sources of variability. The total variation of individual readings (usually in terms of standard deviation) is the one given here.

Reliability of the MSHD Nuclear Measurements. The words "reliability" and "precision" are partially synonymous. Both refer to the reproducibility of estimates made from samples. They are associated with the idea of how well the successive values of a series agree with each other. The less the error of measurement, the more reliable the result. Therefore, the smaller the standard error of the data, the greater the precision of the estimate. The word "accuracy" refers to the closeness of approach to the true values sought. It is a measure of how close individual results come to the "true" value of whatever is being measured. In

the field work, there is no way of knowing a priori the true density or the true moisture of the soil. Neither of the actual measurements made by both the conventional and nuclear methods have shown the desired degree of closeness to a given density or moisture, even in small soil samples. Therefore, the word "precision," which refers to "the clustering of sample values about their own average" as defined by Anderson and Bancroft (4) is used in the present discussion.

Quality Control Charts on Density and Moisture Standards. The main purpose of the control charts is to determine whether the MSHD nuclear gage used for field measurements of moisture and density was stable during the testing period from May 18 to July 11, 1960. In conducting such experiments, two charts must be kept simultaneously, if full control is to be maintained over readings. One, known as the "Mean Chart," is for the average value of the observed counts for a given length of time. The other, known as the "Chi-Square Chart," is for variation of the observed counts accumulated in each of a series of identical measurements.

The Mean Chart. A sample graph which is reasonably effective in showing up trends, jumps, and periodicities, may be constructed as follows:

1. Observations are divided into subgroups of two or four successive readings, and each average, $\bar{x}$, of these subgroups is then plotted as a function of the order in which they were obtained.

2. The grand mean, $\bar{\bar{X}}$, of all these subgroups is plotted as a horizontal line on the chart.

3. Since at high counting rates, observations are very nearly normally distributed, the control limits for the means, $\bar{x}$, of subgroups of n counts can be estimated by the formula:

$$\bar{x} = \bar{\bar{X}} \pm \frac{2.576\sigma}{\sqrt{n}} \qquad (2)$$

where $\bar{\bar{X}}$ = the overall mean
σ = the standard deviation of the total population of data
n = the number of individual counts in each subgroup.

In this case, the upper and lower control limits (UCL and LCL, respectively), are based on a probability of 1 percent rather than 3 σ, since the presence of assignable causes of variation has comparatively little effect on these limits. Then, so long as the instrument is under

control, only one subgroup in a hundred is expected to fall outside the control limits. As an illustration, one may introduce statistical quality control to the series of measurements made on the moisture and density wood standards during the testing period. These count series were obtained routinely on such standards with an actual average counting rate of 31,491 counts per 2 min and a standard deviation of 290 counts per 2 min for density, and an average of 1,477 counts per 2 min and a standard deviation of 82 counts per 2 min for moisture. For an exhaustive analysis of the nuclear device's behavior, the repeated measurements were divided into seven subgroups of 30 successive values. The mean, $\bar{x}$, and control limits of the subgroups are presented in Tables 1 and 2 and the results plotted in Figs. 1 and 3.

TABLE 1
DENSITY STANDARD
(Plotted in Figs. 1 and 2)

Subgroup[1]	Average, $\bar{x}$ counts per 2 min	Chi-Square X^2	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
1	31,870	78.59	2.71
2	31,555	53.44	1.84
3	31,471	33.67	1.16
4	31,494	25.37	0.87
5	31,543	73.00	2.52
6	31,402	41.00	1.41
7	31,280	69.00	2.38

[1] Each subgroup an average of 30 successive readings (n = 30)

Grand Mean = 31,491
1-percent statistical error = $\pm$ 136

Control limits at 99-percent probability level
For Mean Chart, $\bar{x}$: UCL = 31,627; LCL = 31,355
For Sample Variance Ratio Chart, $\frac{s^2}{\bar{x}}$: UCL = 1.71; LCL = 0.49

In this case, limits are based on a probability of 0.01 for both the $\bar{x}$ and $\frac{s^2}{\bar{x}}$ charts, meaning that only 1 in 100 values tested are expected to fall outside limits.

The Chi-Square Chart. This second type of control chart is also commonly used in counting situations. It is highly efficient in determining

whether the counts accumulated in each subgroup may reasonably be supposed to have arisen from the statistical nature of the disintegration process. It is useful to detect trends or jumps in the precision of nuclear instrumentation.

The formula used in this control chart is:

$$\frac{X^2}{(n-1)} \simeq \frac{s^2}{\bar{x}} \quad \text{and} \quad X^2 = \frac{(\bar{x}_1 - \bar{\bar{x}})^2 + (\bar{x}_2 - \bar{\bar{x}})^2 + \ldots}{\bar{\bar{x}}} \tag{3}$$

where $x_1, x_2 \ldots$ = the individual count rates obtained in each subgroup of identical measurements

$\bar{x}$ and s^2 = the mean and variance, respectively, of a subgroup of n counts on the same sample

$(n - 1)$ = the degrees of freedom of the Chi-Square distribution or sample-variance ratio chart.

TABLE 2
MOISTURE STANDARD
(Plotted in Figs. 3 and 4)

Subgroup[1]	Average, $\bar{x}$ counts per 2 min	Chi-Square X^2	Sample Variance Ratio, $\frac{s^2}{\bar{x}}$
1	1,339	301.02	10.38
2	1,506	33.93	1.17
3	1,498	42.92	1.48
4	1,488	24.94	0.86
5	1,506	38.86	1.34
6	1,503	55.10	1.90
7	1,496	53.94	1.86

[1] Each subgroup an average of 30 successive readings (n = 30)

Grand Mean = 1477
1-percent statistical error = $\pm$ 39
Control limits at 99-percent probability level
For Mean Chart, $\bar{x}$: UCL = 1516; LCL = 1438
For Sample Variance Ratio, $\frac{s^2}{\bar{x}}$: UCL = 1.71; LCL = 0.49

It follows that the quantity $s^2/\bar{x}$ has an expected value of unity, and control limits are easily obtained from tabulated values by dividing by $(n - 1)$. The results are presented in Tables 1 and 2 and plotted in Figs. 2 and 4.

Analysis of the Results by the Control-Chart Technique. The expected variation in the counting rate due to the random nature of radioactivity may be affected by disturbing factors which tend to inflate the actual error made in the counting data. The mean control charts, $\bar{x}$, indicate the presence of assignable causes of variation, or some disturbing factor during the series of counting determinations on the same wood standard block (Figs. 1 and 3). Two subgroups out of seven are outside the control limits for the density standard (Fig. 1). Field sheets showed that the first series of 30 consecutive observations included 10 readings which differed considerably from the other value of this series. There was no explanation for the unusual values.

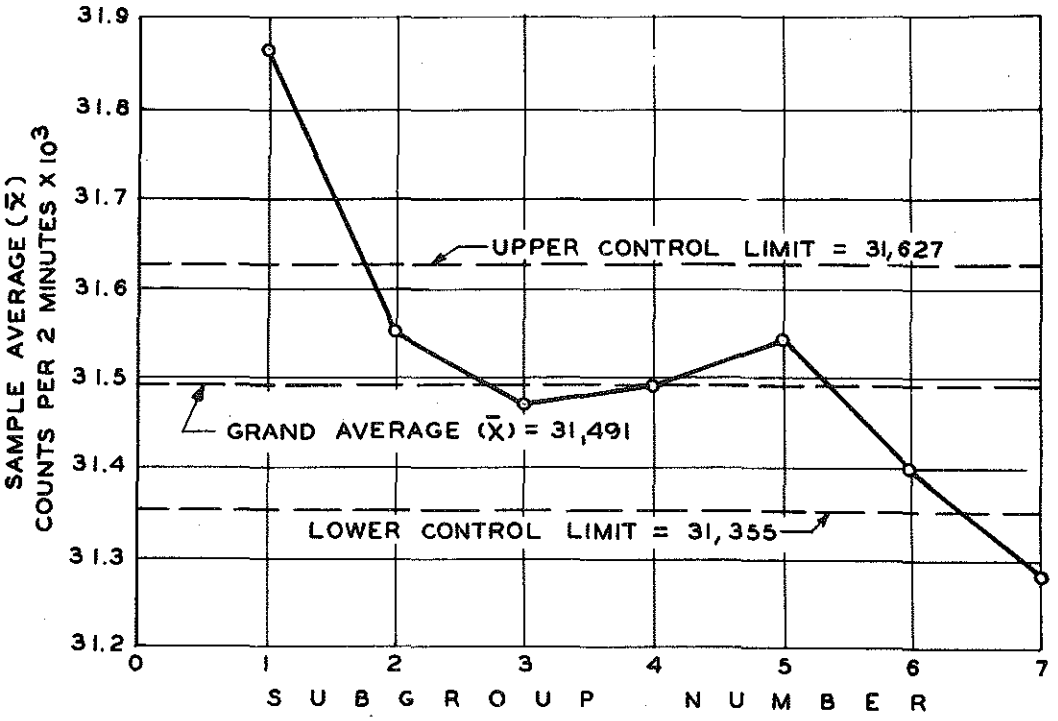

Figure 1. Mean chart for nuclear density wood standard block. Each point (subgroup) represents an average of 30 consecutive readings.

The following facts may be seen also from the control charts:

1. Starting from Subgroup 5 (Fig. 1), the successive measurements decrease steadily in value. It is clear that these values do not display the randomness that is characteristic of nuclear radiation.

2. The Chi-Square Chart ($s^2/\bar{x}$) for both the moisture and density wood standard block (Figs. 2 and 4) shows conclusively that the nuclear gage was not working properly. In both graphs, three subgroups in seven are outside the upper limit.

It was mentioned before that there are several possible sources of variability which may affect the relative precision of the results with the nuclear device. Factors such as extraneous noise from a nearby spark coil or auto ignition system, irregularity in operational procedure (misjudgment of timing, bias in readings, improper placement of instrument for test) may affect the behavior of the nuclear device.

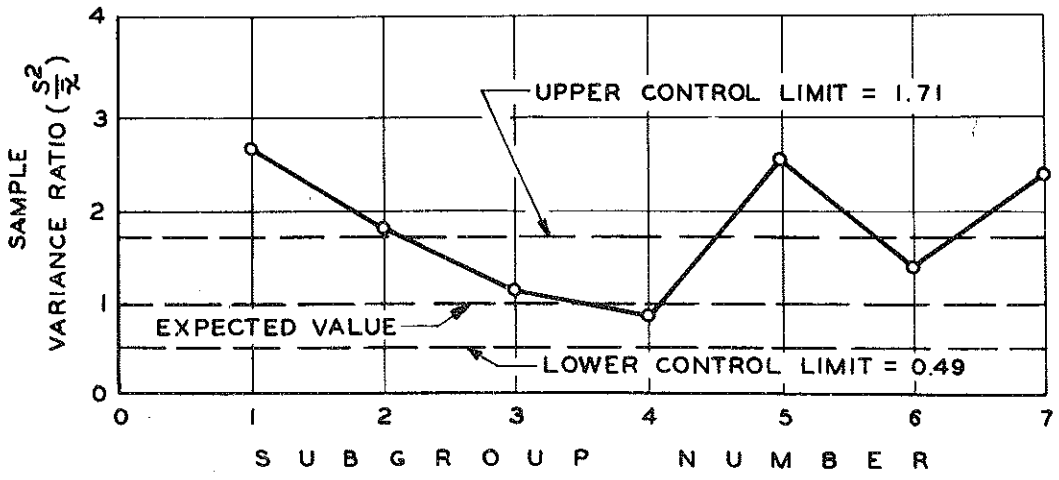

Figure 2. Chi-Square chart for nuclear density wood standard block. Each point (subgroup) represents an average of 30 consecutive readings.

Relative Precision of Conventional Measurements. To determine approximately the relative precision of the conventional balloon method under field conditions, two tests were made at each spot according to the experimental design as described before. The first series of 25 paired values made on sand are shown in Fig. 5. Point M is located by using the average of the 25 first test conventional values or conventional check points. Fig. 5 shows that if there were no source of variation in the experimental values other than sampling variation, all the points should lie exactly on the 45° line, OS, passing through the origin. The amount, OK, by which the line, KD, misses the origin is a measure of the average difference in density between the paired values.

The scatter of all points along the 45° line, KD, reveals the amount of variation or experimental error in the observed measurements. This experimental error includes the variability of the test method and operator. Of course, the degree of precision which can be claimed for a test method depends upon the number of measurements made on homogeneous or uniform material.

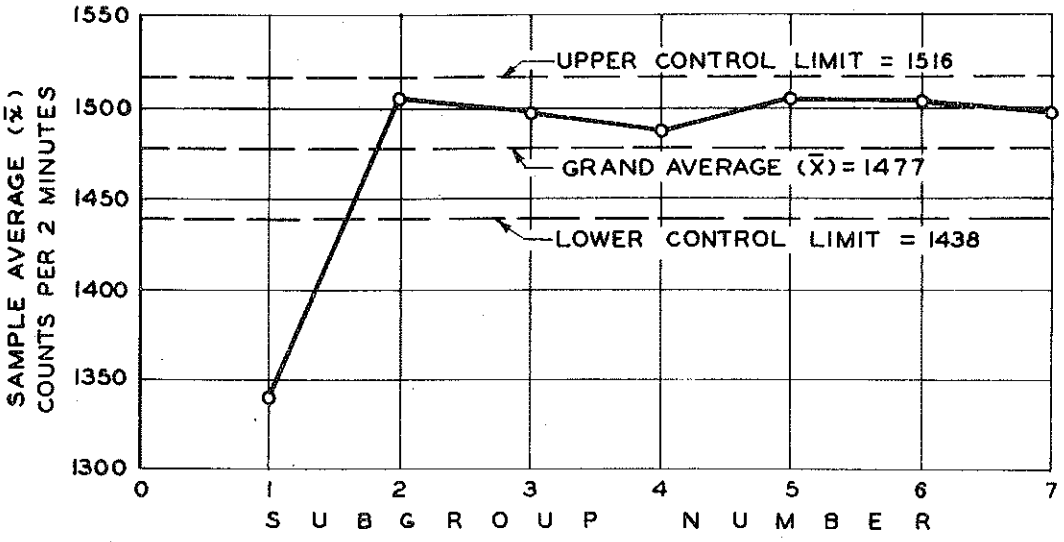

Figure 3. Mean chart for nuclear moisture wood standard block. Each point (subgroup) represents an average of 30 consecutive readings.

Fig. 6 shows another series of paired values taken at a single subsection with the balloon equipment. This case illustrates the extent to which the averages of paired values are subject to fluctuations from one test to another on the same material.

Necessity for Standards. To establish both the precision and accuracy of a sequence of measurements it is necessary to know how close individual results come to the "true" value of whatever is being measured. Such a "true" value could be anything against which comparative observations can be made. That is, an accepted standard which makes it possible to use the results of both methods to establish and compare precision and accuracy between them.

If the required characteristic to be measured is density or moisture in soils, standard samples of known density or moisture should be used to prepare calibration curves for the particular method under study.

Calibration of the nuclear gage should be made outdoors and standard samples of materials similar to those tested in the field should be used.

MSHD Nuclear Versus Conventional Measurements. Since the present field measurements provide no basis for a comparative study of the accuracy of the results, the reader may ask what conclusions can be drawn from them. The statistical analysis based on the field data will answer those questions concerning the relative variability or relative precision of both testing methods and whether the degree of relationship between them is significant.

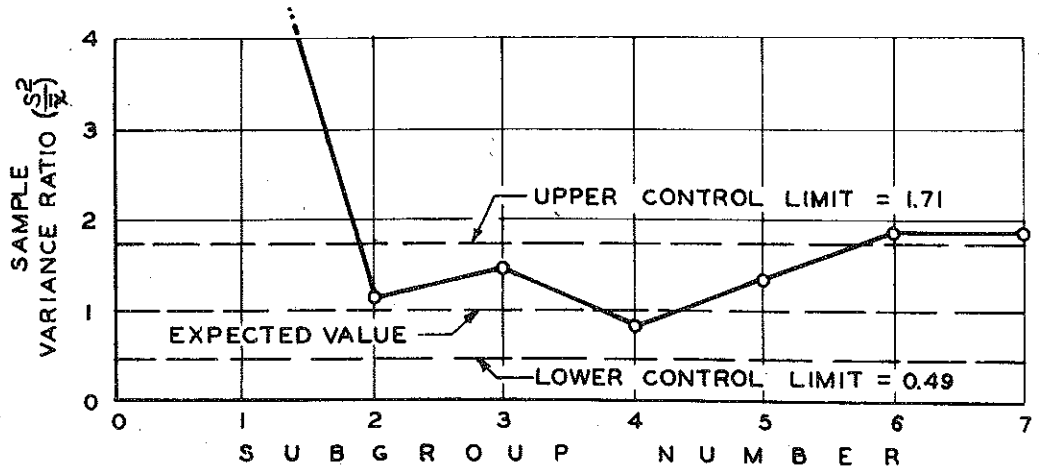

Figure 4. Chi-Square chart for nuclear moisture wood standard block. Each point (subgroup) represents an average of 30 consecutive readings.

The coefficient of variation (or relative standard deviation) is used to compare the relative variability or relative precision of two or more distributions which are expressed in different units. It is expressed as a pure number; that is, divorced from the particular units employed in any testing technique. It is computed from the equation:

$$V = \frac{100s}{\bar{x}} \tag{4}$$

where s = standard deviation of total observations per sample
$\bar{x}$ = average of these observations per sample.

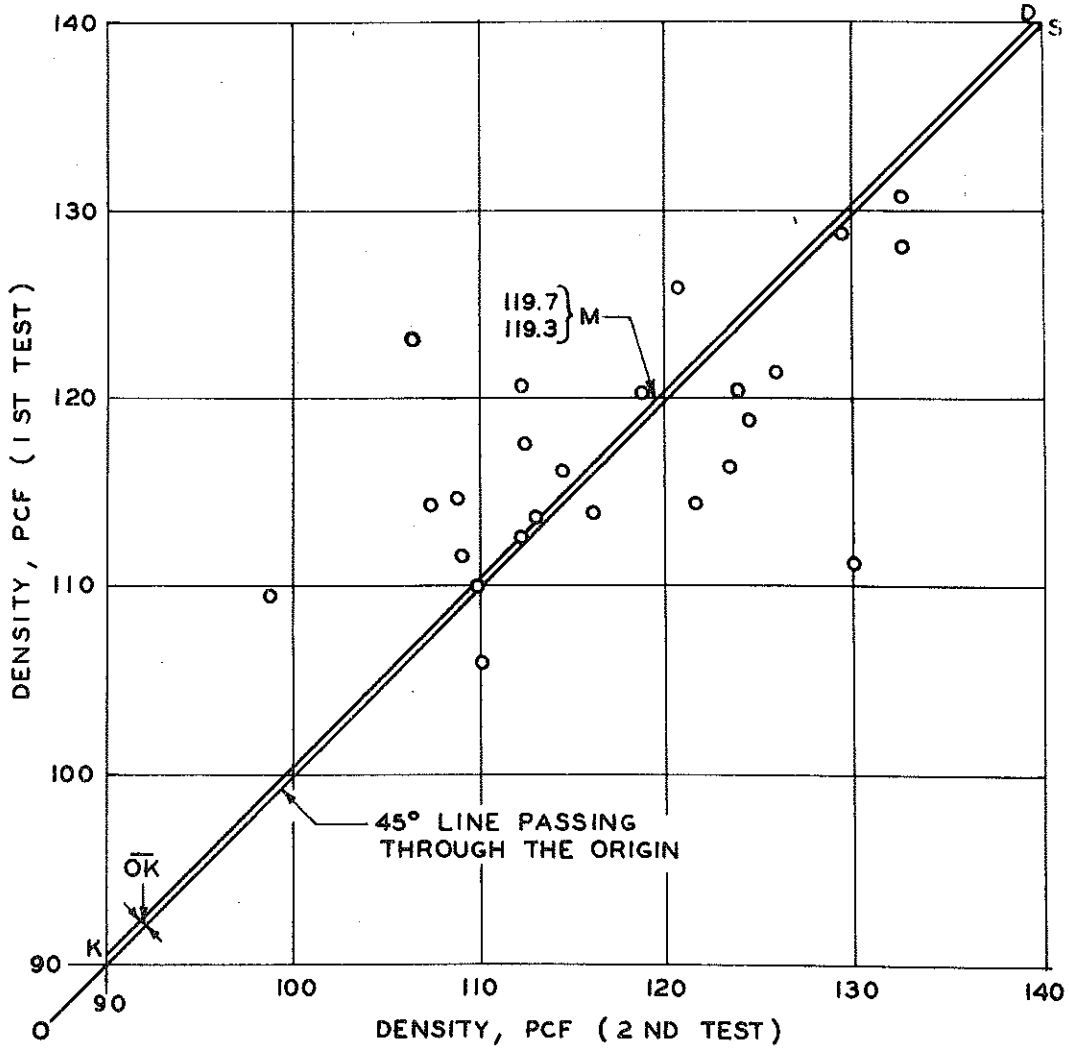

Figure 5. Variability of the balloon method on sand.

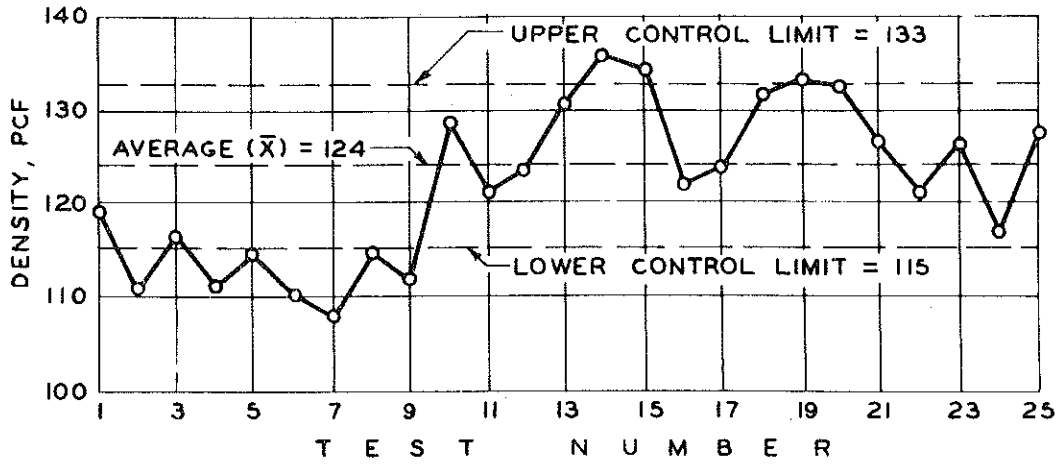

Figure 6. Control chart for averages of paired density measurements in wet sand by balloon method.

Values for density and moisture for six sections of sand, clay, and gravel are presented in the Appendix. Conclusions based on the computed values for the coefficient of variation are as follows:

1. For density measurements in the field, the nuclear and the Rainhart methods are equally good concerning relative precision on sand and clay soil. But on gravel, the experimental data do not provide as much evidence as is necessary for a definite statistical judgment about the relative precision of each method. On this material, more testing is required to decide which method is more reliable.

2. For moisture measurements, the balloon method is relatively more erratic than the nuclear gage. The former showed poor reproducibility on all three tested materials.

Correlation Coefficients. Simple correlation analysis is the method used to analyze the association between any pair of variables. This method performs two functions and produces two results. First, it serves to tell by means of an estimate index how closely two variables tend to be associated; and second, whether the association is adequately described by an equation.

The correlation coefficient computed in this report is a measure of the degree of relationship between the nuclear gage and balloon measurements. It is defined as:

$$r = \frac{\frac{1}{n} \lesssim (x - \bar{x})(y - \bar{y})}{s_x s_y} \qquad (5)$$

where $\bar{x}$ = mean of all the balloon values
$\bar{y}$ = mean of all the nuclear gage values per section
s_x = standard deviations of all the balloon values per section
s_y = standard deviations of all the nuclear values per section
n = total number of paired values (nuclear and balloon) involved in the computations.

With actual data, the correlation coefficient may range in value from +1 to -1. A value of r = +1 denotes perfect functional relationship between y and x, an increasing x being associated with an increasing y. When r = -1, again a perfect relationship is indicated, but in inverse direction; that is, an increasing x is associated with a decreasing y. A value of r = 0 means that there is no relationship at all between the two variables.

Other intermediate values of r indicate a positive or negative trend, as the case may be.

The overall correlation coefficients show the degree of association between nuclear and conventional values, in decreasing order, as follows:

1. Density measurements:
   a) For sand, $r = -0.88$ — Table 3
   b) For clay, $r = -0.65$ — Table 5
   c) For gravel, $r = -0.50$ — Table 7

2. Moisture measurements:
   a) For clay, $r = +0.85$ — Table 11
   b) For gravel, $r = +0.60$ — Table 13
   c) For sand, $r = +0.67$. — Table 9

Although the data exhibit different degrees of significant correlation, they provide no sound basis for computing the best line and its limits, in any case. That is, the experimental values are not sufficiently reliable for two reasons:

1. Experiments conducted with the nuclear gage disclosed the presence of a disturbing influence. The results showed trends and jumps from unknown causes, such as from unfavorable placement of the gage, non-uniformity of the soil, or other factors.

2. Rainhart measurements were affected by testing conditions which were not well controlled.

CONCLUSIONS

Based on the limited conditions imposed by the experimental design, the following conclusions are justified by statistical analysis of the data. These conclusions would not necessarily be pertinent in the case of other experimental designs.

1. The experimental data provided no basis for any assessment concerning the accuracy of either method. They did, however, permit the establishment of the relative precision, or relative variation, of both methods.

2. The relative precision of both methods was equal for density measurements on sand and clay soils. Insufficient data were obtained to support a valid conclusion in the case of gravel.

3. The conventional method was relatively less precise than the nuclear method for moisture measurements. In fact, the conventional technique showed poor reproducibility on all three soil types.

4. Use of the wood standard reference block made possible the application of quality control charts to nuclear density and moisture readings. These charts showed the effect of unwanted variables in changing the precision of values obtained with the nuclear method.

5. Maintenance of similar quality control charts for the conventional method was not possible due to the non-use of a suitable reference standard.

6. Although the experimental data exhibited different degrees of correlation between the two methods, the difficulty of computing the regression line and its limits--hence, of the statistical significance of the correlation--is apparent from Fig. 21 in Research Report No. 358, Field and Laboratory Evaluation of the Michigan Nuclear Gage, by R. C. Mainfort and J. H. DeFoe (5), which covers the same data. Density measurements gave the following order of decreasing correlation: 1) sand, 2) clay, and 3) gravel. Moisture measurements gave: 1) clay, 2) gravel, and 3) sand.

7. Some disturbing influence was present during field measurements made on the wood standard with the nuclear gage, which had the effect of inflating or magnifying the actual error in the readings. The variations produced were far greater than similar variations noted previously in the laboratory. It was not possible to identify the origin of this disturbance by analysis of the data used in the present study, but its existence was clearly demonstrated. Possible causes include operator bias or incorrect operation, intermittent electronic or mechanical failure, inadequate battery charge, unusual background variation, or some as yet unrecognized aspect of the scattering-absorption interrelationships involved.

RECOMMENDATIONS

The following recommendations are made on the basis of the conclusions cited:

1. That an experiment be designed and conducted to establish the magnitude, distribution, and variability of normal background radiation throughout Michigan, together with any abnormal background which may occur during the course of the experiment. This information could then be applied in setting limits of significance, within which background may be neglected when using the gage, and conversely, outside of which it may not be neglected.

2. That experiments be conducted using a nuclear gage with its source removed for determining background magnitude at the exact site of nuclear measurements. If this proves successful, a dummy gage could be used instead of the present standard, in the event that background determinations cannot be neglected.

3. That, routinely, the average of two or four density and moisture readings on the wood standard be checked periodically with control charts furnished by the Research Laboratory Division. Readings found outside the control limits would indicate that the gage is not operating properly, and remedial action should be taken.

4. That calibration curves be established outdoors on materials of known and uniform density and moisture content, and as nearly as possible identical with the materials to be tested.

5. That the use of relative measurements, as contrasted with absolute measurements, be explored. This would greatly simplify the use of the nuclear gage and increase the number of measurements possible within a working day.

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APPENDIX

TABLE 3
SAND DENSITY

Section	Nuclear, counts per 2 min		Conventional, pcf		Correlation Coefficient, r
	Average, $\bar{y}$*	Standard Deviation, s_y	Average, $\bar{x}$**	Standard Deviation, s_x	
1	27,345	889	117.00	4.45	-0.68
2	28,417	884	111.58	5.27	-0.53
3	25,595	1442	121.80	8.15	-0.95
4	24,943	1166	127.05	6.18	-0.94
6	24,161	966	125.56	5.95	-0.71
7	24,384	904	125.79	5.32	-0.90
All Sections	25,808	1889	121.46	8.17	-0.88

* Average of 12 tests per section, and each test the average of 4 counts.
** Average of 16 tests per section.

TABLE 4
RELATIVE PRECISION
OF SAND DENSITY MEASUREMENTS

Section	Coefficient of Variation, percent	
	Nuclear, $\frac{s_y}{\bar{y}}$	Conventional, $\frac{s_x}{\bar{x}}$
1	3.3	3.8
2	3.1	4.7
3	5.6	6.7
4	4.7	4.9
6	4.0	4.7
7	3.7	4.2
All Sections	7.3	6.7

TABLE 5
CLAY DENSITY

Section	Nuclear, counts per 2 min		Conventional, pcf		Correlation Coefficient, r
	Average, $\bar{y}$*	Standard Deviation, s_y	Average, $\bar{x}$**	Standard Deviation, s_x	
5	25,056	1,752	135.61	5.61	-0.61
8	27,031	1,932	122.62	7.59	-0.78
15	22,753	1,196	137.02	3.05	-0.11
16	24,236	2,177	122.73	9.19	-0.80
20	25,692	1,432	126.71	6.39	-0.76
21	25,902	822	119.50	5.57	-0.19
All Sections	25,112	2,135	127.37	10.19	-0.65

* Average of 12 tests per section, and each test the average of 4 counts.
** Average of 16 tests per section.

TABLE 6
RELATIVE PRECISION
OF CLAY DENSITY MEASUREMENTS

Section	Coefficient of Variation, percent	
	Nuclear, $\frac{s_y}{\bar{y}}$	Conventional, $\frac{s_x}{\bar{x}}$
5	7.0	4.1
8	7.1	6.2
15	5.3	2.2
16	9.0	7.5
20	5.6	5.0
21	3.2	4.6
All Sections	8.5	8.0

TABLE 7
GRAVEL DENSITY

Section	Nuclear, counts per 2 min		Conventional, pcf		Correlation Coefficient, r
	Average, $\bar{y}$*	Standard Deviation, s_y	Average, $\bar{x}$**	Standard Deviation, s_x	
11	20,346	1,053	150.87	2.61	-0.79
12	21,341	1,373	145.31	8.33	-0.50
13	22,471	729	146.73	1.38	-0.29
14	23,232	785	145.72	2.07	+0.23
22	25,035	568	138.78	4.92	-0.17
23	25,328	1,330	138.45	2.99	-0.23
All Sections	22,959	1,378	144.31	5.77	-0.50

* Average of 12 tests per section, and each test the average of 4 counts.
** Average of 16 tests per section.

TABLE 8
RELATIVE PRECISION
OF GRAVEL DENSITY MEASUREMENTS

Section	Coefficient of Variation, percent	
	Nuclear, $\frac{s_y}{\bar{y}}$	Conventional, $\frac{s_x}{\bar{x}}$
11	5.2	1.7
12	6.4	5.7
13	3.2	0.9
14	3.4	1.4
22	2.3	3.6
23	5.3	2.2
All Sections	6.0	4.0

TABLE 9
SAND MOISTURE

Section	Nuclear, counts per 2 min		Conventional, pcf		Correlation Coefficient, r
	Average, $\bar{y}$*	Standard Deviation, s_y	Average, $\bar{x}$**	Standard Deviation, s_x	
1	719	97	9.33	1.66	+0.40
2	618	103	8.07	1.33	+0.66
3	665	87	8.58	1.53	+0.92
4	660	109	8.25	1.99	+0.75
6	887	120	9.42	2.39	+0.83
7	790	112	8.73	1.57	+0.80
All Sections	723	139	8.73	1.86	+0.67

* Average of 12 tests per section, and each test the average of 2 counts.
** Average of 16 tests per section.

TABLE 10
RELATIVE PRECISION
OF SAND MOISTURE MEASUREMENTS

Section	Coefficient of Variation, percent	
	Nuclear, $\frac{s_y}{\bar{y}}$	Conventional, $\frac{s_x}{\bar{x}}$
1	13.5	17.8
2	16.6	16.5
3	13.1	17.8
4	16.5	24.1
6	13.5	25.4
7	14.2	18.0
All Sections	19.2	21.3

TABLE 11
CLAY MOISTURE

Section	Nuclear, counts per 2 min		Conventional, pcf		Correlation Coefficient, r
	Average, $\bar{y}$*	Standard Deviation, s_y	Average, $\bar{x}$**	Standard Deviation, s_x	
5	1016	99	11.18	1.50	+0.53
8	1463	196	16.09	2.45	+0.93
15	1162	66	13.76	0.69	+0.64
16	1089	184	13.52	2.32	+0.55
20	1093	188	11.63	2.19	+0.90
21	944	75	11.06	1.28	+0.59
All Sections	1127	186	12.87	2.45	+0.85

* Average of 12 tests per section, and each test the average of 2 counts.
** Average of 16 tests per section.

TABLE 12
RELATIVE PRECISION
OF CLAY MOISTURE MEASUREMENTS

Section	Coefficient of Variation, percent	
	Nuclear, $\frac{s_y}{\bar{y}}$	Conventional, $\frac{s_x}{\bar{x}}$
5	9.7	13.4
8	13.4	15.2
15	5.7	5.0
16	16.9	17.2
20	17.2	18.8
21	7.9	11.6
All Sections	16.5	19.0

TABLE 13
GRAVEL MOISTURE

Section	Nuclear, counts per 2 min		Conventional, pcf		Correlation Coefficient, r
	Average, $\bar{y}$*	Standard Deviation, s_y	Average, $\bar{x}$**	Standard Deviation, s_x	
11	772	67	9.61	2.02	-0.13
12	722	135	7.65	2.55	+0.83
13	566	53	6.42	0.69	+0.44
14	454	25	4.53	0.32	+0.23
22	497	30	5.51	0.65	+0.00
23	477	45	5.36	0.57	+0.77
All Sections	581	138	6.51	2.31	+0.60

* Average of 12 tests per section, and each test the average of 2 counts.
** Average of 16 tests per section.

TABLE 14
RELATIVE PRECISION
OF GRAVEL MOISTURE MEASUREMENTS

Section	Coefficient of Variation, percent	
	Nuclear, $\frac{s_y}{\bar{y}}$	Conventional, $\frac{s_x}{\bar{x}}$
11	8.7	21.0
12	18.6	33.3
13	9.3	10.8
14	5.5	7.1
22	6.1	11.8
23	9.4	10.6
All Sections	23.8	35.5