Department of Civil and Environmental Engineering

The University of Michigan
College of Engineering

Ann Arbor, MI 48109-2125

Testing and Research Section
Construction and Technology Division
Research Report No. RC-1399

CONCRETE MIX DESIGN PROCESS MANUAL

Final Report
Prepared for
Michigan Department of Transportation

by

Gail H. Grove, Ph.D., P.E.

Department of Civil and
Environmental Engineering

The University of Michigan
College of Engineering

Ann Arbor, MI 48109-2125

August, 1998

Addendum

This addendum is being added to thank and give credit to Robert D. Miller for his documentation of two mathematical procedures that have proven benefical to several readers in their understanding of the original report. These are:

1. How to Find the Intersection of Two Lines, and
2. How to Find an Equation of a Line Which Passes Through Two Points.

Gail H. Grove
07/09/2001

How to Find the Intersections of Two Lines

Robert D. Miller

Given two lines whose equations are:

$$a_1x + b_1y + c_1 = 0 \qquad \text{and} \quad a_2x + b_2y + c_2 = 0,$$

we may solve for x and y, their point of intersection, if it exists.

We have two linear equations in two unknowns (x and y):

$$a_1x + b_1y = -c_1 \qquad \text{and} \quad a_2x + b_2y = -c_2.$$

Solving this system of equations with determinants using Cramer's Rule gives:

$$x = \frac{\begin{vmatrix} -c_1 & b_1 \\ -c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_2b_1 - c_1b_2}{a_1b_2 - a_2b_1} \quad \text{and}$$

$$y = \frac{\begin{vmatrix} a_1 & -c_1 \\ a_2 & -c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}.$$

For example, line l_1 passes through $(1,2)$ and $(3,5)$ is described by the equation $3x - 2y + 1 = 0$. Line, l_2 passes through $(4,3)$ and $(7,4)$ has the equation $x - 3y + 5 = 0$. The coordinates of their intersection is:

$$x = \frac{-(-3) - 10}{-9 + 2} = 1, \quad \text{and}$$

$$y = \frac{-15 + 1}{-7} = 2,$$

so their intersection is $(1,2)$. Compute the denominator first; if it is zero, both lines are parallel or coincident and no intersection exists.

How to Find an Equation of a Line Which Passes Through Two Points

Robert D. Miller

To find the equation of a line in the form $ax + by + c = 0$, which passes through two points (x_1, y_1) and (x_2, y_2), where a and b are not both zero, calculate $c = x_2y_1 - x_1y_2$. If $c \neq 0$ then calculate: $a = y_2 - y_1$ and $b = x_1 - x_2$, otherwise the points are coincident and no unique line exists.

To show this method works, we can form a two equations in two unknowns to solve for a and b. The value for c balances the equation. Two conditions must be satisfied:

$$\begin{aligned} ax_1 + by_1 &= 1 \\ ax_2 + by_2 &= 1 \end{aligned}$$

These two equations may be solved evaluating determinants and using Cramer's Rule:

$$a = \frac{\begin{vmatrix} 1 & y_1 \\ 1 & y_2 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}} = \frac{y_2 - y_1}{x_1y_2 - x_2y_1}, \qquad b = \frac{\begin{vmatrix} x_1 & 1 \\ x_2 & 1 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}} = \frac{x_1 - x_2}{x_1y_2 - x_2y_1}.$$

To satisfy the equation of a line $ax + by + c = 0$, let c be the negative of the denominator above:

$$c = -(x_1y_2 - x_2y_1) = x_2y_1 - x_1y_2.$$

For example, given two points $(1, 2)$ and $(3, 5)$, we have

$$(5 - 2)x + (1 - 3)y + 3 \times 2 - 5 \times 1 = 3x - 2y + 1 = 0.$$

Check to see that either point's x- and y- values satisfies the equation. Note: this method is general and works with lines parallel to an axis. For the points parallel to the y-axis, $(2, 1)$ and $(2, 5)$, for example, we get $x = 2$.

Sometimes it is convenient to find the equation of a line in the point-slope form, $y = mx + f$, where m is the slope, (i.e. $m = \frac{y_2 - y_1}{x_2 - x_1}$. and f is the y-intercept. Solving the line's equation for y, when $b \neq 0$, we get: $y = -\frac{a}{b}m - \frac{c}{b}$. If $b = 0$, the equation reduces to $x = -\frac{c}{a}$ which is parallel to the y axis.

Action Plan

1. Construction and Technology Division

    a. Approve report and electronic documentation.

    b. Print copies from electronic media and distribute within MDOT as needed.

2. Engineering Operations Committee

    a. Accept report and electronic documentation.

    b. Authorize 1.b. above.

Executive Summary

In 1997, a new computer program was written by the Materials and Technology Division to generate concrete mix designs using the *Mortar Voids Method.* The previous program was undocumented and its author(s) have long since left the Department. The old program was written in an older style computer code and is no longer supported.

After the new program was placed into production, a User's Manual for its customization and operation was written. Although the User's Manual is sufficient for operation of the new program, the knowledge required to understand the process was not documented. Currently, there is no employee left in the Department who has a solid understanding of the complexities and subtleties of this old, yet extremely valuable method. Providing accurate and reliable concrete mix designs is an important function of the Construction and Technology Division.

Thus, to retain the Department's engineering knowledge of the concrete mix design process, a new document was needed to update, revise, and correct errors and omissions in the 1936 and 1970 reports. Also, an explanation of the working equations and examples of the graphical method in both metric and English measurement units was required. Such examples have been useful in evaluating the new computer program.

Acknowledgements

I would like to thank Robert D. Miller, TAG Chairman, for his project suggestions and guidance and his assistance in producing PostScript files for several of the graphical functions and Thomas Woodhouse, TAG Member, for suggestions on topics to be covered in this report and for allowing me to reference his coarse aggregate unit weight laboratory data.

I would also like to acknowledge the original work of A. Talbot and F. Richart, along with modifications by R. Fulton, and by E. Shehan in adapting the mortar void method to Michigan materials and conditions and to advances by F. Legg, Jr. and R. Vogler.

Lastly, I want to thank Prof. W. Hansen, Director of the Pavement Research Center of Excellence at The University of Michigan for his encouragement and guidance.

Disclaimer

Table of Contents

Appendices:

1. Introduction:

a. Objectives:

The objective of this manual is to provide a detailed discussion of the Michigan Department of Transportation (MDOT) Concrete Mix Design Method, based upon the mortar void theory, for producing Portland Cement Concrete (PCC) for MDOT's existing specifications (10,11). The 1990 Standard Specifications are being used for English measurement unit projects and the 1996 Standard Specifications are being used for metric measurement unit projects. Additional Supplemental Specifications and Special Provisions are utilized where needed.

Since our charge is to combine several previous works (2,3,4,5) which have some differences in designations and terminology, we will use the older term, *proportioning charts*, along with the more modern term, *concrete mix designs*, as the same, eventhough there are some differences.

The *Mortar Void Theory* was first developed by Talbot and Richart (1) at the Illinois Engineering Experiment Station during the early 1920's. With certain modifications, MDOT has used this method for proportioning PCC mixtures for highway construction since about 1928. The method is applicable for pavement mixes, both conventional and slip-form, and for structural concrete mixes.

Today's application of this method is by use of a computer program, although mix designs originally were done by manual calculation and can still be performed by hand calculation as a means of checking the computer program after any modifications are made to the software. The computer program has been revised several times (un-documented) and was completely re-written in a modern programming language in 1997 by Robert D. Miller. His *User's Manual* (6) describes the program and how future modifications should be handled in a very user-friendly fashion.

Several previous manuals and reports by Fulton (2,3,4), Shehan (5) and Legg and Vogler (13) have covered the manual method but nothing has been documented for any of the computer versions, except for Robert Miller's User's Manual. Thus the major task will be to combine and update the work of Fulton, Shehan, Legg and Vogler to include the techniques of the manual calculations and the present-day computer program. This Concrete Mix Design Process Manual should be considered as a handy reference to understanding the concepts and calculations used in the Concrete Mix Design Computer Program.

Worked examples, in English units and metric units are included. Each example has been worked both manually and by use of the menu-driven computer program.

b. Basic Principles:

The principle of the *Mortar Void Theory*, as originally stated by Talbot and Richart, is that the strength of concrete is a function of the ratio between the volume of voids (air plus water) in the concrete and the volume of cement, provided that the spaces between the coarse aggregate particles in the concrete are totally filled with mortar. The primary components of concrete are cement, fine aggregate, coarse aggregate and water. Present-day proportioning of air-entrained concrete recognizes that a well dispersed system of minute air bubbles is another important ingredient. Sufficient water must be added to hydrate the cement and to lubricate the mix for workability during placing and finishing operations. It is recognized that the most dense concrete is the strongest concrete and that the amount of water required to give the most dense mixture is theoretically the proper amount to use. However, it is also known that the strongest concrete may not be the most durable, and in order to improve durability, the MDOT has specified the use of air-entrainment for all exposed concrete since 1942. Currently, the specified entrained air content of most conventionally placed (not slipformed) concrete mixtures used by MDOT is 6.5% with a specification tolerance of plus or minus 1.5%. Thus, the original concept of *Mortar Void Theory* has been modified by MDOT to seek not minimum voids, but to determine the minimum volume of water verses a constant entrained air content.

Later, in the laboratory procedures for determining the proportions of a concrete mixture, the method of determining the volume of water required to obtain the most dense mortar (least voids) for any given sand/cement ratio will be covered. This volume of water is known as the *Basic Water Content* (BWC). However, in actual practice, this Basic Water Content will produce a concrete mixture that is much too dry to place and finish properly. Therefore, in order to obtain suitable workability, the Basic Water Content is adjusted by an empirical factor to increase the volume of water to be used in the mixture. This factor is known as the *Relative Water Content* (RWC) and has been determined from actual field experience with the various grades and uses of concrete specified by the MDOT.

The function of the cement in the mixture is such that the hydrated cementitious materials become the binding agent for the other ingredients in the concrete mixture. The strength of hardened concrete is directly related to the volume of cement in the mixture, provided that all other variables, including the water, remain constant. In the *1996 Standard Specifications For Construction* (10), Tables 601-2, pages 6.18-19, and Table 701-1, pages 7.3-4, list the various grades of PCC mixtures for rigid pavements and structures, respectively.

The function of the coarse aggregate in a concrete mixture is one of economy and reduction of volume change. Sound engineering practice is to use as much coarse aggregate as possible as long as the mixture can be placed and finished properly. The limiting factor on the maximum quantity of coarse aggregate used is that there must be at least the volume of paste components (sand, cement, air, and water) theoretically needed to fill all of the voids between the coarse aggregate particles in the mixture.

The workability factor, b/b_0, is the unit volume of coarse aggregate per unit volume of concrete and defines the amount of coarse aggregate needed in the various grades of concrete such that there will be a definite excess of mortar over that required to fill the voids between the coarse aggregate particles in the mixture. A properly chosen value of b/b_0 will insure satisfactory finishing qualities.

It has been established that the strength of hardened concrete is governed by the strength of the mortar filling the voids between the coarse aggregate particles provided that the volume of coarse aggregate is not excessively high and that all other factors relating to the mixture remains constant. When properly cured concrete is cracked or broken, the break should be through some of the coarse aggregate particles, with numerous particles pulled away from the mortar bond (9).

Since coarse aggregate is composed of solid particles, in workable concrete, there must be more mortar than the minimum which is theoretically necessary to fill all the voids between the coarse aggregate particles. It is assumed that the volume of voids in the concrete is equal to the volume of voids in the mortar for that concrete. Based upon this assumption, the laboratory work involved in proportioning a concrete mixture resolves itself into a trial batch study of the mortar using actual project materials and determining volume relationships existing among the cement, sand, and the voids (air plus water) which make up a unit volume of mortar. Next, determine the properties of these ingredients to meet the required strength, density, and consistency. Finally, combining the designated quantities of sand, cement, and water in the mortar with the proper quantity of coarse aggregate, one obtains the desired mix proportions. These values are calculated by a computer program which generates a proportioning chart or a set of charts for use in the field by the Project Engineer (PE). Using the data provided in this report, the PE only has to determine the actual dry, loose unit weight of the coarse aggregate and the moisture content of the fine and coarse aggregate at the time of batching in order to determine the quantities of each material needed for the batch(es).

It should be noted that we are talking about <u>dry, loose</u> unit weights and workability factors, not <u>dry, rodded</u> when using MDOT's Mix Design Program. See Appendix D for a discussion of the differences between loose and rodded terminology and the relationship between these measures.

2. Principles of Application:

a. Definition of Terms:

Appendix A contains a listing of the significant variables along with definitions.

b. Fundamentals of Portland Cement Concrete (PCC) Design:

Concrete is basically a mixture made up of two components: aggregates and paste. The paste is comprised of Portland cement, possibly other cementitious materials such as flyash, silica fume, and ground granulated blast-furnance slag, water and entrapped or entrained air. All further discussion will be relative to air-entrained concrete. Cement paste normally contributes about 25-40% of the total volume to the concrete. The quality of the PCC depends to a great extent upon the quality of the paste. The paste must coat each aggregate particle and all of the voids between the particles.

Aggregates are generally divided into two groups: coarse and fine. Coarse aggregates for highway projects consist of material predominately retained on the No. 4 (4.75 mm) sieve, with the more prevalent gradations having a top size from 3/4 inch (19.0 mm) to 1 inch (25.0 mm) in size. Fine aggregates consist of particles passing the 3/8 inch (9.5 mm) sieve and predominately retained on the 200 (0.075 mm) sieve. Aggregates usually make up about 60-75% of the total volume of concrete, thus making their selection very important. Aggregates should have adequate strength and resistance to weathering conditions and should not contain materials that will cause deterioration of the concrete.

For air-entrained concrete, the volume range extremes (7,8) for the various components are as follows:

	Lean - Rich	Average Normal Mix
Cementitious Materials	(7% - 15%)	11%
Water	(14% - 18%)	16%
Air	(4% - 8%)	6%
Fine Aggregate	(24% - 28%)	26%
Coarse Aggregate	(51% - 31%)	41%
Total	100% 100%	100%

where the first column of values above represents a lean (less than normal cement content) mix with large size coarse aggregate and the second column of values represents a rich (more than normal cement content) mix with small size coarse aggregate.

c. Factors Affecting the Design:

In preparing a concrete mix design or chart, it is necessary that:

1. Representative samples of sand and coarse aggregate are taken by MDOT approved personnel and are delivered to the MDOT Concrete Testing Lab for evaluation. An average value for the cements available in the area is used for

the cement specific gravity.

2. The aggregates are clean, structurally sound and meet the grading and physical requirements of Section 902 of the 1996 Standard Specifications For Construction (10).

3. The cement is of satisfactory quality, meeting ASTM C 150 (15) or other MDOT approved Specification.

d. Analytical Relationships:

Using the definition of terms from Appendix A, we have:

$a + b + c + v = 1$ unit volume of concrete, and

$a_m + c_m + v_m = 1$ unit volume of mortar.

Since the ratio of the absolute volume of sand, cement and voids to the volume of mortar is the same in a unit of mortar alone as it is in the mortar in concrete, it follows that:

$a = a_m(1 - b)$,

$c = c_m(1 - b)$,

$v = v_m(1 - b)$, and

$w = w_m(1 - b)$.

The derivation of the voids in the mortar equation is given in Appendix D, and provides the value of the voids for different sand/cement ratios for a constant cement content and known voids in coarse aggregate.

3. Laboratory Procedures:

a. Specifications and Materials Tests:

All materials used in the design of concrete mixtures for this report conform to the MDOT Standard Specifications For Construction (10,11). Portland Cement shall conform to Subsection 901.03, Fine aggregate and coarse aggregate shall conform to Subsection 902.09 and 902.03 respectively of the 1996 Standard

Specifications (10). In particular, cement shall meet ASTM C 150, coarse aggregate shall be tested per ASTM C 127 and fine aggregate shall be tested per ASTM C 128 for specific gravity and absorption. Water used in the mortar void tests shall be room temperature, distilled water free of all deleterious substances.

Material samples may be taken from the source, a stockpile or worksite. Sample size shall allow for surplus material to permit repeat testing, if needed, and must be representative of the production material as delivered to the project site. Sufficient time for testing, such as sieve analysis, organic impurity, Los Angeles abrasion, deleterious pick, specific gravity and absorption, and freeze-thaw, must be provided to insure that quality materials are used for each and every project.

b. Effect of Water:

As the term *mortar voids* implies, the concrete mix design is a function of the characteristics of the voids in the mortar. To gain insight, one needs to look at what happens when the constituents of the mixture are varied. Early on, tests were made to determine the volume of voids for several given sand/cement ratios with varing quantities of mix water. According to Talbot and Richart (1), the voids in the mortar for four typical sands, namely: very fine, fine, medium, and very coarse sand is a function, f, of the grading of the fine aggregate. It is noted that the voids equation for a given sand/cement ratio, s_m,

$$v_m = f(w_m, s_m)$$

decreases with added water down to a point, $(v_m)_{min}$, beyond which the voids increase, indicating that the point of minimum voids has been reached. This point is called the *basic mortar voids* and the percent water, by weight, is called the *Basic Water Content*.

Very distinct points of minimum voids are shown for each type of sand, but it should be noted that each point of minimum voids is, in general, reached with different quantities of water. Also, each minimum point is reached with a different void content. The finer sands, requiring more water to produce minimum voids, have a higher void content than the coarser sands. This is true for any ratio of sand to cement.

The sand used by Fulton (2), to develop MDOT's modified mortar void method back in the mid-1930's graded from medium to very coarse, having a fineness modulus (FM) of about 2.75. Present MDOT Standard Specifications (10) allow a range of 2.50 - 3.35 for base fineness modulus with a ±0.20 variation on 2NS fine aggregate. Fineness modulus values within this base FM range have, in practice, worked well over the years when using mortar void proportioning charts based upon FM = 2.75.

c. General Technique of the Mortar-Voids Test (1,2,5):

The apparatus needed for performing the mortar-voids test consists of either a 2 by 4-inch or a 2 1/2 by 4-inch cylindrical brass mold, accurate balance (2000 g capacity), set of standard weights (1 - 1000 g), No. 20 mesh sieve, burette for measuring water, mixing pan, mixing spoon, small trowel, and tamping rod. Comparative tests have shown no appreciable difference in accuracy between results with the 2 by 4-inch size mold (approximately 200 cc.) and much larger molds. The volume of the mold is determined by filling it with water at 20° C, covering the mold with a glass plate, and weighing. The difference in weights of the mold empty and filled with water is equal to the volume of the mold in cubic centimeters. (1 g of water = 1 cc of water). The mold must be weighed carefully. The balance should give weights accurate to the nearest gram. The burette for measuring water should have a capacity of 250 cc. and be graduated in 1 cc. divisions.

The cement to be used should be sieved through a No. 20 sieve to remove any lumps or foreign material. After the sand and cement are weighed out, they should be mixed dry and then add the water. The amount of water for the first mix should be less than the amount required for minimum volume mortar and should produce a dry mixture. The amount of water for the series of mortar mixtures may vary from 8% of the weight of the dry materials for coarse sands and lean mixtures to 20% for fine sands and rich mixtures. The mortar should be mixed thoroughly with the spoon or trowel and placed in the mold in layers of about 1 inch, each layer being tamped firmly (25 strokes) without too much pressure. A flat-ended wooden tamping rod, about 3/4-inch square, is best, since a pointed rod leaves holes in dry mortar instead of compacting it. When the mold has been filled, the top is struck off neatly with the trowel, the outside of the mold wiped clean, and the known volume of mortar is weighed. A check reading is taken by emptying the material back into the pan, refilling the mold and weighing again.

With the same batch of mortar, another increment of water is added, mixed thoroughly, and the weight of mortar in the mold is determined for this water content. The process is repeated with further increments of water, taking several values near the point of minimum voids and increasing the size of increment after this point is passed.

From the weight and volume of the mortar in the mold and the weights of the ingredients, the volume of the mortar batch is calculated by simple proportion, it being assumed that the mortar in the mold is representative of the batch. The ratio of the absolute volume of sand and cement to the volume of the batch is equal to the density of the mortar, or the complement of the voids.

To allow for the loss of water due to evaporation and absorption during the repetition of voids determinations, which may last twenty to thirty minutes, two batches may be used, the first carried just beyond the point of minimum voids and the second started just back of this point. If plotted, the overlapping of these two determinations shows the effect of the lost water near the minimum point. The second determination gives a more accurate value of the water content at the minimum point. The amount of water lost varies with mixing time, temperature, and humidity. It should be noted that an accurate determination of the basic water content is more difficult than the voids determination at this point of minimum voids.

Complete data on one sand can usually be obtained and computed in 3 to 4 hours by one operator and one recorder. Skill is required in the tamping process since this test depends upon producing about the same degree of compactness in the mortar as may be found in the concrete test specimens containing the same proprtions of sand, cement, and water.

d. Specific Considerations:

By experience, it has been found that by using sand/cement ratios of 2, 3, and 4 for s_m, the mortar void tests will provide data needed in the preparation of most MDOT concrete mixtures. However, for rich mixtures (having higher than normal cement content), ratios of 1, 2, and 3 may be needed. Fine aggregate weights used in the tests have been found by experience to provide sufficient mortar for each series of tests (one series for each ratio).

For example, with a sand/cement ratio equal to 2, we can start with 425 grams of surface dry sand with bulk specific gravity of 2.63. Thus, the solid volume of sand to be used is 425/2.63 = 161.6 cc. Therefore, the solid volume of cement equals 161.6/2 = 80.8 cc for $s_m = 2$. Next, the weight of cement to be used equals the solid volume of cement (80.8 cc) times its specific gravity (3.14) to give 254 grams. The total weight of dry ingredients equals 425 + 254 = 679 grams. This procedure is repeated for the sand/cement ratios of 1, 3, and 4.

Having determined the proportions of sand and cement to be used in the mortar void tests, it is now necessary to determine the volume of water to be used with these portions of sand and cement to obtain a mixture with maximum density. Again by experience, it has been determined that for the mortar batch representing $s_m = 2$, 10% by weight of the dry ingredients is usually a good starting weight of water which will generally work with an average fine aggregate and Type I-A cement. This leads to (0.10)(679 g) = 67.9 g. Assuming that 1 cc of water weighs very nearly 1 g, the volume of water to be added to the mix is about 68 cc. Similar calculations are made for 11, 12, 13, and 16% water contents for the sand/cement ratio = 2.

For the ratio = 3, calculations for 9, 10, 11, 12, and 14% water content are required. For the ratio = 4, calculations for 8, 9, 10, 11, and 13% water content are made. For the ratio = 1, calculations for 13, 14, 15, 16, and 20% water content are performed. This eventually provides 5 points to define a piecewise linear approximation of each mortar void function, shown on the next page. The four mortar void functions (Figure 1) are from the data in Table B-1 of Appendix B.

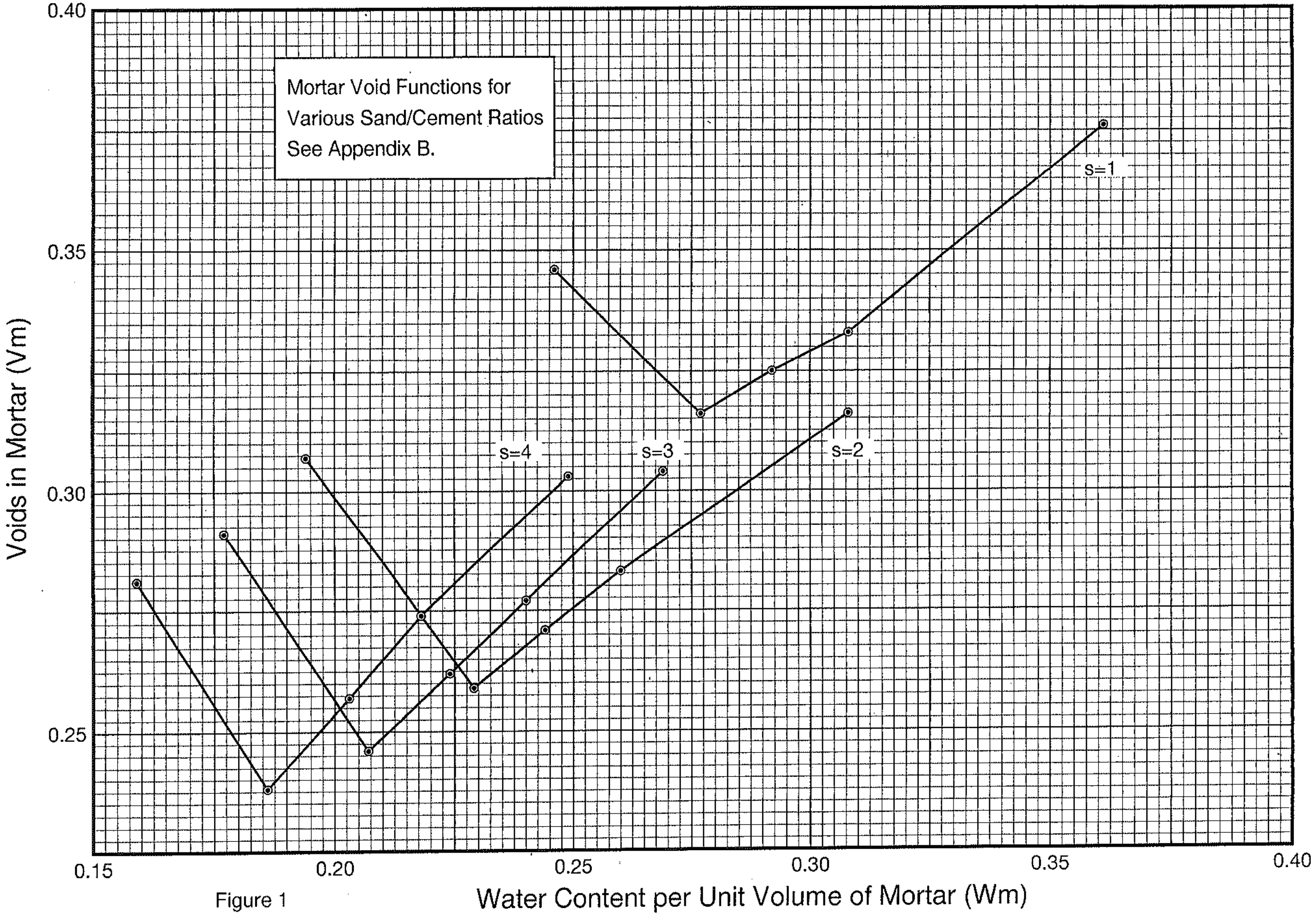
Mortar Void Functions for
Various Sand/Cement Ratios
See Appendix B.
s=1
s=2
s=3
s=4
Voids in Mortar (Vm)
0.40
0.35
0.30
0.25
0.15
0.20
0.25
0.30
0.35
0.40
Water Content per Unit Volume of Mortar (Wm)

Figure 1

4. Development of the Proportioning Chart:

Aggregate specific gravities (G_a, G_b) and absorptions (A_a, A_b), coarse aggregate dry, loose unit weight (u), grade of concrete, cement and flyash content (W_c, W_f) and specific gravities (G_c, G_f), design air content (V_{da}), workability factor (b/b_0), and relative water content (RWC) must be predetermined. Using the computer program, simply input the requested data asked for on each menu. For manually worked problems, proceed through the following steps:

1. Determine whether a metric unit design or an English unit design is required.

2. Calulate (c), the absolute volume of cementitious material per m^3 or ft^3.

$$c = (W_c/G_c + W_f/G_f)/1000 \text{ for metric, or}$$

$$(W_c/G_c + W_f/G_f)/1684.8 \text{ for English,}$$

where the constant (1684.8 lbs/yd^3) = (27 ft^3/yd^3)(62.4 lbs/ft^3).

3. Calculate (b), the absolute volume of coarse aggregate per unit volume of concrete.

$$b = (b/b_0)u/1000G_b \text{ for metric, or}$$

$$(b/b_0)u/62.4G_b \text{ for English.}$$

4. Calculate (1 - b), the absolute volume of mortar per unit volume of concrete.

5. Calculate (b_0), the absolute volume of coarse aggregate per unit volume of coarse aggregate.

$$b_0 = u/1000G_b \text{ for metric, or}$$

$$u/62.4G_b \text{ for English.}$$

6. Calculate (c_m), the absolute volume of cementitious material per volume of mortar. $c_m = c/(1 - b)$.

7. For a pre-determined relative water content, select the water content in the mortar (w_{m2}) and the void content in the mortar (v_{m2}) for the sand/cement ratio, $s = 2$; similarly, select w_{m3} and v_{m3} for $s = 3$ from Table B-2 in Appendix B or

using the sand/cement function graph (Figure 1), follow vertically upward from w_{m2} and w_{m3} to intersect the s = 2 and 3 functions, respectively. From the intersection points, follow left to the dependent axis to read the v_{m2} and v_{m3} values.

8. Plot the void content line (v_{m2}, v_{m3}) and the water content line (w_{m2}, w_{m3}) on Figure 2.

9. Recalculate the voids in the mortar for the actual mix:

$$v_{m2r} = 1 - 3c_m, \text{ and}$$

$$v_{m3r} = 1 - 4c_m,$$

where the constant "3" comes from the sum of 1 + sand/cement ratio of 2 and the constant "4" comes from the sum of 1 + sand/cement ratio of 3.

10. Plot the recalculated voids values (v_{m2r}, v_{m3r}) on Figure 2 and draw a line between these points intersecting the original voids line (v_{m2}, v_{m3}).

11. The point where the recalculated (v_{m2r}, v_{m3r}) voids line intersects the original (v_{m2}, v_{m3}) line is the desired $(v_m)_1$. A line drawn vertically down from this $(v_m)_1$ point intersects the (w_{m2}, w_{m3}) and the sand/cement ratio base line, thus gives the values for $(w_m)_1$, and s_1 at the intersections. If this does not yield intersections, then repeat Steps 7 - 11 for the appropriate adjacent sand/cement functions (ie: s = 1 & 2 or s = 3 & 4).

12. Calculate intermediate results for water, voids, and fine aggregate:

$$w = (w_m)_1(1 - b),$$

$$v = w + 0.01V_{da}, \text{ and}$$

$$a = s_1(c) - v + (v_m)_1(1 - b).$$

13. Verify the yield by adding a + b + c + v = 1 unit volume of concrete.

14. Dry aggregate batch weights are calculated from:

$A = 1000(a)G_a$, and $B = 1000(b)G_b$ for metric, or

$A = 1684.8(a)G_a$, and $B = 1684.8(b)G_b$ for English.

15. Calculate the net water (W_n) from:

$W_n = 1000(w)$ for metric, or

$1684.8(w)$ for English.

16. Calculate the total water (W_t) from:

$$W_t = A(A_a/100) + B(A_b/100) + W_n.$$

17. A Mix Design Chart that will establish design weights for the aggregates and water over a range of coarse aggregate unit weights is created by plotting on Figure 3, the derived weights (Steps 3 through 16) from two different assumed coarse aggregate unit weights, one above and one below the predetermined unit weight (u).

5. Worked Examples (English & Metric Units):

a. Concrete Pavement Mix Design Problem:

A slipform pavement mix design is presented in English and Metric units along with manual calculations for the Metric version in Appendix E.

Table 5a-1 Endpoint Calculations (± 100 kg/m^3):

Unit Weight	Method	FA Weight	CA Weight	Total Water
1470 kg/m^3	Computer	899.	1058.	147.
	Manual	902.	1058.	146.
1670 kg/m^3	Computer	783.	1202.	140.
	Manual	786.	1202.	139.

Table 5a-2 Calculations at Predermined Unit Weight:

Unit Weight	Method	FA Weight	CA Weight	Total Water
1570 kg/m^3	Computer	841.	1130.	144.
	Manual	844.	1130.	143.
	Man.(Sect.6)	841.	1130.	143.

The manually calculated batch weights are slightly in error due to rounding of intermediate results, but still provides a good check when modifying the computer program. The computer results should be accepted as more accurate, after the computer program has been thoroughly checked out after any modification.

b. Structural Concrete Mix Design Problem:

A computer solution of a bridge deck mix design is presented in English and Metric units along with step-by-step manual calculations and Figure 2-2E for the English version in Appendix F.

Table 5b-1 Endpoint Calculations (± 10 lb/ft^3):

Unit Weight	Method	FA Weight	CA Weight	Total Water
76 lb/ft^3	Computer	1409.	1436.	333.
	Manual	1406.	1436.	332.
96 lb/ft^3	Computer	1088.	1814.	313.
	Manual	1087.	1815.	313.

Table 5b-2 Calculations at Predetermined Unit Weight:

Unit Weight	Method	FA Weight	CA Weight	Total Water
86 lb/ft^3	Computer	1249.	1625.	323.
	Manual	1247.	1625.	323.

A blank Concrete Proportioning Worksheet appears in Appendix G from which copies can be made for manual calculations of the reader's choice. Please remember to use original graph sheets, not copies, for making any graphs.

6. Computer Version of Mortar-Voids Equations:

The following is a compilation of equations and general procedure used in the present computer program (6) to perform the mortar-void calculations:

$$\text{If } s = 1:\ \text{Voids}(s,w) = \begin{cases} -0.967741935w + 0.58460645 & \text{if } w \le 0.277 \\ 0.6w + 0.1498 & \text{if } w < 0.292 \\ 0.5w + 0.179 & \text{if } w < 0.308 \\ 0.81132075w + 0.0831132 & \text{if } w \ge 0.308 \end{cases}$$

$$\text{If } s = 2:\ \text{Voids}(s,w) = \begin{cases} -1.3714286w + 0.573057 & \text{if } w \le 0.229 \\ 0.8w + 0.0758 & \text{if } w < 0.244 \\ 0.75w + 0.088 & \text{if } w < 0.260 \\ 0.6875w + 0.10425 & \text{if } w \ge 0.260 \end{cases}$$

$$\text{If } s = 3:\ \text{Voids}(s,w) = \begin{cases} -1.5w + 0.5565 & \text{if } w \le 0.207 \\ 0.93528284w + 0.0524585 & \text{if } w > 0.207 \end{cases}$$

$$\text{If } s = 4:\ \text{Voids}(s,w) = \begin{cases} -1.5357143w + 0.523643 & \text{if } w \le 0.186 \\ 1.117647w + 0.03011765 & \text{if } w < 0.203 \\ 1.13333333w + 0.0269333 & \text{if } w < 0.218 \\ 0.93548387w + 0.0700645 & \text{if } w \ge 0.218 \end{cases}$$

a. Calculate the basic water content, w_{m2} and w_{m3}, for sand/cement ratio 2 & 3:

$w_{m2} = 0.229w$,

$w_{m3} = 0.207w$, where w is the relative water content.

b. Compute voids in mortar for w_{m2} and w_{m3}:

$v_{m2} = \text{voids}(2,w_{m2})$,

$v_{m3} = \text{voids}(3,w_{m3})$.

c. Fit a line w_L through points $(2,w_{m2})$ and $(3,w_{m3})$.

Fit a line v_L through points $(2,v_{m2})$ and $(3,v_{m3})$.

d. Recalculate the voids:

$v_{m2r} = 1 - 3c_m$

$v_{m3r} = 1 - 4c_m$, where c_m is the cement content in the mortar.

e. Fit a line v_R through points $(2, v_{m2r})$ and $(3, v_{m3r})$.

f. Intersect lines v_L and v_R giving $(x,y) = (s_1, (v_m)_1)$.

g. If $x < 2$, recalculate with $s = 1$:

$w_{m1} = 0.277w$,

$v_{m1} = \text{voids}(1, w_{m1})$,

Fit a line v_L through points $(1, v_{m1})$ and $(2, v_{m2})$,

Fit a line w_L through points $(1, w_{m1})$ and $(2, w_{m2})$,

$v_{m1r} = 1 - 2c_m$,

Fit a line v_R through points $(1, v_{m1r})$ and $(2, v_{m2r})$.

Intersect lines v_L and v_R giving (x,y).

h. If intersection in step f corresponds to $s > 3$:

$w_{m4} = 0.186w$,

$v_{m4} = \text{voids}(4, w_{m4})$,

Fit a line v_L through points $(3, v_{m3})$ and $(4, v_{m4})$,

Fit a line w_L through points $(3, w_{m3})$ and $(4, w_{m4})$,

$v_{m4r} = 1 - 5c_m$,

Fit a line v_R through points $(3, v_{m3r})$ and $(4, v_{m4r})$,

Intersect lines v_L and v_R giving (x,y).

i. From step f, voids in mortar, $(v_m)_1 = y$.

Water in mortar, $(w_m)_1$ in terms of coefficients of w_L equation:

$(w_m)_1 = (-a(x)-c)/b$.

The above set of equations used in R. Miller's computer program (6) eliminates the need to use the charts of Figures 1, 2, and 3 originally used in the manual solutions. Use of the computer program provides increased solution accuracy and reduces human calculation error. The remaining portion of the computer solution is similar to steps 12 - 16 of Section 4 of this report. Appendix E contains an illustration of the use of these equations to verify the computer generated batch weights for the predetermined unit weight of 1570 kg/m^3.

7. Use of the Proportioning Chart in the Field:

If the chart does not produce acceptable concrete at some future time, the materials may have changed enough that some adjustments should be made. A common example would be where the coarse aggregate unit weight changes. To correct for this, simply move up or down on the chart to the new unit weight line and pick off the new quantities of fine and coarse aggregate and water for subsequent batches. If this still does not work, consult the Concrete Testing Laboratory in Lansing to see if the aggregate specific gravities and absorptions should be re-checked.

Some sources of aggregate are more variable in specific gravity, absorption, and unit weight than other sources and should be monitored more closely. Their proportioning charts may require adjustment on a more frequent basis.

8. Observations, Comments and Recommendations:

The *Mortar Void Method* of proportioning concrete has been used successfully by MDOT since about 1928. This method has been used for pavements, curb and gutters, base course, sidewalks, bridges and other concrete structures. Each mixture requires different quantities of cement, aggregate and water depending upon its use, strength, the method of placement, or economy required. The relative water content and workability factor for each mix design has been established by feedback from the field over many years of experience.

a. The major appeal of the *Mortar Void Method* is that no trial batches have to be made to obtain the correct yield, unlike other methods such as ACI, where trial batches are required or at least highly recommended. It should be noted that originally, it was thought trial batches were also needed with this method, but has been proven unnecessary based upon many years of acceptable results.

b. This procedure produces acceptable PCC over the range of fineness modulus (2.50 - 3.35) allowed for 2NS by MDOT's Standard Specifications (10,11) even if FM varies from the base value of 2.75 used to generate the sand/cement ratio functions many years ago.

c. In general, PCC pavements should include larger top-size aggregates in more uniform gradations than the Series 6 gradation presently used by MDOT. If large aggregate mixtures are designed properly, some savings in cement cost can be achieved, offsetting possible higher coarse aggregate costs. In addition, aggregate interlock resulting from the larger coarse aggregate can assist the reinforcing steel in holding the slabs together after cracking ocurrs.

However, the larger the top size of the coarse aggregate used, the more important freeze-thaw durability becomes. Only coarse aggregate known to be highly durable in freeze-thaw (maximum of 0.020% dilation per 100 cycles per Michigan Test Method MTM 115) should be allowed when using coarse aggregate having a nominal maximum top size greater than 1 inch (25.0 mm) in a paving mixture. Sources providing coarse aggregate sizes greater than 1 inch (25.0 mm) nominal maximum top size could either be tested in freeze/thaw or the durability (dilation) should be estimated.

If actual dilation test results for greater than 1 inch (25.0 mm) material are unavailable for a given source, the following rule-of-thumb can be used with a reasonable degree of confidence. A measure of acceptable freeze-thaw durability for larger coarse aggregate mixtures is to allow only reasonably uniform sources (± 0.02 in bulk specific gravity throughout the deposit) having a minimum bulk specific gravity (Oven Dry Basis) of 2.75, a maximum absorption of 0.75 as determined from ASTM C 127 (15) and a 0.5% maximum on the sum of soft & chert particles per Michigan Test Method 110 for actual production material.

Alternately, large size (2 inch - 1 inch) material from sources producing 6AA pre-stress quality (maximum of 0.010% dilation per MTM 115) and having a minimum of 2.60 bulk specific gravity could be used.

d. Current MDOT specifications for base course concrete requires more coarse aggregate and less cement than pavement concrete.

e. Similarly, current MDOT specifications for side walk and curb and gutter concrete mixtures require less water and finer aggregates than PCC pavements, with about the same amount of cement.

f. For a given workability factor, the weight of coarse aggregate in a unit volume of concrete is a function of unit weight and is usually ranked in decreasing order from natural gravel, to quarried stone, to slag. This is true for both pavement and structural concrete.

g. MDOT specifies that grades of pavement concrete have a higher workability factor than structural concrete, thus a greater weight of coarse aggregate per unit volume of concrete.

h. If, at some time in the future, materials change enough (ie; cement fineness or alkalinity, fine aggregate fineness modulus, etc.) such that the present computer program does not produce acceptable PCC mixtures after other potential causes have been investigated and eliminated, it is suggested that MDOT consider re-doing the lab test data used to generate the sand/cement functions of Appendix B.This has been done before and was last performed about 1970 and was the basis of Shehan's report (5). The Mix Designer should solicit and rely on feedback from the field to determine if or when updating may be required. However, remember the old saying: "If it isn't broken, don't fix it." It applies here.

i. When using the manual graphical calculation method for Mortar Voids, care should be taken when constructing the Voids and Water Content graph (Figure 2). Slight inaccuracies can lead to significant errors in the final batch weights for the aggregates and total water. It is suggested that when selecting the two assumed unit weight values, one above and one below the predetermined value, select ± 100 kg/m^3 for metric or ± 10 lb/ft^3 for English to increase the accuracy when reading the $(v_m)_1$, $(w_m)_1$, and s_1 values from the proper intersection points. Only original graph paper or computer generated grid sheets should be used and intersection points should be estimated to the nearest 0.0005. Calculations should be recorded to the nearest 0.0001 at intermediate steps to reduce accumulation of rounding errors. The manually calculated coarse aggregate batch weight, B, should be within one unit, however, the fine aggregate batch weight, A, and total water, W_t, may have larger decrepancies, due to more intermediate calculation steps and rounding of results.

j. Since the <u>step size</u> in unit weights is different on the metric chart verses the English chart for the same mix design, <u>only</u> the line for the predetermined unit weight can be cross-checked easily. For checking this line of batch weights, use the following scale factors to obtain an approximate comparison between metric and English quantities:

0.59328 for FA, CA, and Total Water, and

16.01846 for the unit weight.

Example: Consider the example in Appendix E. Compare the two computer output sheets for Mix Design Numbers 1M & 1E. The coarse aggregate from 1M has a typical metric dry, loose unit weight of 1570 kg/m^3. By dividing 1570 by 16.01846, the English equivalent dry, loose unit weight is approximately 98 lbs/ft^3, to the nearest whole number. This number was used as an input value for the English unit computer sheet 1E. The computer generated metric oven-dry fine aggregate, coarse aggregate and total water values are 841, 1130, and 144 kg/m^3, respectively. By dividing these values by 0.59328, one obtains the English values 1418, 1905, and 243 lbs/ft^3, respectively. This checks within ± 1 lb/ft^3 of the computer calculated values on output sheet 1E. The slight differences are due to rounding errors introduced as a result of assuming 98 lbs/ft^3 to be exactly equal to 1570 kg/m^3, when it is only an approximation.

Since the metric step or increment in unit weight of 10 kg/m^3 is not equal to the English increment of 1 lb/ft^3, one can not compare results line-for-line for unit weights above or below the typical line of results.

9. References:

1. Talbot, Arthur N. and Richart, Frank E., *The Strength of Concrete: Its Relation to the Cement, Aggregates, And Water*, University of Illinois Bulletin No. 137, Engineering Experiment Station, October 15, 1923.

2. Fulton, Roy S., *Mortar Voids Method of Design and Field Control of Cement Concrete Mixtures*, MSHD Testing Laboratory, University of Michigan, June 19, 1936.

3. Fulton, Roy S., *A Brief Description of The Mortar Void Method of Concrete Design*, MSHD Testing Laboratory, University of Michigan, August 8, 1938.

4. Fulton, Roy S., *Concrete Pavement Short Course - Concrete Prortioning*, March 25-26, 1942.

5. Shehan, Eugene L., *The Mortar Voids Method of Proportioning Concrete as used by the Michigan Department of State Highways*, September 1, 1970.

6. Miller, Robert D., *MDOT Concrete Mix Design User Manual*, Michigan Department of Transportation, February 14, 1997.

7. Kosmatka, Steven H. and Panarese, William C., *Design and Control of Concrete Mixtures*, Portland Cement Association, Thirteenth Edition, First Printing 1995.

8. Kosmatka, Steven H., et.al., *Design and Control of Concrete Mixtures*, Canadian Portland Cement Association, Sixth Canadian Edition, Second Printing(rev.) 1990.

9. Neville, A. M., *Properties of Concrete*, Pitman Publishing, 1977.

10. *1996 Standard Specifications For Construction*, Michigan Department Of Transportation, Second Printing, May 1, 1996, (Metric Unit Edition).

11. *1990 Standard Specifications For Construction*, Michigan Department Of Transportation, October 10, 1990, (English Unit Edition).

12. Woodhouse, T., "Unit Weight of Coarse Aggregates Comparison", MDOT Concrete Laboratory Memo, September 16, 1992.

13. Vogler, R. H., "Unit Weight of Aggregate", undated written summary based upon reference 12.

14. Legg, F. E., Jr. and Vogler, R. H., *Proportioning Concrete For Michigan State Highway Department Projects*, MSHD Testing Laboratory, June, 1962.

15. *Concrete and Aggregates*, Volume 04.02, ASTM, October, 1997.

Appendix A

Definition of Terms

FA	fine aggregate (sand).
CA	coarse aggregate.
FM	fineness modulus of the fine aggregate.
PCC	Portland Cement Concrete.
BWC	basic water content.
RWC	relative water content.
a	absolute volume of sand per unit volume of concrete.
b	absolute volume of coarse aggregate per unit volume of concrete.
c	absolute volume of cementitious material per unit volume of concrete.
v	absolute volume of voids (air + water) per unit volume of concrete.
w	absolute volume of water per unit volume of concrete.
s	sand/cement ratio in concrete.
$1-b$	absolute volume of mortar per unit volume of concrete.
b_0	absolute volume of coarse aggregate per unit volume of <u>dry, loose</u> coarse aggregate.
b/b_0	unit volume of coarse aggregate per unit volume of concrete.
a_m	absolute volume of sand per unit volume of mortar.
c_m	absolute volume of cement per unit volume of mortar.
v_m	absolute volume of voids (air + water) per unit volume of mortar.
w_m	absolute volume of water per unit volume of mortar.
s_m	sand/cement ratio in the mortar.
G_a	specific gravity of the fine aggregate (sand).
G_b	specific gravity of the coarse aggregate.
G_c	specific gravity of the cement.
G_f	specific gravity of the flyash.
A_a	percent absorption in the fine aggregate (sand).
A_b	percent absorption in the coarse aggregate.
u	unit weight of the coarse aggregate.
V_{da}	percent volume of design air.
W_a	weight of fine aggregate (sand) in the concrete mixture.
W_b	weight of coarse aggregate in the concrete mixture.
W_c	weight of cement in the concrete mixture.
W_f	weight of flyash in the concrete mixture.
A	dry, fine aggregate batch weight.
B	dry, coarse aggregate batch weight.
W_n	net water.
W_t	total water.

Appendix B

Mortar Void Test Data For Various Sand/Cement Ratios:
(Latest, Taken from Shehan's Report (5) and Expanded
For Following Conditions)

Specific Gravity of Cement = 3.15
Specific Gravity of Sand (Surface Dry) = 2.63
Absorption of Sand = 1.34%
Fineness Modulus of Sand = 2.75

Table B-1. Mortar Void Function Data.

s = 1			s = 2			s = 3			s = 4		
%	V_m	W_m	%	V_m	W_m	%	V_m	W_m	%	V_m	W_m
13	0.346	0.246	10	0.307	0.194	9	0.291	0.177	8	0.281	0.158
14	0.316	0.277	11	0.259	0.229	10	0.246	0.207	9	0.238	0.186
15	0.325	0.292	12	0.271	0.244	11	0.262	0.224	10	0.257	0.203
16	0.333	0.308	13	0.283	0.260	12	0.277	0.240	11	0.274	0.218
20	0.376	0.361	16	0.316	0.308	14	0.304	0.269	13	0.303	0.249

Table B-2. Relative Water Content Effect.

		RWC = 1.05		RWC = 1.15		RWC = 1.20		RWC = 1.25	
S	Basic W_m	RWCx Basic W_m	Corr. V_m	RWCx Basic W_m	Corr. V_m	RWCx Basic W_m	Corr. V_m	RWCx Basic W_m	Corr. V_m
1	0.277	0.291	0.323	0.319	0.342	0.332	0.353	0.346	0.364
2	0.229	0.240	0.268	0.263	0.285	0.275	0.294	0.286	0.301
3	0.207	0.217	0.256	0.238	0.275	0.248	0.285	0.259	0.295
4	0.186	0.195	0.248	0.214	0.270	0.223	0.279	0.233	0.288

Corr. = Corresponding.

Appendix C

Derivation of Voids in Mortar Equation.

From basic principles, $a_m + c_m + v_m = 1$ unit volume of mortar, (C.1)

where the variables are defined in Appendix A.

Solving for v_m, we obtain:

$$v_m = -a_m + (1 - c_m), \tag{C.2}$$

$$v_m = -a_m(c_m/c_m) + (1 - c_m), \tag{C.3}$$

$$v_m = -(a_m/c_m)c_m + (1 - c_m), \tag{C.4}$$

$$v_m = -(s_m)c_m + (1 - c_m). \tag{C.5}$$

Let $x = s_m$, thus

$$v_m = -xc_m + (1 - c_m), \text{ and} \tag{C.6}$$

$$\mathbf{v_m = 1 - (x + 1)c_m} \tag{C.7}$$

Then, solve for v_m, by letting $x = s_m = 1, 2, 3,$ and 4,

resulting in:

$$\begin{aligned} v_m &= 1 - 2c_m, \text{ for } x = 1, \\ &= 1 - 3c_m, \text{ for } x = 2, \\ &= 1 - 4c_m, \text{ for } x = 3, \text{ and} \\ &= 1 - 5c_m, \text{ for } x = 4, \text{ respectively.} \end{aligned} \tag{C.8}$$

In general, $v_m = f(x, c_m) = f(s_m, c_m)$. (C.9)

Appendix D

Dry, Loose vs. Dry, Rodded Measure.

The MDOT Concrete Mix Design program is set up to use dry, loose unit weights of coarse aggregate. From R. Vogler (13), if the dry, rodded unit weight is the only value available, the dry, loose unit weight can be estimated from the following:

$$(b/b_0)_i = W/27u_i, \qquad \text{(D.1)}$$

where $i = r$ for rodded and $i = l$ for loose.

Therefore, $$W = 27u_i(b/b_0)_i = \text{constant}. \qquad \text{(D.2)}$$

Thus, $$u_r(b/b_0)_r = u_l(b/b_0)_l \text{ and,} \qquad \text{(D.3)}$$

$$\mathbf{(b/b_0)_l = (u_r/u_l)(b/b_0)_r}. \qquad \text{(D.4)}$$

Based upon T. Woodhouse's lab data (12), it can be shown that the ratio of dry, rodded unit weight to dry, loose unit weight averaged over sixteen typical sources is: $$(u_r/u_l)_{ave} = 1.076 \text{ for sixteen sources.} \qquad \text{(D.5)}$$

Substituting in gives an estimated dry, loose unit weight as a function of dry, rodded unit weight of coarse aggregate:

$$(b/b_0)_l \approx 1.076\ (b/b_0)_r. \qquad \text{(D.6)}$$

Most references (7,8) give tables of dry, rodded workability factors, while MDOT uses dry, loose measure. Thus, one can calculate or construct a corresponding dry, loose workability table from this relationship as follows:

Table D-1: Dry, Loose (b/b_0) For Different Fine Aggregate Fineness Moduli.

Max Size in.(mm)	2.40	2.60	2.80	3.00
1/2(12.5)	0.63	0.61	0.59	0.57
3/4(19.0)	0.71	0.69	0.67	0.65
1(25.0)	0.76	0.74	0.72	0.70
1 1/2(37.5)	0.81	0.79	0.76	0.74
2(50)	0.84	0.82	0.80	0.77

For FM values between those in Table D-1, one can interpolate to determine the desired workability factor.

For pavements, the b/b_0 values may be increased about 10% and for pumped concrete, the b/b_0 values may be reduced up to 10% (7,8).

If the Mix Designer wants an aggregate-specific workability factor instead of the estimated factor from Table D-1, simply run both the dry, rodded and dry, loose unit weights on a representative sample of production material using ASTM C 29 (15) and then calculate the dry, loose workability factor needed for the mix design program from equation (D.4).

Appendix E

Solutions for a Pavement Mix Design Problem

Mix Parameters and Materials Properties

$G_c = 3.12$
$G_a = 2.65$
$A_a = 1.30$
$G_b = 2.89$
$A_b = 0.30$
$b/b_0 = 0.72$
$RWC = 1.05$
$V_{da} = 5.5\%$

Metric Units
P1 (SF)
$W_c = 335$ kg/m^3
$u = 1570$ kg/m^3

English Units
35P (SF)
$W_c = 565$ lb/ft^3
$u = 98$ lb/ft^3

MICHIGAN DEPARTMENT OF TRANSPORTATION

FORM 1830

CONCRETE PROPORTIONING DATA

FILE 300

CONTROL SECTION ID: Ex-1M
JOB NUMBER: 1M
LAB NUMBER: 1M
GRADE OF CONCRETE: P1 (SF)
INTENDED USE OF CONCRETE: Pavement (Slipform)

DATE: 8/23/1998
SPECIFICATION: 1996 STD SPECS
MIX DESIGN NUMBER: 1M

CONCRETE MATERIALS

MATERIAL	SOURCE	PIT NUMBER	CLASS	SPECIFIC GRAVITY	ABSORPTION PERCENT
CEMENT	(SEE REMARKS)		I/IA	3.12	
FINE AGG.	Standard Sand	19-99	2NS	2.65	1.30
COARSE AGG.	Trap Rock	95-99	6AAA	2.89	0.30
FLY ASH					

CEMENT CONTENT, kg/m^3 335
AIR CONTENT (DESIGN): 5.5% (SPECIFIED): 6.5%
R.W.C: 1.05
FLY ASH CONTENT, kg/m^3: 0

B/Bo : 0.72
SPECIFICATION TOLERANCE (±): 1.5%
THEORETICAL YIELD: 100.00%

WEIGHT OF COARSE AGG. (DRY/LOOSE) kg/m^3	AGGREGATE AND WATER PROPORTIONS QUANTITIES, kg/m^3 OF CONCRETE		
	FINE AGG (OVEN DRY)	COARSE AGG (OVEN DRY)	TOTAL WATER
1520	870	1094	145
1530	864	1102	145
1540	858	1109	145
1550	852	1116	144
1560	847	1123	144
1570	841	1130	144
1580	835	1138	143
1590	829	1145	143
1600	823	1152	143
1610	817	1159	142
1620	811	1166	142

REMARKS:
THIS CHART FOR USE WITH CEMENTS OF THE CLASS SHOWN FROM APPROVED SOURCES.

TYPICAL UNIT WEIGHT (DRY, LOOSE) OF COARSE AGGREGATE AS DESCRIBED ABOVE IS 1570 kg/m^3

SPECIAL MESSAGES:

CC:

GAIL H. GROVE
UM RIGID PAVEMENT CENTER

MICHIGAN DEPARTMENT OF TRANSPORTATION

FORM 1830 CONCRETE PROPORTIONING DATA FILE 300

CONTROL SECTION ID: Ex-1E
JOB NUMBER: 1E
LAB NUMBER: 1E
GRADE OF CONCRETE: 35P (SF)
INTENDED USE OF CONCRETE: Pavement (Slipform)

DATE: 8/23/1998
SPECIFICATION: 1996 STD SPECS
MIX DESIGN NUMBER: 1E

CONCRETE MATERIALS

MATERIAL	SOURCE	PIT NUMBER	CLASS	SPECIFIC GRAVITY	ABSORPTION PERCENT
CEMENT	(SEE REMARKS)		I/IA	3.12	
FINE AGG.	Standard Sand	19-99	2NS	2.65	1.30
COARSE AGG.	Trap Rock	95-99	6AAA	2.89	0.30
FLY ASH					

CEMENT CONTENT, lb/cyd 565
AIR CONTENT (DESIGN): 5.5% (SPECIFIED): 6.5%
R.W.C: 1.05
FLY ASH CONTENT, lb/cyd: 0

B/Bo : 0.72
SPECIFICATION TOLERANCE (±): 1.5%
THEORETICAL YIELD: 100.00%

WEIGHT OF COARSE AGG. (DRY/LOOSE) lb/cft	AGGREGATE AND WATER PROPORTIONS QUANTITIES, lb/cyd OF CONCRETE		
	FINE AGG (OVEN DRY)	COARSE AGG (OVEN DRY)	TOTAL WATER
93	1496	1808	247
94	1481	1827	246
95	1465	1847	245
96	1449	1866	244
97	1433	1886	243
98	1417	1905	242
99	1401	1925	241
100	1386	1944	240
101	1370	1963	239
102	1354	1983	238
103	1338	2002	237

REMARKS:
THIS CHART FOR USE WITH CEMENTS OF THE CLASS SHOWN FROM APPROVED SOURCES.

TYPICAL UNIT WEIGHT (DRY, LOOSE) OF COARSE AGGREGATE AS DESCRIBED ABOVE IS 98 lb/cft.

SPECIAL MESSAGES:

CC:

GAIL H. GROVE
UM RIGID PAVEMENT CENTER

CONCRETE PROPORTIONING WORKSHEET

Grade/Use: P1 (SF)/Pavement		b/b_0 = 0.72	RWC = 1.05	08/22/98
G_a = 2.65	G_b = 2.89	G_c = 3.12	G_f = ---	
A_a = 1.30	A_b = 0.30	W_c = 335	W_f = ---	
Design Air = 0.055		Actual (Dry, Loose) Unit Weight = 1570		
Assumed Unit Weight, (above actual) = 1470				
c = 0.1074	b = 0.3362	1-b = 0.6338	b_0 = 0.5087	c_m = 0.1695
w_{m2} = 0.240	v_{m2} = 0.268		w_{m3} = 0.217	v_{m3} = 0.256
v_{m2r} = 0.4915			v_{m3r} = 0.3220	
s_1 = 3.420	$(v_m)_1$ = 0.2515	$(w_m)_1$ = 0.2075		
w = 0.1315	v = 0.1865	a = 0.3402		
check: a + b + c + v = 1.0000			A = 902.	B = 1058.
W_n = 131.5			W_t = 146.0	
Assumed Unit Weight, (below actual) = 1670				
c = 0.1074	b = 0.4160	1-b = 0.5840	b_0 = 0.5779	c_m = 0.1839
w_{m2} = 0.240	v_{m2} = 0.268		w_{m3} = 0.217	v_{m3} = 0.256
v_{m2r} = 0.4483			v_{m3r} = 0.2644	
s_1 = 3.050	$(v_m)_1$ = 0.256	$(w_m)_1$ = 0.215		
w = 0.1256	v = 0.1806	a = 0.2965		
check: a + b + c + v = 1.001			A = 786.	B = 1202.
W_n = 125.6			W_t = 139.0	

Calculated by: ____________

Checked by: ____________

Input:

b/b_0	Workability factor
RWC	Relative water content
G_a, A_a	Sp. Gr. & Abs. - FA
G_b, A_b	Sp. Gr. & Abs. - CA
G_c, W_c	Sp. Gr. & Wt. - cement
G_f, W_f	Sp. Gr. & Wt. - flyash
V_{da}	Design air
u	Unit weight - CA

Output:

A	Batch weight (dry) - FA
B	Batch weight (dry) - CA
W_n	Net water
W_t	Total water

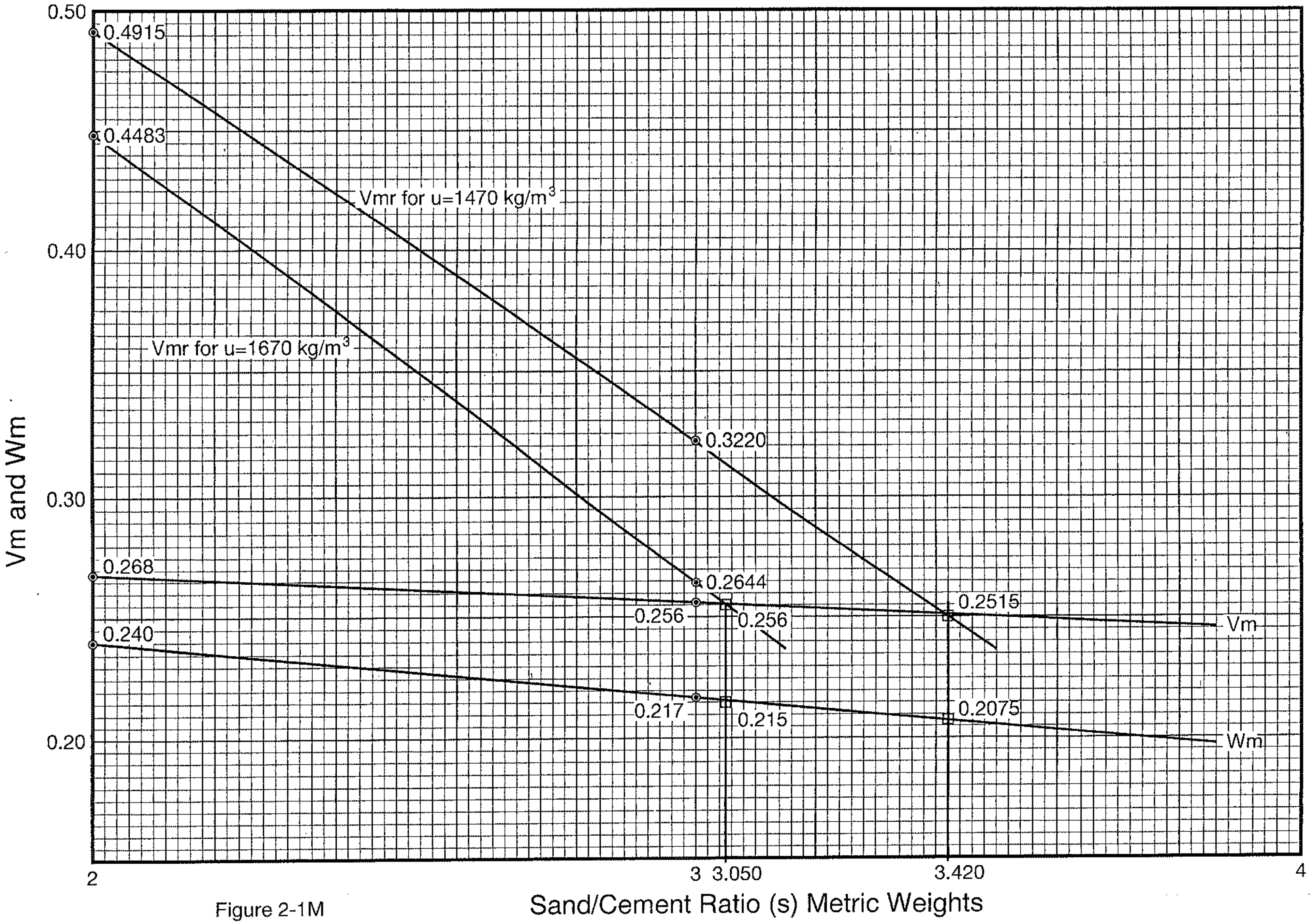
0.50
0.40
0.30
0.20
Vm and Wm
0.4915
0.4483
Vmr for u=1470 kg/m³
Vmr for u=1670 kg/m³
0.3220
0.268
0.2644
0.256
0.256
0.2515
Vm
0.240
0.217
0.215
0.2075
Wm
2
3
3.050
3.420
4
Sand/Cement Ratio (s) Metric Weights

Figure 2-1M

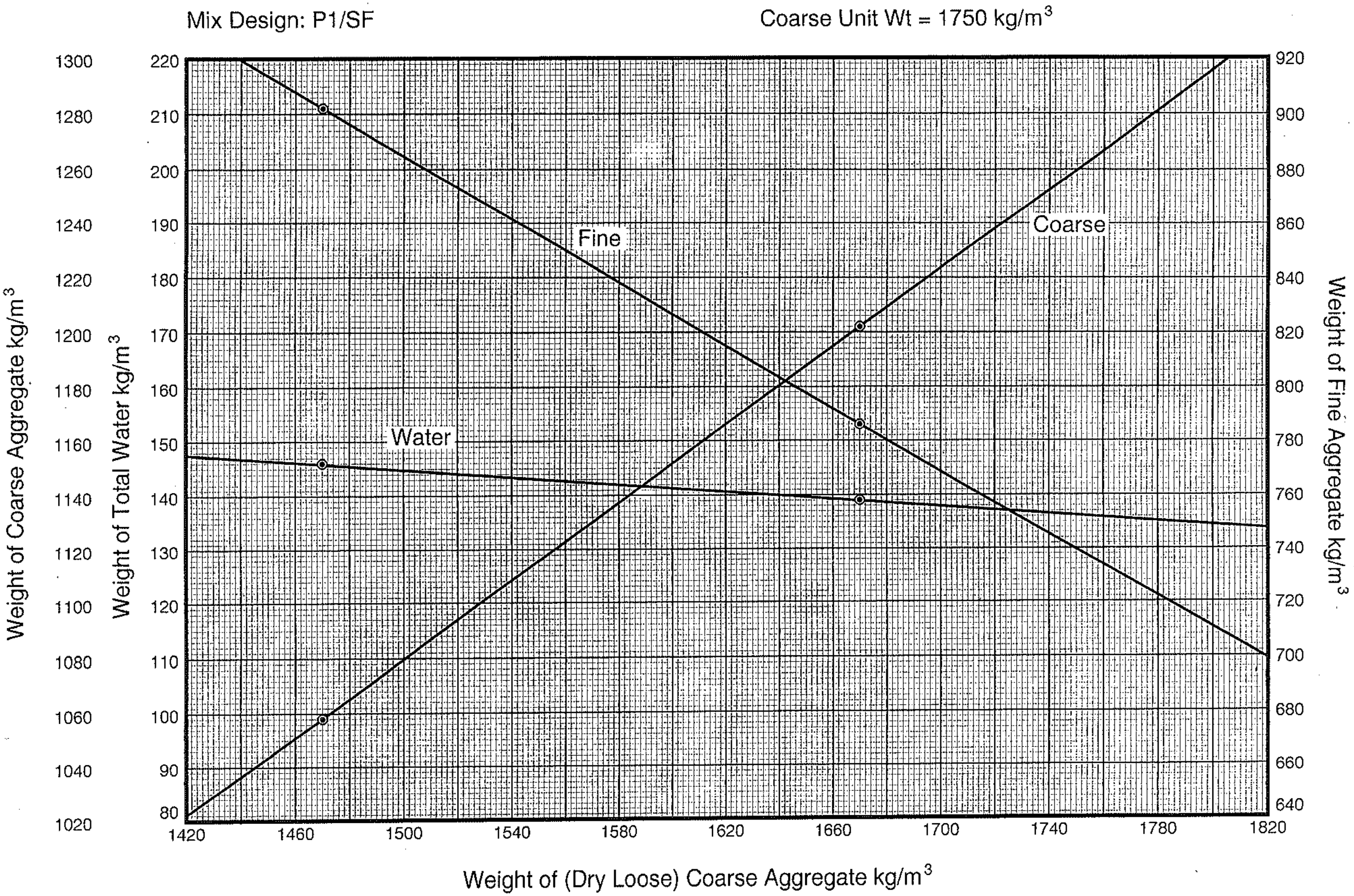

Figure 3-1M Concrete Materials Proportioning Chart (Dry Weights)

Manual Verification of Computer Results:

Using the equations from the new computer program, the batch weights for the unit weight of 1570 kg/m^3 are as determined from Section 6. as follows:

a. $w_{m2} = 0.229(1.05) = 0.24045$

$w_{m3} = 0.207(1.05) = 0.21735$

b. v_{m2} = Voids(2,0.24045) = 0.26816

v_{m3} = Voids(3,0.21735) = 0.25574

c. Pass a line w_L $(ax + by + c = 0)$ through points (2,0.24045) & (3,0.21735).

where $c = x_2y_1 - x_1y_2 = 0.28665$

$a = y_2 - y_1 = -0.02310$

$b = x_1 - x_2 = -1.0$

Thus, $-0.02310x - y + 0.28665 = 0$

Pass a line v_L through points (2,0.26816) & (3,0.25574).

where c = 0.29300, a = -0.01242, and b = -1.0

Thus, $-0.01242x - y + 0.29300 = 0$

d. $c = 335/((1000)(3.12)) = 0.10737$

$b = 0.72(1570)/((1000)(2.89)) = 0.39114$

$1-b = 0.60886$

$c_m = c/(1 - b) = 0.176346$

$v_{m2r} = 1 - 3(0.176346) = 0.47096$

$v_{m3r} = 1 - 4(0.176346) = 0.29462$

e. Pass a line v_R through points (2,0.47096) & (3,0.29462).

where c = 0.82364, a = -0.17634, and b = -1.0

Thus, $-0.17634x - y + 0.82364 = 0$

f. Intersect lines v_L & v_R to obtain $(x,y) = [s_1, (v_m)_1]$.

$d = (-0.01242)(-1.0) - (-0.17634)(-1.0) = -0.16392$

$x = [-(0.29300)(-1.0) + (-0.82364)(-1.0)]/d = 3.23719$

$y = [-(-0.01242)(0.82364) + (0.17634)(0.29300)]/d = 0.25281$

Steps g & h not required.

i. Using the coefficients of w_L,

$(w_m)_1 = -0.02310(3.23719) + 0.28665 = 0.21187$

j. Intermediate calculations for water content, voids, and fine aggregate:

$w = 0.21187(0.60886) = 0.12900$

$v = 0.12900 + 0.055 = 0.18400$

$a = 0.34758 - 0.18400 + 0.15391 = 0.31749$

check: $a + b + c + v = 1.0000$

The batch weights are:

$A = 841.35 \approx 841.$

$B = 1130.39 \approx 1130.$

$W_n = 129.00 \approx 129.$

$W_t = 143.33 \approx 143.$

A, B, and W_t can be compared to the line of computer results for the 1570 unit weight.

Appendix F

Solutions for a Structural Mix Design Problem

Mix Parameters and Material Properties

$G_c = 3.12$
$G_a = 2.65$
$A_a = 1.30$
$G_b = 2.63$
$A_b = 1.86$
$b/b_0 = 0.70$
$RWC = 1.20$
$V_{da} = 6.5\%$

Metric Units
D
$W_c = 390\ \mathrm{kg/m^3}$
$u = 1378\ \mathrm{kg/m^3}$

English Units
45D
$W_c = 657\ \mathrm{lb/ft^3}$
$u = 86\ \mathrm{lb/ft^3}$

MICHIGAN DEPARTMENT OF TRANSPORTATION

FORM 1830

CONCRETE PROPORTIONING DATA

FILE 300

CONTROL SECTION ID: Ex-2M
JOB NUMBER: 2M
LAB NUMBER: 2M
GRADE OF CONCRETE: D
INTENDED USE OF CONCRETE: Bridge, Superstructure

DATE: 8/23/1998
SPECIFICATION: 1996 STD SPECS
MIX DESIGN NUMBER: 2M

CONCRETE MATERIALS

MATERIAL	SOURCE	PIT NUMBER	CLASS	SPECIFIC GRAVITY	ABSORPTION PERCENT
CEMENT	(SEE REMARKS)		I/IA	3.12	
FINE AGG.	Standard Sand	19-99	2NS	2.65	1.30
COARSE AGG.	Quarry	93-99	6AA	2.63	1.86
FLY ASH					

CEMENT CONTENT, kg/m^3 390
AIR CONTENT (DESIGN): 6.5% (SPECIFIED): 6.5%
R.W.C: 1.20
FLY ASH CONTENT, kg/m^3: 0

B/Bo : 0.70
SPECIFICATION TOLERANCE (±): 1.5%
THEORETICAL YIELD: 100.00%

WEIGHT OF COARSE AGG. (DRY/LOOSE) kg/m^3	AGGREGATE AND WATER PROPORTIONS QUANTITIES, kg/m^3 OF CONCRETE		
	FINE AGG (OVEN DRY)	COARSE AGG (OVEN DRY)	TOTAL WATER
1328	770	930	193
1338	764	937	193
1348	758	944	193
1358	752	951	192
1368	746	958	192
1378	740	965	191
1388	734	972	191
1398	728	979	191
1408	723	986	190
1418	717	993	190
1428	711	1000	190

REMARKS:
THIS CHART FOR USE WITH CEMENTS OF THE CLASS SHOWN FROM APPROVED SOURCES.

TYPICAL UNIT WEIGHT (DRY, LOOSE) OF COARSE AGGREGATE AS DESCRIBED ABOVE IS 1378 kg/m^3

SPECIAL MESSAGES:
The Ratio of Net Mixing Water to Cement shall not exceed 0.44 by Weight for Concrete used for bridge superstructure.
Use of a Water Reducing Retarder is required.

CC:

GAIL H. GROVE
UM RIGID PAVEMENT CENTER

MICHIGAN DEPARTMENT OF TRANSPORTATION

FORM 1830

CONCRETE PROPORTIONING DATA

FILE 300

CONTROL SECTION ID: Ex-2E
JOB NUMBER: 2E
LAB NUMBER: 2E
GRADE OF CONCRETE: 45D
INTENDED USE OF CONCRETE: Bridge, Superstructure

DATE: 8/23/1998
SPECIFICATION: 1996 STD SPECS
MIX DESIGN NUMBER: 2E

CONCRETE MATERIALS

MATERIAL	SOURCE	PIT NUMBER	CLASS	SPECIFIC GRAVITY	ABSORPTION PERCENT
CEMENT	(SEE REMARKS)		I/IA	3.12	
FINE AGG.	Standard Sand	19-99	2NS	2.65	1.30
COARSE AGG.	Quarry	93-99	6AA	2.63	1.86
FLY ASH					

CEMENT CONTENT, lb/cyd 657
AIR CONTENT (DESIGN): 6.5% (SPECIFIED): 6.5%
R.W.C: 1.20
FLY ASH CONTENT, lb/cyd: 0

B/Bo : 0.70
SPECIFICATION TOLERANCE (±): 1.5%
THEORETICAL YIELD: 100.00%

WEIGHT OF COARSE AGG. (DRY/LOOSE) lb/cft	AGGREGATE AND WATER PROPORTIONS QUANTITIES, lb/cyd OF CONCRETE		
	FINE AGG (OVEN DRY)	COARSE AGG (OVEN DRY)	TOTAL WATER
81	1329	1531	328
82	1313	1550	327
83	1296	1569	326
84	1280	1588	325
85	1264	1606	324
86	1248	1625	323
87	1232	1644	322
88	1216	1663	321
89	1200	1682	320
90	1184	1701	319
91	1168	1720	318

REMARKS:
THIS CHART FOR USE WITH CEMENTS OF THE CLASS SHOWN FROM APPROVED SOURCES.

TYPICAL UNIT WEIGHT (DRY, LOOSE) OF COARSE AGGREGATE AS DESCRIBED ABOVE IS 86 lb/cft.

SPECIAL MESSAGES:
The Ratio of Net Mixing Water to Cement shall not exceed 0.44 by Weight for Concrete used for bridge superstructure.
Use of a Water Reducing Retarder is required.

CC:

GAIL H. GROVE
UM RIGID PAVEMENT CENTER

CONCRETE PROPORTIONING WORKSHEET

Grade/Use: 45D/Bridge Deck		$b/b_0 = 0.70$	RWC = 1.20	08/22/98
$G_a = 2.65$	$G_b = 2.63$	$G_c = 3.12$	G_f = ---	
$A_a = 1.30$	$A_b = 1.86$	$W_c = 657$	W_f = ---	
Design Air = 0.065		Actual (Dry, Loose) Unit Weight = 86		
Assumed Unit Weight, (above actual) = 76				
$c = 0.1250$	$b = 0.3240$	$1-b = 0.6758$	$b_0 = 0.4631$	$c_m = 0.1850$
$w_{m2} = 0.275$	$v_{m2} = 0.294$		$w_{m3} = 0.248$	$v_{m3} = 0.285$
$v_{m2r} = 0.4450$			$v_{m3r} = 0.2600$	
$s_1 = 2.855$	$(v_m)_1 = 0.286$	$(w_m)_1 = 0.2525$		
$w = 0.1706$	$v = 0.2356$	$a = 0.3149$		
check: $a + b + c + v = 1.0000$			$A = 1406.$	$B = 1436.$
$W_n = 286.9$			$W_t = 332.0$	
Assumed Unit Weight, (below actual) = 96				
$c = 0.1250$	$b = 0.4095$	$1-b = 0.5905$	$b_0 = 0.5850$	$c_m = 0.2117$
$w_{m2} = 0.275$	$v_{m2} = 0.294$		$w_{m3} = 0.248$	$v_{m3} = 0.285$
$v_{m2r} = 0.3649$			$v_{m3r} = 0.1532$	
$s_1 = 2.345$	$(v_m)_1 = 0.292$	$(w_m)_1 = 0.266$		
$w = 0.1571$	$v = 0.2221$	$a = 0.2434$		
check: $a + b + c + v = 1.000$			$A = 1087.$	$B = 1815.$
$W_n = 264.7$			$W_t = 313.0$	

Calculated by: ____________

Checked by: ____________

Input:

b/b_0	Workability factor
RWC	Relative water content
G_a, A_a	Sp. Gr. & Abs. - FA
G_b, A_b	Sp. Gr. & Abs. - CA
G_c, W_c	Sp. Gr. & Wt. - cement
G_f, W_f	Sp. Gr. & Wt. - flyash
V_{da}	Design air
u	Unit weight - CA

Output:

A	Batch weight (dry) - FA
B	Batch weight (dry) - CA
W_n	Net water
W_t	Total water

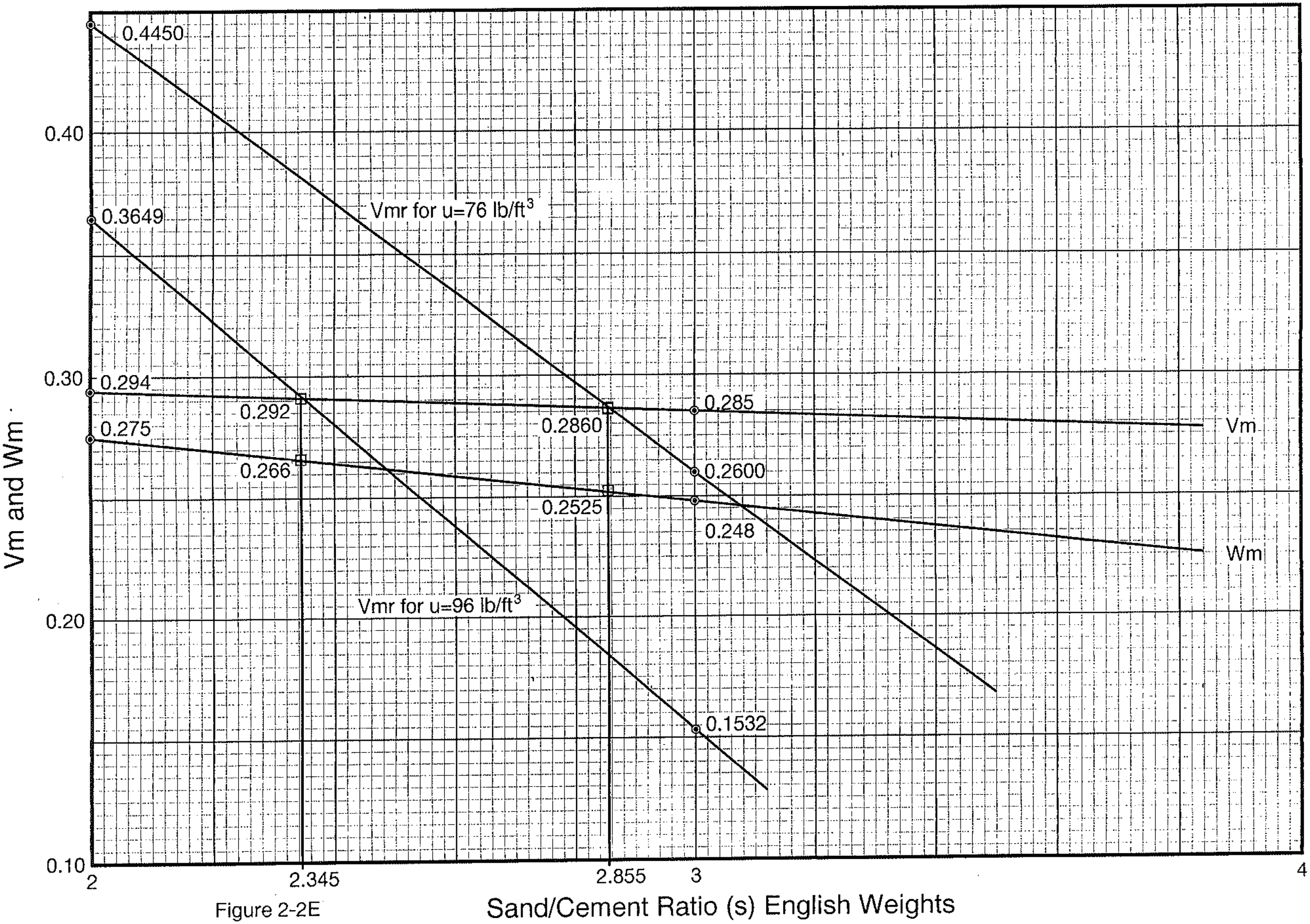
0.4450
0.40
Vmr for u=76 lb/ft^3
0.3649
0.30
0.294
0.292
0.2860
0.285
Vm
0.275
0.266
0.2600
0.2525
0.248
Wm
0.20
Vmr for u=96 lb/ft^3
0.1532
0.10
2
2.345
2.855
3
4
Vm and Wm
Sand/Cement Ratio (s) English Weights

Figure 2-2E

Mix Design: 45D Bridge Deck

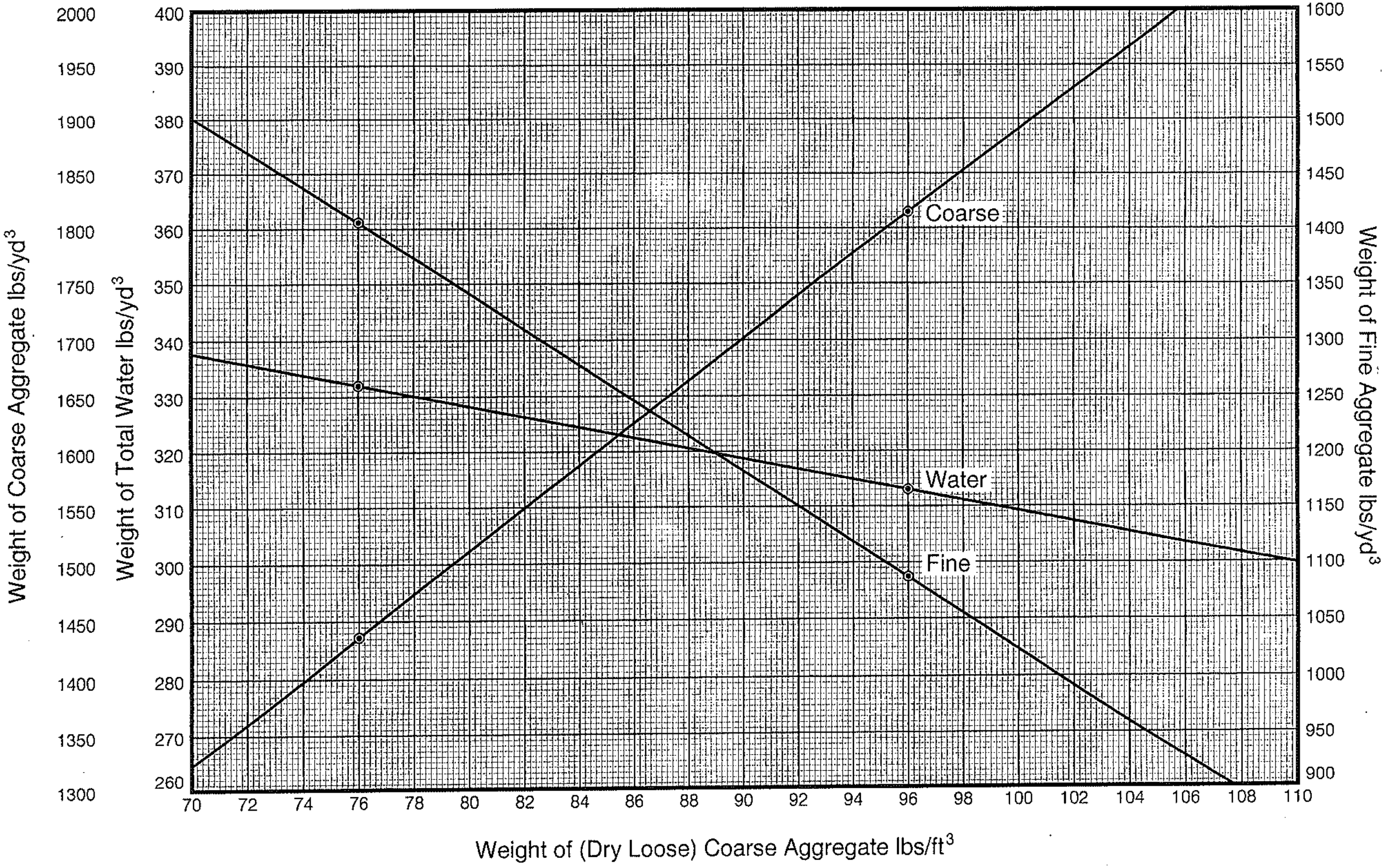

Figure 3-2E

Concrete Materials Proportioning Chart (Dry Weights)

Appendix G

CONCRETE PROPORTIONING WORKSHEET

Grade/Use: 45D/Bridge Deck		b/b_0 = 0.	RWC =	/ /
G_a = 2.	G_b = 2.	G_c = 3.1	G_f =	
A_a = .	A_b = .	W_c =	W_f =	
Design Air = 0.0		Actual (Dry, Loose) Unit Weight =		
Assumed Unit Weight, (above actual) =				
c = 0.	b = 0.	1-b = 0.	b_0 = 0.	c_m = 0.
w_{m2} = 0.	v_{m2} = 0.		w_{m3} = 0.	v_{m3} = 0.
v_{m2r} = 0.			v_{m3r} = 0.	
s_1 = .	$(v_m)_1$ = 0.	$(w_m)_1$ = 0.		
w = 0.	v = 0.	a = 0.		
check: a + b + c + v = .			A = .	B = .
W_n = .			W_t = .	
Assumed Unit Weight, (below actual) =				
c = 0.	b = 0.	1-b = 0.	b_0 = 0.	c_m = 0.
w_{m2} = 0.	v_{m2} = 0.		w_{m3} = 0.	v_{m3} = 0.
v_{m2r} = 0.			v_{m3r} = 0.	
s_1 = .	$(v_m)_1$ = 0.	$(w_m)_1$ = 0.		
w = 0.	v = 0.	a = 0.		
check: a + b + c + v = .			A = .	B = .
W_n = .			W_t = .	

Calculated by: ____________________
Checked by: ____________________

Input:

b/b_0	Workability factor
RWC	Relative water content
G_a, A_a	Sp. Gr. & Abs. - FA
G_b, A_b	Sp. Gr. & Abs. - CA
G_c, W_c	Sp. Gr. & Wt. - cement
G_f, W_f	Sp. Gr. & Wt. - flyash
V_{da}	Design air
u	Unit weight - CA

Output:

A	Batch weight (dry) - FA
B	Batch weight (dry) - CA
W_n	Net water
W_t	Total water