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# DESIGN CONSIDERATIONS FOR STEEL REINFORCEMENT IN CONTINUOUSLY REINFORCED PAVEMENT

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# DESIGN CONSIDERATIONS FOR STEEL REINFORCEMENT IN CONTINUOUSLY REINFORCED PAVEMENT

This study concerns the effects of concrete shrinkage and temperature change on the selection of steel reinforcement for a maximum allowable crack opening in continuously reinforced pavements. This report does not prescribe any actual design procedure, but rather points out some of the factors, and their relationships, which influence the choice of steel reinforcement. Emphasis is placed on the magnitude of crack openings in the concrete as a primary consideration affecting design.

In particular, the discussion involves the influence of the size of deformed bars for deformed bar mat reinforcement and crosswire spacing for welded wire mesh reinforcement, the effects of the variation of the ultimate tensile strength of the concrete, and the variability of climate conditions on the selection of the steel ratio.

The following presentation is concerned with the stresses and deformations in steel reinforcement in continuously reinforced concrete pavements. Both deformed bar and welded wire mesh reinforcement are considered. Equations are developed, based on a theoretical stress distribution in the concrete and the steel due to shrinkage and to temperature drop.

The purpose of the steel reinforcement is to resist the induced stresses set up by the restraint of the pavement in resisting deformation due to shrinkage and temperature change. It is assumed that in the center region of a continuously reinforced pavement--about 300 ft from the free ends--the steel reinforcement is completely restrained from any movement, and the only deformation taking place is due to concrete slippage at the cracks. Further, since these movements are small and restricted, subgrade friction is assumed to be zero. In addition, it is assumed that the thermal coefficients of linear expansion for steel and concrete are equal, and that temperature is uniformly distributed throughout the slab.

## DEFORMED BAR REINFORCEMENT

Consider a reinforced concrete slab segment of length $L$ subjected to shrinkage and a temperature drop. If unrestrained, this segment would shorten by an amount $\Delta_t + \Delta_{sh}$, representing the temperature and shrinkage deformation. If a force were now applied to the steel at the ends of the segment, causing a deformation $\frac{\Delta_t + \Delta_{sh}}{2}$ at each end, the steel would return to its original position. The concrete, however, because of slippage, or relative steel-to-concrete displacement, would not quite return to its original position. The assumed stress distributions and deformations are shown in Figure 1.

Because the steel is restrained, a stress $f_{so}$ is produced in the steel at the ends of the segment, causing the concrete to slip an amount $s$ from its original position. This sets up a uniform bond stress $\mu$ between the steel and concrete, which is distributed over some length $a$. The concrete stress and steel stress distribution for both shrinkage and temperature drop varies linearly as shown, where $f_s$ is the steel stress, and $f_c$ the concrete stress at the distance $a$ from the face of the segment. With this stress distribution, the relative steel-to-concrete deformation would vary parabolically from $s$ to zero as shown.

Considering temperature drop and shrinkage individually, the following equations are developed.

### Temperature Drop

Referring to Figure 1, in the region $L-2a$ the relative steel-to-concrete deformation is zero, and the steel strain $\varepsilon_s$ is equal to the concrete strain $\varepsilon_c$; thus, $\varepsilon_s = \varepsilon_c$ or $f_s = nf_c$ where $n = \frac{E_s}{E_c}$ and $E_s$ and $E_c$ are the moduli of elasticity for steel and concrete.

For equilibrium, $f_c A_c + f_s A_s = f_{so} A_s$ where $A_c$ and $A_s$ are the concrete and steel areas, or

$$f_{so} = \frac{f_c}{p}(1 + np) = \frac{f_s}{np}(1 + np)$$

where $p$ is the steel ratio $\frac{A_s}{A_c}$.

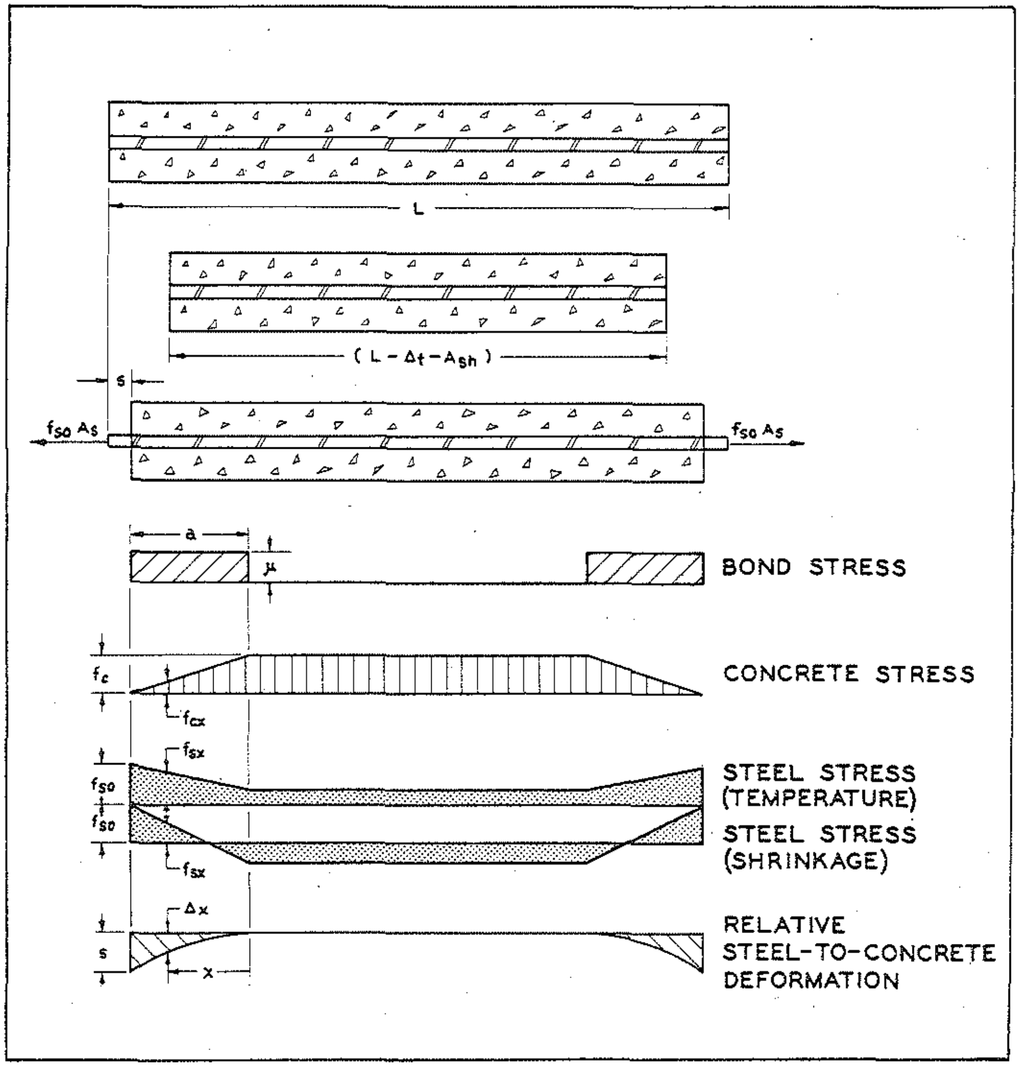


Figure 1. Stress distribution for deformed bars

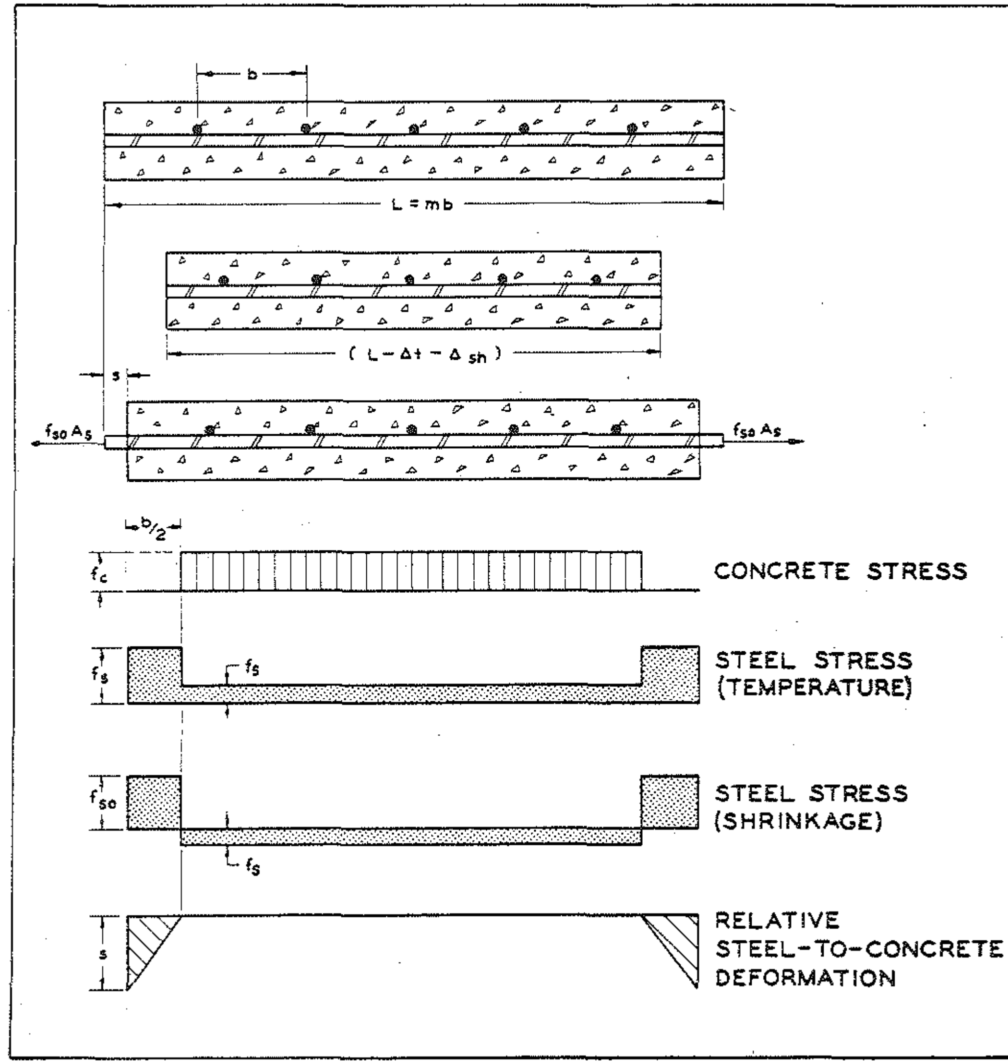


Figure 2. Stress distribution for welded wire mesh

Also $A_c f_c = \mu \Sigma_0 a$ or $a = \frac{A_c f_c}{\mu \Sigma_0}$ where $\Sigma_0$ is the bar perimeter.

In the region $a$ at either end, the relative steel-to-concrete deformation at a point $x$ would be equal to the steel deformation minus the concrete deformation at the point $x$, or

$$\Delta_x = \Delta_{sx} - \Delta_{cx} = \int_0^x \frac{f_{sx}\, dx}{E_s} - \int_0^x \frac{f_{cx}\, dx}{E_c} .$$

Since

$$f_{sx} = \frac{x}{a}(f_{so} - f_s) + f_s$$

and

$$f_{cx} = f_c\left(\frac{a - x}{a}\right),$$

the value of $\Delta_x$ at $x$ equal to $a$ would be

$$s = \frac{f_{so}\, a}{2E_s} + \frac{f_s\, a}{2E_s} - \frac{f_c\, a}{2E_c},$$

or by substitution from above

$$s = \frac{f_{so}\, a}{2E_s} .$$

From the condition that the steel length remains unchanged,

$$\int_0^{\frac{L}{2}-a} \frac{f_s\, dx}{E_s} + \int_0^a \frac{f_{sx}\, dx}{E_s} = \frac{\alpha t L}{2},$$

where $\alpha$ is the coefficient of thermal expansion and $t$ is the temperature drop. Integrating and reducing, one finds

$$f_{so} = \frac{(np + 1)(E_s \alpha t L)}{a + npL}$$

and

$$f_c = \frac{E_s \alpha t L p}{a + npL} .$$

Shrinkage

Again referring to Figure 1, in the region $L-2a$ the relative steel-to-concrete deformation is zero, and the concrete strain is $\varepsilon_c = \varepsilon_{sh} - \varepsilon_s$, where $\varepsilon_{sh}$ is the shrinkage strain. Then, $\frac{f_c}{E_c} = \varepsilon_{sh} - \frac{f_s}{E_s}$ or

$$f_s = E_s\,\varepsilon_{sh} - nf_c .$$

For equilibrium, $f_{so}\,A_s = f_c\,A_c - f_s\,A_s$

or

$$f_{so} = \frac{f_c}{p} - f_s = \frac{f_c}{p}(1+np) - E_s\,\varepsilon_{sh}$$

$$= \frac{E_s\,\varepsilon_{sh} - f_s(1+np)}{np}$$

and $a = \frac{A_c f_c}{\mu \Sigma_o}$ as before.

In the region $a$ at either end

$$\Delta_x = \Delta_{sx} - \Delta_{cx} = \int_0^x \frac{f_{sx}\,d_x}{E_s} - \int_0^x \frac{f_{cx}\,d_x}{E_c} ,$$

where

$$f_{sx} = \left(\frac{f_{so}+f_s}{a}\right) x - f_s$$

and

$$f_{cx} = \left(\frac{a-x}{a}\right) f_c .$$

The value of $\Delta_x$ at $x = a$ would be

$$s = \frac{a}{2E_s}(f_{so} - f_s) - \frac{a\,\varepsilon_{sh}}{2} + \frac{f_s\,a}{2\,E_s}$$

or by substitution from above, $s = \frac{f_{so}\,a}{2\,E_s} - \frac{a\,\varepsilon_{sh}}{2}$ .

Since the steel length remains unchanged, the shortening in the region $\frac{L}{2}-a$ would be equal to the elongation in the region $a$ or

$$\int_0^{\frac{L}{2}-a} \frac{f_s\, d_x}{E_s} = \int_0^{a} \frac{f_{sx}\, d_x}{E_s} .$$

Substituting and integrating, one finds

$$f_{so} = f_s \left( \frac{L-a}{a} \right),$$

and by further substitution,

$$f_{so} = \frac{E_s\, \varepsilon_{sh}\, ( L - a )}{L np + a}$$

and

$$f_c = \frac{E_c\, \varepsilon_{sh}\, (npL)}{npL + a} .$$

Combined Temperature Drop and Shrinkage

Combining the effects of both temperature and shrinkage, one gets

$$f_{so} = \frac{E_s}{a + npL} \left\{ \left[ \alpha t L ( np + 1 ) \right] + \varepsilon_{sh} ( L - a ) \right\}$$

and

$$f_c = \frac{E_s\, L p\, ( \alpha t + \varepsilon_{sh} )}{a + npL}$$

Solving for $L$ in each of the above two equations, one finds

$$L = \frac{f_c\, a}{p ( E_s\, \alpha t + E_s\, \varepsilon_{sh} - n f_c )}$$

and

$$L = \frac{a ( f_{so} + E_s\, \varepsilon_{sh} )}{\alpha t E_s ( np + 1 ) + E_s\, \varepsilon_{sh} - np f_{so}} .$$

Equating the above two equations and solving for $p$, one finds

$$p = \frac{f_c}{f_{so} - nf_c + E_s \varepsilon_{sh}} .$$

Further,

$$a = \frac{A_c f_c}{\mu \Sigma_o}$$

and

$$s = \frac{f_{so}\, a}{2E_s} - \frac{\varepsilon_{sh}\, a}{2} .$$

Since

$$f_{so} = \frac{f_c}{p}(1 + np) - E_s \varepsilon_{sh} ,$$

then

$$f_c = \frac{p}{1+np}(f_{so} + E_s \varepsilon_{sh}) .$$

Substituting for $f_c$ and $a$ in the above, one gets

$$s = \frac{d\left\{(f_{so})^2 - (E_s \varepsilon_{sh})^2\right\}}{(1+np)\, 8\, \mu E_s}$$

where $d$ is the bar diameter. Since $(E_s \varepsilon_{sh})^2$ is small compared to $(f_{so})^2$ and $1 + np \simeq 1$, $s \simeq \dfrac{(f_{so})^2 d}{8 \mu E_s}$ .

## WELDED WIRE MESH REINFORCEMENT

Consider a concrete slab segment containing welded wire mesh reinforcement, with transverse wires spaced at a distance $b$, subjected to shrinkage and a temperature drop. If unrestrained, this segment would shorten an amount $\Delta t + \Delta_{sh}$, representing the temperature and shrinkage deformation. It is assumed that the force in the longitudinal wires is transferred to the concrete only at the anchorages provided by the transverse wires, and that no bond stress exists along the longitudinal wires. The stress distribution and deformations are shown in Figure 2, for slab segment $L$ equal to $mb$ where $m$ is the number of transverse wires in the length $L$. Further, the first transverse wire is assumed to be at a distance $\frac{b}{2}$ from the end face of the segment as shown.

As in the case of the deformed bar reinforcement, the steel is restrained, and a stress $f_{so}$ is produced in the steel at the ends of the segment, causing the concrete to slip an amount $s$ from its original position. This causes the concrete stress, steel stress, and relative steel-to-concrete deformation distribution shown in Figure 2.

Again, considering the effects of shrinkage and temperature drop separately, the following equations are developed.

Temperature Drop

In the center region of the segment $\Delta_s - \Delta_c = 0$, $\varepsilon_s = \varepsilon_c$, or $f_s = nf_c$.

For equilibrium,

$$f_{so} A_s = f_s A_s + f_c A_c$$

or

$$f_{so} = \frac{f_s}{np}(1 + np)$$

$$= \frac{f_c}{p}(1 + np).$$

In the region $\frac{b}{2}$ at either end, the relative steel-to-concrete deformation at the end face would be equal to $s = \frac{f_{so}\, b}{2E_s}$.

Since the steel length remains unchanged,

$$\frac{f_{so}\, b}{2E_s} + \frac{f_s\,(mb)}{2E_s} = \frac{\alpha\, tmb}{2}.$$

Substituting and reducing

$$f_{so} = \frac{(1+np)(E_s \alpha tm)}{1+np+mnp}$$

and

$$f_c = \frac{E_s \alpha tmp}{1+np+mnp} .$$

## Shrinkage

In the center region

$$\Delta_s - \Delta_c = 0 \text{ , } \varepsilon_c = \varepsilon_{sh} - \varepsilon_s \quad \text{or} \quad f_s = E_s \varepsilon_{sh} - nf_c .$$

For equilibrium,

$$f_{so} A_s = f_c A_c - f_s A_s$$

or

$$f_{so} = \frac{f_c}{p}(1+np) - E_s \varepsilon_{sh}$$

$$= \frac{E_s \varepsilon_{sh} - f_s(1+np)}{np} .$$

In the region $\frac{b}{2}$ at each end, the relative steel-to-concrete deformation would be equal to $s = \frac{f_{so} b}{2E_s}$ .

Since the steel length remains unchanged,

$$\frac{f_{so} b}{2E_s} - \frac{f_s mb}{2E_s} = 0$$

or

$$f_{so} = mf_s .$$

Upon substituting from above

$$f_{so} = \frac{E_s \varepsilon_{sh} m}{mnp + np + 1}$$

and

$$f_c = \frac{E_s \varepsilon_{sh} p(m+1)}{mnp + np + 1}.$$

## Combined Temperature Drop and Shrinkage

Combining the effects of both temperature and shrinkage,

$$f_{so} = \frac{(1+np)(E_s \alpha tm) + E_s \varepsilon_{sh} m}{mnp + np + 1}$$

and

$$f_c = \frac{E_s \alpha tmp + E_s \varepsilon_{sh} p(m+1)}{mnp + np + 1}.$$

Solving the above two equations for $m$ and substituting $m = \frac{L}{b}$, one gets

$$L = \frac{b\{f_c(np+1) - E_s \varepsilon_{sh} p\}}{p(E_s \alpha t + E_s \varepsilon_{sh} - nf_c)}$$

and

$$L = \frac{b f_{so}(np+1)}{E_s(\alpha t + \varepsilon_{sh}) + np(E_s \alpha t - f_{so})}.$$

Equating the above two equations and solving for $p$,

$$p = \frac{f_c}{f_{so} + E_s \varepsilon_{sh} - nf_c}.$$

Also,

$$s = \frac{f_{so} b}{2E_s}.$$

## DISCUSSION

If a continuously reinforced concrete pavement is to function satisfactorily, the numerous cracks which occur must be held tightly closed by the reinforcing steel under whatever conditions the pavement encounters. It would seem more desirable to have closely spaced cracks with minute crack openings, rather than greater spacings with large crack openings. If the crack openings are small, the continuity of the slab will not be sufficiently impaired to affect the strength intended in the design of the pavement slab.

For deformed bar reinforcement, the slippage $S$ at the ends of the slab segment $L$ was given by $S \simeq \frac{(f_{SO})^2 d}{8 \mu E_S}$. This equation reveals $S$ as a function of the bar size $d$ as well as the maximum bar stress. In actual pull-out bond tests, the bond stress $\mu$ varies with the bar size and probably with the concrete quality as well. It is possible, then, with the same steel percentage $p$ and bar stress $f_{SO}$, to get different slippage deformations for bars with different diameters.

Referring to bond slip curves found in <u>Design Considerations for Distributed Reinforcement for Crack Control,</u> ACI Comm. 325 Report of Subcomm. IV (Feb. 1955), the following stresses are shown for an end slip of 0.01 in., along with bond lengths computed from derived equations:

| Bar Size | Steel Stress, psi | in. |
|---|---|---|
| #3 | 70,000 | 6.3 |
| #4 | 54,000 | 10.0 |
| #5 | 41,000 | 12.8 |
| #6 | 36,000 | 15.0 |

On the basis of the above limiting stresses, and assuming $\varepsilon_{sh} = 0.0002$, $\alpha = 6 \times 10^{-6}/F°$, $E_S = 30 \times 10^{-6}$ psi, and $n = 6$, Figure 3 shows the steel ratio $p$ plotted against the ultimate concrete tensile strength for the four sizes of deformed bars. Figure 3 also shows minimum crack spacing versus temperature drop, based on steel percentages for the various bar sizes for an ultimate concrete tensile strength of 400 psi.

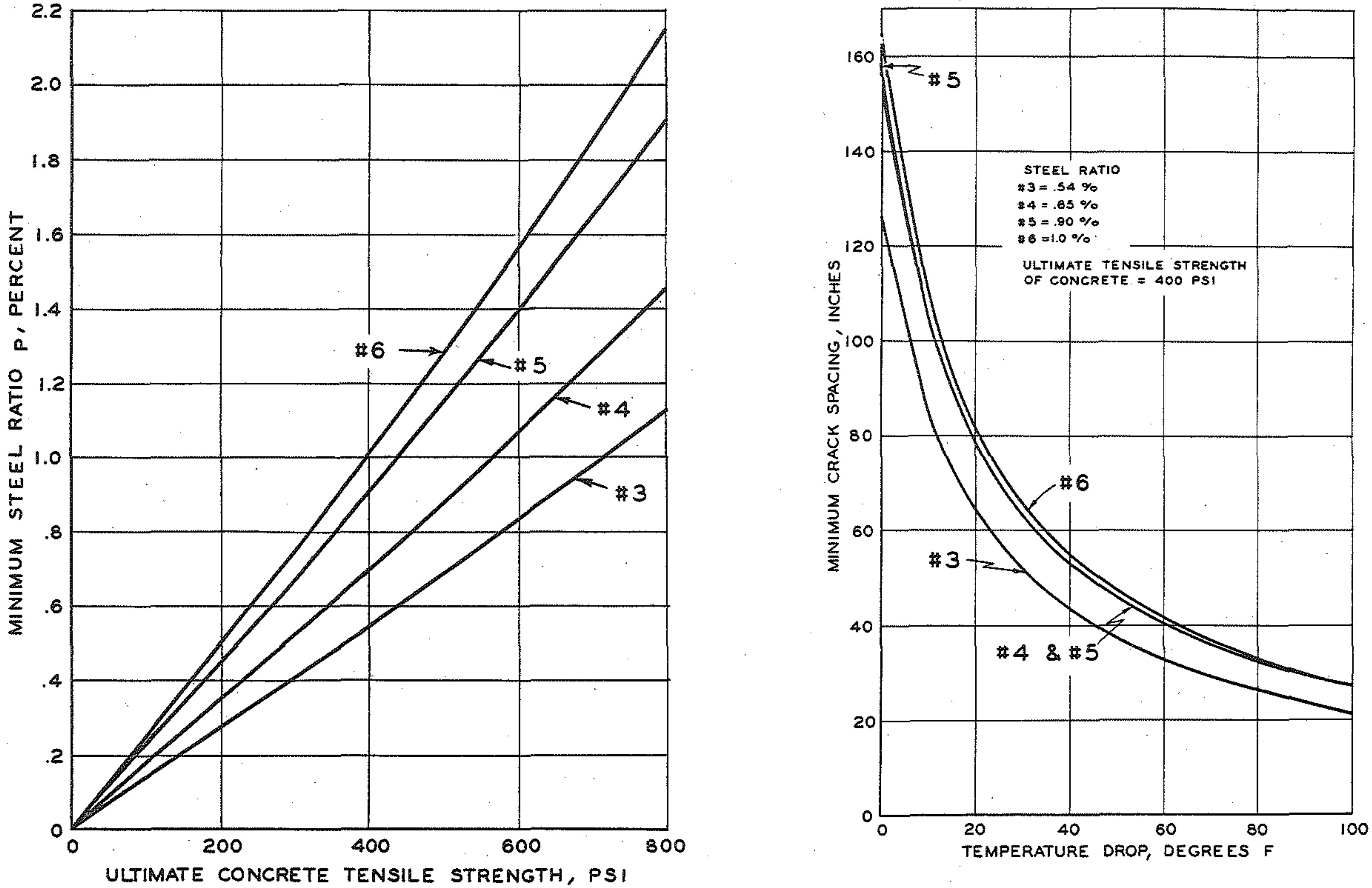


Figure 3. Design variables for four sizes of deformed bar reinforcement.

For welded wire mesh reinforcement, the slippage $s$ is given by $s = \frac{f_{so}\, b}{2E_s}$ Here, $s$ is a function of the transverse wire spacing $b$ as well as the maximum wire stress. Again, it is possible with the same steel ratio $p$ and wire stress $f_{so}$ to get different slippage deformations with different cross-wire spacings. In this discussion, the deformation of the anchorage cross-wires is considered to be zero. Referring again to the ACI discussion, the cross-wire is assumed analagous to a beam on an elastic foundation. Based on this analysis, and using a value for $G$, the modulus of bearing, of $2 \times 10^{6} \frac{psi}{in}$, and an average cross-wire diameter of 0.35 in., the following steel stresses $f_{so}$ and cross-wire spacings $b$ were deduced which would produce an end slip of 0.01 inch:

| Cross-wire spacing, in. | Steel Stress, psi |
|---|---|
| 6 | 70,000 |
| 9 | 54,000 |
| 12 | 42,000 |

On the basis of these limiting stresses, and again assuming

$$\varepsilon_{sh} = 0.0002\,,\ \alpha = 6 \times 10^{-6}/F^{\circ},\ E_s = 30 \times 10^{-6} psi, \text{ and } n = 6$$

Figure 4 shows the steel ratio $p$ plotted against the ultimate concrete tensile strength for the three different cross-wire spacings. Figure 4 also shows the minimum crack spacing plotted against temperature drop, based on steel percentages shown for an ultimate concrete tensile strength of 400 psi.

The emphasis of this discussion centers on the magnitude of crack opening as the foremost consideration in selecting steel reinforcement for continuously reinforced pavements. Since this opening is a function of the concrete slippage at the crack, it has been pointed out that greater allowable steel stresses, and thus lower steel ratios may be used for the smaller deformed bars and for welded wire mesh with closer cross-wire spacings.

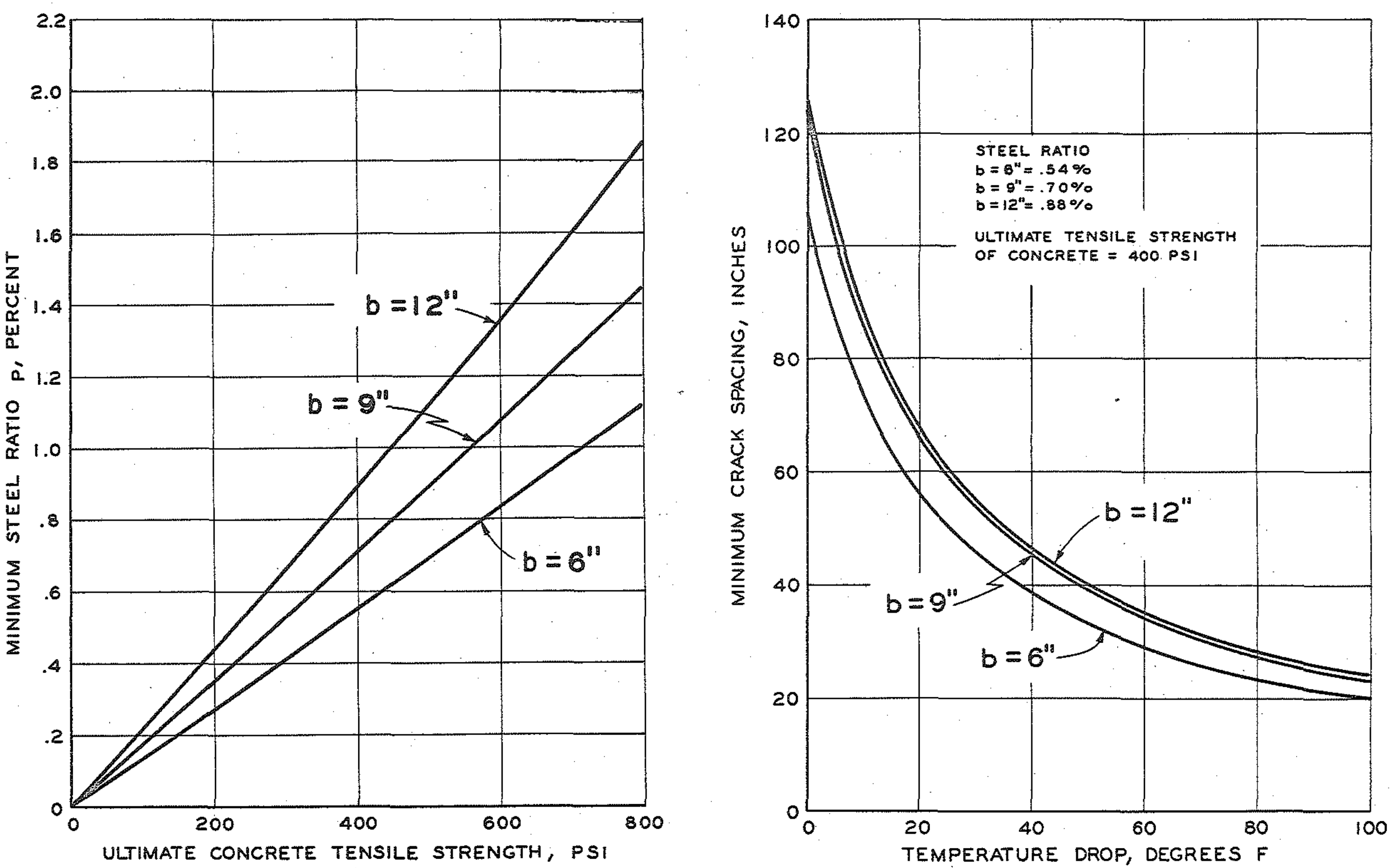


Figure 4. Design variables for three cross-wire spacings of welded wire mesh reinforcement.

For either type of reinforcement, the resulting equations for the minimum steel percentage are the same, and are independent of the temperature drop. In a properly functioning pavement, as the temperature falls, the steel stress at the crack will approach the allowable stress, based on a limiting slippage value. When this stress is reached, the tensile stress in the concrete will just exceed its ultimate tensile strength, and a new crack will form. Thus, the effect of temperature will be to cause closer crack spacings in areas with large temperature ranges, and greater crack spacings in less severe climates, for the same steel percentage and crack opening.

Since the steel percentage is directly proportional to the concrete tensile stress, producing weaker concrete with lower ultimate tensile strength would be a way of utilizing lower steel percentages, and still effect the close crack openings desired.

In this discussion, a crack opening of 0.02 in., as well as the values of $\varepsilon_{sh}$ and $n$ were arbitrarily used as representative values in order to draw the curves. It should also be pointed out that in the graphs of minimum crack spacing, which were based on the crack occurring at the center of the segment, new cracks could form at a spacing equal to $a$ for deformed bars, and to the cross-wire spacing $b$ for welded wire fabric.

Also, this discussion does not consider such factors as reversals of stress, temperature gradients in the slab, safety factors, plastic flow, and the differences in thermal coefficients of expansion of concrete and steel, which would be pertinent to complete design criteria for steel reinforcement in continuously reinforced pavements.

## CONCLUSIONS

On the basis of this analysis it may be stated that:

1. The minimum percentage of steel varies with the bar size for deformed reinforcement, and with the transverse or cross-wire spacing for welded wire mesh.

2. The minimum steel percentage varies directly with the ultimate tensile strength of the concrete, and providing concrete with lower ulti-

mate tensile strength would result in a lower steel percentage requirement.

3. The minimum steel percentage does not depend upon climatic conditions insofar as temperature change is concerned; the more severe climates produce closer crack spacings for the same steel percentage.

# APPENDIX

NOTATION:

| | |
|---|---|
| $L$ | uncracked slab segment |
| $\Delta t$ | temperature deformation |
| $\Delta_{sh}$ | shrinkage deformation |
| $s$ | slippage |
| $a$ | length over which bond stress is distributed |
| $f_s$ | steel stress at the center of a bar or wire |
| $f_{so}$ | steel stress at the end of a bar or wire |
| $f_c$ | concrete tensile stress |
| $\mu$ | bond stress |
| $\varepsilon_s$ | steel strain |
| $\varepsilon_c$ | concrete strain |
| $\varepsilon_{sh}$ | shrinkage strain |
| $E_s$ | steel modulus of elasticity |
| $E_c$ | concrete modulus of elasticity |
| $G$ | modulus of bearing |
| $A_s$ | steel area |
| $A_c$ | concrete area |
| $p$ | steel ratio |
| $\Sigma_o$ | bar perimeter |
| $d$ | bar diameter |
| $b$ | transverse wire spacing |
| $m$ | number of transverse wires in length $L$ |
| $\alpha$ | coefficient of thermal expansion |
| $t$ | temperature drop, deg F |