Prestressing Design of Beams

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Project Manager: Bradley Wagner, P.E. and David Juntunen, P.E.

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MDOT

Submitted by

Haluk Aktan, Ph.D., P.E.
Professor
Western Michigan University
(269) 276 – 3206
haluk.aktan@wmich.edu

Upul Attanayake, Ph.D., P.E.
Associate Professor
Western Michigan University
(269) 276 – 3217
upul.attanayake@wmich.edu

Funda Yavuz, MSE.
Graduate Research Assistant
Western Michigan University
(269) 276 – 3210
funda.yavuz@wmich.edu

Western Michigan University
Department of Civil & Construction Engineering
College of Engineering and Applied Sciences
Kalamazoo, MI 49008-5316
Fax: (269) 276 – 3211

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NOTATION

Text in Italics designates MDOT policies and AASHTO LRFD Specifications.

Text in Italics with shading designates MDOT practice.

A	= cross-sectional area of the beam (noncomposite section)	Step 5.1.1
a	= depth of equivalent stress block	Step 8.4.2
A_b	= area of noncomposite section	Step 3.1
A_c	= area of concrete on the flexural side of the member	Step 8.7.3
	= total area of transformed section	Step 3.2.1
A_{cv}	= area of concrete section resisting interface shear transfer	Step 8.8
A_{ps}	= area of one strand	Step 2.2
	= area of prestressing steel at the tension side of the section	Step 8.9
A_{pst}	= total area of prestressing strands	Step 8.3.2
A_s	= area of mild steel tension reinforcement	Step 8.4.2
	= total area of vertical reinforcement located within a distance h/4 from the end of the beam	Step 8.10
A_s’	= area of mild steel compression reinforcement	Step 8.4.2
A_{sv}	= area of vertical shear reinforcement	Step 8.8
A_v	= area of shear reinforcement within a distance	Step 8.7.4
A_{vf}	= area of shear reinforcement crossing the shear plane	Step 8.8
b_{eff}	= effective flange width	Step 3.2.1
b_{tf}	= top flange width	Step 3.1
b_v	= effective web width taken as the minimum web width within the depth d_v	Step 8.7.3
c	= cohesion factor	Step 8.8
	= distance between the neutral axis and extreme compressive fiber for rectangular section	Step 8.4.2
c_1	= parameter for skewed supports	Step 5.3.1
COEFF	= coefficient	Step 8.11.2
C_u	= factor used in long-term beam deflection calculation due to slab and haunch weight	Step 8.11.1
C_V	= correction factor for live load shear	Step 5.3.2
d_b	= nominal strand diameter	Step 8.3.1
d_c	= thickness of concrete cover measured from extreme tension fiber to center of the flexural reinforcement located closest thereto	Step 8.6.3
d_e	= roadway part of the overhang	Step 4.1
d_e	= effective depth from extreme compression fiber to centroid of the tensile force in the tensile reinforcement	Step 8.7.1

DFD	= distribution factor for deflection	Step 8.11.2
DFM	= distribution factor for moment in interior beam	Step 5.1.1
DFM_E	= distribution factor for moment in exterior beam	Step 5.2.1
DFM_{EB}	= distribution factor for moment in exterior beam adjusted for skew	Step 5.4
DFM_I	= controlling distribution factor for moment in interior beam	Step 5.1.1
DFM_{IB}	= distribution factor for moment in interior beam adjusted for skew	Step 5.4
DFV	= distribution factor for shear in interior beam	Step 5.1.2
DFV_E	= distribution factor for shear in exterior beam	Step 5.2.2
DFV_{EB}	= distribution factor for shear in exterior beam adjusted for skew	Step 5.4
DFV_I	= controlling distribution factor for shear in interior beam	Step 5.1.2
DFV_{IB}	= distribution factor for shear in interior beam adjusted for skew	Step 5.4
d_p	= distance from extreme compressive fiber to centroid of prestressing strands	Step 8.4.2
d_s	= distance from extreme compression fiber to the centroid of nonprestressed tensile reinforcement	Step 8.4.3
d_s’	= distance from extreme compression fiber to the centroid of compression reinforcement	Step 8.4.3
d_v	= effective shear depth (distance between resultants of tensile and compressive forces)	Step 8.7.1
E_c	= modulus of elasticity of concrete	Step 2.1
E_{cb}	= modulus of elasticity of beam in service	Step 2.1
E_{ci}	= modulus of elasticity of precast beam at transfer	Step 2.1
e_{end}	= eccentricity of strands at end of beam	Step 8.3.1
e_g	= distance between center of gravity of beam and slab	Step 5.1.1
E_p	= modulus of elasticity of prestressing strand	Step 2.2
e_{pg}	= eccentricity of strands	Step 8.3.2
e_{pge}	= eccentricity of prestresssing force at beam end	Step 8.11.1
e_{pgm}	= eccentricity of prestresssing force at mid-span	Step 8.11.1
E_s	= modulus of elasticity of steel	Step 2.2
FACTDB	= debonded factor	Step 8.3.1
f_b	= concrete tensile stress due to applied loads at bottom fiber of the beam	Step 8.2.1
f_{bottom}	= beam bottom fiber compression stress at transfer location	Step 8.2.4
$f_c{}'$	= 28-day strength of concrete	Step 2.1
	= specified strength of concrete	Step 2.1
$f_{ci}{}'$	= strength at release	Step 2.1
f_{cgp}	= the sum of concrete stress at the center of gravity of prestressing strands due to prestressing force and the self-weight of the beam	Step 8.3.1
	= the sum of concrete stress at the center of gravity of prestressing strands due to prestressing force at transfer and beam self-weight at maximum moment location	Step 8.3.2

f_{cpe}	= compressive stress in concrete due to effective prestress (after allowance for all losses) at the extreme fiber of the section where tensile stress is generated by externally applied loads	Step 8.4.6
f_{ct}'	= concrete compressive strength for lifting stress check	Step 8.5
F_e	= prestressing force after all losses	Step 8.2.2
F_i	= prestressing force at release	Step 8.2.2
f_{pb}	= required precompressive stress at the bottom fiber of the beam	Step 8.2.2
f_{pbt}	= stress in prestressing steel immediately prior to transfer	Step 2.2
f_{pe}	= effective stress in the prestressing steel after losses	Step 2.2
f_{pi}	= stress in prestressing steel immediately prior to transfer	Step 8.1
	= initial stress in strands	Step 8.3.1
f_{po}	= parameter taken as modulus of elasticity of prestressing strands multiplied by the locked-in difference in strain between the prestressing strands and the surrounding concrete	Step 8.7.3
f_{ps}	= average stress in prestressing steel	Step 8.4.2
f_{pu}	= ultimate strength of prestressing strand	Step 2.2
f_{py}	= yield strength of prestressing strand	Step 2.2
f_r	= concrete modulus of rupture	Step 8.4.6
f_s	= stress in mild steel tension reinforcement at nominal flexural resistance	Step 8.4.2
f_s'	= stress in mild steel compression reinforcement at nominal flexural resistance	Step 8.4.2
f_{ss}	= tensile stress in mild steel reinforcement at the service limit state	Step 8.6.3
f_{top}	= beam top fiber tensile stress at transfer location	Step 8.2.4
FWS	= future wearing surface	Step 6.3
f_y	= yield strength of steel	Step 2.2
f_{yh}	= specified yield strength of shear reinforcement	Step 8.7.4
$\bar{f}_{ci}$	= allowable concrete compressive stress at release	Step 2.3
$\bar{f}_{cpl}$	= allowable compressive stress for concrete subjected to effective prestress and permanent loads	Step 2.3
$\bar{f}_{ct}$	= allowable concrete compressive stress	Step 8.5
$\bar{f}_{ctl}$	= allowable compressive stress for concrete subjected to effective prestress, permanent loads, and transient loads	Step 2.3
$\bar{f}_{ti}$	= allowable concrete tensile stress at release	Step 2.3
$\bar{f}_{ts}$	= allowable tensile stress for concrete with bonded steel and subjected to not worse than moderate corrosion condition	Step 2.3
$\bar{f}_{tt}$	= allowable concrete tensile stress	Step 8.5
$g_{interior}$	= distribution factor for moment in interior beam	Step 5.2.1
	= distribution factor for shear in interior beam	Step 5.2.2
H	= distance between temporary support or lifting points and the beam end	Step 8.5
	= relative humidity	Step 8.3.2

h	= beam depth	Step 1.1
	= overall thickness or depth of the component	Step 8.6.3
h_c	= overall depth	Step 3.2.1
I	= moment of inertia of the beam (noncomposite section) ($I = I_b$)	Step 5.1.1
I_b	= moment of inertia of noncomposite section	Step 3.1
I_c	= moment of inertia of composite section	Step 3.2.1
IM	= dynamic allowance factor	Step 4.2
K_1	= correction factor for source of aggregate	Step 2.1
	= fraction of concrete strength available to resist interface shear	Step 8.8
K_2	= limiting interface shear resistance	Step 8.8
K_g	= longitudinal stiffness parameter	Step 5.1.1
L	= beam span	Step 5.1.1
L_1	= distance between truck load resultant and first interior beam	Step 5.2.1
L_b	= beam length	Step 1.1
L_{bcl}	= distance from beam end to bearing centerline	Step 1.1
L_{deb}	= distance between temporary support/lifting point and debonding point	Step 8.5
L_{dra}	= distance between temporary support/lifting point and draping point	Step 8.5
L_{ds}	= design span	Step 1.1
m	= multiple presence factor	Step 5.2.1
M_b	= moment in composite section due to barrier weight	Step 6.3.1
M_{cr}	= cracking moment strength	Step 8.4.6
M_D	= moment in noncomposite section due to deck and haunch weight	Step 6.2
M_{dnc}	= total unfactored dead load moment acting on the monolithic or noncomposite section	Step 8.4.6
M_g	= moment in noncomposite section due to beam weight	Step 6.2
M_{gr}	= moment due to beam weight at prestress release	Step 6.1
M_{HL-M}	= moment due to HL – 93 Mod with impact	Step 7.3
M_{LA}	= moment per beam due to axle load	Step 7.1
M_{LL}	= moment per beam due to design lane load	Step 7.2
M_{LT}	= moment per beam due to design truck load	Step 7.1
M_n	= nominal flexure resistance	Step 8.4.3
M_r	= factored flexural resistance	Step 8.4.4
M_s	= moment due to settlement	Step 7.3
M_u	= ultimate moment for Strength I limit state	Step 8.4.1
M_{ws}	= moment in composite section due to FWS weight	Step 6.3.1
n	= modular ratio between slab and beam concrete	Step 3.2.1
	= modular ratio between beam and slab material	Step 5.1.1
N_b	= number of beams	Step 1.1
NC	= noncomposite section	Step 7.3

N_u	= applied factored normal force at critical section	Step 8.7.3
P_c	= permanent net compressive force normal to the shear plane	Step 8.8
P_i	= prestress force at transfer	Step 8.3.1
P_{pi}	= force in the strands prior to transfer	Step 8.10
P_r	= bursting resistance	Step 8.10
R_M	= skew reduction factor	Step 5.3.1
S	= beam spacing	Step 1.1
s	= spacing of mild steel reinforcement in the layer closest to the tension face	Step 8.6.3
	= spacing of shear reinforcement	Step 8.7.4
S_b	= section modulus for bottom fiber of noncomposite section	Step 3.1
S_{bc}	= section modulus for bottom fiber of the composite section	Step 3.2.1
S_c	= S_{bc} = section modulus for the extreme fiber of the composite section where tensile stress is generated by externally applied loads	Step 8.4.6
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t_s	= deck slab thickness	Step 1.1
t_w	= web thickness	Step 3.1
V_b	= shear in composite section due to barrier weight	Step 6.3.1
V_c	= shear strength provided by concrete	Step 8.7
V_D	= shear in noncomposite section due to deck and haunch weight	Step 6.2
V_g	= shear in noncomposite section due to beam weight	Step 6.2
V_{gr}	= shear due to beam weight at prestress release	Step 6.1
V_{hi}	= horizontal factored shear force per unit length of the beam at the strength limit state	Step 8.8
$V_{HL\text{-}M}$	= shear due to HL – 93 Mod with impact	Step 7.3
V_{LA}	= shear per beam due to design axle load	Step 7.1
V_{LL}	= shear per beam due to design lane load	Step 7.2
V_{LT}	= shear per beam due to design truck load	Step 7.1
V_n	= nominal shear resistance	Step 8.7.4
V_{ni}	= nominal interface shear resistance	Step 8.8
V_p	= component of the effective prestressing force in the direction of the applied shear	Step 8.7
V_s	= shear due to settlement	Step 7.3
	= shear strength provided by reinforcement	Step 8.7.4

V_u	= total factored shear force	Step 8.7
V_{ws}	= shear in composite section due to FWS weight	Step 6.3.1
w	= clear roadway width	Step 5
w_b	= barrier or railing weight	Step 4.1
w_{bar}	= barrier weight on one beam	Step 4.1
w_c	= unit weight of concrete	Step 2.1
w_g	= beam weight	Step 4.1
w_s	= future wearing surface weight	Step 4.1
w_{ws}	= wearing surface weight on one beam	Step 4.1
x_{cr}	= critical section for shear	Step 8.7.1
y_b	= distance from the centroid to extreme bottom fiber	Step 3.1
y_{bc}	= distance from the centroid to extreme bottom fiber in composite section	Step 3.2.1
y_{bs}	= distance between center of gravity of bottom strands and the bottom fiber of the beam at midspan	Step 8.2.3
y_t	= distance from the centroid to extreme top fiber	Step 3.1
y_{tc}	= distance from the centroid to extreme top fiber in composite section	Step 3.2.1
α	= angle of inclination of shear reinforcement to longitudinal axis	Step 8.7.4
α_s	= factor used in long-term beam deflection calculation due to slab and haunch weight	Step 8.11.1
β	= factor indicating the ability of diagonally cracked concrete to transmit tension and shear	Step 8.7.3
β_1	= stress factor of compression block	Step 8.4.2
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γ_h	= correction factor for relative humidity of the ambient air	Step 8.3.2
γ_c	= exposure factor	Step 8.6.3
γ_{SE}	= load factor for settlement	Step 8.7.2
γ_{st}	= correction factor for specified concrete strength at the time of prestress transfer	Step 8.3.2
Δ_b	= deflection due to beam weight at transfer	Step 8.11.1
Δ_{bfws}	= deflection due to barrier and future wearing surface weight	Step 8.11.1
Δf_{pES}	= elastic shortening loss	Step 8.3.1
	= sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads	Step 8.3.2
Δf_{PLT}	= losses due to long-term shrinkage and creep of concrete, and relaxation of steel	Step 8.3.2
Δf_{pR}	= relaxation loss	Step 8.3.2

Δf_{pR_bt}	= relaxation loss before transfer	Step 8.3.1
Δf_{pT}	= total losses	Step 8.3.2
Δ_{fr}	= deflection due to forms and reinforcement weight	Step 8.11.1
Δ_{init}	= camber at transfer	Step 8.12
Δ_{Lb}	= long term camber due to beam weight	Step 8.11.1
Δ_{LL}	= deflection due to design lane load	Step 8.11.2
Δ_{Lp}	= long term camber due to prestress	Step 8.11.1
Δ_{LT}	= deflection due to design truck plus impact	Step 8.11.2
Δ_p	= camber due to prestressing force at transfer	Step 8.11.1
Δ_s	= long term beam deflection due to slab and haunch weight	Step 8.11.1
Δw	= overhang width	Step 1.1
ε_s	= strain in flexural reinforcement	Step 8.7.3
η	= load modifying factor for ductility, redundancy, and operational importance	Step 4.2
	= ratio of effective prestress after losses to stress prior to transfer	Step 8.3.2
	= ratio of effective stress to stress in prestressing steel after losses prior to transfer	Step 8.1
θ	= skew	Step 1.1
	= angle of inclination of diagonal compressive stress	Step 8.7.3
μ	= coefficient of friction	Step 8.8
ϕ	= resistance factor	Step 8.4.4
ϕ_c	= axial resistance factor	Step 8.9
ϕ_{cr}	= creep factor	Step 6.4
ϕ_f	= moment resistance factor	Step 8.9
ϕ_v	= shear resistance factor	Step 8.9
Ψ	= drape angle	Step 8.7.3

Prestressing Design of Bulb-tee Beam

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INTRODUCTION

Design of a bulb-tee prestressed concrete beam of a two-span continuous for live load (CLL) highway bridge is demonstrated in this example.

The purpose of this example is to illustrate the design of a typical interior bridge beam for flexure, shear, and deflection under dead and live loads. A zero skew, 167 ft-6 in. long, two span continuous for live load, bulb-tee prestressed beam bridge is considered. The bridge superstructure consists of two similar spans. Each span consists of five beams spaced at 6 ft-10¾ in. on centers (**Figure 1**). Cross-sectional dimensions of the beam are shown in **Figure 2**. Beams are designed for composite behavior with a 9-in. thick cast-in-place concrete deck to resist superimposed dead, live, and impact loads. Superstructure includes one interior steel diaphragm (C10 × 15.3) at midspan. Steel diaphragm weight is not considered due to its negligible effect on girder design.

The design is implemented in accordance with the Michigan Department of Transportation (MDOT) policies documented in the Bridge Design Manual (BDM) and Bridge Design Guides (BDG) as of 04/30/2017. As needed, the stipulations in the AASHTO LRFD *Bridge Design Specifications*, 7th Edition, with 2015 and 2016 interims are considered. Certain material and design parameters are selected to be in compliance with MDOT practice reflected in the Bridge Design System (BDS), the MDOT legacy software.

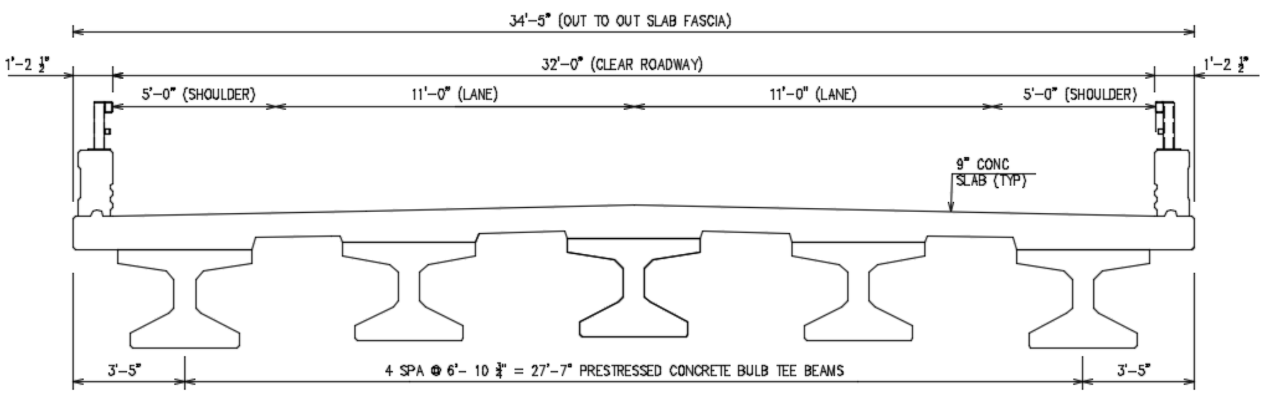


Figure 1. Bridge cross-section

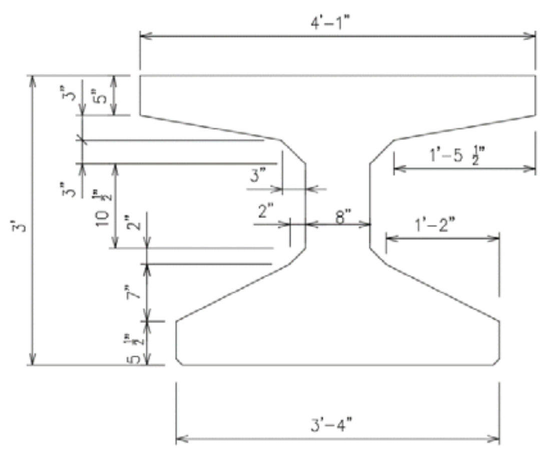


Figure 2. Beam cross-section (36 in. × 49 in.)

STEP 1. GEOMETRY

Step 1.1. Superstructure

Beam length, L_b		= 85.25 ft	
Distance from beam end to bearing centerline, L_{bcl}		= 9 in. = 0.75 ft	
Design span, L_{ds}	$= L_b - 2\times L_{bcl}$	= 83.75 ft	
Skew, θ		= 0 deg.	
Skew is the angle measured from line perpendicular to bridge centerline to support reference line. When skew exceeds 30 degrees, refined analysis methods should be utilized for load calculations.			BDM Art. 7.01.14
Number of lanes		= 2	
Lane width		= 11 ft	
10 ft ≤ Lane width ≤ 12 ft			MDOT (2017c)
Shoulder width		= 5 ft	BDG 6.05.01A
Overhang width, Δw		= 3 ft – 5 in.	
Barrier (aesthetic parapet tube) width		= 12 in.	BDG 6.29.10
Distance from slab fascia to outside face of the barrier		= 2.5 in.	BDG 6.29.10
Deck width		= 34 ft – 5 in.	BDG 6.05
Beam depth, h		= 36 in.	
Bulb Tee 36 in. × 49 in.			BDG 6.60.3
Number of beams, N_b		= 5	
Designers should layout beam spacing to accommodate future part width reconstruction. In most cases beams at centerline of structure should be avoided.			BDM Art. 7.01.15
Beam spacing, S		= 6 ft – 10 ¾ in.	
S ≤ 10 ft for all the beams			BDM Art. 7.02.02
Deck slab thickness, t_s		= 9 in.	
Standard deck slab thickness is 9 in.			BDG 6.41.01 BDG 6.41.02
Full slab thickness is included in dead load calculation as well as in the composite section calculations for the beam design. The deck slab design is based on 7.5 in. thickness, excluding the top 1.5 in. integrated wearing surface.			BDM Art. 7.02.08 B BDM Art. 7.02.19 A4
Haunch thickness, t_h		= 2 in.	
$t_h \geq 2$ in.			BDM Art. 7.02.19-C

STEP 2. MATERIAL PROPERTIES AND STRESS LIMITS

Step 2.1. Concrete

Unit weight, w_c = 0.145 kip/ft^3

MDOT standard concrete density designated as 0.145 kip/ft^3 is used in modulus of elasticity calculations.

Cast-in-place concrete slab, 28-day strength, f'_c = 4.0 ksi

f'_c = 4 ksi, Grade D concrete — BDM Art. 7.01.03

Precast beam, 28-day strength, f'_c = 7.5 ksi

5 ksi ≤ f'_c ≤ 8 ksi — BDM Art. 07.02.03-A

Strength at release, f_{ci}' = 6.3 ksi

f'_{ci} ≤ 7000 psi — BDM Art. 7.01.03

If f'_{ci} is unknown, $f'_{ci} = 0.8f'_c$ can be assumed — LRFD Art. 5.4.2.3.2

Modulus of elasticity, $E_c = 120{,}000\ K_1 (w_c)^{2.0}(f'_c)^{0.33}$ — LRFD Eq. 5.4.2.4-1

where,

w_c = unit weight of concrete, kcf — LRFD Table 3.5.1.1

f'_c = specified strength of concrete, ksi

K_1 = correction factor for source of aggregate = 1 — LRFD Art. 5.4.2.4

Elasticity Modulus of:

Cast-in-place slab, $E_c = 120{,}000(0.145)^{2.0}(4)^{0.33}$ = 3,987 ksi

Precast beam at transfer, $E_{ci} = 120{,}000(0.145)^{2.0}(6.30)^{0.33}$ = 4,631 ksi

Precast beam at service, $E_{cb} = 120{,}000(0.145)^{2.0}(7.50)^{0.33}$ = 4,906 ksi

MDOT modulus of elasticity calculation is different from AASHTO LRFD 7th edition, 2016.

Step 2.2. Steel and Prestressing Strand

Prestressing strands: 0.6 in. dia., seven-wire, low relaxation

The design and detail sheets shall specify only ASTM A416 (AASHTO M 203) Grade 270 low relaxation strands. Strands shall be 0.6 in. in diameter with a 44 kips release force. — BDM Art. 7.02.18.A1

Area of one strand, A_{ps} = 0.217 in.^{2}

Ultimate strength, f_{pu} = 270 ksi

Yield strength, $f_{py} = 0.9f_{pu}$ = 243 ksi — LRFD Table 5.4.4.1-1

Stress limits of prestressing strands: — LRFD Table 5.9.3-1

prior to transfer, $f_{pbt} \le 0.75f_{pu}$ = 202.5 ksi

at service limit state (after losses) $f_{pe} \le 0.80f_{py}$ = 194.4 ksi

Modulus of elasticity, E_p = 28,500 ksi — LRFD Art. 5.4.4.2

Reinforcing steel:

Yield strength, f_y = 60 ksi — BDM Art. 7.01.03

Modulus of elasticity, E_s = 29,000 ksi — LFRD Art. 5.4.3.2

Step 2.3. Stress Limits

Allowable concrete tensile stress at release, $\bar{f}_{ti} = 0.24\sqrt{f'_{ci}}$ — LRFD Table 5.9.4.1.2-1

Allowable concrete compressive stress at release, $\bar{f}_{ci} = 0.6\ f'_{ci}$ — LRFD Art. 5.9.4.1.1

Allowable tensile stress for concrete with bonded steel and subjected to not worse than moderate corrosion condition, $\bar{f}_{ts} = 0.19\sqrt{f'_c}$ — LRFD Table 5.9.4.2.2-1

Allowable compressive stress for concrete subjected to effective prestress and permanent loads, $\bar{f}_{cpl} = 0.45\ f'_c$ — LRFD Table 5.9.4.2.1-1

Allowable compressive stress for concrete subjected to effective prestress, permanent loads, and transient loads, $\bar{f}_{ctl} = 0.6\ f'_c$ — LRFD Table 5.9.4.2.1-1

STEP 3. SECTION PROPERTIES

Step 3.1. Noncomposite Section Properties — BDG 6.60.03

Area A_b = 878.30 in.^{2}

Top flange width b_{tf} = 49.00 in.

Web thickness t_w = 8.00 in.

Distance from the centroid to extreme top fiber y_t = 17.80 in.

Distance from the centroid to extreme bottom fiber y_b = 18.20 in.

Moment of inertia I_b = 145,592 in.^{4}

Section modulus for top fiber $S_t = I_b/y_t$ = 8,179 in.^{3}

Section modulus for bottom fiber $S_b = I_b/y_b$ = 8,000 in.^{3}

Step 3.2. Composite Section Properties

Haunch thickness varies along the beam length; 2 in. thickness is assumed for the design.

Step 3.2.1. Interior Beam

Effective flange width, b_{eff} = 6.896 ft — LRFD Art. 4.6.2.6.1

Modular ratio between slab and beam concrete, n, is needed to calculate transformed section width of deck slab for composite section properties.

$$n = \frac{E_c(\text{slab})}{E_c(\text{beam})} = \frac{E_c}{E_{cb}} = \frac{3{,}987}{4{,}906}$$

Transformed flange width $= n \times b_{eff} = \frac{3{,}987}{4{,}906} \times 6.896 \times 12$ = 67.25 in.

Transformed haunch width $= n \times b_{tf}$ = 39.82 in.

Overall depth, h_c $h + t_s + t_h$ = 47.00 in.

Total area of the transformed section, A_c = 1,563 in.^{2}

Distance from centroid to extreme bottom fiber, y_{bc} = 28.57 in.

Distance from centroid to extreme top fiber, y_{tc} = 18.43 in.

Moment of inertia, I_c = 367,259 in.^{4}

Section modulus for top fiber, $S_{tc} = I_c/y_{tc}$ = 19,927 in.^{3}

Section modulus for bottom fiber, $S_{bc} = I_c/y_{bc}$ = 12,855 in.^{3}

Step 3.2.2. Exterior Beam

Effective flange width, b_{eff}	$= S/2 + \Delta w$	= 6.865 ft	LRFD Art. 4.6.2.6.1
Transformed flange width	$= n \times b_{eff} = \frac{3,987}{4,906} \times 6.865 \times 12$	= 66.95 in.	
Overall depth, h_c	$h + t_s + t_h$	= 47.00 in.	
Total area of the transformed section, A_c		= 1,560 in.^{2}	
Distance from centroid to extreme bottom fiber, y_{bc}		= 28.54 in.	
Distance from centroid to extreme top fiber, y_{tc}		= 18.46 in.	
Moment of inertia, I_c		= 366,716 in.^{4}	
Section modulus for top fiber, S_{tc}	$= I_c/y_{tc}$	= 19,865 in.^{3}	
Section modulus for bottom fiber, S_{bc}	$= I_c/y_{bc}$	= 12,849 in.^{3}	

Figure 3 and **Figure 4** show the geometry of the transformed interior and exterior beam cross-sections. A summary of cross-section properties are presented in **Table 1** and **Table 2**.

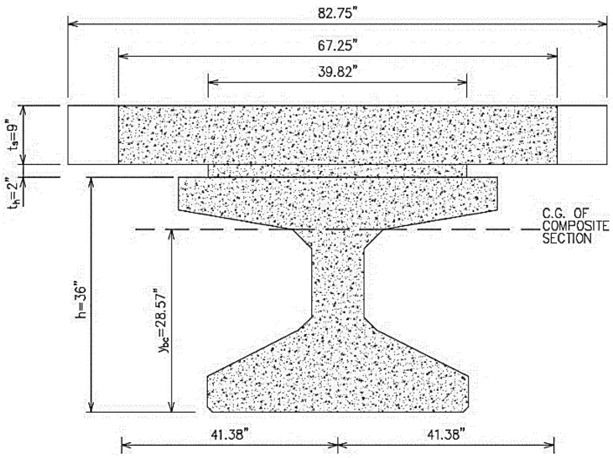


Figure 3. Dimensions of the interior beam

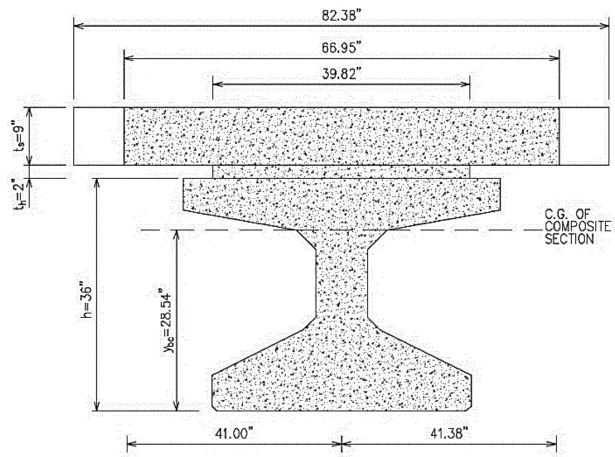


Figure 4. Dimensions of the exterior beam

Table 1. Interior Beam Properties

	Area, in.^{2}	y_b, in.	Ay_b, in.^{3}	$A(y_{bc} - y_b)^2$, in.^{4}	I, in.^{4}	$I + A(y_{bc} - y_b)^2$, in.^{4}
Beam	878.30	18.20	15,985.06	94,449.66	145,592.00	240,041.66
Haunch	79.64	37.00	2,946.68	5,659.61	26.55	5,686.16
Deck	605.25	42.50	25,723.13	117,445.68	4,085.44	121,531.12
Σ	1,563.19		44,654.87			367,258.94

Table 2. Exterior Beam Properties

	Area, in.^{2}	y_b, in.	Ay_b, in.^{3}	$A(y_{bc} - y_b)^2$, in.^{4}	I, in.^{4}	$I + A(y_{bc} - y_b)^2$, in.^{4}
Beam	878.30	18.20	15,985.06	93,903.97	145,592.00	239,495.97
Haunch	79.64	37.00	2,946.68	5,699.96	26.55	5,726.51
Deck	602.55	42.50	25,608.38	117,425.91	4,067.21	121,493.13
Σ	1,560.49		44,540.12			366,715.61

STEP 4. LOADS

Step 4.1. Dead Loads

Dead loads on the composite structure: LRFD Art. 4.6.2.2.1

Permanent loads (curbs and future wearing surface) may be distributed uniformly among all beams if the following criteria are met:

- Width of the deck is constant **O.K.**
- Number of beams, $N_b \geq 4$ **O.K.**
- The roadway part of the overhang, $d_e \leq 3.0$ ft.

  d_e = 41 in. – 14.5 in. = 2 ft – 2.5 in. **O.K.**
- Effect of curvature in plan can be ignored because LRFD Art. 4.6.1.2.4b
  - Girders are concentric
  - Bearing lines are not skewed more than 10 degrees from radial
  - The stiffness of the girders are similar
  - The arc span divided by the girder radius in feet is less than 0.06 radians
- Cross-section of the bridge is consistent with LRFD Table 4.6.2.2.1-1 **O.K.**

The criteria are satisfied and the loads can be distributed equally to all 5 beams.

Beam weight, $w_g = A_b w_c$ =878.30 in.^{2}/12^2 × 0.145 kip/ft^3 = 0.884 kip/ft

Cast-in-place concrete deck and haunch weight on interior beam

= $[(b_{eff}\ t_s) + (b_{tf}\ t_h)]\ w_c$

= [(82.75 in. × 9 in.) + (49 in. × 2 in.)]/12^2 × 0.145 kip/ft^3 = 0.849 kip/ft

Cast-in-place concrete deck and haunch weight on exterior beam

= $[(b_{eff}\ t_s) + (b_{tf}\ t_h)]\ w_c$

= [(82.38 in. × 9 in.) + (49 in. × 2 in.)]/12^2 × 0.145 kip/ft^3 = 0.845 kip/ft

Future wearing surface weight, w_s = 0.025 ksf

New bridges and bridge replacements shall be designed for a future wearing surface load of 25 LBS/SFT BDM Art. 7.01.04-H

Barrier or railing weight, w_b = 0.32 kip/ft BDG 6.29 Series

Wearing surface weight on one beam, w_{ws}

= (0.025 ksf)(32.0 ft)/(5 beams) = 0.160 kip/ft per beam

where, clear roadway width is 32 ft.

Barrier weight, w_{bar} = (2 barriers)(0.320 kip/ft)/(5 beams) = 0.128 kip/ft per beam

Steel diaphragm weight is not considered due to its negligible effect on girder design. There is no utility attached to this bridge, and the deck formwork is removed after construction; thus, their weight is not included in this example.

Step 4.2. Live Loads

Load modifying factor for ductility, redundancy, and operational importance, η = 1.0

The load modifying factor, η (eta), related to ductility, redundancy, and operational importance, shall be considered for less important roads.

BDM Art. 7.01.04-B
Typically 1.0 is used

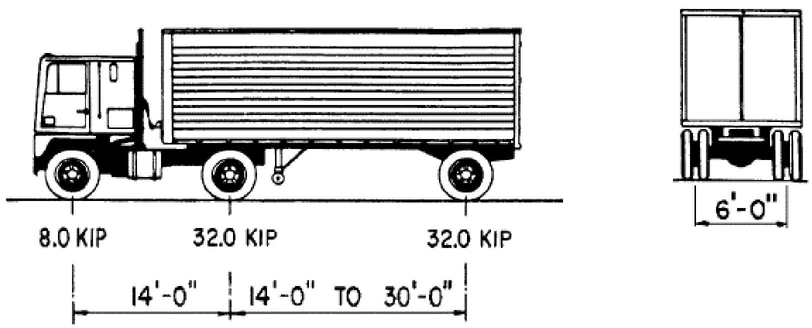
Vehicular live loading on the roadways of bridges designated HL-93 Mod, shall consist of 1.2 times the combination of the:

BDM Art. 7.01.04-A

- *Design truck or single 60 kip load*
- *Design lane load*

Design truck is shown in **Figure 5**. Design lane load is 0.64 kip/ft.

LRFD Art. 3.6.1.2.1



LRFD
Figure 3.6.1.2.2-1

Figure 5. Characteristics of the design truck

Where 90% of two design trucks are combined with 90% of the effect of a lane load for both negative moment and pier reactions per A.3.6.1.3, a 1.2 multiplier shall be applied to the resulting moment or load. Each design lane under consideration shall be occupied by either the design truck or single 60 kip load, coincident with the lane load, where applicable. The loads shall be assumed to occupy 10.0 ft transversely within a design lane.

BDM Art. 7.01.04-A

Vehicular live loading on the roadways of bridges designated HL-93 Mod, shall consist of 1.2 times the combination of the:

- *Design truck for continuity design*
- *Design lane load*

Design truck configuration is shown in **Figure 6**. Design lane load is 0.64 kip/ft.

LRFD Art. 3.6.1.3.1

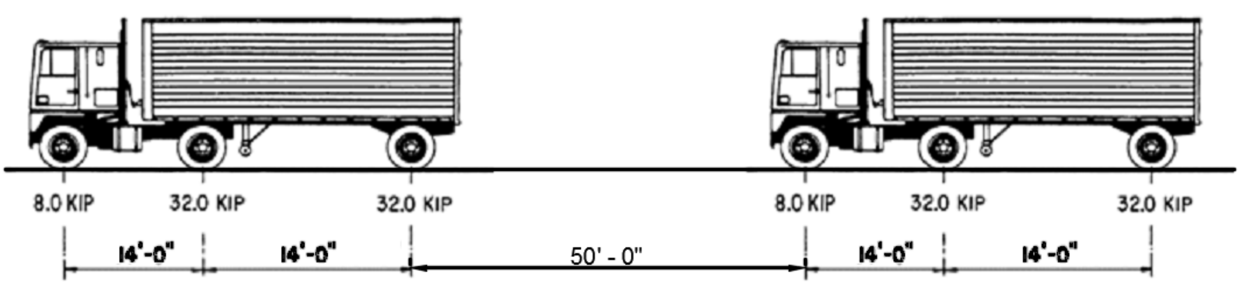


Figure 6. Characteristics of the design truck for continuity design

Design truck load is increased by a dynamic allowance factor

LRFD Table 3.6.2.1-1

Dynamic allowance, IM = 33%

STEP 5. LIVE LOAD DISTRIBUTION FACTORS

The bridge geometry satisfies the conditions stipulated, as described in **Step 4.1**. Thus, allows the determination of live load moments and shears using the simplified factor formulas. LRFD Art. 4.6.2.2 LRFD Art. 4.6.2.2.1

Bridge type for precast concrete I- or bulb-tee beams with a cast-in-place concrete deck is (k). LRFD Table 4.6.2.2.1-1

Number of 12 ft wide design lanes = the integer part of the ratio of (w/12) LRFD Art 3.6.1.1.1

where (w) is the clear roadway width, in ft, between the curbs.

From **Figure 1**, w = 32 ft

Number of design lanes = integer part of (32/12) = 2 lanes.

Hence, the above calculation shows the number of 12 ft wide lanes that can be accommodated within the clear roadway width.

Step 5.1. Interior Beam Live Load Distribution Factor

Distribution factor calculations for moment and shear in interior beams include multiple presence factor. LRFD Table 4.6.2.2.2b-1 LRFD Table 4.6.2.2.3a-1

Step 5.1.1. Distribution Factor for Moment

For all limit states, except fatigue:

For two or more lanes loaded:

$$\text{DFM} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$ LRFD Table 4.6.2.2.2b-1

where, DFM = distribution factor for moment in interior beam

S = beam spacing, ft

L (=L_{ds}) = beam span, ft

t_s = depth of concrete slab, in.

K_g = longitudinal stiffness parameter, in.^{4} = n (I + Ae_g^2) LRFD Eq. 4.6.2.2.1-1

n = modular ratio between beam and slab material LRFD Eq. 4.6.2.2.1-2

$$= \frac{E_c(\text{beam})}{E_c(\text{deck})} = \frac{E_{cb}}{E_c} = \frac{4906}{3987} = 1.231$$

LRFD Eq. 4.6.2.2.1-2 defines modular ratio between the beam and deck slab material, n. This modular ratio is the inverse of the modular ratio utilized in Step 3.2.1.

A = cross-sectional area of the beam (noncomposite section), in.^{2}

I = moment of inertia of the beam (noncomposite section), in.^{4}

e_g = distance between centers of gravity of the beam and slab, in.

= $t_s/2 + t_h + y_t$ = 9/2 + 2 + 17.80 = 24.30 in.

K_g = 1.231 [145,592 + 878.30(24.30)^2] = 817,654 in.^{4}

To use DFM equations, the criteria below need to be satisfied:

Range of Applicability LRFD Table 4.6.2.2.2b-1 and Table 4.6.2.2.3a-1	**MDOT Policy BDG 6.41.01 and BDM Art. 7.02.02**	**Bridge Attributes**
3.5 ft ≤ S ≤ 16 ft	S ≤ 10 ft for I beams	S = 6 ft – 10.75 in. **O.K.**
4.5 in. ≤ t_s ≤ 12 in.	t_s = 9 in.	t_s = 9 in. **O.K.**
20 ft ≤ L ≤ 240 ft		L_{ds} = 83.75 ft **O.K.**
N_b ≥ 4		N_b = 5 **O.K.**
10,000 in.^{4} ≤ K_g ≤ 7,000,000 in.^{4}		K_g = 817,654 in.^{4} **O.K.**

For two or more lanes loaded:

$$\mathrm{DFM} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

$$= 0.075 + \left(\frac{6.896}{9.5}\right)^{0.6}\left(\frac{6.896}{83.75}\right)^{0.2}\left(\frac{817{,}654}{12.0(83.75)(9)^3}\right)^{0.1}$$

= 0.581 lanes/beam

For one design lane loaded:

LRFD Table 4.6.2.2.2b-1

$$\mathrm{DFM} = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

$$= 0.06 + \left(\frac{6.896}{14}\right)^{0.4}\left(\frac{6.896}{83.75}\right)^{0.3}\left(\frac{817{,}654}{12.0(83.75)(9)^3}\right)^{0.1}$$

= 0.420 lanes/beam

Thus, the scenario with two or more lanes loaded controls, and

DFM_I = 0.581 lanes/beam.

Fatigue of the reinforcement need not be checked for prestressed components designed for extreme fiber tensile stress under Service III Limit State within the tensile stress limit specified in Table 5.9.4.2.2-1. LRFD Art. 5.5.3.1

In this example, tensile stresses are maintained below the limits specified in Table 5.9.4.2.2-1 with Service III Limit State. Hence, fatigue will not be considered.

Step 5.1.2. Distribution Factor for Shear

For two or more lanes loaded:

LRFD Table 4.6.2.2.3a-1

$$\mathrm{DFV} = 0.2 + \left(\frac{S}{12}\right) - \left(\frac{S}{35}\right)^2$$

where,

DFV = distribution factor for shear in interior beam

S = beam spacing, ft

Distribution factor for shear is

$$\text{DFV} = 0.2 + \left(\frac{6.896}{12}\right) - \left(\frac{6.896}{35}\right)^2 = 0.736 \text{ lanes/beam}$$

For one design lane loaded:

$$\text{DFV} = 0.36 + \left(\frac{S}{25.0}\right) = 0.36 + \left(\frac{6.896}{25.0}\right) = 0.636 \text{ lanes/beam}$$

Thus, the scenario with two or more lanes loaded controls, and

DFV_I = 0.736 lanes/beam

Step 5.2. Exterior Beam Live Load Distribution Factor

Step 5.2.1. Distribution Factor for Moment

For two or more lanes loaded:

$$g = e\, g_{interior}$$ LRFD Table 4.6.2.2.2d-1

$$e = 0.77 + \frac{d_e}{9.1}$$

where, $g_{interior}$ = distribution factor for moment in interior beam

d_e = horizontal distance from the exterior beam web centerline to the interior edge of curb or traffic barrier = 2.208 ft $-1 \le d_e \le 5.5$

e = 0.77 + 2.208/9.1 = 1.01

$g_{interior}$ = 0.581 lanes/beam

g = (1.01) (0.581) = 0.587 lanes/beam

Live load distribution factor is calculated using the lever rule with one design lane loaded. LRFD Table 4.6.2.2.2d-1

Live load distribution factor can be calculated with the three rules shown below:

- Lever rule based on the resultant of truck load
- Lever rule based on the wheel load
- AASHTO LRFD Eq. C4.6.2.2.2d-1 for steel girder bridges. LRFD Art. 4.6.2.2.2d

When both wheel lines are located between the barrier and the 1st interior beam, as shown in **Figure 7**, lever rule based on the resultant of the truck load is applied. In bridges with narrow beam spacing, there is a possibility to have one of the wheel lines located between the 1st and 2nd interior beams; thus, lever rule based on the wheel load is applied.

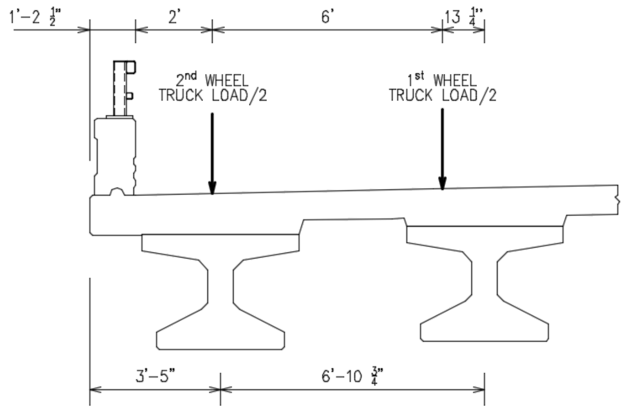


Figure 7. Position of the wheel load

Lever rule based on the resultant of truck load:

Associated data is presented in **Figure 5, Figure 8, and Figure 9**. LRFD Art. 3.6.1.3.1

Distance between wheel load and barrier = 2 ft

Distance between wheel and centerline of the design truck = 3 ft

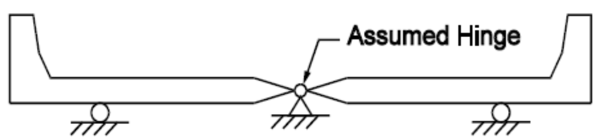
Beam spacing, S = 6.869 ft

Distance between truck load resultant and first interior beam, L_1 = 4.104 ft

Multiple presence factor for one design lane, m = 1.2 LRFD Table 3.6.1.1.2-1

Distribution factor for one design lane loaded from lever rule based on the resultant of truck load = $(L_1/S) \times m = (4.104/6.896) \times 1.2 = 0.714$ lanes/beam

Single lane loaded controls, and $DFM_E = 0.714$ lanes/beam



LRFD Figure C4.6.2.2.1-1

Figure 8. Notional model for applying lever rule to three-girder bridges

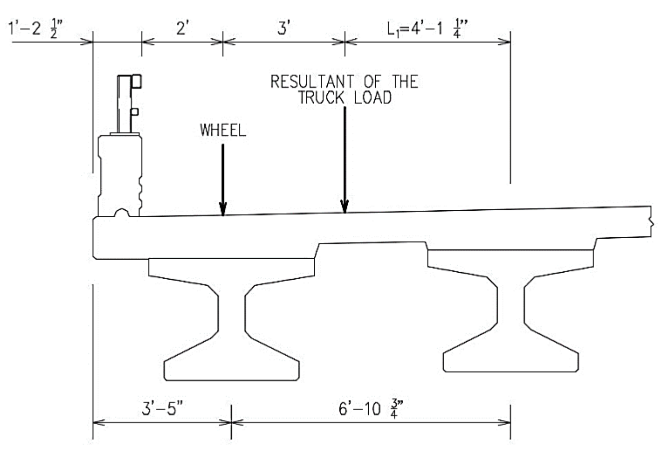


Figure 9. Dimensions required for lever rule based on the resultant of truck load

Step 5.2.2. Distribution Factors for Shear

For two or more lanes loaded:

$$g = e\, g_{interior}$$

$$e = 0.6 + \frac{d_e}{10}$$

LRFD Table 4.6.2.2.3b-1
$-1 \leq d_e \leq 5.5$

where, $g_{interior}$ = distribution factor for shear in interior beam

d_e = horizontal distance from the exterior beam web centerline to the interior edge of curb or traffic barrier = 2.208 ft

e = 0.6 + 2.208/10 = 0.82

$g_{interior}$ = 0.736 lanes/beam

g = (0.82) (0.736) = 0.604 lanes/beam

Live load distribution factor is calculated using the lever rule with one design lane loaded.

From **Step 5.2.1**, distribution factor for one design lane loaded, DFM_E = 0.714 lanes/beam

Thus, the case of single lane loaded controls, and DFV_E = 0.714 lanes/beam

Step 5.3. Skew Reduction/Correction Factors

Skew reduction/correction factor calculation is not needed for a bridge of zero skew. Calculations shown in this step are for illustration purposes only.

Step 5.3.1. Reduction Factor for Live Load Moment

$R_M = 1 - c_1(\tan\theta)^{1.5}$ LRFD Table 4.6.2.2.2e-1

where

R_M = Reduction factor

$$c_1 = 0.25\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.25}\left(\frac{S}{L}\right)^{0.5}$$

If $\theta < 30°$ then $c_1 = 0.0$

If $\theta > 60°$ use $\theta = 60°$

Range of Applicability **LRFD Table 4.6.2.2.2e-1**	**MDOT Policy** **BDM Art. 7.01.14 and 7.02.02**	**Bridge Attributes**
$30° \leq \theta \leq 60°$	$\theta \leq 30°$ (Approximate method) $30° < \theta \leq 45°$ (Refined method) $\theta > 45°$ (Need approval)	$\theta = 0°$
3.5 ft ≤ S ≤ 16 ft	S ≤ 10 ft for I beams	S = 6 ft – 10.75 in.
20 ft ≤ L ≤ 240 ft		L_{ds} = 83.75 ft
$N_b \geq 4$		N_b = 5

Since $\theta < 30°$, $c_1 = 0.0$ and R_M = 1

Step 5.3.2. Correction Factor for Support Shear at Obtuse Corner

LRFD Table 4.6.2.2.3c-1

$$C_V = 1.0 + 0.20\left(\frac{12.0Lt_s^3}{K_g}\right)^{0.3} \tan\theta$$

where

C_V = Correction factor for live load shear

Use of this correction factor is subjected to the same criteria shown above; except, θ range is between 0^o and 60^o.

Since $\theta = 0°$, $C_V = 1$

Step 5.4. Moment and Shear Distribution Factors Adjusted for Skew

Distribution factor for moment in interior beam,

$DFM_{IB} = (DFM_I)(R_M) = 0.581$ lanes/beam

Distribution factor for moment in exterior beam,

$DFM_{EB} = (DFM_E)(R_M) = 0.714$ lanes/beam

Distribution factor for shear in interior beam,

$DFV_{IB} = (DFV_I) = 0.736$ lanes/beam

Distribution factor for shear in exterior beam,

$DFV_{EB} = (DFV_E)(C_V) = 0.714$ lanes/beam

Fatigue of the reinforcement need not be checked for fully prestressed components designed with extreme fiber tensile stress under Service III Limit State and tensile stress limit specified in Table 5.9.4.2.2-1.

LRFD Art. 5.5.3.1

In this example, tensile stresses are maintained below the limits specified in Table 5.9.4.2.2-1 with Service III Limit State. Hence, fatigue is not considered.

STEP 6. MOMENT AND SHEAR DUE TO DEAD LOADS AND SUPPORT SETTLEMENT

Shear (V_x) and moment (M_x) of a simply supported beam with a span (L) under a uniformly distributed load (w) are:

$$V_x = w(0.5L - x)$$
$$M_x = 0.5wx(L - x)$$

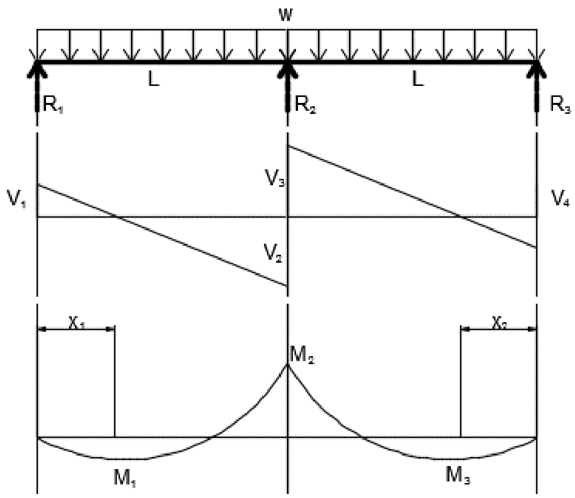
where x is the distance from the support.

Shear (V_x) and moment (M_x) of a beam with two equal spans (L) and a uniformly distributed load (w) are:

$$V_x = w(3L/8 - x)$$
$$M_x = wx(3L/8 - x/2)$$

where x is the distance measured from the outer support to the middle support.

Since the bridge spans are equal, above equations are applicable for both spans. Shear and moment diagrams are shown below.



$$R_1 = R_3 = \frac{3wL}{8} \qquad V_1 = V_4 = R_1 = R_3$$

$$R_2 = \frac{10wL}{8} \qquad V_2 = V_3 = \left|\frac{5wL}{8}\right|$$

$$M_1\left(\text{when } x_1 = \frac{3L}{8}\right) = \frac{9wL^2}{128}$$

$$M_2 = -\frac{wL^2}{8}$$

$$M_3\left(\text{when } x_2 = \frac{3L}{8}\right) = \frac{9wL^2}{128}$$

Step 6.1. Moment and Shear at Prestress Release due to Beam Weight (Simple Span)

Full beam length (L_b) of 85.25 ft is used at the time of prestress release. Beam self-weight is the only load.

As an example, moment and shear due to a beam weight of 0.884 kip/ft at 4.9375 ft from the beam end are calculated as follows:

$$M_{gr} = 0.5wx(L - x) = 0.5(0.884)(4.9375)(85.25 – 4.9375) \qquad = 175.27 \text{ kip-ft}$$
$$V_{gr} = w(0.5L - x) = (0.884)(0.5 \times 85.25 – 4.9375) \qquad = 33.32 \text{ kip}$$

Moment and shear due to beam self-weight at release are shown in **Table 3 - Table 6**.

Step 6.2. Moment and Shear in Noncomposite Section (Simple Span)

In this analysis, span length is now the design span (L_{ds}) of 83.75 ft, distance between bearing centerlines. The self-weight of beam, deck, haunch, and diaphragm loads are applied to the noncomposite, simple span structure.

As an example, moment and shear at 4.1875 ft from support due to beam weight of 0.884 kip/ft and deck and haunch weight of 0.849 kip/ft are calculated as follows:

Moment and shear due to beam self-weight

$M_g = 0.5wx(L - x) = 0.5\,(0.884)(4.1875)(83.75 - 4.1875) = 147.26$ kip-ft

$V_g = w(0.5L - x) = (0.884)(0.5 \times 83.75 - 4.1875) = 33.32$ kip

Moment and shear due to deck and haunch self-weight

$M_D = 0.5wx(L - x) = 0.5(0.849)(4.1875)(83.75 - 4.1875) = 141.43$ kip-ft

$V_D = w(0.5L - x) = (0.849)(0.5 \times 83.75 - 4.1875) = 32.00$ kip

The respective moment and shear values are shown in **Table 3** - **Table 6.**

Step 6.3. Moment and Shear in Composite Section due to Barrier and FWS Weight

Continuous for live load prestressed concrete beams shall be designed as simple span beams for all positive dead load and live load moments. BDM Art.7.02.18.A6

Moment and shear due to barrier and future wearing surface (FWS) weight are also calculated for simple and continuous spans.

Step 6.3.1. Moment and Shear in Composite Section (Simple Span)

Barrier and FWS loads on the composite, simple span structure are applied to calculate the maximum positive moments.

As an example, moment and shear at 4.1875 ft from a support due to a barrier weight of 0.128 kip/ft and FWS weight of 0.160 kip/ft are calculated as follows:

Moment and shear due to barrier weight

$M_b = 0.5wx(L - x) = 0.5\,(0.128)(4.1875)(83.75 - 4.1875) = 21.32$ kip-ft

$V_b = w(0.5L - x) = (0.128)(0.5 \times 83.75 - 4.1875) = 4.82$ kip

Moment and shear due to FWS weight

$M_{ws} = 0.5wx(L - x) = 0.5\,(0.160)(4.1875)(83.75 - 4.1875) = 26.65$ kip-ft

$V_{ws} = w(0.5L - x) = (0.160)(0.5 \times 83.75 - 4.1875) = 6.03$ kip

The respective simple span moment and shear values are shown in **Table 3** - **Table 6.**

Step 6.3.2. Moment and Shear in Composite Section (Continuous Span)

Barrier and FWS loads acting on the composite, continuous structure are applied to calculate the maximum negative moment at the intermediate pier. The design span (L_{ds}) of 83.75 ft plus 1 ft (i.e., 84.75 ft) is used in this calculation, assuming that the distance between bearing centerlines at the intermediate pier is 2 ft.

As an example, moment and shear at 4.1875 ft from an abutment support due to a barrier weight of 0.128 kip/ft and FWS weight of 0.160 kip/ft are calculated as follows:

Moment and shear due to barrier weight

M_b = wx(3L/8 - x/2)= (0.128)(4.1875)(3 × 84.75/8 – 4.1875/2) = 15.91 kip-ft

V_b = w(3L/8 - x) = (0.128)(3 × 84.75/8 – 4.1875) = 3.53 kip

Moment and shear due to FWS weight

M_{ws} = wx(3L/8 - x/2)= (0.160)(4.1875)(3 × 84.75/8 – 4.1875/2) = 19.89 kip-ft

V_{ws} = w(3L/8 - x) = (0.160)(3 × 84.75/8 – 4.1875) = 4.42 kip

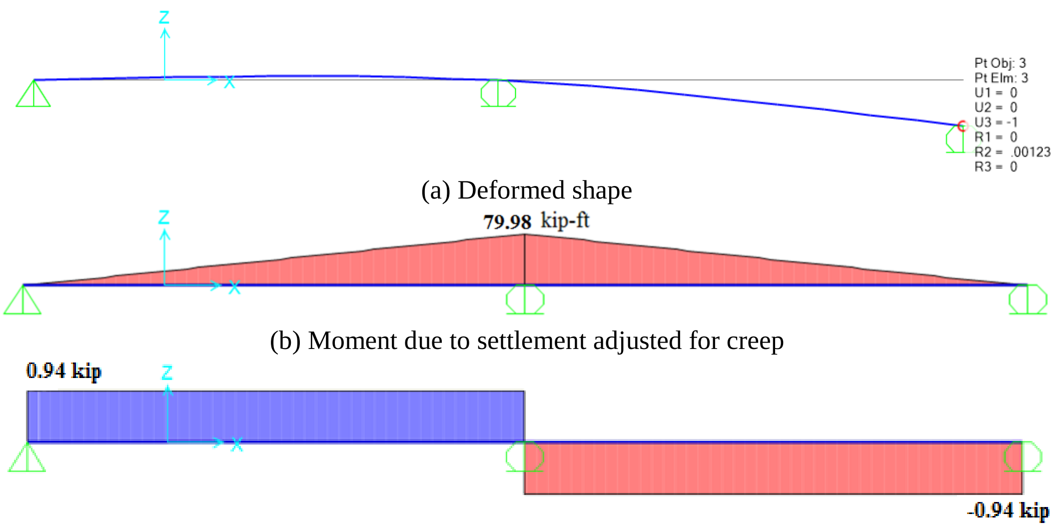
The respective continuous span moment and shear values are shown in **Table 3 - Table 6.**

Step 6.4. Moment and Shear in Composite Section due to Support Settlement

A negative moment caused by a 1 in. settlement of a substructure unit is imposed when non-spread footings are used. A settlement of 2 in. is imposed when the bridge is on spread footings.

The negative moment due to settlement is reduced for creep. A creep factor, ϕ_{cr}, of 0.368 is used. PCI (1978)

Moment and shear developed along interior and exterior beams are calculated by imposing a 1 in. settlement at the right abutment as shown in **Figure 10a**. The resultant moment and shear values are multiplied by ϕ_{cr} and shown in **Figure 10b** and **c**. The results are also presented in **Table 3** - **Table 6**. Similarly, effect of settlement at the left abutment needs to be considered for design. This will only change the sign of shear force developed in the beam because of equal spans. This calculation can be lengthy with bridges of three or more spans, and settlement at each support needs to be considered.



(a) Deformed shape

(b) Moment due to settlement adjusted for creep

(c) Shear due to support settlement adjusted for creep

Figure 10. Moment and shear in an interior beam due to a 1 in. settlement at the right abutment

STEP 7. MOMENT AND SHEAR DUE TO LIVE LOADS

Moment and shear envelopes per lane are usually calculated using structural analysis software.

A simple span is used to calculate maximum positive moment due to live loads, while a continuous span is used to calculate maximum negative moment due to live loads at an intermediate pier.

The following equations can be used to calculate <u>simple span maximum bending moment per lane for HS20 truck loading.</u> FHWA (2003)

Load type	x/L	Formula for maximum bending moment, ft-kips	Minimum	
			x,* ft	L, ft
HS20 Truck	0 - 0.333	$\frac{72(x)[(L-x)-9.33]}{L}$	0	28
	0.333 - 0.500	$\frac{72(x)[(L-x)-4.67]}{L}-112$	14	28

* x is the distance from the left support to the section being considered, ft
L is the span length, note that there is a minimum span length requirement to use the listed equations.
The equations from the above table are used for service and strength limit states.
Rear axle spacing variation from 14 ft to 30 ft is considered.
<u>Do not</u> use the above equations for fatigue limit state calculations.

The following equations can be used to calculate <u>simple span maximum shear force per lane for HS20 truck loading.</u> FHWA (2003)

Load type	x/L	Formula for maximum shear force, kips	Minimum		Maximum
			x,* ft	L, ft	L, ft
HS20 Truck	0 - 0.500	$\frac{72[(L-x)-4.67]}{L}-8$	14	28	42
	0 - 0.500	$\frac{72[(L-x)-9.33]}{L}$	0	42	-

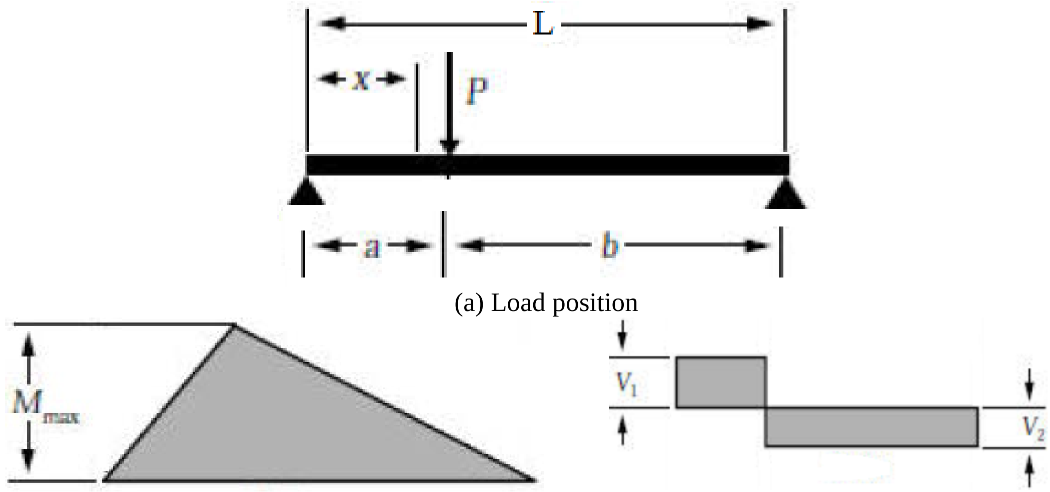
* x is the distance from the left support to the section being considered, ft
L is the span length, note that there is a minimum and maximum span length requirement to use the listed equations.

The following equations can be used to calculate <u>simple span maximum bending moment per lane for HL-93 fatigue truck loading.</u> FHWA (2003)

Load type	x/L	Formula for maximum bending moment, ft-kips	Minimum	
			x,* ft	L, ft
Fatigue Truck Loading (LRFD)	0 - 0.241	$\frac{72(x)[(L-x)-18.22]}{L}$	0	44
	0.241 - 0.500	$\frac{72(x)[(L-x)-11.78]}{L}-112$	14	28

* x is the distance from the left support to the section being considered, ft
L is the span length, note that there is a minimum span length requirement to use the listed equations.
Equations in the above table are applicable only for a rear axle spacing of 30 ft.

The following equations can be used to calculate simple span maximum bending moment and shear per lane for an axle loading of P.



(a) Load position

$M_{max} = Pab/L$

$M_x\ (x < a) = Pbx/L$

(b) Moment diagram for the given position of P

$V_1 = Pb/L$

$V_2 = Pa/L$

(c) Shear diagram for the given position of P

Figure 11. Simple span moment and shear force for a single axle load

Step 7.1. Moment and Shear due to Design Truck or Axle Load

For all limit states except fatigue:

The moment per beam due to truck (M_{LT}) or axle load (M_{LA}) and shear per beam due to truck (V_{LT}) or axle load (V_{LA}) are expressed as follows:

Interior beam,

M_{LT} or M_{LA} = (moment per lane)(factor for HL-93 Mod)(DFM_{IB})(1+IM)
= (moment per lane)(1.2)(0.581)(1+0.33)
= (moment per lane)(0.927) kip-ft

V_{LT} or V_{LA} = (shear force per lane)(factor for HL-93 Mod)(DFV_{IB})(1+IM)
= (shear force per lane)(1.2)(0.736)(1+0.33)
= (shear force per lane)(1.175) kips

Exterior beam,

M_{LT} or M_{LA} = (moment per lane)(factor for HL-93 Mod)(DFM_{EB})(1+IM)
= (moment per lane)(1.2)(0.714)(1+0.33)
= (moment per lane)(1.140) kip-ft

V_{LT} or V_{LA} = (shear force per lane)(factor for HL-93 Mod)(DFV_{EB})(1+IM)
= (shear force per lane)(1.2)(0.714)(1+0.33)
= (shear force per lane)(1.140) kips

The following example shows moment, M_{LT} and M_{LA}, calculation of an interior beam at midspan:

Design span, L_{ds} = 83.75 ft

Distance from support to mid span, x = 0.5L_{ds} = 41.875 ft

L = 83.75 ft > 28 ft, x = 41.875 ft > 14 ft, x/L = 0.5

Moment due to HS-20 truck $= \frac{72(x)[(L-x)-4.67]}{L} - 112$

Moment due to HS-20 truck at midspan

= 72 (41.875)[(83.75 - 41.875) – 4.67]/83.75 – 112

= 1227.38 kip – ft

Moment in an interior beam due to HS-20 truck, M_{LT}, at midspan

= 0.927 × 1227.38 = **1137.78 kip- ft**

Moment due to 60 kip axle load at midspan

= PL/4 = 60 × 83.75/4 = 1256.25 kip – ft

Moment in an interior beam due to 60 kip axle load, M_{LA}, at midspan

= 0.927 × 1256.25 = **1164.54 kip- ft** ***Controls***

Therefore, the moment due to 60 kip axle load (M_{LA}) at midspan and the lane load moment (M_{LL}) at midspan are combined to calculate live load moment for design.

The following example shows shear, V_{LT} and V_{LA}, calculation of an interior beam at bearing centerline:

L = 83.75 ft > 42 ft, x = 0 ft , x/L = 0

Shear due to HS-20 truck $= \frac{72[(L-x)-9.33]}{L}$

Shear due to HS-20 truck at bearing centerline

= 72 [(83.75 - 0) - 9.33]/83.75 = 63.98 kip

Shear in an interior beam due to HS-20 truck, V_{LT}, at bearing centerline

= 1.175 × 63.98 = **75.18 kip** ***Controls***

Shear at bearing centerline due to 60 kip axle load

= P = 60 kips

Shear in an interior beam due to 60 kip axle load, V_{LA}, at bearing centerline

= 1.175 × 60 = **70.50 kip**

Therefore, shear due to HS-20 truck load (V_{LT}) at bearing centerline and the lane load shear (V_{LL}) at bearing centerline are combined to calculate live load shear for design.

M_{LT}, M_{LA}, V_{LT}, and V_{LA} at selected sections for interior and exterior beams are given in **Appendix A**.

Step 7.2. Moment and Shear due to Design Lane Load

Moment (M_x) and shear force (V_x) per lane at a section located a distance of x from the left support due to a uniformly distributed load of 0.64 kip/ft (**Figure 12**) can be calculated using the following equations:

$$M_x = \frac{0.64}{2}(x)(L - x)$$

$$V_x = \frac{0.64}{2}\frac{(L - x)^2}{L} \text{ for } x \le 0.5L$$

where, M_x is in kip-ft/lane, V_x is in kip/lane, and L and x are in ft.

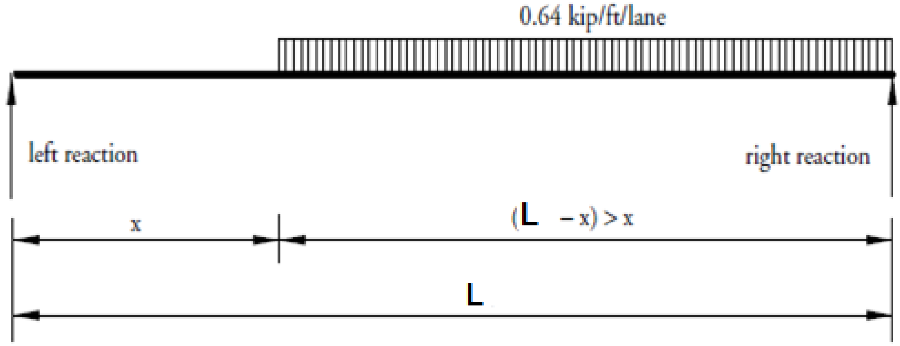


Figure 12. Design lane load for shear force calculation

Lane load shear and moment per beam are as follows:

Interior beam,

M_{LL} = (moment per lane)(factor for HL-93 Mod)(DFM_{IB})
= (moment per lane)(1.2)(0.581)
= (moment per lane)(0.697) ft-kips

V_{LL} = (lane load shear force)(factor for HL-93 Mod)(DFV_{IB})
= (lane load shear force)(1.2)(0.736)
= (lane load shear force)(0.883) kips

Exterior beam,

M_{LL} = (moment per lane)(factor for HL-93 Mod)(DFM_{EB})
= (moment per lane)(1.2)(0.714)
= (moment per lane)(0.857) ft-kips

V_{LL} = (lane load shear force)(factor for HL-93 Mod)(DFV_{EB})
= (lane load shear force)(1.2)(0.714)
= (lane load shear force)(0.857) kips

The following example shows moment due to lane load, M_{LL}, calculation at midspan of an interior beam:

L = 83.75 ft and x = 41.875 ft

Moment due to lane load $M_x = \frac{0.64}{2}(x)(L - x)$

Moment at midspan due to lane load

= (0.64/2)(41.875)(83.75 - 41.875)

= 561.13 kip-ft

Moment in an interior beam due to lane load, M_{LL}, at midspan

= (0.697)(561.13) = **391.11 kip-ft**

The following example shows shear, V_{LL}, calculation at the bearing centerline of an interior beam:

L = 83.75 ft and x = 0 ft

Shear at bearing centerline due to lane load, $V_x = \frac{0.64}{2}\frac{(L - x)^2}{L}$

Shear at bearing centerline due to lane load

= $(0.64/2)(83.75 - 0)^2/83.75$

= 26.80 kip

Shear in an interior beam due to lane load, V_{LL}, at bearing centerline

= 0.883 × 26.80 = **23.66 kip**

M_{LL} and V_{LL} at selected sections for interior and exterior beams are given in **Appendix A**.

Step 7.3. Moment and Shear Summary

Simple span live load moment, HL – 93 Mod with impact (M_{HL-M}), at midspan

M_{LA} at midspan > M_{LT} at midspan

Therefore, M_{LA} and M_{LL} at midspan are combined to calculate the live load moment, HL – 93 Mod with impact (M_{HL-M}), at midspan.

From **Step 7.1**,

Moment in an interior beam due to 60 kip axle load, M_{LA}, at midspan

= 1164.54 kip- ft

From **Step 7.2**,

Moment in an interior beam due to lane load, M_{LL}, at midspan

= 391.11 kip-ft

Therefore,

M_{HL-M} = 1164.54 + 391.11 = **1555.65 kip-ft**

Simple span live load moment, M_{HL-M}, of interior and exterior beams are given in **Table 3** and **Table 5**, respectively.

Simple span live load shear, HL – 93 Mod with impact (V_{HL-M}), at bearing centerline

V_{LT} at bearing centerline > V_{LA} at bearing centerline

Therefore, V_{LT} and V_{LL} at the bearing centerline are combined to calculate the live load shear, HL – 93 Mod with impact (V_{HL-M}), at the bearing centerline.

From **Step 7.1**,

Shear in an interior beam due to HS-20 truck, V_{LT}, at bearing centerline

= 75.18 kip

From **Step 7.2**,

Shear in an interior beam due to lane load, V_{LL}, at bearing centerline

= 23.66 kip

Therefore,

V_{HL-M} = 75.18 + 23.66 = **98.84 kip**

Simple span live load shear, V_{HL-M}, of interior and exterior beams are given in **Table 4** and **Table 6**, respectively.

Continuous span live load moment, HL – 93 Mod with impact (M_{HL-M}), over pier

For negative moment between points of contraflexure under a uniform load on all spans, 90 percent of the effect of two design trucks spaced a minimum of 50.0 ft between the lead axle of one truck and the rear axle of the other truck, combined with 90 percent of the effect of the design lane load is used. LRFD Art. 3.6.1.3.1

Where multiple lanes of heavier vehicles are considered probable, consideration should be given to investigating negative moment at interior supports for pairs of the design tandem spaced from 26.0 ft to 40.0 ft apart, combined with the design lane load. LRFD C 3.6.1.3.1

In this example, 60 kip axle load is not included, assuming the bridge is subjected to normal traffic load.

Step 6.3.2, design span for continuous span = 84.75 ft

Live load, described in **Figure 6** in **Step 4.2**, is applied and a maximum moment of **– 1,103 kip-ft** is calculated over the pier.

Moment in an interior beam due to 90% of the combined HS-20 truck,

M_{LT}, over the pier

= (0.927)(-1,103)(0.90) = - 920.23 kip- ft

Using the equation in **Step 6**, the moment over the pier due to 0.64 kip/ft lane load

= $-wL^2/8 = -(0.64)(84.75)^2/8$ = -574.61 kip- ft

Moment in an interior beam due to 90% of the lane load,

M_{LL}, over the pier

= (0.697)(-574.61)(0.90) = -360.45 kip- ft

M_{LT} and M_{LL} over the pier are combined to calculate the live load moment, HL – 93 Mod with impact (M_{HL-M}), over the pier.

Therefore,

$$M_{HL-M} = (-920.23) + (-360.45) = \mathbf{-1280.68\ kip\text{-}ft}$$

Continuous span live load moment, M_{HL-M}, of interior and exterior beams are given in **Table 3** and **Table 5**, respectively.

Continuous span live load shear, HL – 93 Mod with impact (V_{HL-M}), over pier

In this example, 60 kip axle load is not included, assuming the bridge is subjected to normal traffic load.

Step 6.3.2, design span for continuous span = 84.75 ft

Live load, described in **Figure 6** in **Step 4.2**, is applied and a maximum shear of **-69.90 kip** is calculated over the pier.

Shear in an interior beam due to 90% of the combined HS-20 truck,
V_{LT}, over the pier

$$= (1.175)(-69.90)(0.90) = -73.92 \text{ kip}$$

Using the equation in **Step 6**, the shear over the pier due to 0.64 kip/ft lane load

$$= -5wL/8 = -5(0.64)(84.75)/8 = -33.90 \text{ kip}$$

Shear in an interior beam due to 90% of the lane load,
V_{LL}, over the pier

$$= (0.883)(-33.90)(0.90) = -26.94 \text{ kip}$$

V_{LT} and V_{LL} over the pier are combined to calculate the live load shear, HL – 93 Mod with impact (V_{HL-M}), over the pier.

Therefore,

$$V_{HL-M} = (-73.92) + (-26.94) = \mathbf{-100.86\ kip}$$

Continuous span live load shear, V_{HL-M}, of interior and exterior beams are given in **Table 4** and **Table 6**, respectively.

Table 3. Unfactored Moment in Interior Beam (kip-ft)

Location	At release	For service and strength limit state checks								
		Noncomposite section (simple span)		Composite section						
				Simple span			Continuous span[+]			
	Beam, M_{gr}	Beam, M_g	Deck and haunch, M_D	Barrier, M_b	Future wearing surface, M_{ws}	HL – 93 Mod with impact, M_{HL-M}	Settlement, M_s	Barrier, M_b	Future wearing surface, M_{ws}	HL – 93 Mod with impact, M_{HL-M}
Beam end	0.00									
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0
Transfer location*	109.11	81.09	77.81	11.74	14.67	224.85	0.00	11.36	14.21	19.18
0.10 × L_{ds}	307.17	279.15	267.85	40.40	50.50	581.48	-0.02	29.48	36.85	46.49
0.20 × L_{ds}	524.28	496.26	476.18	71.82	89.78	1020.39	-0.24	49.99	62.48	64.86
0.30 × L_{ds}	679.36	651.34	624.99	94.27	117.84	1315.49	-1.20	61.51	76.89	55.11
0.40 × L_{ds}	772.41	744.39	714.27	107.74	134.67	1493.87**	-4.08	64.06	80.07	17.21
0.50 × L_{ds} (Midspan)	803.43	775.40	744.03	112.23	140.28	1555.65**	-10.39	57.63	72.04	-48.80
0.60 × L_{ds}	772.41	744.39	714.27	107.74	134.67	1493.87**	-21.16	42.22	52.78	-142.94
0.70 × L_{ds}	679.36	651.34	624.99	94.27	117.84	1315.49	-35.90	17.83	22.29	-265.21
0.80 × L_{ds}	524.28	496.26	476.18	71.82	89.78	1020.39	-52.20	-15.53	-19.41	-415.61
0.90 × L_{ds}	307.17	279.15	267.85	40.40	50.50	581.48	-66.42	-57.87	-72.34	-727.71
Transfer location*	109.11	81.09	77.81	11.74	14.67	224.85	-75.12	-101.60	-126.99	-1156.70
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	-78.57	-109.19	-136.49	-1230.26
Span 2 - 0	-	0.00	0.00	0.00	0.00	0.00	-79.98	-114.23	-142.78	-1280.68

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

** 60 kip axle load moment is higher than HS-20 design truck load moment.

\+ 60 kip axle load is not included in continuous span analysis.

Note: The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

Table 4. Unfactored Shear in Interior Beam (kip)

Location	At release	For service and strength limit state checks								
		Noncomposite section (simple span)		Composite section						
				Simple span			Continuous span+			
	Beam, V_{gr}	Beam, V_g	Deck and haunch, V_D	Barrier, V_b	Future wearing surface, V_{ws}	HL – 93 Mod with impact, $V_{HL\text{-}M}$	Settlement, V_s	Barrier, V_b	Future wearing surface, V_{ws}	HL – 93 Mod with impact, $V_{HL\text{-}M}$
Beam end	37.70									
CL of bearing	37.03	37.03	35.54	5.36	6.70	98.84	0.94	4.06	5.07	9.18
Transfer location*	35.04	35.04	33.63	5.07	6.34	94.04	0.94	3.68	4.59	7.65
0.10 × L_{ds}	29.63	29.63	28.43	4.29	5.36	85.82	0.94	2.98	3.73	4.92
0.20 × L_{ds}	22.22	22.22	21.32	3.22	4.02	73.30	0.94	1.91	2.39	-2.01
0.30 × L_{ds}	14.81	14.81	14.21	2.14	2.68	61.34	0.94	0.84	1.05	-17.4
0.40 × L_{ds}	7.41	7.41	7.11	1.07	1.34	50.74**	0.94	-0.23	-0.29	-32.46
0.50 × L_{ds} (Midspan)	0.00	0.00	0.00	0.00	0.00	41.13**	0.94	-1.30	-1.63	-46.79
0.60 × L_{ds}	-7.41	-7.41	-7.11	-1.07	-1.34	-50.74**	0.94	-2.38	-2.97	-59.82
0.70 × L_{ds}	-14.81	-14.81	-14.21	-2.14	-2.68	-61.34	0.94	-3.45	-4.31	-71.73
0.80 × L_{ds}	-22.22	-22.22	-21.32	-3.22	-4.02	-73.30	0.94	-4.52	-5.65	-82.36
0.90 × L_{ds}	-29.63	-29.63	-28.43	-4.29	-5.36	-85.82	0.94	-5.59	-6.99	-91.84
Transfer location*	-35.04	-35.04	-33.63	-5.07	-6.34	-94.04	0.94	-6.51	-8.14	-98.95
CL of bearing	-37.03	-37.03	-35.54	-5.36	-6.70	-98.84	0.94	-6.66	-8.33	-100.07
Span 2 - 0		0.00	0.00	0.00	0.00	0.00	0.94	-6.76	-8.45	-100.86

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

** 60 kip axle load shear is higher than HS-20 design truck load shear.

+ 60 kip axle load is not included in continuous span analysis.

Note: The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

Table 5. Unfactored Moment in Exterior Beam (kip-ft)

Location	At release	For service and strength limit state checks								
		Noncomposite section (simple span)		Composite section						
				Simple span			Continuous span+			
	Beam, M_{gr}	Beam, M_g	Deck and haunch, M_D	Barrier, M_b	Future wearing surface, M_{ws}	HL – 93 Mod with impact, $M_{HL\text{-}M}$	Settlement, M_s	Barrier, M_b	Future wearing surface, M_{ws}	HL – 93 Mod with impact, $M_{HL\text{-}M}$
Beam end	0.00									
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Transfer location*	109.11	81.09	77.50	11.74	14.67	276.32	0.00	11.36	14.21	23.76
$0.10 \times L_{ds}$	307.17	279.15	266.79	40.40	50.50	714.08	-0.02	29.48	36.85	57.64
$0.20 \times L_{ds}$	524.28	496.26	474.29	71.82	89.78	1257.40	-0.24	49.99	62.48	80.47
$0.30 \times L_{ds}$	679.36	651.34	622.51	94.27	117.84	1625.39	-1.20	61.51	76.89	68.46
$0.40 \times L_{ds}$	772.41	744.39	711.44	107.74	134.67	1846.91**	-4.08	64.06	80.07	21.63
$0.50 \times L_{ds}$ (Midspan)	803.43	775.40	741.08	112.23	140.28	1923.79**	-10.39	57.63	72.04	-60.02
$0.60 \times L_{ds}$	772.41	744.39	711.44	107.74	134.67	1846.91**	-21.16	42.22	52.78	-176.48
$0.70 \times L_{ds}$	679.36	651.34	622.51	94.27	117.84	1625.39	-35.90	17.83	22.29	-327.79
$0.80 \times L_{ds}$	524.28	496.26	474.29	71.82	89.78	1257.40	-52.20	-15.53	-19.41	-513.91
$0.90 \times L_{ds}$	307.17	279.15	266.79	40.40	50.50	714.08	-66.42	-57.87	-72.34	-900.00
Transfer location*	109.11	81.09	77.50	11.74	14.67	276.32	-75.12	-109.19	-136.49	-1291.70
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	-78.57	-109.19	-136.49	-1521.58
Span 2 - 0	-	0.00	0.00	0.00	0.00	0.00	-79.98	-114.23	-142.78	-1581.07

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

** 60 kip axle load moment is higher than HS-20 design truck load moment.

+ 60 kip axle load is not included in continuous span analysis.

Note: The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

Table 6. Unfactored Shear in Exterior Beam (kip)

Location	At release	For service and strength limit state checks									
		Noncomposite section (simple span)		Composite section							
				Simple span			Continuous span+				
	Beam, V_{gr}	Beam, V_g	Deck and haunch, V_D	Barrier, V_b	Future wearing surface, V_{ws}	HL – 93 Mod with impact, $V_{HL\text{-}M}$	Settlement, V_s	Barrier, V_b	Future wearing surface, V_{ws}	HL – 93 Mod with impact, $V_{HL\text{-}M}$
Beam end	37.70									
CL of bearing	37.03	37.03	35.39	5.36	6.70	95.45	0.94	4.06	5.07	8.95
Transfer location*	35.04	35.04	33.49	5.07	6.34	91.36	0.94	3.68	4.59	7.46
0.10 × L_{ds}	29.63	29.63	28.32	4.29	5.36	83.19	0.94	2.98	3.73	4.80
0.20 × L_{ds}	22.22	22.22	21.24	3.22	4.02	71.37	0.94	1.91	2.39	-1.96
0.30 × L_{ds}	14.81	14.81	14.16	2.14	2.68	59.44**	0.94	0.84	1.05	-16.99
0.40 × L_{ds}	7.41	7.41	7.08	1.07	1.34	49.52**	0.94	-0.23	-0.29	-31.68
0.50 × L_{ds} (Midspan)	0.00	0.00	0.00	0.00	0.00	40.13**	0.94	-1.30	-1.63	-45.68
0.60 × L_{ds}	-7.41	-7.41	-7.08	-1.07	-1.34	-49.52**	0.94	-2.38	-2.97	-58.40
0.70 × L_{ds}	-14.81	-14.81	-14.16	-2.14	-2.68	-59.44**	0.94	-3.45	-4.31	-70.02
0.80 × L_{ds}	-22.22	-22.22	-21.24	-3.22	-4.02	-71.37	0.94	-4.52	-5.65	-80.40
0.90 × L_{ds}	-29.63	-29.63	-28.32	-4.29	-5.36	-83.19	0.94	-5.59	-6.99	-89.65
Transfer location*	-35.04	-35.04	-33.49	-5.07	-6.34	-91.36	0.94	-6.51	-8.14	-94.91
CL of bearing	-37.03	-37.03	-35.39	-5.36	-6.70	-95.45	0.94	-6.66	-8.33	-97.69
Span 2 - 0	-	0.00	0.00	0.00	0.00	0.00	0.94	-6.76	-8.45	-98.34

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

** 60 kip axle load shear is higher than HS-20 design truck load shear.

\+ 60 kip axle load is not included in continuous span analysis.

Note: The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

Combined loads for service and strength limit states – Simple span

Service I Limit State Moment for Noncomposite Section

$1.0\ (M_g + M_D)$

1.0 (775.40 + 744.03) = 1519.43 kip-ft

Service I Limit State Moment for Composite Section

$1.0\ (M_b + M_{ws}) + 1.0\ (M_{HL\text{-}M})$

1.0 (112.23 + 140.28) + 1.0 (1555.65) = 1808.16 kip-ft

Service III Limit State Moment for Noncomposite Section

$1.0\ (M_g + M_D)$

1.0 (775.40 + 744.03) = 1519.43 kip-ft

Service III Limit State Moment for Composite Section

$1.0\ (M_b + M_{ws}) + 0.8\ (M_{HL\text{-}M})$

1.0 (112.23 + 140.28) + 0.8 (1555.65) = 1497.03 kip-ft

Strength I Limit State Moment

$1.25\ (M_g + M_D + M_b) + 1.50\ (M_{ws}) + 1.75\ (M_{HL\text{-}M})$

1.25 (775.40 + 744.03 + 112.23) + 1.50 (140.28) + 1.75 (1555.65) = 4972.38 kip-ft

Load combinations and load factors are given in **Table 7**. Load factors for permanent loads are given in **Table 8**.

Simple span service and strength limit state moment and shear values of interior and exterior beams are given in **Table 9** to **Table 12**.

Table 7. Load Combinations and Load Factors (LRFD Table 3.4.1-1)

Load Combination Limit State	*DC* *DD* *DW* *EH* *EV* *ES* *EL* *PS* *CR* *SH*	*LL* *IM* *CE* *BR* *PL* *LS*	*WA*	*WS*	*WL*	*FR*	*TU*	*TG*	*SE*	Use One of These at a Time				
										EQ	*BL*	*IC*	*CT*	*CV*
Strength I (unless noted)	γ_p	1.75	1.00	—	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Strength II	γ_p	1.35	1.00	—	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Strength III	γ_p	—	1.00	1.40	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Strength IV	γ_p	—	1.00	—	—	1.00	0.50/1.20	—	—	—	—	—	—	—
Strength V	γ_p	1.35	1.00	0.40	1.0	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Extreme Event I	γ_p	γ_{EQ}	1.00	—	—	1.00	—	—	—	1.00	—	—	—	—
Extreme Event II	γ_p	0.50	1.00	—	—	1.00	—	—	—	—	1.00	1.00	1.00	1.00
Service I	1.00	1.00	1.00	0.30	1.0	1.00	1.00/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Service II	1.00	1.30	1.00	—	—	1.00	1.00/1.20	—	—	—	—	—	—	—
Service III	1.00	0.80	1.00	—	—	1.00	1.00/1.20	γ_{TG}	γ_{SE}	—	—	—	—	—
Service IV	1.00	—	1.00	0.70	—	1.00	1.00/1.20	—	1.0	—	—	—	—	—
Fatigue I— *LL*, *IM* & *CE* only	—	1.50	—	—	—	—	—	—	—	—	—	—	—	—
Fatigue II— *LL*, *IM* & *CE* only	—	0.75	—	—	—	—	—	—	—	—	—	—	—	—

Table 8. Load Factors for Permanent Loads, γ_p (LRFD Table 3.4.1-2)

Type of Load, Foundation Type, and Method Used to Calculate Downdrag		Load Factor	
		Maximum	Minimum
DC: Component and Attachments		1.25	0.90
DC: Strength IV only		1.50	0.90
DD: Downdrag	Piles, α Tomlinson Method	1.4	0.25
	Piles, λ Method	1.05	0.30
	Drilled shafts, O'Neill and Reese (1999) Method	1.25	0.35
DW: Wearing Surfaces and Utilities		1.50	0.65
EH: Horizontal Earth Pressure			
• Active		1.50	0.90
• At-Rest		1.35	0.90
• *AEP* for anchored walls		1.35	N/A
EL: Locked-in Construction Stresses		1.00	1.00
EV: Vertical Earth Pressure			
• Overall Stability		1.00	N/A
• Retaining Walls and Abutments		1.35	1.00
• Rigid Buried Structure		1.30	0.90
• Rigid Frames		1.35	0.90
• Flexible Buried Structures			
○ Metal Box Culverts and Structural Plate Culverts with Deep Corrugations		1.5	0.9
○ Thermoplastic culverts		1.3	0.9
○ All others		1.95	0.9
ES: Earth Surcharge		1.50	0.75

Table 9. Factored Moment in Interior Beam (kip-ft)

Location	Load Combinations – Simple Span				
	Service I		**Service III**		**Strength I**
	Noncomp	Comp	Noncomp	Comp	
Beam end					
CL of bearing	0.00	0.00	0.00	0.00	0.00
Transfer length	158.90	251.26	158.90	206.29	628.79
$0.10 \times L_{ds}$	547.00	672.38	547.00	556.08	1827.59
$0.20 \times L_{ds}$	972.44	1181.99	972.44	977.91	3225.68
$0.30 \times L_{ds}$	1276.33	1527.60	1276.33	1264.50	4192.12
$0.40 \times L_{ds}$	1458.66	1736.28	1458.66	1437.51	4774.28
$0.50 \times L_{ds}$ (Midspan)	1519.43	1808.16	1519.43	1497.03	4972.38
$0.60 \times L_{ds}$	1458.66	1736.28	1458.66	1437.51	4774.28
$0.70 \times L_{ds}$	1276.33	1527.60	1276.33	1264.50	4192.12
$0.80 \times L_{ds}$	972.44	1181.99	972.44	977.91	3225.68
$0.90 \times L_{ds}$	547.00	672.38	547.00	556.08	1827.59
Transfer length	158.59	251.26	158.59	206.29	628.41
CL of bearing	0.00	0.00	0.00	0.00	0.00
Span 2 - 0	0.00	0.00	0.00	0.00	0.00

Table 10. Factored Shear in Interior Beam (kip)

Location	Load Combinations – Simple Span				
	Service I		**Service III**		**Strength I**
	Noncomp	Comp	Noncomp	Comp	
Beam end					
CL of bearing	72.57	110.90	72.57	91.13	278.09
Transfer length	68.67	105.45	68.67	86.64	266.26
$0.10 \times L_{ds}$	58.06	95.47	58.06	78.31	236.16
$0.20 \times L_{ds}$	43.54	80.54	43.54	65.88	192.76
$0.30 \times L_{ds}$	29.02	66.16	29.02	53.89	150.32
$0.40 \times L_{ds}$	14.52	53.15	14.52	43.00	110.29
$0.50 \times L_{ds}$ (Midspan)	0.00	41.13	0.00	32.90	71.98
$0.60 \times L_{ds}$	-14.52	-53.15	-14.52	-43.00	-110.29
$0.70 \times L_{ds}$	-29.02	-66.16	-29.02	-53.89	-150.32
$0.80 \times L_{ds}$	-43.54	-80.54	-43.54	-65.88	-192.76
$0.90 \times L_{ds}$	-58.06	-95.47	-58.06	--78.31	-236.16
Transfer length	-68.67	-105.45	-68.67	-86.64	-266.26
CL of bearing	-72.57	-110.90	-72.57	-91.13	-278.09
Span 2 - 0	0.00	0.00	0.00	0.00	0.00

The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

Table 11. Factored Moment in Exterior Beam (kip-ft)

Location	**Load Combinations – Simple Span**				
	Service I		**Service III**		**Strength I**
	Noncomp	Comp	Noncomp	Comp	
Beam end					
CL of bearing	0.00	0.00	0.00	0.00	0.00
Transfer length	158.59	302.73	158.59	247.47	718.48
0.10 × L_{ds}	545.94	804.98	545.94	662.16	2058.32
0.20 × L_{ds}	970.55	1419.00	970.55	1167.52	3638.08
0.30 × L_{ds}	1273.85	1837.50	1273.85	1512.42	4731.34
0.40 × L_{ds}	1455.83	2089.32	1455.83	1719.94	5388.56
0.50 × L_{ds} (Midspan)	1516.48	2176.30	1516.48	1791.54	5612.94
0.60 × L_{ds}	1455.83	2089.32	1455.83	1719.94	5388.56
0.70 × L_{ds}	1273.85	1837.50	1273.85	1512.42	4731.34
0.80 × L_{ds}	970.55	1419.00	970.55	1167.52	3638.08
0.90 × L_{ds}	545.94	804.98	545.94	662.16	2058.32
Transfer length	158.59	302.73	158.59	247.47	718.48
CL of bearing	0.00	0.00	0.00	0.00	0.00
Span 2 - 0	0.00	0.00	0.00	0.00	0.00

Table 12. Factored Shear in Exterior Beam (kip)

Location	**Load Combinations – Simple Span**				
	Service I		**Service III**		**Strength I**
	Noncomp	Comp	Noncomp	Comp	
Beam end					
CL of bearing	72.42	107.51	72.42	88.42	274.31
Transfer length	68.53	102.77	68.53	84.50	261.39
0.10 × L_{ds}	57.95	92.84	57.95	76.20	231.42
0.20 × L_{ds}	43.46	78.61	43.46	64.34	189.28
0.30 × L_{ds}	28.97	64.26	28.97	52.37	146.93
0.40 × L_{ds}	14.49	51.93	14.49	42.03	108.12
0.50 × L_{ds} (Midspan)	0.00	40.13	0.00	32.10	70.23
0.60 × L_{ds}	-14.49	-51.93	-14.49	-42.03	-108.12
0.70 × L_{ds}	-28.97	-64.26	-28.97	-52.37	-146.93
0.80 × L_{ds}	-43.46	-78.61	-43.46	-64.34	-189.28
0.90 × L_{ds}	-57.95	-92.84	-57.95	-76.20	-231.42
Transfer length	-68.53	-102.77	-68.53	-84.50	-261.39
CL of bearing	-72.42	-107.51	-72.42	-88.42	-274.31
Span 2 - 0	0.00	0.00	0.00	0.00	0.00

The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

STEP 8. INTERIOR BEAM PRESTRESS DESIGN

Based on moment values shown in **Table 3** and **Table 5**, interior and exterior beams are designed. This example demonstrates the design of an interior beam. The required number of strands is usually governed by concrete tensile stresses at the bottom fiber for Service III Limit State load combination at the section of maximum moment (midspan). The required number of strands is calculated for the midspan.

Step 8.1. Lump Sum Prestress Loss Estimation

Stress in prestressing steel immediately prior to transfer, f_{pi} $= 0.75f_{pu} = 202.5$ ksi

$f_{pi} \leq 0.75f_{pu}$; LRFD Table 5.9.3-1

Prestress losses range from 15% to 25%. An initial lump sum loss of 20% is assumed. The loss assumption will be reviewed upon calculating detailed losses following the strand design.

Naaman (2012)

The ratio of effective stress to stress in prestressing steel after losses, but prior to transfer, η $= 0.80$

Effective stress in prestressing steel after losses, $f_{pe} = \eta f_{pi} = 162.0$ ksi

Step 8.2. Prestressing Strand Design

Step 8.2.1. Bottom Tensile Stress at Midspan

Bottom tensile stress due to applied dead and live loads using <u>Service III Limit State</u> load combination

$$f_b = \frac{(M_g + M_D)}{S_b} + \frac{(M_b + M_{ws} + 0.8M_{HL-M})}{S_{bc}}$$

where,

f_b = concrete tensile stress due to applied loads at bottom fiber of the beam (ksi)

M_g = moment due to beam weight (kip-in)

M_D = moment due to deck and haunch weight (kip-in)

S_b = section modulus of a noncomposite beam for bottom fiber (in.^{3})

M_b = moment due to barrier weight (kip-in)

M_{ws} = moment due to future wearing surface (kip-in)

M_{HL-M} = moment due to HL-93 Mod live load (kip-in)

S_{bc} = section modulus for a bottom fiber of the composite beam (in.^{3})

Using moments from **Table 3**, bottom tensile stress due to applied loads at midspan are calculated, as follows:

LRFD Table 5.9.4.2.2-1

$$f_b = -\frac{(775.40 + 744.03) \times 12}{8000} - \frac{(112.23 + 140.28 + 0.8 \times 1555.65) \times 12}{12855}$$

$$= -3.68 \text{ ksi}$$

Step 8.2.2. Required Number of Strands

Allowable tensile stress for concrete with bonded steel and subjected to not worse than moderate corrosion condition, LRFD Table 5.9.4.2.2-1

$$\bar{f}_{ts} = -\,0.19\sqrt{f'_c} = -\,0.19\sqrt{7.5} = -0.52 \text{ ksi}$$

The required precompressive stress at the bottom fiber of the beam is the difference between allowable tensile stress for concrete and bottom tensile stress due to applied loads:

$$f_{pb} = \bar{f}_{ts} - f_b = -0.52 - (-3.68) = 3.16 \text{ ksi}$$

The strand center of gravity location, measured from the bottom of the beam, is 5% to 15% of the beam depth. As an initial assumption, location is assumed as 6% of beam depth,

$$e_{pg} = y_b - 0.06h = 18.20 - 0.06 \times 36 \cong 16.00 \text{ in.}$$

The required precompressive stress at the bottom fiber (f_{pb}) due to total prestressing force after all losses, F_e:

$$f_{pb} = \frac{F_e}{A_b} + \frac{F_e e_{pg}}{S_b}$$

$$3.16 = \frac{F_e}{878.30} + \frac{F_e \times 16.00}{8000}$$

$$3.16 = \left(\frac{1}{878.30} + \frac{16.00}{8000}\right) F_e$$

Solving for F_e,

The required F_e = 1006.83 kip

The prestressing force per strand = $A_{ps}f_{pe}$

= 0.217 × 162 = 35.15 kip

The required number of strands = 1006.83/35.15 = 28.64 ≅ 29 (rounded up)

An iterative procedure is implemented to arrive at a satisfactory strand configuration and associated eccentricity. Twenty-nine (29) strands with an eccentricity of 16 in. is the initial trial. As eccentricity decreases, the required number of strands increases. After the iterative process, 32 strands with an eccentricity of 15.14 in. are calculated. See **Step 8.2.3** for strand eccentricity (e_{pg}) calculation process.

$F_e = \eta F_i$ = prestressing force after all losses (kip)

F_i = prestressing force at release (kip)

Therefore,

$$F_e = 32 \times 35.15 = 1{,}124.80 \text{ kip}$$

$$F_i = F_e/\eta = 1{,}124.8/0.8 = 1{,}406.00 \text{ kip}$$

Step 8.2.3. Strand Arrangement at Midspan

Strand arrangement at midspan is shown in **Figure 13**. Based on the arrangement, distance between center of gravity of the strands and the bottom fiber of the beam at midspan (y_{bs}) is calculated.

$y_{bs} = (17 \times 2 + 13 \times 4 + 2 \times 6)/32 = 3.06$ in.

$e_{pg} = y_b - y_{bs} = 18.20 - 3.06 = 15.14$ in.

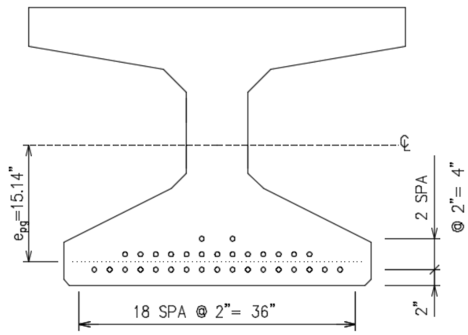


Figure 13. Strand arrangement at midspan

Step 8.2.4. Strand Arrangement at Beam End

Beam end stresses need to be checked at transfer because this stage almost always controls. Also, losses with time will reduce the concrete stresses.

Transfer length = 60 × (Strand dia.) = 60 × 0.6 = 36 in. — LRFD Art. 5.11.4

As shown below, the beam top fiber tensile stress (f_{top}) at the transfer location is calculated using moments from **Table 3**. Since beam end strand pattern is yet to be designed, the beam stresses at transfer length are checked with the strand pattern at midspan. — LRFD Table 5.9.4.1.2-1

$$f_{top} = \frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M_{gr}}{S_t}$$

$$f_{top} = \frac{1406}{878.30} - \frac{1406 \times 15.14}{8179} + \frac{109.11 \times 12}{8179} = -0.84 \text{ ksi}$$

Allowable concrete tensile stress at release, — LRFD Table 5.9.4.1.2-1

$$\bar{f}_{ti} = 0.24\sqrt{f'_{ci}} = 0.24 \times \sqrt{6.3} = -0.60 \text{ ksi}$$

$$|f_{top}| > |\bar{f}_{ti}| \quad \textbf{NOT O.K.}$$

As shown below, the beam bottom fiber compression stress (f_{bottom}) at the transfer location is calculated using moments from **Table 3**. — LRFD Art. 5.9.4.1.1

$$f_{bottom} = \frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M_{gr}}{S_b}$$

$$f_{bottom} = \frac{1406}{878.30} + \frac{1406 \times 15.14}{8000} - \frac{109.11 \times 12}{8000} = 4.10 \text{ ksi}$$

Allowable concrete compressive stress at release, — LRFD Art. 5.9.4.1.1

$$\bar{f}_{ci} = 0.6 f'_{ci} = 0.6 \times 6.3 = 3.78 \text{ ksi}$$

$$f_{bottom} > \bar{f}_{ci} \quad \textbf{NOT O.K.}$$

The top and the bottom stresses exceed stress limits at the transfer location. Stresses need to be reduced by debonding, draping, or a combination thereof.

MDOT preference is debonding. However, draping is allowed for bulb tee sections.

In this example, draped strands are designed.

The longitudinal strand profile along the span is designed and shown in **Figure 14** for the half length of the beam. In this example, draping point is located at 0.4 L_b. This position of draping is most often valid due to the relationship between standard section depth and span.

The principle process and comprehensive design procedure, using a Magnel diagram, for draping or debonding in order to satisfy the stress limits along the beam length is described in **Appendix B**.

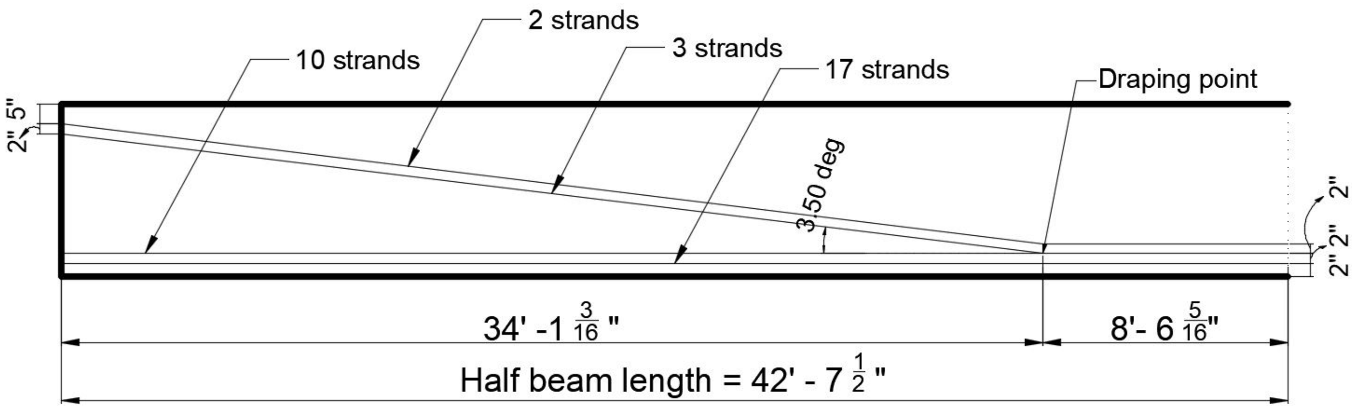


Figure 14. Longitudinal strand profile along half span of the beam

Strand arrangement at beam end is shown in **Figure 15**.

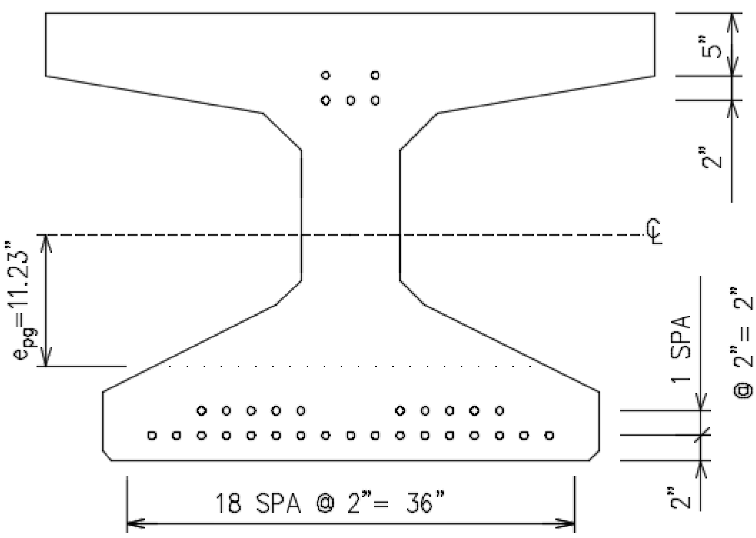


Figure 15. Strand arrangement at the end of beam

Step 8.2.5. Stress Checks along Beam Length

Stress checks are performed at multiple locations along the beam as shown in the **Table 13**.

Table 13. Stress Checks along the Half Beam Length

Location	**Stress at release (ksi)**		**Stress in service (ksi)**			**Condition**
	Top	Bottom	Top: Service I Limit State — Permanent and transient loads (DL and LL with impact)	Top: Service I Limit State — Permanent loads (DL only)	Bottom: Service III Limit State	
Beam end	0.00	0.00				O.K.
CL of bearing	-0.05	0.86	-0.07	-0.07	0.72	O.K.
Transfer length	-0.23	3.47	0.07	-0.08	2.48	O.K.
$0.10 \times L_{ds}$	-0.06	3.30	0.80	0.40	1.67	O.K.
$0.20 \times L_{ds}$	0.09	3.14	1.60	0.89	0.77	O.K.
$0.30 \times L_{ds}$	0.16	3.08	2.12	1.21	0.18	O.K.
$0.40 \times L_{ds}$	0.13	3.10	2.38	1.35	-0.12	O.K.
Midspan	0.18	3.06	2.52	1.44	-0.27	O.K.

- Tension + Compression

Stress in strands is assumed to vary linearly along the transfer length for service limit state as per LRFD Art. 5.11.4.1.

The last column in the above table shows if stress limits are satisfied.

Step 8.3. Loss of Prestress

Step 8.3.1. Initial Loss at Beam End

Before strand release, two short-term losses occur: relaxation and elastic shortening.

Relaxation Loss

Note: The equation for Δf_{pR_bt} is no longer in the AASHTO LRFD 7th edition, 2016.

$$\Delta f_{pR\_bt} = \frac{\log(24.0t)}{40.0}\left[\frac{f_{pj}}{f_{py}} - 0.55\right] f_{pj}$$ MDOT (2002)

where,

Δf_{pR_bt} = relaxation loss before transfer, ksi

t = duration of transfer, days

f_{pj} = $f_{pi} + \Delta f_{pR\_bt}$ = initial stress in strands, ksi

The tendon stress limits are specified in Table 5.9.3-1 LRFD Art .5.9.3

f_{pi} = stress in strands prior to transfer = $0.75f_{pu}$ = 202.5 ksi LRFD Table 5.9.3-1

f_{pu} = ultimate tensile strength of prestressing strands = 270 ksi BDM Art. 7.02.18.A1

f_{py} = yield strength of prestressing strands = $0.9f_{pu}$ = 243 ksi LRFD Table 5.4.4.1-1

The relaxation losses are controlled by the fabrication schedule.

Assumption:

Δf_{pR_bt1} = Initial relaxation loss at one day (t = 1 day) = 2 ksi

$f_{pj} = f_{pi} + \Delta f_{pR\_bt} = 202.50 + 2 = 204.50$ ksi

$$\Delta f_{pR\_bt2} = \frac{\log(24.0 \times 1)}{40.0}\left[\frac{204.50}{243} - 0.55\right]204.50 = 2.06 \text{ ksi}$$

The difference between assumed initial loss (Δf_{pR_bt1}) and the second iteration (Δf_{pR_bt2}) is small. Hence, Δf_{pR_bt2} is used without performing further iterations.

$$\frac{|\Delta f_{pR\_bt1} - \Delta f_{pR\_bt2}|}{\Delta f_{pR\_bt1}} \times 100 = 3.0\% \rightarrow \Delta f_{pR\_bt} = \Delta f_{pR\_bt2}$$

Elastic Shortening Loss

LRFD Art. 5.9.5.2.3a
LRFD Eq. 5.9.5.2.3a-1

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

where,

Δf_{pES} = elastic shortening loss, ksi

E_p = modulus of elasticity of prestressing steel = 28,500 ksi

E_{ci} = modulus of elasticity of beam at transfer = 4,631 ksi

f_{cgp} = the sum of concrete stress at the center of gravity of prestressing strands due to the prestressing force and self-weight of the beam

$$= \frac{P_i}{A_b} + \frac{P_i e_{end}^2}{I_b} - \frac{M_{gr} e_{end}}{I_b}$$

where:

P_i = prestress force at transfer

Relaxation loss (Δf_{pR_bt}), calculated in **Step 8.3.1**, is a time dependent loss of prestress when a tendon is held at a constant strain. Since the stress in strands prior to transfer is 202.50 ksi (i.e., $0.75f_{pu}$), and Δ_{fpR_bt} is 2.06 ksi, strands are pulled to achieve a stress of 204.56 ksi. At the time the strands are cut, the relaxation losses would take place and the remaining stress in strands would be 202.50 ksi.

Elastic shortening loss takes place with beam shortening when the strands are cut. With this loss, the stress in strands at transfer will fall below $0.75f_{pu}$.

Since, elastic shortening losses are not known, AASHTO LRFD suggests assuming a 10% loss (i.e., 0.75 × 0.9 = 0.68). LRFD C5.9.5.2.3a

Hence, $0.70f_{pu}$ is initially assumed for stress at transfer.

P_i = (area of strand) × (stress at transfer)

= A_{ps} × Number of strands × $0.70f_{pu}$

= 0.217 × 32 × 189 = 1312.42 kip

e_{end} = eccentricity of strands at end of beam

Eccentricity at the end of the transfer length is used for e_{end}.

Transfer length = 60 × (diameter of a strand) = 60 × 0.6 in. = 36 in. LRFD Art. 5.11.4.2

Thus, e_{end} is calculated at a distance of 36 in. from beam end.

Distance from beam end to bearing centerline = 9 in.

Distance to centroid of strands from the beam bottom at 36 in.

= (17 × 2 + 10 × 4 + 3 × 26.80 + 2 × 28.80)/32

= 6.63 in.

e_{end} = y_b – 6.63 = 18.20 – 6.63

= 11.57 in.

A_b = area of beam = 878.30 in.^{2}

I_b = moment of inertia of the beam = 145,592 in.^{4}

Moment at 36 in. from the beam end at release due to beam self-weight

= 109.11 kip-ft = 1,309.32 kip-in

$$f_{cgp_1} = \frac{1312.42}{878.30} + \frac{1312.42 \times 11.57^2}{145592} - \frac{1309.32 \times 11.57}{145592} = 2.60 \text{ ksi}$$

$$\Delta f_{pES\_1} = \frac{E_p}{E_{ci}} f_{cgp} = \frac{28500}{4631} \times 2.60 = 16.00 \text{ ksi}$$ LRFD Eq. 5.9.5.2.3a-1

Elastic shortening reduces the prestressing force, which in return reduces elastic shortening. Thus, an iterative process is required as shown below:

1st iteration:

P_i = force at transfer = A_{ps} × Number of strands × (f_{pi} - Δf_{pES_1})

= 0.217 × 32 × (202.50 – 16.00) = 1295.06 kip

$$f_{cgp_2} = \frac{1295.06}{878.30} + \frac{1295.06 \times 11.57^2}{145592} - \frac{1309.32 \times 11.57}{145592} = 2.56 \text{ ksi}$$

$$\Delta f_{pES\_2} = \frac{28500}{4631} \times 2.56 = 15.75 \text{ ksi}$$

2nd iteration:

P_i = force at transfer = A_{ps} × Number of strands × (f_{pi} - Δf_{pES_2})

= 0.217 × 32 × (202.50 – 15.75) = 1296.79 kip

$$f_{cgp_3} = \frac{1296.79}{878.30} + \frac{1296.79 \times 11.57^2}{145592} - \frac{1309.32 \times 11.57}{145592} = 2.56 \text{ ksi}$$

$$\Delta f_{pES\_3} = \frac{28500}{4631} \times 2.56 = 15.75 \text{ ksi}$$

With two iterations the solution converges. Thus, Δf_{pES} = 15.75 ksi

Initial losses at beam end $= \Delta f_{pES} + \Delta f_{pR_{bt}}$

$= 15.75 + 2.06 = 17.81$ ksi

Step 8.3.2. Losses at Midspan

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$$

LRFD Art. 5.9.5.1
LRFD Eq. 5.9.5.1-1

where,

Δf_{pT} = total losses, ksi

Δf_{pES} = sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads, ksi

Δf_{pLT} = losses due to long-term shrinkage and creep of concrete, and relaxation of steel, ksi

Elastic Shortening Losses

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

LRFD Eq. 5.9.5.2.3a-1

where,

Δf_{pES} = elastic shortening loss, ksi

E_p = modulus of elasticity of prestressing steel = 28,500 ksi

E_{ci} = modulus of elasticity of beam at transfer = 4,631 ksi

f_{cgp} = the sum of concrete stress at the center of gravity of prestressing strands due to the prestressing force at transfer and the self-weight of the beam <u>at maximum moment location</u>

$$= \frac{P_i}{A_b} + \frac{P_i e_{pg}^2}{I_b} - \frac{M_{gr} e_{pg}}{I_b}$$

e_{pg} = eccentricity of strands at midspan at transfer = 15.14 in.

M_{gr} = moment due to beam weight at release at midspan = 803.43 kip-ft

A_b = area of beam = 878.30 in.^{2}

I_b = moment of inertia of the beam = 145,592 in.^{4}

P_i = prestressing force at transfer = A_{ps} × Number of strands × ($0.70f_{pu}$)
= 0.217 × 32 × 189 = 1312.42 kip

Since, elastic shortening losses are not known, AASHTO LRFD suggests assuming a 10% loss (i.e., 0.75 × 0.9 = 0.68).

LRFD C5.9.5.2.3a

Hence, $0.70f_{pu}$ is the initial assumption for stress at transfer.

$$f_{cgp_1} = \frac{1312.42}{878.30} + \frac{1312.42 \times 15.14^2}{145592} - \frac{803.43 \times 12 \times 15.14}{145592}$$

$$= 2.56 \text{ ksi}$$

Initial elastic losses, $\Delta f_{pES\_1} = \frac{28500}{4631} \times 2.56 = 15.75 \text{ ksi}$

LRFD Eq. 5.9.5.2.3a-1

1st iteration:

P_i = force at transfer = A_{ps} × Number of strands × (f_{pi} - Δf_{pES_1})
= 0.217 × 32 × (202.50 - 15.75) = 1296.80 kip

$$f_{cgp_2} = \frac{1296.80}{878.30} + \frac{1296.80 \times 15.14^2}{145592} - \frac{803.43 \times 12 \times 15.14}{145592} = 2.52 \text{ ksi}$$

$$\Delta f_{pES\_2} = \frac{28500}{4631} \times 2.52 = 15.51 \text{ ksi}$$

2nd iteration:

P_i = force at transfer = A_{ps} × Number of strands × (f_{pi}-Δf_{pES_2})
= 0.217 × 32 × (202.50 - 15.51) = 1298.50 kip

$$f_{cgp_3} = \frac{1298.50}{878.30} + \frac{1298.50 \times 15.14^2}{145592} - \frac{803.43 \times 12 \times 15.14}{145592} = 2.52 \text{ ksi}$$

$$\Delta f_{pES\_3} = \frac{28500}{4631} \times 2.52 = 15.51 \text{ ksi}$$

Iterations converge to losses presented below;

Parameter	Initial assumptions	Results of the iterative process	
		1st	2nd
Δf_{pES}	15.75 ksi	15.51 ksi	15.51 ksi
f_{cgp}	2.56 ksi	2.52 ksi	2.52 ksi

Elastic shortening losses:

Δf_{pES}= 15.51 ksi

Time-Dependent Losses (Approximate Estimate)

The losses due to long-term creep and shrinkage of concrete, and relaxation of steel, Δf_{PLT}, shall be estimated using the following equation:

$$\Delta f_{PLT} = 10.0 \frac{f_{pi} A_{ps}}{A_b} \gamma_h \gamma_{st} + 12.0 \gamma_h \gamma_{st} + \Delta f_{PR}$$ LRFD Eq. 5.9.5.3-1

where,

f_{pi} = stress in prestressing steel immediately prior to transfer
= 202.50 ksi

A_{ps} = A_{pst} = total area of prestressing strands = 0.217 × 32
= 6.944 in.^{2}

A_b = area of beam= 878.30 in.^{2}

H = relative humidity = 75% (Michigan climate) LRFD Fig.5.4.2.3.3-1

γ_h = correction factor for relative humidity of the ambient air LRFD Eq. 5.9.5.3-2
= 1.7 – 0.01H = 1.7 – 0.01 × 75 = 0.95

γ_{st} = correction factor for specified concrete strength at time of prestress transfer — LRFD Eq. 5.9.5.3-3

$$= \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 6.3} = 0.68$$

Δf_{pR} = relaxation loss = 2.40 ksi — LRFD Art. 5.9.5.3

Therefore,

$$\Delta f_{PLT} = 10.0\frac{202.50 \times 6.944}{878.30} \times 0.95 \times 0.68 + 12.0 \times 0.95 \times 0.68 + 2.40$$

$$\Delta f_{PLT} = 20.49 \text{ ksi}$$

Total losses at midspan

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$$ LRFD Eq. 5.9.5.1-1

$$\Delta f_{pT} = 15.51 + 20.49 = 36 \text{ ksi}$$

Ratio of effective prestress after losses to stress prior to transfer, η

$$= \frac{f_{pi} - \Delta f_{pT}}{f_{pi}} = \frac{202.50 - 36}{202.50} = 0.82$$

The losses calculated with the approximate estimate are 18%, and sufficiently close to the lump sum estimate of 20%. Hence, the stress limit checks will not be repeated.

Detailed estimates of time-dependent losses are included in **Appendix C**.

Step 8.4. Flexural Design Check for Strength Limit State

Step 8.4.1. Ultimate Moment

Ultimate moment for Strength I limit state, M_u — LRFD Table 3.4.1-1 and Table 3.4.1-2

= 1.25(DC) + 1.5(DW) + 1.75(LL + IM)

The ultimate moment at midspan is calculated from moments given in **Table 3**.

$M_u = 1.25(M_g + M_D + M_b) + 1.5(M_{ws}) + 1.75(M_{HL\text{-}M})$

= 1.25(775.40 + 744.03 + 112.23) + 1.5(140.28) + 1.75(1555.65)

= 4972.38 kip-ft

Step 8.4.2. Average Stress in Prestressing Steel

Average stress in prestressing steel when $f_{pe} \geq 0.5f_{pu}$ — LRFD Art. 5.7.3.1.1

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$ LRFD Eq. 5.7.3.1.1-1

where,

f_{pu} = specified tensile strength of prestressing steel = 270 ksi

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$$ LRFD Eq. 5.7.3.1.1-2

= 0.28 for low relaxation strands — LRFD Table C5.7.3.1.1-1

d_p = distance from extreme compressive fiber to centroid of prestressing strands
= $h_c - y_{bs}$ = 47 – 3.06 = 43.94 in.

c = distance between the neutral axis and extreme compressive fiber for rectangular section

$$= \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b_{eff} + kA_{ps}\frac{f_{pu}}{d_p}}$$ LRFD Eq. 5.7.3.1.1-4

A_{ps} = A_{pst} = total area of prestressing steel = 32 × 0.217 = 6.944 in.^{2}
A_s = area of mild steel tension reinforcement = 0 in.^{2}
A'_s = area of mild steel compression reinforcement = 0 in.^{2}
f'_c = compressive strength of deck concrete = 4.0 ksi
f_s = stress in mild steel tension reinforcement at nominal flexural resistance, ksi
f'_s = stress in mild steel compression reinforcement at nominal flexural resistance, ksi
b_{eff} = effective width of compression flange = 82.752 in.
β_1 = stress factor of compression block = 0.85 LRFD Art. 5.7.2.2

Therefore,

$$c = \frac{6.944 \times 270 + 0 - 0}{0.85(4.0)(0.85)(82.752) + 0.28(6.944)\left(\frac{270}{43.94}\right)} = 7.47 \text{ in.}$$

Since c < t_s = 9 in., rectangular section assumption is valid.

When c extends below the deck thickness, t_s, rectangular section assumption will not be valid and a T-section analysis is required.

The beam is tension controlled if $c/d_p \leq 0.375$ LRFD Art. 5.7.2.1.

$$\frac{c}{d_p} = \frac{7.47}{43.94} = 0.17 < 0.375$$ **O.K.**

Depth of the equivalent stress block, a = $\beta_1 c$ = 0.85 × 7.47 = 6.35 in.

The average stress in prestressing steel, f_{ps} $= 270\left(1 - 0.28\frac{7.47}{43.94}\right) = 257.15$ ksi

Step 8.4.3. Nominal Flexural Resistance LRFD Eq. 5.7.3.2.2-1

Nominal flexural resistance, M_n

$$= \left[A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) - A`_s f`_s\left(d`_s - \frac{a}{2}\right)\right]$$

$$= \left[6.944 \times 257.15 \times \left(43.94 - \frac{6.35}{2}\right) + 0 - 0\right]/12$$

$$= 6066.00 \text{ kip} - \text{ft}$$

Deck reinforcement can be included in nominal flexural resistance calculation if amounts are known. In most cases, the deck reinforcement contribution is very small and can be omitted.

Step 8.4.4. Factored Flexural Resistance

Factored flexural resistance, M_r = ϕM_n — LRFD Eq.5.7.3.2.1-1

Resistance factor, ϕ = 1.00 — LRFD Art. 5.5.4.2.1, for a tension-controlled section

M_r (6066.00 kip – ft) > M_u (4972.38 kip – ft)….. **O.K.** — LRFD Eq. 5.7.3.2.1-1

Step 8.4.5. Maximum Reinforcement — LRFD Art. 5.7.3.3.1 and Art. 5.5.4.2.1

The check for maximum reinforcement limits was discontinued by AASHTO in 2005. Adequate ductility is ensured with a tension controlled design of the beam (See **Step 8.4.2**). When beam flexural resistance is not tension-controlled, the resistance factor for the Strength Limits State I is decreased.

Step 8.4.6. Minimum Reinforcement — LRFD Art. 5.7.3.3.2

At any section, the amount of prestressed and nonprestressed tensile reinforcement should be adequate to develop a factored flexural resistance, M_r, equal to the lesser of:

- *1.2 times the cracking strength determined on the basis of elastic stress distribution and the modulus of rupture, and,*
- *1.33 times the factored moment required by the applicable strength load combination.*

Check at midspan:

$$M_{cr} = \gamma_3\left[(\gamma_1 f_r + \gamma_2 f_{cpe})S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$$

LRFD Eq. 5.7.3.3.2-1

where,

γ_1 = flexural cracking variability factor =1.6 — LRFD Art. 5.7.3.3.2

γ_2 = prestress variability factor =1.1, for bonded tendons — LRFD Art. 5.7.3.3.2,

γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement = 1.0, for prestressed concrete structures. — LRFD Art. 5.7.3.3.2,

f_r = concrete modulus of rupture $= 0.24\sqrt{f'_c}$ — LRFD Art. 5.4.2.6

$= 0.24\sqrt{7.5} = 0.657$ ksi

f_{cpe} = compressive stress in concrete due to effective prestress (after allowance for all losses) at the extreme fiber of the section where tensile stress is generated by externally applied loads

$$= \frac{\eta F_i}{A_b} + \frac{\eta F_i e_{pg}}{S_b} = \frac{0.8\times1406}{878.30} + \frac{0.8\times1406\times15.14}{8000} = 3.41 \text{ ksi}$$

M_{dnc} = total unfactored dead load moment acting on the monolithic or noncomposite section

$= M_g + M_D$ = (775.40 kip-ft + 744.03 kip-ft) × 12 in./ft = 18,233.16 kip-in.

S_c = S_{bc} = section modulus for the extreme fiber of the composite section where tensile stress is generated by externally applied loads

= 12,855 in.^{3}

S_{nc} = S_b = section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is generated by externally applied loads

= 8,000 in.^{3}

$$M_{cr} = 1.0 \times \left[(1.6 \times 0.657 + 1.1 \times 3.41) \times 12855 - 18233.16 \times \left(\frac{12855}{8000} - 1\right)\right]/12$$
$$= 4222.25 \text{ kip} - \text{ft}$$

$1.2M_{cr}$ = 5066.70 kip-ft LRFD Art. 5.7.3.3.2

$1.33M_u$ = 6613.27 kip-ft

Min ($1.2M_{cr}$; $1.33M_u$) =$1.2M_{cr}$

M_r (= 6066.00 kip-ft) ≥ $1.2M_{cr}$ (=5066.70 kip-ft) **O.K.**

Step 8.5. Lifting Stress Check

*MDOT practice is to check stresses at draping point and over the supports/lifting points due to the cantilever moment. The check is preformed by assuming support/lifting point locations at 3.0 ft from the beam end (**Figure 16**). If the stress conditions are not satisfied, the supports are moved towards the beam end. MDOT procedure does not consider the dynamic effects, and a dead load multiplier of 1 is used.*

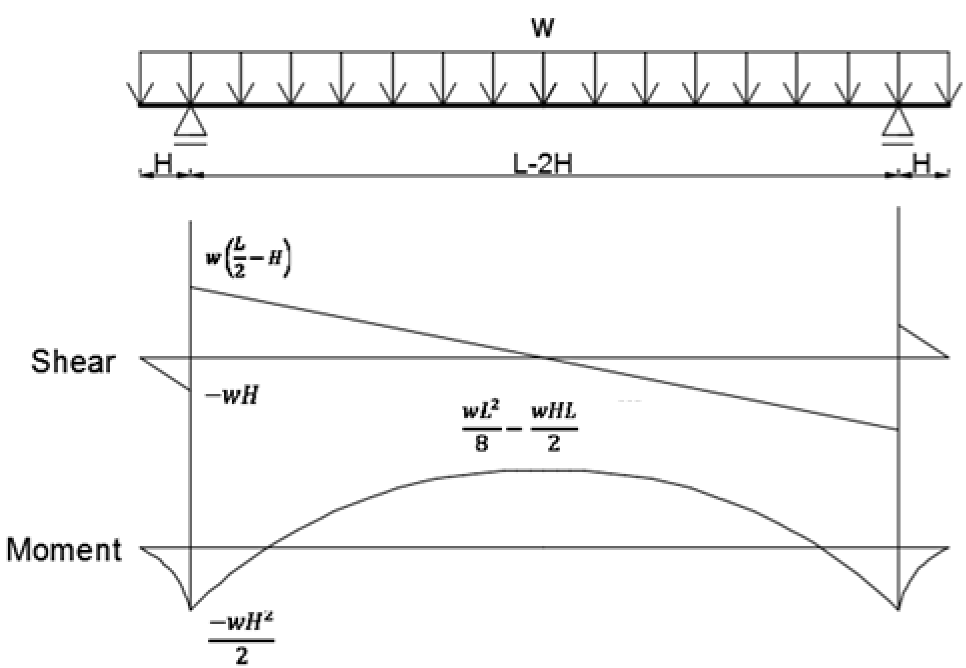


Figure 16. Moment and shear diagrams for lifting stress check

Moment at a distance of x from a support $= \frac{wLx}{2} - \frac{w}{2}(H + x)^2$

Distance between temporary support or lifting points and beam end, H = 3 ft

Stresses over Temporary Support

Moment at temporary support/lifting, M= $\frac{-wH^2}{2}$ where w = w_g

$$M = \frac{-w_g H^2}{2} = \frac{-0.884 \times 3^2}{2}$$
$$= -3.98\ \text{kip} - \text{ft}$$

Allowable concrete tensile stress, $\bar{f}_{tt}$ $= 0.24\sqrt{f'_{ct}}$

Allowable concrete compressive stress, $\bar{f}_{ct}$ = 0.6 f_{ct}'

where,

f_{ct}' = concrete compressive strength for lifting stress check

$$= \frac{f`_c + f`_{ci}}{2} = \frac{7.5 + 6.3}{2} = 6.9\ \text{ksi}$$

Hence,

$\bar{f}_{tt} = 0.24\sqrt{6.9}$ = 0.630 ksi

$\bar{f}_{ct}$ = 0.60 × 6.9 = 4.140 ksi

Prestressing strand eccentricity (e_{pg}) at 3 ft = 11.57 in.

The tensile stresses are calculated and compared to the allowable stresses at top:

$$\frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M}{S_t} \geq (-f_{tt})$$

$$\frac{1406}{878.30} - \frac{1406 \times 11.57}{8179} + \frac{-3.98 \times 12}{8179} = -0.394\ \text{ksi} > -0.630\ \text{ksi} \qquad \textbf{O.K.}$$

The compressive stresses are calculated and compared to the allowable stresses at bottom:

$$\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M}{S_b} \leq (f_{ct})$$

$$\frac{1406}{878.30} + \frac{1406 \times 11.57}{8000} - \frac{-3.98 \times 12}{8000} = 3.640\ \text{ksi} < 4.140\ \text{ksi} \qquad \textbf{O.K.}$$

Stresses at Draping Point

According to **Figure 14** and **Figure 16** the distance between temporary support/lifting point and draping point, L_{dra} = 34.1 – 3.0 = 31.1 ft

Moment at draping point, M

$$= w_g \frac{L}{2} L_{dra} - \frac{w_g}{2}(H + L_{dra})^2$$

where,

L = L_b

$$M = 0.884 \times \frac{85.25}{2} \times 31.1 - \frac{0.884}{2} \times (3 + 31.1)^2$$
$$= 657.90\ \text{kip} - \text{ft}$$

According to **Figure 13**

Prestressing strand eccentricity (e_{pg}) at draping point (i.e., at 34.1 ft) = 15.14 in.

The tensile stresses are calculated and compared to the allowable stresses at top:

$$\frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M}{S_t} \geq (-f_{tt})$$

$$\frac{1406}{878.30} - \frac{1406 \times 15.14}{8179} + \frac{657.90 \times 12}{8179} = -0.037 \text{ ksi} > -0.630 \text{ ksi} \qquad \textbf{O.K.}$$

The compressive stresses are calculated and compared to the allowable stresses at bottom:

$$\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M}{S_b} \leq (f_{ct})$$

$$\frac{1406}{878.30} + \frac{1406 \times 15.14}{8000} - \frac{657.90 \times 12}{8000} = 3.275 \text{ ksi} < 4.140 \text{ ksi} \qquad \textbf{O.K.}$$

Some highway agencies require shear checks at the support and lifting points. However, as per MDOT practice, shear check is omitted.

Step 8.6. Continuity Connection Check

Step 8.6.1. Negative Moment Connection at Strength Limit State

The scope of this example is limited to prestressed concrete beam design. Hence, the continuity connection check calculations are not included. The steps below can be followed to complete the continuity connection check:

a) Calculate the nominal flexural resistance (M_n) using reinforcement amount and spacing in the cast-in-place deck slab over pier. — LRFD Eq. 5.7.3.2.2-1

b) Select the resistance factor, ϕ — LRFD Art. 5.5.4.2.1

c) Calculate the factored flexural resistance ($M_r = \phi M_n$) — LRFD Eq. 5.7.3.2.1-1

d) Calculate the ultimate moment over the pier (M_u) using Strength I limit state.

e) Check if $M_u < M_r$ — LRFD Eq. 5.7.3.2.1-1

Step 8.6.2. Service State – Compressive Strength Check at Negative Moment Region

In the negative moment region, girder bottom flange is subjected to additional compressive stress proportional to the negative moment at the section. The critical section for compression check is established as the section in the negative moment region where compression due to prestressing is the greatest. Hence, the critical section is at a distance equal to the transfer length measured from the beam end. Analysis is performed by assuming elastic cracked section to check if compressive strength on beam bottom flange exceeds $0.6f`_c$.

Step 8.6.3. Crack Control LRFD Art. 5.7.3.4

Tension reinforcement needs to be distributed to control flexural cracking. LRFD Art. C5.7.3.4

(a) Calculate the overall thickness of the component, h.

(b) Calculate the thickness of concrete cover measured from extreme tension fiber to center of the flexural reinforcement located closest thereto, d_c.

(c) Calculate $\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$

(d) Determine the exposure factor, γ_e

(e) Calculate the tensile stress in the mild steel reinforcement at the service limit state, f_{ss}

(f) Check if $f_{ss} \leq 0.6f_y$. Otherwise, change the amount of steel to satisfy the condition.

(g) Maintain the spacing, s, of mild steel reinforcement in the layer closest to the tension face such that $s \leq \frac{700\,\gamma_c}{\beta_s f_{ss}} - 2d_c$ LRFD Eq. 5.7.3.4-1

Step 8.6.4. Positive Moment Connection at Strength Limit State

A positive moment may develop at intermediate piers under the effect of prestressing, permanent loads, and creep and shrinkage (if creep and shrinkage is considered in the design). Reinforcement is provided at beam bottom flange at intermediate piers to resist the factored positive moment (M_u) at negative moment locations.

NCHRP (2004)

FHWA (2003)

MDOT currently does not perform this calculation.

Step 8.7. Shear Design

Transverse shear reinforcement is required when $V_u > 0.5\phi(V_c + V_p)$ LRFD Eq. 5.8.2.4-1

where,

V_u = total factored shear force

V_c = shear strength provided by concrete

V_p = component of the effective prestressing force in the direction of the applied shear

ϕ = resistance factor for shear = 0.9 LRFD Art. 5.5.4.2.1

Transverse shear is evaluated to identify the length of beam requiring reinforcement. Even if $V_u \leq 0.5\phi(V_c + V_p)$, shear reinforcement is provided in beams at the maximum spacing.

In this example, transverse shear design procedure is demonstrated for the critical section near the support.

Step 8.7.1. Critical Section for Shear

The critical section near the support is taken as the effective shear depth, d_v, from the internal face of the support. LRFD Art. 5.8.3.2

d_v = effective shear depth (i.e., distance between resultants of tensile and compressive forces) LRFD Art. 5.8.2.9

= $(d_e – a/2) \geq$ greater of $0.9d_e$ or $0.72h_c$

d_e = effective depth from extreme compression fiber to centroid of the tensile force in the tensile reinforcement

a = depth of compression block

h_c = overall depth = 47 in.

For flexural members, the distance between the resultants of the tensile and compressive forces due to flexure can be determined from: LRFD Eq. C5.8.2.9-1

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}}$$

where, M_n = 6066.00 kip-ft = 72,792 kip-in.

$A_{ps} = A_{pst}$ = 6.944 in.^{2}

f_{ps} = 257.15 ksi (From **Step 8.4.2)**

A_s = 0 in^2.

Therefore, d_v = 40.77 in.

Check if $d_v \geq$ Max $(0.9d_e, 0.72h_c)$ LRFD Art. 5.8.2.9

Since only prestressing strands are present,

$d_e = d_p$ = 43.94 in. **(Step 8.4.2)**, and

$d_v \geq$ Max (39.55 in., 33.84 in.) **O.K.**

Bearing width is not yet established, and conservatively, the distance to the critical section for shear can be measured from the bearing centerline.

Distance to critical section for shear from bearing centerline, x_{cr}

= d_v = 40.77 in. = $0.04L_{ds}$

Step 8.7.2. Factored Moment and Shear at Critical Location

In designing continuous for live load (CLL) bridges, the larger value for shear represents the required strength calculated for both simply supported and continuous spans.

As shown in **Table 4**, shear in continuous span, V_{HL-M}, governs. Hence, the design is performed accordingly. The following tables show moment and shear at the critical seciton for shear, $0.04L_{ds}$.

Location	Moment (kip – ft)					
	Simple span		Continuous span			
	M_g	M_D	M_s	M_b	M_{ws}	M_{HL-M}
0.04 L_{ds}	113.18	108.60	-76.95	-93.40	-116.70	-1078.45

Location	Shear (kips)					
	Simple span		Continuous span			
	V_g	V_D	V_s	V_b	V_{ws}	V_{HL-M}
0.04 L_{ds}	-34.23	-32.84	-0.94	-6.4	-7.9	-97.8

The load factor for settlement, γ_{SE}, should be considered on a project-specific basis. In lieu of project specific information to the contrary, γ_{SE}, may be taken as 1.0. Load combinations which include settlement shall also be applied without settlement. LRFD Art. 3.4.1

Without project specific requirements, shear design is performed with γ_{SE} = 1.0

Factored moment at critical section (0.04L_{ds}), M_u

$= 1.25(M_g + M_D + M_b) + 1.5(M_{ws}) + 1.75(M_{HL-M}) + 1.0(M_S)$ LRFD Table 3.4.1.1 (Strength I)

$= 1.25(113.18 + 108.60 - 93.40) + 1.5(-116.70) + 1.75(-1078.45) + 1.0(-76.95)$

= -1978.81 kip-ft

Factored shear at critical section (0.04L_{ds}), V_u

$= 1.25\ (V_g + V_D + V_b) + 1.5\ (V_{ws}) + 1.75\ (V_{HL-M}) + 1.0\ (V_S)$ LRFD Table 3.4.1.1 (Strength I)

$= 1.25\ (-34.23 - 32.84 - 6.4) + 1.5\ (-7.9) + 1.75\ (-97.8) + 1.0\ (-0.94)$

= -275.78 kip

Step 8.7.3. Shear Strength Provided by Concrete

Concrete contribution to nominal shear resistance, V_c LRFD Eq. 5.8.3.3-3

$$= 0.0316\beta\sqrt{f'_c}b_v d_v$$

where,

β = factor indicating the ability of diagonally cracked concrete to transmit tension and shear

b_v = effective web width taken as the minimum web width within the depth d_v

The following calculations are performed for evaluating β.

Calculation of Strain in Flexural Reinforcement

Strain in flexural reinforcement, ε_s

$$= \frac{\left|\frac{M_u}{d_v}\right| + 0.5N_u + \left|V_u - V_p\right| - A_{ps}f_{po}}{(E_sA_s + E_pA_{ps})}$$ LRFD Eq. 5.8.3.4.2-4

where,

N_u = applied factored normal force at critical section = 0 kip

V_p = component of the effective prestressing force in the direction of the applied shear,

= (Force per strand)(Number of draped strands)(sin ψ)

Ψ = drape angle = 3.50°

$$V_p = 35.154 \times 5 \times \sin(3.50°) = 10.73 \text{ kip}$$

A_{ps} = area of prestressing strands on the flexural tension side of the member

= 27 × 0.217 = 5.86 in.^{2}

f_{po} = parameter taken as modulus of elasticity of prestressing strands multiplied by the locked-in difference in strain between the prestressing strands and the surrounding concrete

For prestressed members, f_{po} can be assumed $0.7f_{pu}$ — LRFD Art. 5.8.3.4.2

= 0.7 × 270 = 189 ksi

Within the transfer length, f_{po} shall be increased linearly from zero at the location where the bond between the strands and concrete commences to its full value at the end of the transfer length. — LRFD Art. 5.8.3.4.2

In this example, d_v > transfer length of 36 in. (i.e., 60 × Strand dia.)

$|M_u| > |V_u - V_p|\, d_v$

$|M_u|$ = 1978.81 × 12 = 23,737.44 kip-in

$||V_u - V_p||\, d_v$ = |275.78 – 10.73| × 40.77 = 10806.09 kip-in — LRFD Art. 5.8.3.4.2

$|M_u| > |V_u - V_p|\, d_v$ → **O.K.**

$$\varepsilon_s = \frac{\frac{|-1978.81 \times 12|}{40.77} + 0 + |275.78 - 10.73| - 5.86 \times 189}{(0 + 28500 \times 5.86)} = -0.0016 < 0$$

If ε_s calculated from Eq. 5.8.3.4.2-4 is negative, it can be taken as zero or recalculated with the denominator of Eq. 5.8.3.4.2-4 replaced by ($E_sA_s + E_pA_{ps} + E_cA_c$). However, ε_s should not be taken as less than -0.40×10^{-3}. — LRFD Art.5.8.3.4.2

where, — LRFD Fig. 5.8.3.4.2.2

A_c = area of concrete on the flexural side of the member (**Figure 17**)

= 493.44 in.^{2}

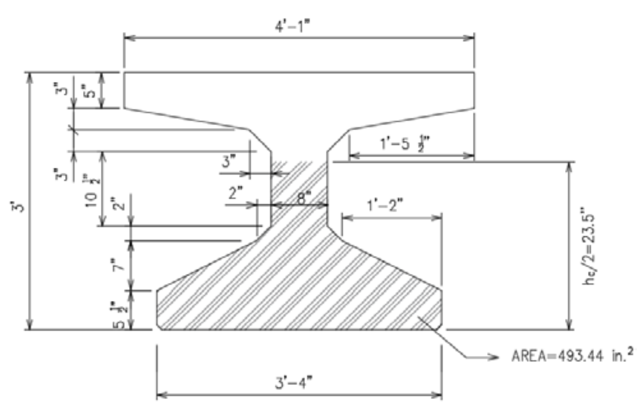


Figure 17. Area of the concrete on the flexural tension side

The flexural tension side of the member shall be taken as the half-depth containing the flexural tension zone. LRFD Fig. 5.8.3.4.2-1

$$\varepsilon_s = \frac{\frac{|-1978.81 \times 12|}{40.77} + 0 + |275.78 - 10.73| - 5.86 \times 189}{(4906 \times 493.44 + 28500 \times 5.86)}$$

$$= -0.0001 > -0.00040 \quad \textbf{O.K.}$$

β and θ Calculation

Assume that the section contains at least the minimum transverse reinforcement.

β = factor indicating ability of diagonally cracked concrete to transmit tension and shear

$= \frac{4.8}{1+750\,\varepsilon_s}$ LRFD Eq.5.8.3.4.2-1

$= \frac{4.8}{1+750\times(-1\times10^{-4})} = 5.19$

θ = angle of inclination of diagonal compressive stress

$= 29 + 3500 \times \varepsilon_s = 28.65°$ LRFD Eq. 5.8.3.4.2-3

MDOT procedure is to iterate for β and θ since the AASHTO LRFD 7th edition (2016) equations are not yet implemented in the BDS.

Shear is carried by the web.

Thickness of the web, t_w = 8 in.

Shear strength provided by concrete, V_c

$= 0.0316\beta\sqrt{f'_c}t_w d_v$ LRFD Eq. 5.8.3.3-3

$= 0.0316 \times 5.19 \times \sqrt{7.5} \times 8 \times 40.77 = 146.49$ kip

Step 8.7.4. Shear Reinforcement Requirements

Check if the following condition is satisfied; LRFD Eq. 5.8.2.4-1

$V_u > 0.5\phi\,(V_c + V_p)$

ϕ = 0.9 for normal weight concrete LRFD Art. 5.5.4.2.1

275.78 kip > 0.5 × 0.9 × (146.49 + 10.73) = 70.75 kip **O.K.**

Therefore, the transverse shear reinforcement is required.

Area of Shear Reinforcement

$$\frac{V_u}{\phi} \le V_n = V_c + V_s + V_p$$ LRFD Eq. 5.8.3.3-1

Strength required from shear reinforcement,

$$V_s = \frac{V_u}{\phi} - V_c - V_p = \frac{275.78}{0.9} - 146.49 - 10.73 = 149.20 \text{ kip}$$

Shear strength provided by reinforcement; LRFD Eq. 5.8.3.3-4

$$V_s = \frac{A_v f_{yh} d_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$$

where,

A_v = area of shear reinforcement within a distance of s

s = spacing of shear reinforcements

f_{yh} = specified yield strength of shear reinforcement = 60 ksi

α = angle of inclination of shear reinforcement to longitudinal axis

= 90° (i.e., vertical shear reinforcement)

Area of shear reinforcement for a spacing s

$$A_v = \frac{V_s s}{f_{yh} d_v \cot\theta} = \frac{149.20 \times s}{60 \times 40.77 \times \cot 28.65°} = 0.033(s)\ \text{in.}^2$$

Therefore, s = A_v /0.033

Select, #4-2 leg stirrups. Thus, A_v = 0.4 in.^{2}

Spacing, s = (0.4)/(0.033) = 12.12 in.

Spacing of Shear Reinforcement

Maximum allowable spacing of shear reinforcement shall be checked. LRFD Art. 5.8.2.7

Check if the concrete shear stress limitation is satisfied; LRFD Eq. 5.8.2.7-1

$v_u < 0.125f'_c$

$$v_u = \frac{|V_u - \phi V_p|}{\phi t_w d_v} = \frac{|275.78 - 0.9 \times 10.73|}{0.9 \times 8 \times 40.77} = 0.907\ \text{ksi}$$ LRFD Eq. 5.8.2.9-1

$0.125f'_c = 0.125 \times 7.5 = 0.94$ ksi LRFD Eq. 5.8.2.7-1

0.907 ksi < 0.94 ksi

Then, LRFD Eq. 5.8.2.7-2

$s_{max} \leq$ min (0.8d_v, 24 in.)

$s_{max} \leq$ min (32.7 in., 24 in.) = 24 in.

Use #4 – 2 leg stirrups at 9 in. spacing

Shear resistance provided by shear reinforcement, V_s LRFD Eq.5.8.3.3-4

$$= \frac{A_v f_{yh} d_v \cot\theta}{s} = \frac{0.40 \times 60 \times 40.77 \times \cot 28.65°}{9} = 198.99\ \text{kip}$$

A larger spacing of shear reinforcement could have been specified. However, minimum interface shear reinforcement requirements will necessitate additional steel. (See **Step 8.8**)

Check the adequacy of the section with shear reinforcement;

$$\frac{V_u}{\phi} \leq V_n = V_c + V_s + V_p$$ LRFD Eq. 5.8.3.3-1

where, ϕ = 0.9 for normal weight concrete LRFD Art. 5.5.4.2.1

$$V_n = 146.49 + 198.99 + 10.73 = 356.21\ \text{kip}$$

$$\frac{V_u}{\phi} = \frac{275.78}{0.9} = 306.42\ \text{kip} \leq V_n = 356.21\ \text{kip}$$ **O.K.**

Minimum Reinforcement Requirement

The area of shear reinforcement should be more than

$0.0316\sqrt{f'_c}\frac{t_w s}{f_{yh}} = 0.0316\sqrt{7.5}\frac{(8)(9)}{60} = 0.104 \text{ in.}^2 < A_v$ provided — LRFD Eq. 5.8.2.5-1

O.K.

Step 8.7.5. Maximum Nominal Shear Resistance

In order to ensure that shear reinforcement yields before web crushing, an upper limit of V_n is defined.

Since $V_n = 0.25f'_c t_w d_v + V_p$ and $V_n = V_c + V_s + V_p$ — LRFD Eq. 5.8.3.3-2, LRFD Eq. 5.8.3.3-1

$V_c + V_s \leq 0.25f'_c t_w d_v$

$V_c + V_s = 146.49 + 198.99 = 345.48$ kip

$0.25f'_c t_w d_v = 0.25 \times 7.5 \times 8 \times 40.77 = 611.55$ kip

$V_c + V_s \leq 0.25f'_c t_w d_v$ **O.K.**

The procedure shown in **Step 8.7** is repeated at the end of **Step 8.8** to calculate the changing stirrup spacing along the beam.

Step 8.8. Interface Shear Transfer

Factored Horizontal Shear — LRFD Art. 5.8.4

$$V_{hi} = \frac{V_u}{d_v}$$ LRFD Eq. C5.8.4.2-7

where,

V_{hi} = horizontal factored shear force per unit length of the beam at the strength limit state

V_u = factored shear force at a specified section due to superimposed loads

d_v = distance between tensile and compressive force resultants
= 40.77 in.

The critical section location is the same location where the critical section for vertical shear is defined (i.e., $0.04L_{ds}$).

Factored shear at critical section ($0.04L_{ds}$), V_u

$= 1.25\ (V_g + V_D + V_b) + 1.5\ (V_{ws}) + 1.75\ (V_{HL\text{-}M}) + 1.0\ (V_S)$ — LRFD Table 3.4.1.1 (Strength I)

$= 1.25\ (-34.23 - 32.84 - 6.4)$

$+ 1.5\ (-7.9) + 1.75\ (-97.8) + 1.0\ (-0.94) = -275.78$ kip

Therefore, the applied horizontal shear, $V_{hi} = \frac{V_u}{d_v} = \frac{275.78}{40.77} = 6.76\frac{\text{kip}}{\text{in}}$ — LRFD Eq. C5.8.4.2-7

Required Nominal Resistance

$$V_{ni} = \frac{V_{hi}}{\phi} = \frac{6.76}{0.9} = 7.51\frac{\text{kip}}{\text{in.}}$$ LRFD Eq. 5.8.4.1-1

Required Interface Shear Reinforcement

The nominal shear resistance at the interface, V_{ni}

$= cA_{cv} + \mu[A_{vf} f_{yh} + P_c]$ LRFD Eq. 5.8.4.1-3

where,

c = cohesion factor

= 0.28 (for an intentionally roughened surface) LRFD Art.5.8.4.3

μ = coefficient of friction = 1.0 LRFD Art. 5.8.4.3

b_{tf} = top flange width = 49 in.

A_{cv} = area of concrete section resisting interface shear

= $b_{tf} \times 1$ in.= 49 in.^{2} (for 1 in. longer section along the span)

A_{vf} = area of shear reinforcement crossing the shear plane

P_c = permanent net compressive force normal to the shear plane

It is conservative to neglect P_c. Hence, P_c = 0 kip LRFD C5.8.4.1

Solving for A_{vf}

$V_{ni} = cA_{cv} + \mu[A_{vf} f_{yh} + P_c]$ LRFD Eq. 5.8.4.1-3

$7.51 = 0.28 \times 49 + 1.0 (A_{vf} \times 60 + 0) \rightarrow A_{vf} = -0.104 \text{ in.}^2 < 0$

Hence, the resistance provided by cohesion is greater than the stress developed under the applied force, and the minimum required interface reinforcement is provided.

Minimum Interface Shear Reinforcement

Minimum $A_{vf} \geq (0.05A_{cv})/f_{yh}$ LRFD Eq. 5.8.4.4-1

Vertical shear reinforcement of #4 – 2 leg stirrup (i.e., A_v = 0.4 in.^{2}) at 9-in. spacing is provided from the beam extending into the deck.

Therefore, $A_{vf} = A_v \times 12/s$ = 0.533 in.^{2}/ft

$0.05 A_{cv}/f_{yh} = (0.05\times b_{tf}\cdot\times 1) /f_{yh} = 0.05(49 \times 1)/60 = 0.041 \text{ in.}^2/\text{in.} = 0.492 \text{ in.}^2/\text{ft}$

$A_{vf} > 0.05 A_{cv}/f_{yh}$ **O.K**.

Maximum Nominal Shear Resistance

$V_{ni} \leq \min (K_1 f'_c A_{cv}; K_2 A_{cv})$ LRFD Eq.5.8.4.1-4 and Eq. 5.8.4.1-5

where,

K_1 = fraction of concrete strength available to resist interface shear

= 0.3 (for a cast-in-place concrete slab) LRFD 5.8.4.3

K_2 = limiting interface shear resistance LRFD 5.8.4.3

=1.8 ksi (for a cast-in-place concrete slab with normal weight concrete)

$K_1 f'_c A_{cv} = 0.3 \times 4 \times 49 = 58.80$ kip/in.

K_2A_{cv} = 1.8 × 49 = 88.20 kip/in.

$V_{ni,\ provided}$ = $cA_{cv} + \mu[(A_{vf}/12)f_{yh} + P_c]$; where, A_{vf} = 0.533 in.^{2}/ft   LRFD Eq. 5.8.4.1-3

= 0.28 × 49 +1.0 [(0.533/12) × 60 + 0) =16.39 kip/in.

16.39 kip/in. < Min (58.80 kip/in.; 88.20 kip/in.)   **O.K.**

The shear reinforcement spacing at $0.1L_{ds}$ along the half beam length are given in **Table 14**. Standard shear details are presented in **Appendix F**.

Table 14. The Shear Reinforcement Spacing along the Beam

Location	V_u/ϕ (kip)	β	V_c (kip)	V_p (kip)	V_s (kip)	s (in.)	s_{max} (in.)	$s_{provided}$ (in.)
Critical section for shear	-306.42	5.19	146.49	10.73	149.20	12.12	24	9
0.10 × L_{ds}	-277.59	5.82	152.72	10.73	114.14	14.88	24	9
0.20 × L_{ds}	-235.27	6.22	167.35	10.73	57.19	30.76	24	24
0.30 × L_{ds}	-190.71	5.81	160.21	10.73	19.77	90.22	24	24
0.40 × L_{ds}	-143.70	5.58	157.50	0.00	0.00	NR*	24	24
Midspan	-94.46	5.50	155.20	0.00	0.00	NR*	24	24

* NR: Not required

Since shear reinforcement placement is symmetrical about the midspan, continuous for live load moment and shear are assumed equal at both ends of the beam.

s_{max} is calculated using LRFD Eq. 5.8.2.7-1 and LRFD Eq. 5.8.2.7-2.

Step 8.9. Minimum Longitudinal Reinforcement Requirement

Longitudinal reinforcement should be proportioned such that the following condition is satified at each section:

$$A_{ps}f_{ps} + A_sf_y \geq \frac{M_u}{d_v\phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$ LRFD Eq. 5.8.3.5-1

where,

A_s = area of nonprestressed tension reinforcement

f_y = specified minimum yield strength of reinforcing steel

A_{ps} = area of prestressing steel at the tension side of the section

f_{ps} = average stress in prestressing steel at the location for which the nominal resistance is required

M_u = factored moment at the section corresponding to the factored shear force

N_u = applied factored axial force

V_u = factored shear force at section

V_s = shear resistance provided by shear reinforcement ($V_s < V_u/\phi$)

V_p = component of the effective prestressing force in the direction of the applied shear

d_v = effective shear depth

ϕ = resistance factor as appropriate for moment, shear, and axial resistance

θ = angle of inclination of diagonal compressive stresses

Required Reinforcement at Face of Bearing

For simple supports with M_u=0 kip-ft and N_u=0 kip, the longitudinal reinforcement on the flexural tension side of the beam at the inside edge of the bearing shall satisfy the following:

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\phi} - 0.5V_s - V_p\right)\cot\theta$$ LRFD Eq. 5.8.3.5-2

When the bearing size is not yet established, the above equation can be applied at the bearing centerline for conservative results. Consequently, the assumed failure crack for this analysis radiates from the centerline of the bearing (i.e., 9 in. from the end of the beam).

Factored shear force at the bearing centerline, V_u = -278.09 kip — LRFD Table 3.4.1.1 (Strength I)

Shear resistance, V_s, provided by shear reinforcement with 9 in. stirrup spacing is calculated:

$$V_s = \frac{A_v f_{yh} d_v \cot\theta}{s} = \frac{0.40 \times 60 \times 40.77 \times \cot 28.65}{9} = 198.99 \text{ kip}$$

where

$\theta = 28.65^{o}$ at the centerline of the bearing

In determining the tensile force that the reinforcement is expected to resist at the inside edge of the bearing area, the values of V_u, V_s, V_p, and θ, calculated for the section d_v from the face of the support may be used. — LRFD C5.8.3.5

$$\left(\frac{V_u}{\phi} - 0.5V_s - V_p\right)\cot\theta = \left(\frac{278.09}{0.9} - 0.5 \times 198.99 - 10.73\right)\cot 28.65° = 363.80 \text{ kip}$$

The assumed crack plane crosses the centroid of the group of 27 straight strands at a distance of 14 in. (i.e., 9 + 2.74 in. × cot 28.65°) from the beam end.

The transfer length = 36 in. from the beam end (60 × strand diameter). — LRFD 5.11.4.1

Five (5) draped strands do not contribute to the tensile capacity since they are not located within the flexural tension zone of the beam.

The available prestress (f_{ps}) from 27 straight strands at 14 in. is a fraction of the effective prestress, f_{pe}, of 162 ksi (i.e., 0.8 × 0.75f_{pu}).

$$A_{ps} f_{ps} + A_s f_y = 27 \times 0.217 \times 162 \times \left(\frac{14}{36}\right) + 0$$
$$= 369.12 \text{ kip} > 363.80 \text{ kip} \quad \textbf{O.K.}$$ LRFD Eq.5.8.3.5-2

If the bearing width is 9 in., the failure crack will extend from the edge of the bearing, and the assumed crack plane crosses the centroid of the group of 27 straight strands at a distance of 18.37 in. (i.e., 9 + 9/2 + 2.74 in. × cot 28.65°) from the beam end.

$$A_{ps} f_{ps} + A_s f_y = 27 \times 0.217 \times 162 \times \left(\frac{18.37}{36}\right) + 0$$ LRFD Eq.5.8.3.5-2
$$= 484.33 \text{ kip} > 363.80 \text{ kip} \quad \textbf{O.K.}$$

Step 8.10. Anchorage Zone Reinforcement LRFD Art. 5.10.10.1

Design of the anchorage zone reinforcement is based on the force in the strands just prior to transfer.

P_{pi} = force in the strands prior to transfer

= 32 × 0.217 × 202.50 = 1,406 kip

The bursting resistance, $P_r = f_sA_s$ LRFD Eq. 5.10.10.1 - 1

where,

A_s = total area of vertical reinforcement located within a distance of h/4, from the end of the beam

f_s = stress in steel ≤ 20 ksi

The bursting resistance, $P_r \geq 0.04P_{pi}$ LRFD Art. 5.10.10.1

$$P_r \geq 0.04\ (1{,}406) = 56.25 \text{ kip}$$

Solving for the required area of steel, $A_{s,req} = P_r/f_s$

$= 56.25/20 = 2.81 \text{ in.}^2$

At least 2.81 in.^{2} of vertical transverse reinforcement is required within a distance of

h/4 = 36/4 = 9 in.

Starting at 3 in. from the beam end, use three #4 – 2 leg stirrups at 3 in. spacing. BDG 7.11.01

$$A_s = 3 \times 2 \times 0.20 \text{ in.}^2 = 1.2 \text{ in.}^2$$

$A_s < A_{s,req}$ **NOT O.K.**

In most cases, it is difficult to satisfy the anchorage zone requirement. Therefore, until more clarity is brought to the anchorage zone design (such as the use of strut and tie models), designer should be aware that the time of prestress release is most critical. Specifically, end zone reinforcement that is less than the required steel area is consistently used in actual production without objectionable cracking at the member end. PCI (2011) Section 8.3.1.6.

Confinement Reinforcement

Reinforcement is provided to confine the prestressing steel in the bottom flange for a distance of 1.5d from the beam end. The reinforcement may not be less than #3 deformed bars with spacing not exceeding 6 in. The reinforcement should be of a shape that will confine the strands. LRFD Art. 5.10.10.2

For the bulb-tee, this confining steel shall be provided for a distance of

1.5h = 1.5 × 36 = 54 in.

Step 8.11. Deflection and Camber

Please note that camber sign convention is positive when deflections are negative.

Long-term deflection may be taken as the instantaneous deflection multiplied by a factor 4.0, if the instantaneous deflection is based on gross moment of inertia of the beam. LRFD Art. 5.7.3.6.2

However, a factor 4.0 is not appropriate for this type of precast construction. It is recommended that the designer follow the guidelines of the owner agency for which the bridge is being designed or follow a rigorous, time-dependent analysis. PCI (2011) Ch. 9.4

This example demonstrates MDOT deflection calculation practices.

Step 8.11.1. Deflection due to Static Loads

Deflection is calculated using the modulus of elasticity of concrete and the gross cross-section properties of the noncomposite precast beam.

Camber due to prestressing force at transfer, Δ_p

$$\Delta_p = \frac{F_i(0.098e_{pgm} + 0.027e_{pge})L_b^2}{E_{ci}I_b}$$

Libby (1977) Chapter 6-3

where,

F_i = total prestressing force after transfer = 1,406 kip

e_{pgm} = eccentricity of prestresssing force at midspan = 15.14 in.

e_{pge} = eccentricity of prestresssing force at beam end = 11.23 in.

L_b = beam length at transfer = 85.25 ft

L_{ds} = beam length at erection = 83.75 ft

E_{ci} = modulus of elasticity of beam at transfer = 4,631 ksi

E_{cb} = modulus of elasticity of beam in service = 4,906 ksi

I_b = moment of inertia of the beam = 145,592 in.^{4}

$$\Delta_p = \frac{1406 \times (0.098 \times 15.14 + 0.027 \times 11.23) \times (85.25 \times 12)^2}{4631 \times 145592}$$
$$= 3.90 \text{ in.}\uparrow$$

MDOT (2002)

Deflection due to beam weight at transfer, Δ_b

$$\Delta_b = \frac{0.104 M_{gr} L_b^2}{E_{ci}I_b}$$

MDOT (2002)

where, M_{gr} = moment due to beam weight = 803.43 kip-ft
= 9641.16 kip-in

$$\Delta_b = \frac{0.104 \times 9641.16 \times (85.25 \times 12)^2}{4631 \times 145592} = 1.56 \text{ in.}\downarrow$$

Resultant camber at transfer, $\Delta_{int} = \Delta_p + (-\Delta_b)$
$= 3.90 - 1.56 = 2.34$ in. ↑

Long term camber due to prestress, Δ_{Lp}

$$\Delta_{Lp} = \Delta_p[1.9 + 0.6(I_b/I_c)]$$

where, I_c = moment of inertia of composite section

$$\Delta_{Lp} = 3.90\ [1.9 + 0.6 \times (145{,}592/367{,}259)]$$
$= 8.34$ in. ↑

Long term camber due to beam weight, Δ_{Lb}

$$\Delta_{Lb} = \Delta_b\ [2.1 + 0.7(I_b/I_c)]$$
$= 1.56\ [2.1 + 0.7 \times (145{,}592/367{,}259)]$
$= 3.71$ in. ↓

Net ultimate beam camber $= \Delta_{Lp} + (-\Delta_{Lb})$
$= 8.34 – 3.71 = 4.63$ in. ↑

Long term beam deflection due to slab and haunch weight (ult), Δ_s

$$\Delta_s = \frac{5wL_b^4}{384E_{cb}I_b}\left[1 + \alpha_s C_u \frac{I_b}{I_c}\right]$$

where, $\alpha_s = 0.60$
$C_u = 1.8$
$E_{cb} = 4{,}906$ ksi
w = slab and haunch weight = 0.878 kip/ft

$$\Delta_s = \frac{5 \times \frac{0.878}{12} \times (85.25 \times 12)^4}{384 \times 4906 \times 145592}\left[1 + 0.60 \times 1.8 \times \frac{145592}{367259}\right]$$

$= 2.09$ in. ↓

Deflection due to forms and reinforcement weight, Δ_{fr}

$$\Delta_{fr} = \frac{5wL_b^4}{384E_{cb}I_b}$$

where, w = forms and reinforcement weight = 20 × (82.75/12) = 137.92 lb/ft

Weight of forms and reinforcement is assumed to be 20 lb/ft^2
Beam spacing = 82.75 in.

$$\Delta_{fr} = \frac{5 \times \frac{0.13792}{12} \times (85.25 \times 12)^4}{384 \times 4906 \times 145592}$$

$= 0.23$ in. ↓

Deflection due to barrier and future wearing surface weight, Δ_{bfws}

$$\Delta_{bfws} = 2.3 \times \frac{5wL_b^4}{384E_{cb}I_c}$$

where, $w = w_{bar} + w_{ws} = 0.128 + 0.16 = 0.288$ kip/ft

w_{bar} = barrier weight = 0.128 kip/ft

w_{ws} = wearing surface weight = 0.16 kip/ft

$$\Delta_{bfws} = 2.3 \times \frac{5 \times \frac{0.288}{12} \times (85.25 \times 12)^4}{384 \times 4906 \times 367259}$$

= 0.44 in. ↓

Step 8.11.2. Deflections due to Live Load and Impact

Live load deflection limit = $L_{ds}/800$ — LRFD Art. 2.5.2.6.2

= 83.75 × 12/800=1.26 in.

If owner requires the optional live load criteria, the deflection is the greater of: — LRFD Art. 2.5.2.6.2, LRFD Art. 3.6.1.3.2

- That resulting from the design truck plus impact, Δ_{LT}, or
- That resulting from 25% of the design truck plus impact, Δ_{LT}, taken together with the design lane load, Δ_{LL}.

The dynamic load allowance must be included in the calculation of live load deflection. — LRFD Art. 2.5.2.6.2

All the beams are assumed to deflect equally due to the applied live load. Therefore, the distribution factor for deflection, DFD — LRFD Art. 2.5.2.6.2

= Number of lanes/Number of beams

= 2/5 = 0.4 lanes/beam

Deflection = $\frac{COEFF}{I_c}\left(\frac{E_s}{E_c}\right)\left(\frac{\text{No.of Design Lanes}}{\text{No.of Beams}}\right)$(Multiple Presence Factor) — MDOT (2002)

COEFF for lane load and truck load are derived for steel beams with $E_s = 29 \times 10^6$ psi. Hence, when the deflection is calculated using the above equation, $E_s = 29 \times 10^6$ psi should be used.

For lane load, COEFF = $0.000496L^4$ where, L = Span in ft. — MDOT (2002)

For truck load, COEFF = $0.0894\,(L^3 - 555L + 4780)$ OR

= $0.0794(L^3 - 368L + 2400)$

Multiple presence factor of 1.0 is used for the bridge with 2 design lanes. — LRFD Table 3.6.1.1.2- 1

Deflection due to Lane Load

COEFF = $0.000496L^4 = 0.000496\,(83.75)^4 = 24{,}402$

Deflection, $\Delta_{LL} = \frac{24402}{367259}\left(\frac{29\times10^6}{4.906\times10^6}\right)\left(\frac{2}{5}\right)1.0 = 0.16$ in.

Deflection due to Design Truck and Impact

COEFF = 0.0894 (L^3 – 555L + 4780) = 0.0894 (83.75^3 – 555 × 83.75 + 4780)

= 48,788

OR

= 0.0794 (L^3 – 368L + 2400) = 0.0794 (83.75^3 –368 × 83.75 + 2400)

= 44,385

With impact, COEFF = 48,788 × 1.33 = 64,888

$$\text{Deflection, } \Delta_{LT} = \frac{64888}{367259}\left(\frac{29\times10^6}{4.906\times10^6}\right)\left(\frac{2}{5}\right)1.0 = 0.42 \text{ in.}$$

Live Load Deflection

Live load deflection = Max (Δ_{LT}, 0.25Δ_{LT} + Δ_{LL})

= Max (0.42, 0.25 × 0.42 + 0.16) = 0.42 in.

0.42 in. < L_{ds}/800 = 1.26 in. **O.K.**

Step 8.12. Bottom Flange Shortening During Stress Transfer

$$\text{Bottom flange shortening} = \frac{F_i L}{A_b E_{ci}} + \frac{8\Delta_{int}}{L}\left(\frac{I_b}{S_b}\right)$$

where,

F_i = total prestressing force at transfer = 1,406 kip

L = beam length = 85.25 ft

A_b = area of beam = 878.30 in.^{2}

E_{ci} = modulus of elasticity of beam at transfer = 4,631 ksi

Δ_{int} = camber at transfer = 2.34 in.

I_b = moment of inertia of the beam = 145,592 in.^{4}

S_b = section modulus for bottom fiber = 8,000 in.^{3}

$$\text{Bottom flange shortening} = \frac{1406 \times (85.25 \times 12)}{878.30 \times 4631} + \frac{8 \times 2.34}{85.25 \times 12}\left(\frac{145592}{8000}\right) = 0.687 \text{ in.}$$

Prestressing Design of Box Beam

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INTRODUCTION

Design of a spread box beam of a four span highway bridge is demonstrated in this example.

The purpose of this example is to illustrate the design of a typical interior bridge beam for flexure, shear, and deflection under dead and live loads. The total length of this 25 degree skew, four span, continuous for live load bridge is, 220 ft and 6 in. The bridge is symmetrical with inner two of 55 ft and outer two of 55 ft – 3 in. spans. Each span consists of twelve box beams spaced at 6 ft - 3 in. on centers (**Figure 1**). Cross-sectional dimensions of the beam are shown in **Figure 2**. Beams are designed for composite behavior with a 9-in. thick cast-in-place concrete deck. Superstructure includes one interior concrete diaphragm of 8 in. × 20 in. at the middle of each span.

The design is implemented in accordance with the Michigan Department of Transportation (MDOT) policies documented in the Bridge Design Manual (BDM) and Bridge Design Guides (BDG) as of 04/30/2017. As needed, the stipulations in the AASHTO LRFD *Bridge Design Specifications*, 7th Edition, with 2015 and 2016 interims are considered. Certain material and design parameters are selected to be in compliance with MDOT practices reflected in the Bridge Design System (BDS), the MDOT legacy software.

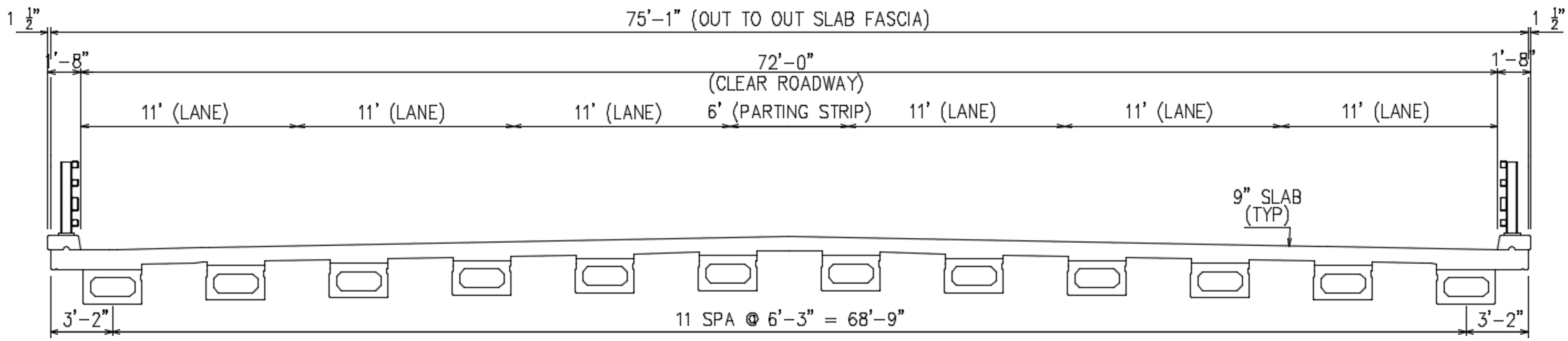


Figure 1. Bridge cross-section

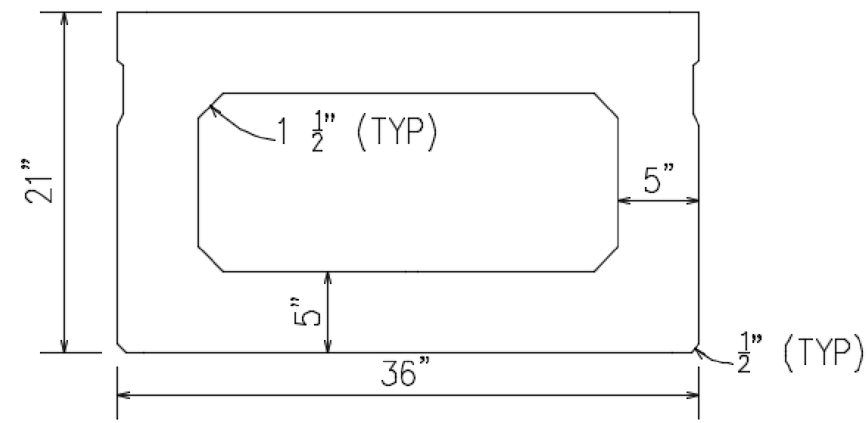


Figure 2. 21 in. × 36 in. beam cross-section

STEP 1. GEOMETRY

Step 1.1. Superstructure

Beam length, L_b		= 54.625 ft	
Distance from beam end to bearing centerline, L_{bcl}		= 8.75 in. = 0.729 ft	
Design span, L_{ds}	$=L_b - 2\times L_{bcl}$	= 53.167 ft	
Skew, θ		= 25 deg.	
Skew is the angle measured from line perpendicular to bridge centerline to support reference line. When skew exceeds 30 degrees, refined methods should be utilized for load calculations.			BDM Art. 7.01.14
Number of lanes		= 4	
Lane width		= 11 ft	
10 ft ≤ Lane width ≤ 12 ft			MDOT (2017c)
Shoulder width		= 11 ft	BDG 6.05.01A
Overhang width, Δw		= 3 ft – 2 in.	
Barrier (4 tube - bicycle railing option) width		= 20 in.	BDG 6.29.17A
Distance from slab fascia to outside face of the barrier		= 1.5 in.	BDG 6.29.17A
Deck width		= 75 ft – 1 in.	BDG 6.05
Beam depth, h		= 21 in.	
Box beam 21 in. × 36 in.			BDG 6.65.02A
Number of beams, N_b		= 12	
Designers should layout beam spacing to accommodate future part width reconstruction. In most cases beams at centerline of structure should be avoided.			BDM Art. 7.01.15
Beam spacing, S		= 6 ft – 3 in.	
S ≤ 10 ft for all the beams			BDM Art. 7.02.02
Deck slab thickness, t_s		= 9 in.	

Full slab thickness is included in dead load calculation as well as in the composite section calculations for the beam design. The deck slab design is based on 7.5 in. thickness, excluding the top 1.5 in. integrated wearing surface. — BDM Art. 7.02.08 B; BDM Art. 7.02.19 A4

Haunch thickness, t_h	= 2 in.	
$t_h \geq 2$ in.		BDM Art. 7.02.19-C
Cross-section area of concrete diaphragm, A_{cd}	$= 8 \times 20 = 160$ in.^{2}	

STEP 2. MATERIAL PROPERTIES AND STRESS LIMITS

Step 2.1. Concrete

Unit weight, w_c = 0.145 kip/ft^3

MDOT standard concrete density designated as 0.145 kip/ft^3 is used in modulus of elasticity calculations.

Cast-in-place concrete slab, 28-day strength, f'_c = 4.0 ksi

f'_c = 4 ksi, Grade D concrete — BDM Art. 7.01.03

Precast beam, 28-day strength, f'_c = 6.8 ksi

5 ksi $\leq f'_c \leq$ 8 ksi — BDM Art. 07.02.03-A

Strength at release, f_{ci}' = 6.4 ksi

$f'_{ci} \leq$ 7000 psi — BDM Art. 7.01.03

If f'_{ci} is unknown, $f'_{ci} = 0.8f'_c$ can be assumed — LRFD Art. 5.4.2.3.2

Modulus of elasticity, $E_c = 120{,}000\ K_1(w_c)^{2.0}(f'_c)^{0.33}$ — LRFD Eq. 5.4.2.4-1

where,

w_c = unit weight of concrete, kcf — LRFD Table 3.5.1.1

f'_c = specified strength of concrete, ksi

K_1 = correction factor for source of aggregate = 1 — LRFD Art. 5.4.2.4

Elasticity modulus of:

Cast-in-place slab, $E_c = 120{,}000(0.145)^{2.0}(4)^{0.33}$ = 3,987 ksi

Precast beam at transfer, $E_{ci} = 120{,}000(0.145)^{2.0}(6.40)^{0.33}$ = 4,655 ksi

Precast beam at service, $E_{cb} = 120{,}000(0.145)^{2.0}(6.80)^{0.33}$ = 4,749 ksi

MDOT modulus of elasticity calculation is different from the AASHTO LRFD 7th edition, 2016

Step 2.2. Steel and Prestressing Strand

Prestressing strands: 0.6 in. dia., seven-wire, low relaxation

The design and detail sheets shall specify only ASTM A416 (AASHTO M 203) Grade 270 low relaxation strands. Strands shall be 0.6 in. in diameter with a 44 kips release force. — BDM Art. 7.02.18.A1

Area of one strand, A_{ps} = 0.217 in.^{2}

Ultimate strength, f_{pu} = 270 ksi

Yield strength, $f_{py} = 0.9f_{pu}$ = 243 ksi — LRFD Table 5.4.4.1-1

Stress limits of prestressing strands: — LRFD Table 5.9.3-1

prior to transfer, $f_{pbt} \leq 0.75f_{pu}$ = 202.5 ksi

at service limit state (after losses) $f_{pe} \leq 0.80f_{py}$ = 194.4 ksi

Modulus of elasticity, E_p = 28,500 ksi — LRFD Art. 5.4.4.2

Reinforcing steel:

Yield strength, f_y = 60 ksi — BDM Art. 7.01.03

Modulus of elasticity, E_s = 29,000 ksi — LFRD Art. 5.4.3.2

Step 2.3. Stress Limits

Allowable concrete tensile stress at release, $\bar{f}_{ti} = 0.24\sqrt{f'_{ci}}$ — LRFD Table 5.9.4.1.2-1

Allowable concrete compressive stress at release, $\bar{f}_{ci} = 0.6f'_{ci}$ — LRFD Art. 5.9.4.1.1

Allowable tensile stress for concrete with bonded steel and subjected to not worse than moderate corrosion condition, $\bar{f}_{ts} = 0.19\sqrt{f'_c}$ — LRFD Table 5.9.4.2.2-1

Allowable compressive stress for concrete subjected to effective prestress and permanent loads, $\bar{f}_{cpl} = 0.45f'_c$ — LRFD Table 5.9.4.2.1-1

Allowable compressive stress for concrete subjected to effective prestress, permanent loads, and transient loads, $\bar{f}_{ctl} = 0.6f'_c$ — LRFD Table 5.9.4.2.1-1

STEP 3. SECTION PROPERTIES

Step 3.1. Noncomposite Section Properties

BDG 6.65.02A

Area	A_b = 467 in.^{2}
Top flange width	b_{tf} = 36.00 in.
Web thickness	t_w = 5.00 in.
Width of two webs	t_{ws} = 10.00 in.
Distance from the centroid to extreme top fiber	y_t = 10.60 in.
Distance from the centroid to extreme bottom fiber	y_b = 10.40 in.
Moment of inertia	I_b = 24,600 in.^{4}
Section modulus for top fiber	S_t = 2,320 in.^{3}
Section modulus for bottom fiber	S_b = 2,360 in.^{3}

Noncomposite section properties are from BDG 6.65.02A version date 09/22/14.

Step 3.2. Composite Section Properties

Haunch thickness varies along the beam length; a uniform thickness of 2 in. is assumed for the design.

Step 3.2.1. Interior Beam

Effective flange width, b_{eff} = 6.25 ft — LRFD Art. 4.6.2.6.1

Modular ratio between slab and beam concrete,

$$n = \frac{E_c(\text{slab})}{E_c(\text{beam})} = \frac{E_c}{E_cb} = \frac{3{,}987}{4{,}749} = 0.84$$

Modular ratio between slab and beam concrete, n, is needed to calculate transformed section width of deck slab for composite section properties.

Transformed flange width = $n \times b_{eff}$ = 0.84 × 6.25 × 12 = 63.00 in.

Transformed haunch width = $n \times b_{tf}$ = 30.24 in.

Overall depth, h_c = $h + t_s + t_h$ = 21 + 9 + 2 = 32.00 in.

Total area of the transformed section, A_c = 1,094 in.^{2}

Distance from centroid to extreme bottom fiber, y_{bc} = 19.90 in.

Distance from centroid to extreme top fiber, y_{tc} = 12.10 in.

Moment of inertia, I_c = 103,611 in.^{4}

Section modulus for top fiber, S_{tc} = I_c/y_{tc} = 8,563 in.^{3}

Section modulus for bottom fiber, S_{bc} = I_c/y_{bc} = 5,207 in.^{3}

Figure 3 shows the geometry of the transformed interior beam cross-section. A summary of cross-section properties are presented in **Table 1**.

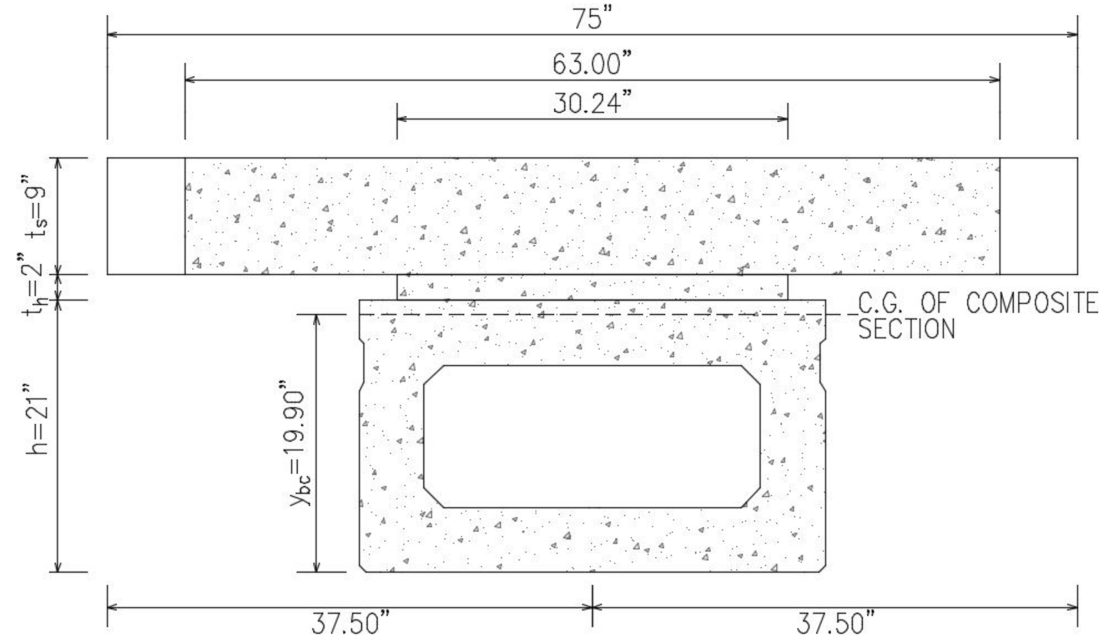


Figure 3. Dimensions of the interior beam

Table 1. Interior Beam Properties

	Area, in.^{2}	y_b, in.	Ay_b, in.^{3}	$A(y_{bc} - y_b)^2$, in.^{4}	I, in.^{4}	$I + A(y_{bc} - y_b)^2$, in.^{4}
Beam	467.00	10.40	4,856.80	42,146.75	24,600.00	66,746.75
Haunch	60.48	22.00	1,330.56	266.72	20.16	286.88
Deck	567.00	27.50	15,592.50	32,749.92	3,827.25	36,577.17
Σ	1,094.48		21,779.86			103,610.80

STEP 4. LOADS

Step 4.1. Dead Loads

Dead loads on the composite structure: LRFD Art. 4.6.2.2.1

Permanent loads (curbs and future wearing surface) may be distributed uniformly among all beams if the following criteria are met:

- Width of the deck is constant **O.K.**
- Number of beams, $N_b \geq 4$ **O.K.**
- The roadway part of the overhang, $d_e \leq 3.0$ ft.

  d_e = 38 in. – 20 in. = 1 ft – 6 in. **O.K.**
- Effect of curvature in the plan can be ignored because LRFD Art. 4.6.1.2.4b
  - Girders are concentric
  - Bearing lines are not skewed more than 10 degrees from radial
  - The stiffness of the girders are similar
  - The arc span divided by the girder radius in feet is less than 0.06 radians.
- Cross-section of the bridge is consistent with LRFD Table 4.6.2.2.1-1 **O.K.**

The criteria are satisfied and the loads can be distributed equally to all 12 beams.

Beam weight, w_g = 0.486 kip/ft BDG 6.65.02A

Cast-in-place concrete deck and haunch weight on interior beam

= $[(b_{eff}\ t_s) + (b_{tf}\ t_h)]w_c$ = [(75 in. × 9 in.) + (36 in. × 2 in.)]/12^2 × 0.145 kip/ft^3

= 0.752 kip/ft

Future wearing surface weight, w_s = 0.025 ksf

New bridges and bridge replacements shall be designed for a future wearing surface load of 25 LBS/SFT BDM Art. 7.01.04-H

Barrier or railing weight, w_b = 0.265 kip/ft BDG 6.29.17

Wearing surface weight on one beam, w_{ws}

= (0.025 ksf)(72.0 ft)/(12 beams) = 0.150 kip/ft per beam

where, clear roadway width is 72 ft.

Barrier weight on one beam, w_{bar}

= (2 barriers)(0.265 kip/ft)/(12 beams) = 0.044 kip/ft per beam

Concrete diaphragm weight on an interior beam, w_{cd}

= $A_{cd}Sw_c$ = (160/12^2) × 6.25 × 0.145 = 1.007 kip

There is no utility attached to this bridge, and the deck formwork is removed after construction; thus, their weight is not included in this example.

Step 4.2. Live Loads

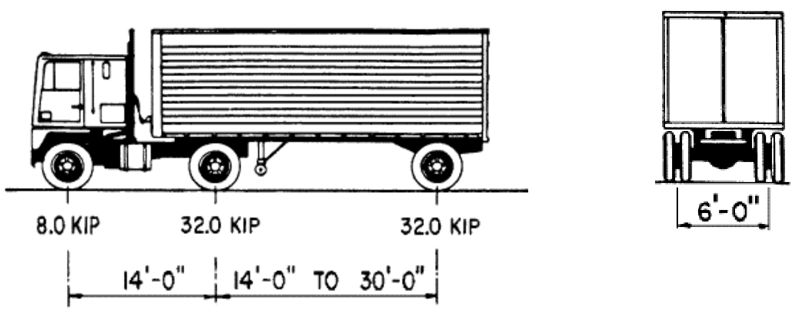
Load modifying factor for ductility, redundancy, and operational importance, η = 1.0

The load modifying factor, η (eta), related to ductility, redundancy, and operational importance, shall be considered for less important roads. BDM Art. 7.01.04-B Typically 1.0 is used

Vehicular live loading on the roadways of bridges designated HL-93 Mod, shall consist of 1.2 times the combination of the: BDM Art. 7.01.04-A

- *Design truck or single 60 kip load*
- *Design lane load*

Design truck is shown in **Figure 4**. Design lane load is 0.64 kip/ft. LRFD Art. 3.6.1.2.1



LRFD Figure 3.6.1.2.2-1

Figure 4. Characteristics of the design truck

When 90% of two design trucks are combined with 90% of the effect of a lane load for both negative moment and pier reactions per A.3.6.1.3, a 1.2 multiplier shall be applied to the resulting moment or load. Each design lane under consideration shall be occupied by either the design truck or single 60 kip load, coincident with the lane load, where applicable. The loads shall be assumed to occupy 10.0 ft transversely within a design lane. BDM Art. 7.01.04-A

Design truck configuration for continuity design is shown in **Figure 5**. LRFD Art. 3.6.1.3.1

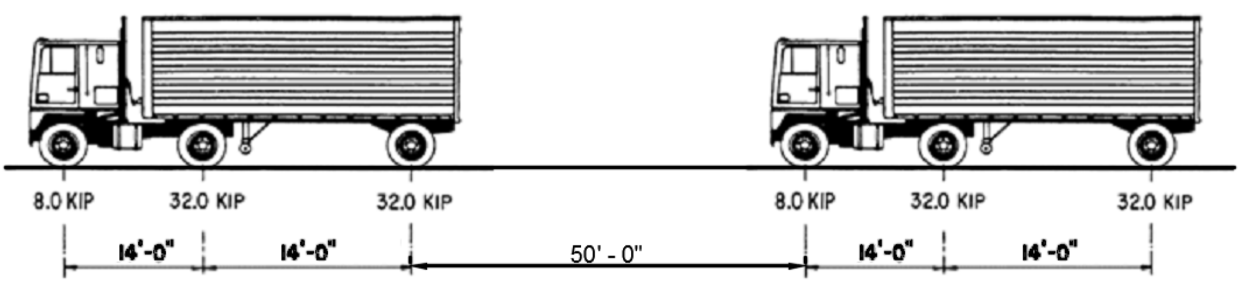


Figure 5. Characteristics of the design truck for continuity design

Design truck load is increased by a dynamic allowance factor LRFD Table 3.6.2.1-1

Dynamic allowance, IM = 33%

STEP 5. LIVE LOAD DISTRIBUTION FACTORS

The bridge geometry satisfies the conditions stipulated, as described in Step 4.1. Thus, allows the determination of live load moments and shears using the simplified factor formulas. LRFD Art. 4.6.2.2 LRFD Art. 4.6.2.2.1

Bridge type for precast concrete spread box beams with a cast-in-place concrete deck is (b). LRFD Table 4.6.2.2.1-1

Number of 12 ft wide design lanes = the integer part of the ratio of (w/12) where (w) is the clear roadway width, in ft, between the curbs. LRFD Art 3.6.1.1.1

From **Figure 1**, w = 72 ft

Number of design lanes = 72 ft/12 ft = 6 lanes.

Step 5.1. Interior Beam Live Load Distribution Factor

Distribution factor calculations for moment and shear in interior beams include multiple presence factor. LRFD Table 4.6.2.2.2b-1 LRFD Table 4.6.2.2.3a-1

Step 5.1.1. Distribution Factor for Moment

For all limit states, except fatigue:

With two or more lanes loaded:

$$\mathrm{DFM} = \left(\frac{S}{6.3}\right)^{0.6}\left(\frac{Sd}{12.0L^2}\right)^{0.125}$$ LRFD Table 4.6.2.2.2b-1

where

DFM = distribution factor for moment in interior beam

S = beam spacing, ft

L (=L_{ds}) = beam span, ft

d = depth of beam, in.

To use DFM equations, the criteria below need to be satisfied:

Range of Applicability LRFD Table 4.6.2.2.2b-1 and Table 4.6.2.2.3a-1	MDOT Policy BDG 6.41.01 and BDM Art. 7.02.02	Bridge Attributes
6.0 ft ≤ S ≤ 18.0 ft	6 ft ≤ S ≤ 10 ft for Spread box beams	S = 6 ft – 3 in. **O.K.**
20 ft ≤ L ≤ 140 ft		L_{ds} = 53.167 ft **O.K.**
18 in. ≤ d ≤ 65 in.		d = 21 in. **O.K.**
$N_b \geq 3$		N_b = 12 **O.K.**

With two or more lanes loaded:

$$\mathrm{DFM} = \left(\frac{S}{6.3}\right)^{0.6}\left(\frac{Sd}{12.0L^2}\right)^{0.125}$$

$$= \left(\frac{6.25}{6.3}\right)^{0.6}\left(\frac{6.25\times 21}{12.0\times(53.167)^2}\right)^{0.125} = 0.497 \text{ lanes/beam}$$

With one design lane loaded:

$$\text{DFM} = \left(\frac{S}{3.0}\right)^{0.35}\left(\frac{Sd}{12.0L^2}\right)^{0.25}$$

LRFD Table 4.6.2.2.2b-1

$$= \left(\frac{6.25}{3.0}\right)^{0.35}\left(\frac{6.25\times 21}{12.0\times(53.167)^2}\right)^{0.25} = 0.322 \text{ lanes/beam}$$

Thus, two or more lanes loaded case controls, and

DFM_I = 0.497 lanes/beam.

Fatigue of the reinforcement need not be checked for prestressed components designed for extreme fiber tensile stress under Service III Limit State within the tensile stress limit specified in Table 5.9.4.2.2-1. LRFD Art. 5.5.3.1

In this example, tensile stresses are maintained below the limits specified in Table 5.9.4.2.2-1 with Service III Limit State. Hence, fatigue will not be considered.

Step 5.1.2. Distribution Factor for Shear

With two or more lanes loaded:

LRFD Table 4.6.2.2.3a-1

$$\text{DFV} = \left(\frac{S}{7.4}\right)^{0.8}\left(\frac{d}{12.0L}\right)^{0.1}$$

where,

DFV = distribution factor for shear in interior beam

S = beam spacing, ft

L (=L_{ds}) = beam span, ft

d = depth of beam, in.

Distribution factor for shear is

$$\text{DFV} = \left(\frac{6.25}{7.4}\right)^{0.8}\left(\frac{21}{12.0\times(53.167)}\right)^{0.1} = 0.621 \text{ lanes/beam}$$

For one design lane loaded:

LRFD Table 4.6.2.2.3a-1

$$\text{DFV} = \left(\frac{S}{10}\right)^{0.6}\left(\frac{d}{12.0L}\right)^{0.1}$$

$$= \left(\frac{6.25}{10}\right)^{0.6}\left(\frac{21}{12.0\times(53.167)}\right)^{0.1} = 0.536 \text{ lanes/beam}$$

Thus, two or more lanes loaded case controls, and

DFV_I = 0.621 lanes/beam

Step 5.2. Skew Reduction/Correction Factors

Step 5.2.1. Reduction Factor for Live Load Moment

$R_M = 1.05 - 0.25\tan\theta \leq 1.0$ LRFD Table 4.6.2.2.2e-1

where

R_M = Reduction factor

If $\theta > 60^o$ use $\theta = 60^o$

Range of Applicability LRFD Table 4.6.2.2.2e-1	**MDOT Policy BDM Art. 7.01.14 and 7.02.02**	**Bridge Attributes**
$0° \leq \theta \leq 60°$	$\theta \leq 30°$ (Approximate method) $30° < \theta \leq 45°$ (Refined method) $\theta > 45°$ (Need approval)	$\theta = 25°$

$R_M = 1.05 - 0.25\tan 25°$

$= 1.05 - 0.25 \times \tan 25° = 0.933 \leq 1.0$ **O.K.**

Step 5.3. Moment and Shear Distribution Factors Adjusted for Skew

Distribution factor for moment in interior beam,

$DFM_{IB} = (DFM_I)(R_M)$

$= (0.497)(0.933) = 0.464$ lanes/beam

Distribution factor for shear in interior beam,

$DFV_{IB} = (DFV_I) = 0.621$ lanes/beam

Fatigue of the reinforcement need not be checked for fully prestressed components designed with extreme fiber tensile stress under Service III Limit State and tensile stress limit specified in Table 5.9.4.2.2-1. LRFD Art. 5.5.3.1

In this example, tensile stresses are maintained below the limits specified in Table 5.9.4.2.2-1 with Service III Limit State. Hence, fatigue is not considered.

STEP 6. MOMENT AND SHEAR DUE TO DEAD LOADS AND SUPPORT SETTLEMENT

Shear (V_x) and moment (M_x) of a simply supported beam with a span (L) under a uniformly distributed load (w) are:

$$V_x = w(0.5L - x)$$
$$M_x = 0.5wx(L - x)$$

where, x is the distance from the support.

Interior beam moment and shear of this four span continuous for live load structure are calculated and given in **Table 2** and **Table 3**, respectively.

Step 6.1. Moment and Shear at Prestress Release due to Beam Weight (Simple Span)

Full beam length (L_b) of 54.625 ft is used at the time of prestress release. Beam self-weight is the only load.

As an example, moment and shear due to a beam weight of 0.486 kip/ft at 6.0458 ft from the beam end are calculated as follows:

$$M_{gr} = 0.5wx(L - x) = 0.5(0.486)(6.0458)(54.625 - 6.0458) \quad = 71.37 \text{ kip-ft}$$
$$V_{gr} = w(0.5L - x) = (0.486)(0.5 \times 54.625 - 6.0458) \quad = 10.34 \text{ kip}$$

Moment and shear due to beam self-weight at release are shown in **Table 2** - **Table 3**.

Step 6.2. Moment and Shear in Noncomposite Section (Simple Span)

In this analysis, span length is now the design span (L_{ds}) of 53.167 ft, distance between bearing centerlines. The self-weight of beam, deck, haunch, and diaphragm loads are applied to the noncomposite, simple span structure.

As an example, moment and shear at 5.3167 ft from support due to beam weight of 0.486 kip/ft and deck and haunch weight of 0.752 kip/ft are calculated as follows:

Moment and shear due to beam self-weight

$$M_g = 0.5wx\,(L - x) = 0.5(0.486)(5.3167)(53.167 - 5.3167) \quad = 61.82 \text{ kip-ft}$$
$$V_g = w(0.5L - x) = (0.486)(0.5 \times 53.167 - 5.3167) \quad = 10.34 \text{ kip}$$

Moment and shear due to deck and haunch self-weight

$$M_D = 0.5wx(L - x) = 0.5(0.752)(5.3167)(53.167 - 5.3167) \quad = 95.66 \text{ kip-ft}$$
$$V_D = w(0.5L - x) = (0.752)(0.5 \times 53.167 - 5.3167) \quad = 15.99 \text{ kip}$$

The respective moment and shear values are shown in **Table 2** - **Table 3.**

Step 6.3. Moment and Shear in Composite Section due to Barrier and FWS Weight

Continuous for live load prestressed concrete beams shall be designed as simple span beams for all positive dead load and live load moments. BDM Art. 7.02.18A-6

Moment and shear due to barrier and future wearing surface (FWS) weight are also calculated for simple and continuous spans.

Step 6.3.1. Moment and Shear in Composite Section (Simple Span)

Barrier and FWS loads on the composite, simple span structure are applied to calculate the maximum positive moments.

As an example, moment and shear at 5.3167 ft from a support due to a barrier weight of 0.044 kip/ft and a FWS weight of 0.150 kip/ft are calculated as follows:

Moment and shear due to barrier weight

$M_b = 0.5wx(L - x) = 0.5(0.044)(5.3167)(53.167 – 5.3167) = 5.60$ kip-ft

$V_b = w(0.5L - x) = (0.044)(0.5 \times 53.167 – 5.3167) = 0.94$ kip

Moment and shear due to FWS weight

$M_{ws} = 0.5wx(L - x) = 0.5(0.150)(5.3167)(53.167 – 5.3167) = 19.08$ kip-ft

$V_{ws} = w(0.5L - x) = (0.150)(0.5 \times 53.167 – 5.3167) = 3.19$ kip

The respective simple span moment and shear values are given in **Table 2 - Table 3.**

Step 6.3.2. Moment and Shear in Composite Section (Continuous Span)

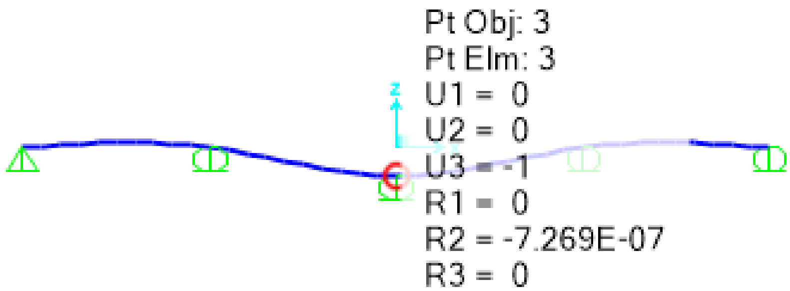
The respective continuous span moment and shear values are calculated and shown in **Table 2 - Table 3.**

Step 6.4. Moment and Shear in Composite Section due to Support Settlement

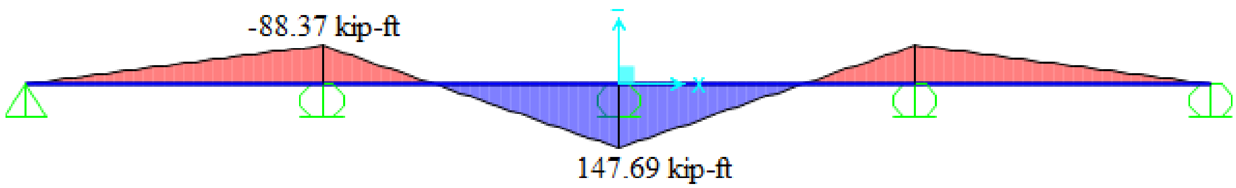
A negative moment caused by a 1 in. settlement of a substructure unit is imposed when non-spread footings are used. A settlement of 2 in. is imposed when the bridge is on spread footings.

The negative moment due to settlement is reduced for creep. A creep factor, ϕ_{cr}, of 0.368 is used. PCI (1978)

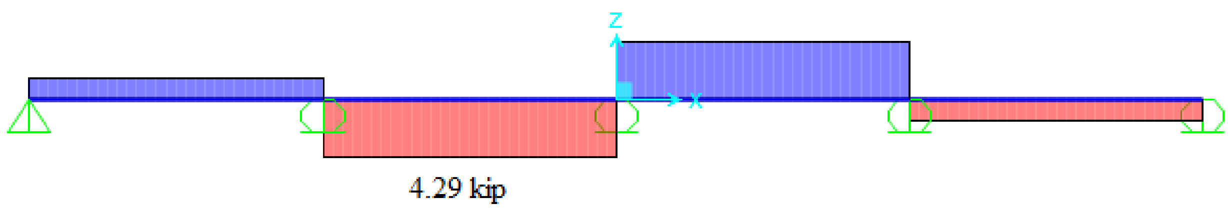
Moment and shear due to settlement at abutments and intermediate supports are evaluated to identify the controlling case. In this example, 1 in. settlement at the middle pier develops the greatest moment and shear along the span. Interior beam resultant moment and shear values are multiplied by ϕ_{cr} as shown in **Figure 6b** and **c**. The moment and shear values are also presented in **Table 2 - Table 3**.



(a) Deformed shape



(b) Moment due to settlement adjusted for creep (M_s)



(c) Shear due to support settlement adjusted for creep (V_s)

Figure 6. Moment and shear of an interior beam due to a 1 in. settlement at the middle pier

STEP 7. MOMENT AND SHEAR DUE TO LIVE LOADS

Please refer to *Bulb-tee Beam Design Example* for truck and axle load moment and shear calculation procedures.

Step 7.1. Moment and Shear due to Design Truck or Axle Load

For all limit states except fatigue:

The interior beam moment due to truck (M_{LT}) or axle load (M_{LA}), and the interior beam shear due to truck (V_{LT}) or axle load (V_{LA}) are expressed as follows:

$$M_{LT} \text{ or } M_{LA} = (\text{moment per lane})(\text{factor for HL-93 Mod})(DFM_{IB})(1 + IM)$$
$$= (\text{moment per lane})(1.2)(0.464)(1 + 0.33)$$
$$= (\text{moment per lane})(0.741) \text{ kip-ft}$$

$$V_{LT} \text{ or } V_{LA} = (\text{shear force per lane})(\text{factor for HL-93 Mod})(DFV_{IB})(1 + IM)$$
$$= (\text{shear foce per lane})(1.2)(0.621)(1 + 0.33)$$
$$= (\text{shear force per lane})(0.991) \text{ kip}$$

M_{LT}, M_{LA}, V_{LT}, and V_{LA} at selected sections for an interior beam are given in **Appendix D**.

Step 7.2. Moment and Shear due to Design Lane Load

Please refer to *Bulb-tee Beam Design Example* for lane load moment and shear calculation procedures.

The interior beam moment and shear due to lane load are as follows:

$$M_{LL} = (\text{moment per lane})(\text{factor for HL-93 Mod})(DFM_{IB})$$
$$= (\text{moment per lane})(1.2)(0.464) \text{ kip-ft}$$
$$= (\text{moment per lane})(0.557) \text{ kip-ft}$$

$$V_{LL} = (\text{lane load shear force})(\text{factor for HL-93 Mod})(DFV_{IB})$$
$$= (\text{lane load shear force})(1.2)(0.621) \text{ kip}$$
$$= (\text{lane load shear force})(0.745) \text{ kip}$$

M_{LL} and V_{LL} at selected sections of an interior beam are given in **Appendix D**.

Step 7.3. Moment and Shear Summary

Unfactored moment and shear of interior beam are given in **Table 2** and **Table 3**, respectively.

Simple span service and strength limit state moment and shear values of interior beam are given in **Table 4** and **Table 5**. Refer to *Bulb-tee Beam Design Example* for the calculation procedure of combined loads.

Table 2. Unfactored Moment in Interior Beam (kip-ft)

Location	At release	For service and strength limit state checks									
		Noncomposite section (simple span)			Composite section						
					Simple span			Continuous span+			
	Beam, M_{gr}	Beam, M_g	Deck and haunch, M_D	Concrete Diaphragm, M_{cd}	Barrier, M_b	Future wearing surface, M_{ws}	HL – 93 Mod with impact, M_{HL-M} **	Settlement, M_s	Barrier, M_b	Future wearing surface, M_{ws}	HL – 93 Mod with impact, M_{HL-M}
Pier 1 CL								-88.37	-14.33	-48.84	-396.23
Beam end	0.00							-87.57	-14.09	-48.03	-392.39
CL of bearing	9.33	0.00	0.00	0.00	0.00	0.00	0.00	-84.44	-13.17	-44.91	-377.54
Transfer location*	37.40	27.96	43.27	1.15	2.53	8.63	107.89	-74.69	-10.43	-35.55	-332.08
$0.10 \times L_{ds}$	71.37	61.82	95.66	2.68	5.60	19.08	250.35	-61.62	-7.11	-24.24	-273.75
$0.20 \times L_{ds}$	119.32	109.68	169.70	5.37	9.93	33.85	445.10	-38.80	-2.29	-7.80	-194.89
$0.30 \times L_{ds}$	153.57	144.16	223.06	8.05	13.05	44.49	584.21	-15.98	1.29	4.40	-144.69
$0.40 \times L_{ds}$	174.22	164.70	254.85	10.74	14.91	50.83	667.68	6.93	3.63	12.37	-108.43
$0.50 \times L_{ds}$ (Midspan)	181.27	171.49	265.34	13.42	15.53	52.93	695.51	29.66	4.72	16.10	-83.08
$0.60 \times L_{ds}$	174.22	164.70	254.85	10.74	14.91	50.83	667.68	52.48	4.56	15.55	-88.24
$0.70 \times L_{ds}$	153.57	144.16	223.06	8.05	13.05	44.49	584.21	75.30	3.16	10.76	-106.41
$0.80 \times L_{ds}$	119.32	109.68	169.70	5.37	9.93	33.85	445.10	98.12	0.51	1.73	-144.01
$0.90 \times L_{ds}$	71.37	61.82	95.66	2.68	5.60	19.08	250.35	120.94	-3.38	-11.52	-211.61
Transfer location*	37.40	27.96	43.27	1.15	2.53	8.63	107.89	134.01	-6.16	-21.02	-264.10
CL of bearing	9.33	0.00	0.00	0.00	0.00	0.00	0.00	143.76	-8.51	-29.02	-305.21
Beam end	0.00							146.89	-9.30	-31.71	-318.67
Pier 2 CL								147.69	-9.50	-32.40	-322.19

L_{ds} – Design span

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

** 60 kip axle load moment is higher than HS-20 design truck load moment.

+ 60 kip axle load is not included in continuous span analysis.

Note: The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

Table 3. Unfactored Shear in Interior Beam (kip)

Location	At release	For service and strength limit state checks									
		Noncomposite section (simple span)			Composite section						
					Simple span			Continuous span[+]			
	Beam, V_{gr}	Beam, V_g	Deck and haunch, V_D	Concrete Diaphragm, V_{cd}	Barrier, V_b	Future wearing surface, V_{ws}	HL – 93 Mod with impact, V_{HL-M} **	Settlement, V_s	Barrier, V_b	Future wearing surface, V_{ws}	HL – 93 Mod with impact, V_{HL-M}
Pier 1 CL								4.29	1.30	4.42	66.82
Beam end	13.27							4.29	1.29	4.40	66.54
CL of bearing	12.92	12.92	19.99	0.51	1.17	4.00	72.14	4.29	1.26	4.29	65.42
Transfer location*	11.82	11.82	18.28	0.51	1.07	3.65	68.55	4.29	1.16	3.95	61.93
$0.10 \times L_{ds}$	10.34	10.34	15.99	0.51	0.94	3.19	63.79	4.29	1.02	3.49	57.24
$0.20 \times L_{ds}$	7.75	7.75	11.99	0.51	0.70	2.39	55.69	4.29	0.79	2.69	49.14
$0.30 \times L_{ds}$	5.17	5.17	8.00	0.51	0.47	1.60	47.84	4.29	0.56	1.89	41.12
$0.40 \times L_{ds}$	2.58	2.58	4.00	0.51	0.23	0.80	40.24	4.29	0.32	1.10	33.32
$0.50 \times L_{ds}$ (Midspan)	0.00	0.00	0.00	0.51	0.00	0.00	32.90	4.29	0.09	0.30	25.59
$0.60 \times L_{ds}$	-2.58	-2.58	-4.00	-0.51	-0.23	-0.80	-40.24	4.29	-0.15	-0.50	18.82
$0.70 \times L_{ds}$	-5.17	-5.17	-8.00	-0.51	-0.47	-1.60	-47.84	4.29	-0.38	-1.30	12.32
$0.80 \times L_{ds}$	-7.75	-7.75	-11.99	-0.51	-0.70	-2.39	-55.69	4.29	-0.61	-2.09	6.53
$0.90 \times L_{ds}$	-10.34	-10.34	-15.99	-0.51	-0.94	-3.19	-63.79	4.29	-0.85	-2.89	1.48
Transfer location*	-11.82	-11.82	-18.28	-0.51	-1.07	-3.65	-68.55	4.29	-0.98	-3.35	-1.07
CL of bearing	-12.92	-12.92	-19.99	-0.51	-1.17	-4.00	-72.14	4.29	-1.08	-3.69	-2.97
Beam end	-13.27							4.29	-1.11	-3.80	-3.58
Pier 2 CL								4.29	-1.12	-3.83	-3.73

L_{ds} – Design span

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

** 60 kip axle load shear is higher than HS-20 design truck load shear.

\+ 60 kip axle load is not included in continuous span analysis.

Note: The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

Table 4. Factored Moment in Interior Beam (kip-ft)

Location	Load Combinations – Simple Span				
	Service I		Service III		Strength I
	Noncomp	Comp	Noncomp	Comp	
Pier 1 CL					
Beam end					
CL of bearing	0.00	0.00	0.00	0.00	0.00
Transfer location	72.38	119.05	72.38	97.47	295.39
$0.10 \times L_{ds}$	160.16	275.75	160.16	225.68	673.93
$0.20 \times L_{ds}$	284.75	488.88	284.75	399.86	1198.05
$0.30 \times L_{ds}$	375.27	641.75	375.27	524.91	1574.50
$0.40 \times L_{ds}$	430.29	733.42	430.29	599.88	1801.19
$0.50 \times L_{ds}$ (Midspan)	450.25	763.97	450.25	624.87	1878.76
$0.60 \times L_{ds}$	430.29	733.42	430.29	599.88	1801.19
$0.70 \times L_{ds}$	375.27	641.75	375.27	524.91	1574.50
$0.80 \times L_{ds}$	284.75	488.88	284.75	399.86	1198.05
$0.90 \times L_{ds}$	160.16	275.75	160.16	225.68	673.93
Transfer location	72.38	119.05	72.38	97.47	295.39
CL of bearing	0.00	0.00	0.00	0.00	0.00
Beam end					
Pier 2 CL					

Table 5. Factored Shear in Interior Beam (kip)

Location	Load Combinations – Simple Span				
	Service I		Service III		Strength I
	Noncomp	Comp	Noncomp	Comp	
Pier 1 CL					
Beam end					
CL of bearing	33.42	77.31	33.42	62.88	175.48
Transfer location	30.61	73.27	30.61	59.56	165.04
$0.10 \times L_{ds}$	26.84	67.92	26.84	55.16	151.14
$0.20 \times L_{ds}$	20.25	58.78	20.25	47.64	127.23
$0.30 \times L_{ds}$	13.68	49.91	13.68	40.34	103.81
$0.40 \times L_{ds}$	7.09	41.27	7.09	33.22	80.77
$0.50 \times L_{ds}$ (Midspan)	0.51	32.90	0.51	26.32	58.21
$0.60 \times L_{ds}$	-7.09	-41.27	-7.09	-33.22	-80.77
$0.70 \times L_{ds}$	-13.68	-49.91	-13.68	-40.34	-103.81
$0.80 \times L_{ds}$	-20.25	-58.78	-20.25	-47.64	-127.23
$0.90 \times L_{ds}$	-26.84	-67.92	-26.84	-55.16	-151.14
Transfer location	-30.61	-73.27	-30.61	-59.56	-165.04
CL of bearing	-33.42	-77.31	-33.42	-62.88	-175.48
Beam end					
Pier 2 CL					

The critical section for shear is not shown since it is yet to be determined. See **Step 8.7** for shear calculation details.

STEP 8. INTERIOR BEAM PRESTRESS DESIGN

This example demonstrates the design of an interior beam. The required number of strands is usually governed by concrete tensile stresses at the bottom fiber for Service III Limit State load combination at the section of maximum moment (midspan). The required number of strands is calculated for the midspan.

Step 8.1. Lump Sum Prestress Loss Estimation

Stress in prestressing steel immediately prior to transfer, f_{pi} = $0.75f_{pu}$ = 202.5 ksi

$f_{pi} \leq 0.75f_{pu}$; LRFD Table 5.9.3-1

Prestress losses range from 15% to 25%. An initial lump sum loss of 20% is assumed. The loss assumption will be reviewed upon calculating detailed losses following the strand design.

Naaman (2012)

The ratio of effective stress to stress in prestressing steel after losses but prior to transfer, η = 0.80

Effective stress in prestressing steel after losses, f_{pe} = ηf_{pi} = 162.0 ksi

Step 8.2. Prestressing Strand Design

Step 8.2.1. Bottom Tensile Stress at Midspan

Bottom tensile stress due to applied dead and live loads using a Service III Limit State load combination

$$f_b = \frac{(M_g + M_D + M_{cd})}{S_b} + \frac{(M_b + M_{ws} + 0.8M_{HL-M})}{S_{bc}}$$

where,

- f_b = concrete tensile stress due to applied loads at bottom fiber of the beam (ksi)
- M_g = moment due to beam weight (kip-in)
- M_D = moment due to deck and haunch weight (kip-in)
- M_{cd} = moment due to concrete diaphragm weight (kip-in)
- S_b = section modulus of a noncomposite beam for bottom fiber (in.^{3})
- M_b = moment due to barrier weight (kip-in)
- M_{ws} = moment due to future wearing surface (kip-in)
- M_{HL-M} = moment due to HL-93 Mod live load (kip-in)
- S_{bc} = section modulus for bottom fiber of the composite beam (in.^{3})

Using moments from **Table 2**, bottom tensile stress due to applied loads at midspan are calculated as follows:

LRFD Table 5.9.4.2.2-1

$$f_b = -\frac{(171.49 + 265.34 + 13.42) \times 12}{2360} - \frac{(15.53 + 52.93 + 0.8 \times 695.51) \times 12}{5207}$$

$$= -3.73 \text{ ksi}$$

Step 8.2.2. Required Number of Strands

Allowable tensile stress for concrete with bonded steel and subjected to not worse than moderate corrosion condition,

LRFD Table 5.9.4.2.2-1

$$\bar{f}_{ts} = -\,0.19\sqrt{f'_c} = -\,0.19\sqrt{6.8} = -0.495 \text{ ksi}$$

The required precompressive stress at the bottom fiber of the beam is the difference between allowable tensile stress for concrete and bottom tensile stress due to applied loads:

$$f_{pb} = \bar{f}_{ts} - f_b = -0.495 - (-3.730) = 3.235 \text{ ksi}$$

The strand center of gravity location, measured from the bottom of the beam, is 5% to 15% of the beam depth. As an initial assumption, location is assumed as 10% of beam depth due to the shallow profile,

$$e_{pg} = y_b - 0.10h = 10.40 - 0.10 \times 21 = 8.30 \text{ in.}$$

The required precompressive stress at the bottom fiber (f_{pb}) due to total prestressing force after all losses, F_e:

$$f_{pb} = \frac{F_e}{A_b} + \frac{F_e e_{pg}}{S_b}$$

$$3.235 = \frac{F_e}{467} + \frac{F_e \times 8.30}{2360}$$

$$3.235 = \left(\frac{1}{467} + \frac{8.30}{2360}\right) \times F_e$$

Solving for F_e,

The required F_e = 571.73 kip

The prestressing force per strand = $A_{ps}f_{pe}$

= 0.217 × 162 = 35.15 kip

The required number of strands = 571.73/35.15 = 16.27 ≅ 17 (rounded up)

An iterative procedure is implemented to arrive at a satisfactory strand configuration and associated eccentricity. Seventeen (17) strands with an eccentricity of 8.30 in. is the initial trial. As eccentricity decreases, the required number of strands will increase. After the iterative process, 20 strands with an eccentricity of 7.60 in. are calculated. See **Step 8.2.3** for strand eccentricity (e_{pg}) calculation.

$F_e = \eta F_i$ = prestressing force after all losses (kip)

F_i = prestressing force at release (kip)

Therefore,

$$F_e = 20 \times 35.15 = 703.00 \text{ kip}$$

$$F_i = F_e/\eta = 703.00/0.8 = 878.75 \text{ kip}$$

Step 8.2.3. Strand Arrangement at Midspan

Strand arrangement at midspan is shown in **Figure 7**. Based on the arrangement, distance between center of gravity of the strands and the bottom fiber of the beam at midspan (y_{bs}) is calculated.

$y_{bs} = (12 \times 2 + 8 \times 4)/20$ = 2.80 in.

$e_{pg} = y_b - y_{bs} = 10.40 - 2.80$ = 7.60 in.

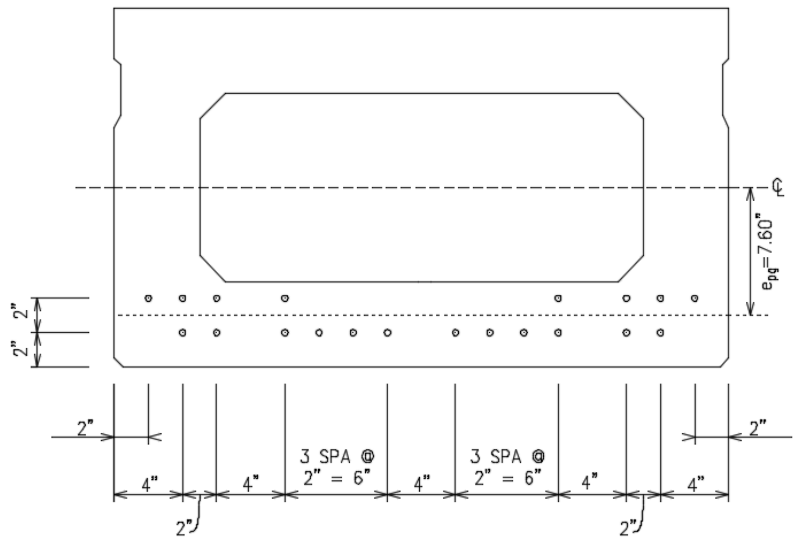


Figure 7. Strand arrangement at midspan

Step 8.2.4. Strand Arrangement at Beam End

Beam end stresses need to be checked only at transfer because this stage almost always controls the design. Also, losses with time will reduce the concrete stresses.

Transfer length = 60 × (Strand dia.) = 60 × 0.6 = 36 in. — LRFD Art. 5.11.4

As shown below, the beam top fiber tensile stress (f_{top}) at the transfer location is calculated using moments from **Table 2**. Since beam end strand pattern is yet to be designed, the beam stresses at transfer length are checked with the strand pattern at midspan. — LRFD Table 5.9.4.1.2-1

$$f_{top} = \frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M_{gr}}{S_t}$$

$$f_{top} = \frac{878.75}{467} - \frac{878.75 \times 7.60}{2320} + \frac{37.40 \times 12}{2320} = -0.80 \text{ ksi}$$

Allowable concrete tensile stress at release, — LRFD Table 5.9.4.1.2-1

$$\bar{f}_{ti} = 0.24\sqrt{f'_{ci}} = 0.24\sqrt{6.4} = -0.607$$

$$|f_{top}| > |\bar{f}_{ti}| \quad \textbf{NOT O.K.}$$

As shown below, the beam bottom fiber compression stress (f_{bottom}) at the transfer location is calculated using moments from **Table 2**. — LRFD Art. 5.9.4.1.1

$$f_{bottom} = \frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M_{gr}}{S_b}$$

$$f_{bottom} = \frac{878.75}{467} + \frac{878.75 \times 7.60}{2360} - \frac{37.40 \times 12}{2360} = 4.52 \text{ ksi}$$

Allowable concrete compressive stress at release, LRFD Art. 5.9.4.1.1

$$\bar{f}_{ci} = 0.6\, f'_{ci} = 0.6 \times 6.4 = 3.84 \text{ ksi}$$

$$f_{bottom} > \bar{f}_{ci}$$ **NOT O.K.**

The top and the bottom stresses exceed stress limits at the transfer location. Stresses need to be reduced by debonding a selected number of strands or adding strands to beam top flange (cut strands). For debonding, the criteria given in **Table 6** needs to be satisfied.

Table 6. Debonding Criteria

LRFD Art. 5.11.4.3	**MDOT Policy BDM Art. 7.02.18-A2**	**Bridge Attributes**
# of partially debonded strands < 25% of total number of strands	# of partially debonded strands <40% of total number of strands	8% at end **O.K.** 17% at mid **O.K.**
# of partially debonded strands < 40% of the strands in that row		25% **O.K.**
Debonded strands shall be symmetrically distributed about the centerline of the member	Strands should be debonded in pairs.	Symmetrical **O.K.**
Exterior strands in each horizontal row should be fully bonded	Where possible, debonding shall not be placed on peripheral strands.	Exterior strands are not debonded **O.K.**
The length of debonding of any strand shall be in a position which does not violate the stress limits	Debonding locations are not specified	See **Appendix E**

Please refer to **Appendix E** for the process and a comprehensive design procedure for strand debonding in order to satisfy the stress limits.

The longitudinal strand profile along the span is designed. **Figure 8** and **Figure 9** shows the strand profile at release and in service, respectively.

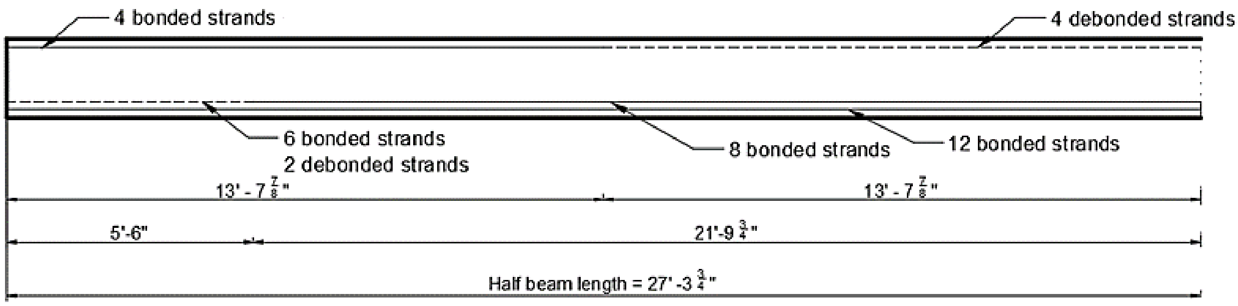


Figure 8. Longitudinal strand profile along half span of the beam at release

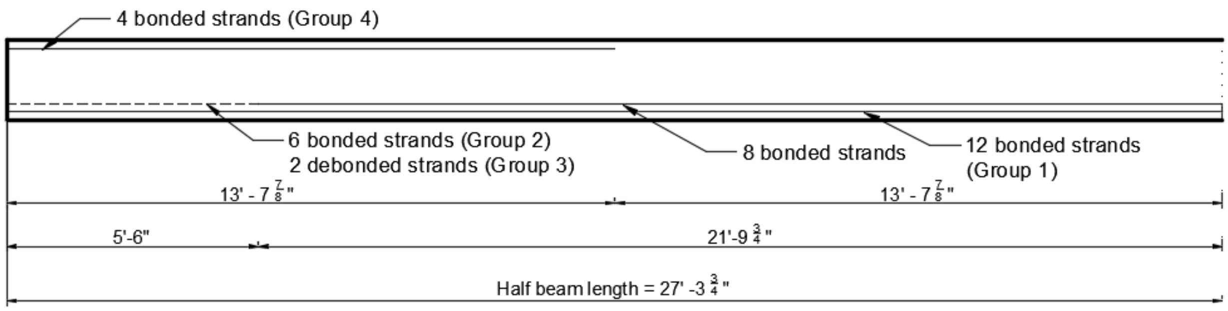


Figure 9. Longitudinal strand profile along half span of the beam in service

Strand arrangement at beam end is shown in **Figure 10**.

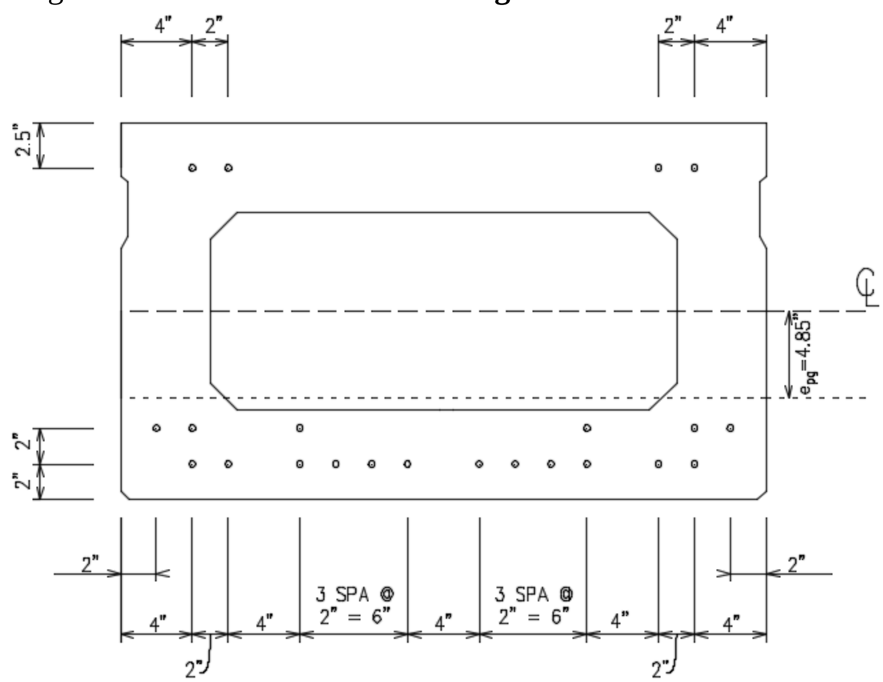


Figure 10. Strand arrangement at the end of beam

Step 8.2.5. Stress Checks along Beam Length

Stress checks are performed along the beam to evaluate the adequacy of prestressing force after debonding. Four groups of prestressing strands are defined in **Figure 8** and **Figure 9**. **Figure 11** and **Figure 12** show prestressing force in each strand group at release and in service. **Table 7** shows prestressing force and eccentricity at release and in service.

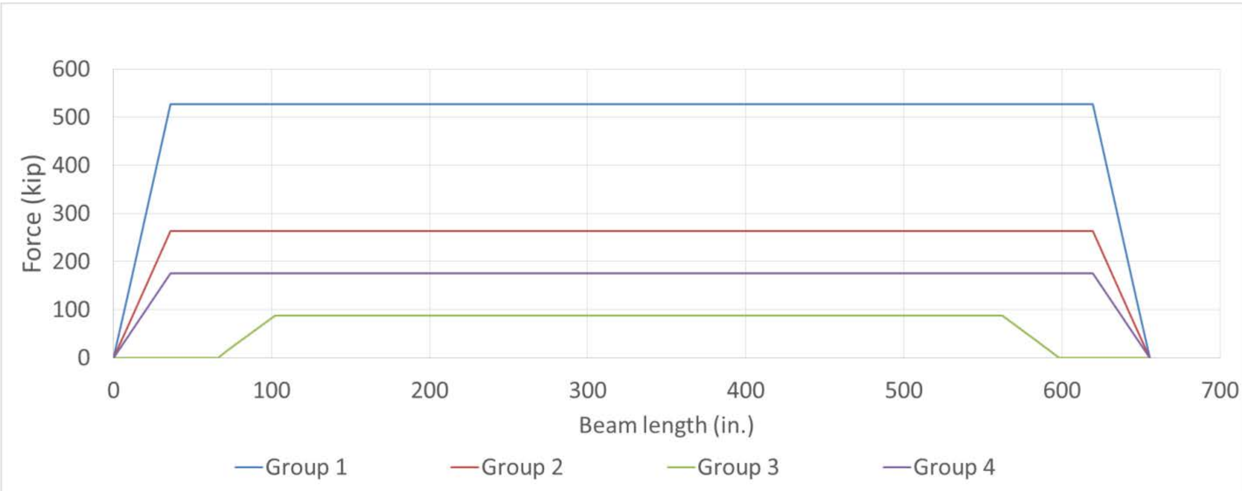


Figure 11. Change in prestressing force at release

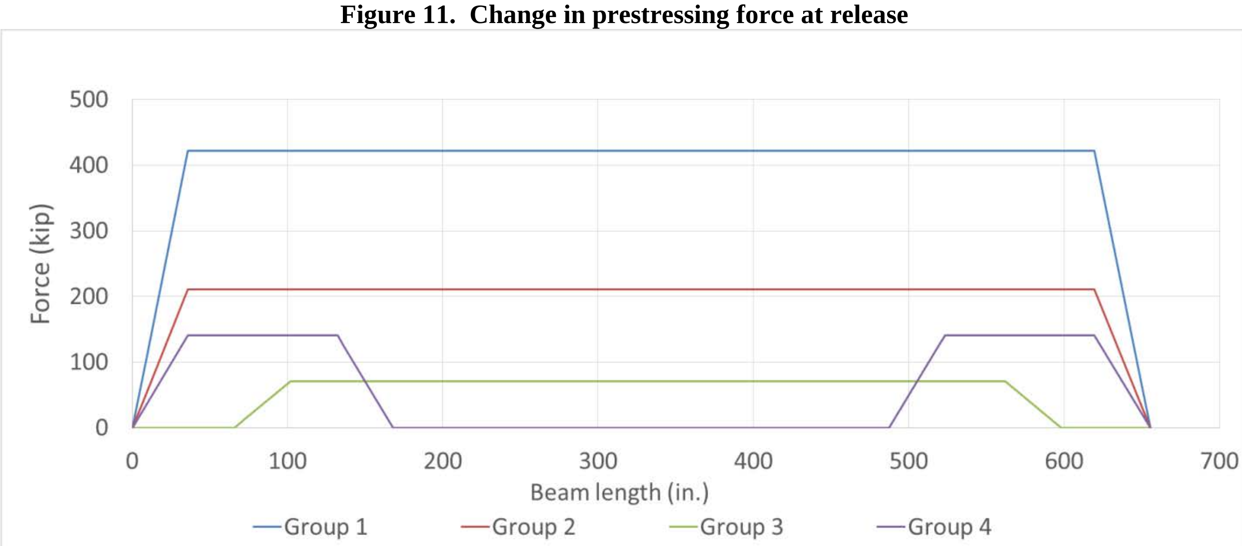


Figure 12. Change in prestressing force in service

Table 7. Prestressing Force and Eccentricity along the Half Beam Length

Location	**At release**		**In service**	
	F_i (kip)	e_{pg} (in.)	F_e (kip)	e_{pg} (in)
Beam end	0	4.86	0	4.86
CL of bearing	235	4.86	188	4.86
Transfer length	967	4.86	773	4.86
0.10 × L_{ds}	983	4.98	786	4.98
0.20 × L_{ds}	1055	4.98	703	4.98
0.30 × L_{ds}	1055	4.98	703	7.60
0.40 × L_{ds}	1055	4.98	703	7.60
0.50 × L_{ds} (Midspan)	1055	4.98	703	7.60

Stress checks are performed at multiple locations along the beam and the results are shown in **Table 8**.

Table 8. Stress Checks along the Half Beam Length

Location	Stress at release (ksi)		Stress in service (ksi)			Condition
			Top		Bottom	
			Service I Limit State		Service III Limit State	
	Top	Bottom	Permanent and transient loads (DL and LL with impact)	Permanent loads (DL only)		
Beam end	0.00	0.00				O.K.
CL of bearing	0.06	0.94	0.01	0.01	0.79	O.K.
Transfer length	0.24	3.87	0.58	0.43	2.66	O.K.
0.10 × L_{ds}	0.36	3.82	1.21	0.86	2.01	O.K.
0.20 × L_{ds}	0.61	3.88	2.15	1.53	0.62	O.K.
0.30 × L_{ds}	0.79	3.70	2.04	1.22	0.65	O.K.
0.40 × L_{ds}	0.90	3.60	2.46	1.52	0.20	O.K.
Midspan	0.93	3.56	2.60	1.63	0.04	O.K.

- Tension + Compression

Stress in strands is assumed to vary linearly along the transfer length for service limit state as per LRFD Art. 5.11.4.1.

The last column in the above table shows if stress limits are satisfied.

Step 8.3. Loss of Prestress

Step 8.3.1. Initial Losses at Beam End

Before strand release, two short-term losses occur: relaxation and elastic shortening

Relaxation Loss

Note: The equation for Δf_{pR_bt} is no longer in the AASHTO LRFD 7th edition, 2016.

$$\Delta f_{pR\_bt} = \frac{\log(24.0t)}{40.0}\left[\frac{f_{pj}}{f_{py}} - 0.55\right] f_{pj}$$ MDOT (2002)

where,

Δf_{pR_bt} = relaxation loss before transfer, ksi

t = duration of transfer, days

f_{pj} = $f_{pi} + \Delta f_{pR\_bt}$ = initial stress in strands, ksi

The tendon stress limits are specified in Table 5.9.3-1 LRFD Art .5.9.3

f_{pi} = stress in strands prior to transfer = $0.75f_{pu}$ = 202.50 ksi LRFD Table 5.9.3-1

f_{pu} = ultimate tensile strength of prestressing strands = 270 ksi BDM Art. 7.02.18.A1

f_{py} = yield strength of prestressing strands = $0.9f_{pu}$ = 243 ksi LRFD Table 5.4.4.1-1

The relaxation losses are controlled by the fabrication schedule.

Assumption:

Δf_{pR_bt1} = initial relaxation loss at one day (t = 1 day) = 2 ksi

$f_{pj} = f_{pi} + \Delta f_{pR\_bt} = 202.50 + 2 = 204.50$ ksi

$$\Delta f_{pR\_bt2} = \frac{\log(24.0 \times 1)}{40.0}\left[\frac{204.50}{243} - 0.55\right] 204.50 = 2.06 \text{ ksi}$$

The difference between assumed initial loss (Δf_{pR_bt1}) and the second iteration (Δf_{pR_bt2}) is small. Hence, Δf_{pR_bt2} is used without performing further iterations.

$$\frac{|\Delta f_{pR\_bt1} - \Delta f_{pR\_bt2}|}{\Delta f_{pR\_bt1}} \times 100 = 3\% \rightarrow \Delta f_{pR\_bt} = \Delta f_{pR\_bt2}$$

Elastic Shortening Loss

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$

LRFD Art. 5.9.5.2.3a
LRFD Eq. 5.9.5.2.3a-1

where,

Δf_{pES} = elastic shortening loss, ksi

E_p = modulus of elasticity of prestressing steel =28,500 ksi

E_{ci} = modulus of elasticity of beam at transfer = 4,631 ksi

f_{cgp} = the sum of concrete stress at the center of gravity of prestressing strands due to prestressing force and the self-weight of the beam

$$= \frac{P_i}{A_b} + \frac{P_i e_{end}^2}{I_b} - \frac{M_{gr} e_{end}}{I_b}$$

where:

P_i = prestress force at transfer

Relaxation loss (Δf_{pR_bt}) calculated in **Step 8.3.1** is a time dependent loss of prestress when a tendon is held at a constant strain. Since the stress in strands prior to transfer is 202.50 ksi (i.e., $0.75f_{pu}$), and Δ_{fpR_bt} is 2.06 ksi, strands are stressed to 204.56 ksi. At the time the strands are cut, the relaxation losses would take place and the remaining stress in strands would be 202.50 ksi.

Elastic shortening loss takes place with beam shortening when the strands are cut. With this loss, the stress in strands at transfer will be below $0.75f_{pu}$.

Since, elastic shortening losses are not known, AASHTO LRFD suggests assuming a 10% loss (i.e., 0.75 × 0.9 = 0.68).

LRFD C5.9.5.2.3a

Hence, $0.7f_{pu}$ is assumed for initial stress at transfer.

P_i = (area of strand) × (prestress stress at transfer)

= A_{ps} × Number of strands × $0.70f_{pu}$

= 0.217 × 22 × 189 = 902.29 kip

(2 out of 24 strands are debonded at the beam end)

e_{end} = eccentricity of strands at end of beam

Transfer length = 60 × (diameter of a strand) = 60 × 0.6 in. = 36 in. LRFD Art. 5.11.4.2

Thus, e_{end} is calculated at a distance of 36 in. from beam end.

Distance from beam end to bearing centerline = 8.75 in.

Distance to centroid of strands from the beam bottom at 36 in.

= (12 × 2 + 6 × 4 + 4 × 18.50)/22

= 5.55 in.

e_{end} = y_b – 5.55= 10.40 – 5.55

= 4.85 in.

A_b = area of beam = 467 in.^{2}

I_b = moment of inertia of beam = 24,600 in.^{4}

Moment at 36 in. from the beam end at release due to beam self-weight

= 37.40 kip-ft = 448.80 kip-in

$$f_{cgp} = \frac{902.29}{467} + \frac{902.29 \times 4.85^2}{24600} - \frac{448.80 \times 4.85}{24600} = 2.71 \text{ ksi}$$

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp} = \frac{28500}{4655} \times 2.71 = 16.59 \text{ ksi}$$

LRFD Eq. 5.9.5.2.3a-1

Elastic shortening reduces the prestressing force, which in turn reduces elastic shortening. This effect is evaluated through an iterative process.

Parameter	Initial assumptions	Results of the iterative process			
		1st	2nd	3rd	4th
Δf_{pES}	16.59 ksi	16.29 ksi	16.32 ksi	16.31 ksi	16.31 ksi
f_{cgp}	2.71 ksi	2.66 ksi	2.67 ksi	2.66 ksi	2.66 ksi

With four iterations, the solution converges. Thus, Δf_{pES} = 16.31 ksi

Initial losses at beam end = $\Delta f_{pES} + \Delta f_{pR_{bt}}$

$= 16.31 + 2.06 = 18.37$ ksi

Step 8.3.2. Losses at Midspan

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$$

LRFD Art. 5.9.5.1

LRFD Eq. 5.9.5.1-1

where,

Δf_{pT} = total losses, ksi

Δf_{pES} = sum of all losses or gains due to elastic shortening or extension at the time of application of prestress and/or external loads, ksi

Δf_{pLT} = losses due to long-term shrinkage and creep of concrete, and relaxation of steel, ksi

Elastic Shortening Losses

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp}$$ LRFD Eq. 5.9.5.2.3a-1

where,

Δf_{pES} = elastic shortening loss, ksi

E_p = modulus of elasticity of prestressing steel = 28,500 ksi

E_{ci} = modulus of elasticity of beam at transfer = 4,655 ksi

f_{cgp} = the sum of concrete stress at the center of gravity of prestressing strands due to the prestressing force at transfer and the self-weight of the beam at maximum moment location

$$= \frac{P_i}{A_b} + \frac{P_i e_{pg}^2}{I_b} - \frac{M_{gr} e_{pg}}{I_b}$$

e_{pg} = eccentricity of strands at midspan at transfer = 4.98 in.

M_{gr} = moment due to beam weight at release at midspan =181.27 kip-ft

A_b = area of beam = 467 in.^{2}

I_b = moment of inertia of beam = 24,600 in.^{4}

Since, elastic shortening losses are not known, AASHTO LRFD suggests assuming a 10% loss (i.e., 0.75 × 0.9 = 0.68). LRFD C5.9.5.2.3a

Hence, $0.7f_{pu}$ is used as the initial assumption for the stress at transfer.

P_i = prestressing force at transfer = A_{ps} × Number of strands × $0.70f_{pu}$

= 0.217 × 24 × 189 = 984.31 kip

(24 strands include the top strands)

$$f_{cgp_1} = \frac{984.31}{467} + \frac{984.31 \times 4.98^2}{24600} - \frac{181.27 \times 12 \times 4.98}{24600}$$

$$= 2.66 \text{ ksi}$$

Initial elastic losses, $\Delta f_{pES\_1} = \frac{28500}{4655} \times 2.66 = 16.29$ ksi LRFD Eq. 5.9.5.2.3a-1

1st iteration:

P_i = force at transfer = A_{ps} × Number of strands × (f_{pi} - Δf_{pES_1})

= 0.217 × 24 × (202.50 - 16.29) = 969.78 kip

$$f_{cgp_2} = \frac{969.78}{467} + \frac{969.78 \times 4.98^2}{24600} - \frac{181.27 \times 12 \times 4.98}{24600}$$

$$= 2.61 \text{ ksi}$$

$$\Delta f_{pES\_2} = \frac{28500}{4655} \times 2.61 = 15.98 \text{ ksi}$$

2nd iteration:

P_i = force at transfer = A_{ps} × Number of strands × (f_{pi} - Δf_{pES_2})

= 0.217 × 24 × (202.50 - 15.98) = 971.40 kip

$$f_{cgp_3} = \frac{971.40}{467} + \frac{971.40 \times 4.98^2}{24600} - \frac{181.27 \times 12 \times 4.98}{24600}$$

$$= 2.62 \text{ ksi}$$

$$\Delta f_{pES\_3} = \frac{28500}{4655} \times 2.62 = 16.04 \text{ ksi}$$

Iterations converge to losses as shown below;

Parameter	Initial assumptions	Results of the iterative process	
		1st	2nd
Δf_{pES}	16.29 ksi	15.98 ksi	16.04 ksi
f_{cgp}	2.66 ksi	2.61 ksi	2.62 ksi

Elastic shortening losses:

$$\Delta f_{pES} = 16.04 \text{ ksi}$$

Time-Dependent Losses (Approximate Estimate)

The long-term prestress loss, Δf_{PLT}, due to creep of concrete, shrinkage of concrete, and relaxation of steel shall be estimated using the following equation:

$$\Delta f_{PLT} = 10.0 \frac{f_{pi} A_{ps}}{A_b} \gamma_h \gamma_{st} + 12.0 \gamma_h \gamma_{st} + \Delta f_{PR}$$ LRFD Eq. 5.9.5.3-1

where,

f_{pi} = stress in prestressing steel immediately prior to transfer
= 202.50 ksi

A_{ps} = A_{pst} = total area of prestressing strands = 0.217 × 24
= 5.208 in.^{2}

A_b = area of beam= 467 in.^{2}

H = relative humidity = 75% (Michigan climate) LRFD Fig.5.4.2.3.3-1

γ_h = correction factor for relative humidity of the ambient air LRFD Eq. 5.9.5.3-2
= 1.7 – 0.01H = 1.7 – 0.01 × 75 = 0.95

γ_{st} = correction factor for specified concrete strength at time of prestress transfer LRFD Eq. 5.9.5.3-3

$$= \frac{5}{1 + f'_{ci}} = \frac{5}{1 + 6.4} = 0.68$$

Δf_{pR} = relaxation loss = 2.40 ksi LRFD Art. 5.9.5.3

Therefore,

$$\Delta f_{PLT} = 10.0 \frac{202.50 \times 5.208}{467} \times 0.95 \times 0.68 + 12.0 \times 0.95 \times 0.68 + 2.40$$

$$\Delta f_{PLT} = 24.74 \text{ ksi}$$

Total losses at midspan

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$$ LRFD Eq. 5.9.5.1-1

$$\Delta f_{pT} = 16.04 + 24.74 = 40.78 \text{ ksi}$$

Ratio of effective prestress after losses to stress prior to transfer, η

$$= \frac{f_{pi} - \Delta f_{pT}}{f_{pi}} = \frac{202.50 - 40.78}{202.50} = 0.80$$

The losses calculated with the approximate estimate are 20%, and is equal to the lump sum estimate. Hence, the stress limit check will not be repeated.

Step 8.4. Flexural Design Check for Strength Limit State

Step 8.4.1. Ultimate Moment

Ultimate moment for Strength I limit state, M_u — LRFD Table 3.4.1-1 and Table 3.4.1-2

$$= 1.25(DC) + 1.5(DW) + 1.75(LL + IM)$$

The ultimate moment at midspan is calculated from moments given in **Table 3**.

$$M_u = 1.25(M_g + M_D + M_{cd} + M_b) + 1.5(M_{ws}) + 1.75(M_{HL\text{-}M})$$
$$= 1.25(171.49 + 265.34 + 13.42 + 15.53) + 1.5(52.93) + 1.75(695.51)$$
$$= 1878.76 \text{ kip-ft}$$

Step 8.4.2. Average Stress in Prestressing Steel

Average stress in prestressing steel when $f_{pe} \geq 0.5 f_{pu}$ — LRFD Art. 5.7.3.1.1

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right)$$ LRFD Eq. 5.7.3.1.1-1

where,

f_{pu} = specified tensile strength of prestressing steel = 270 ksi

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$$ LRFD Eq. 5.7.3.1.1-2

= 0.28 for low relaxation strands — LRFD Table C5.7.3.1.1-1

d_p = distance from extreme compressive fiber to centroid of prestressing strands
= $h_c - y_{bs}$ = 32 – 2.8 = 29.2 in.

c = distance between the neutral axis and extreme compressive fiber for a rectangular section

$$= \frac{A_{ps}f_{pu} + A_s f_s - A'_s f'_s}{0.85 f'_c \beta_1 b_{eff} + kA_{ps}\frac{f_{pu}}{d_p}}$$ LRFD Eq. 5.7.3.1.1-4

$A_{ps} = A_{pst}$ = total area of prestressing steel = 20 × 0.217 = 4.34 in.^{2}

A_s = area of mild steel tension reinforcement = 0 in.^{2}

A'_s = area of mild steel compression reinforcement = 0 in.^{2}

f'_c = compressive strength of deck concrete = 4.0 ksi

f_s = stress in mild steel tension reinforcement at nominal flexural resistance, ksi

f'_s = stress in mild steel compression reinforcement at nominal flexural resistance, ksi

b_{eff} = effective width of compression flange = 6.25 ft = 75 in.

β_1 = stress factor of compression block = 0.85 LRFD Art. 5.7.2.2

Therefore,

$$c = \frac{4.34 \times 270 + 0 - 0}{0.85(4.0)(0.85)(75) + (0.28)(4.34)\left(\frac{270}{29.2}\right)} = 5.14 \text{ in.}$$

Since c < t_s = 9 in., rectangular section assumption is valid.

The beam is tension controlled if $c/d_p \leq 0.375$ LRFD Art. 5.7.2.1.

$$\frac{c}{d_p} = \frac{5.14}{29.2} = 0.176 < 0.375 \;\; \textbf{O.K.}$$

Depth of the equivalent stress block, a = $\beta_1 c$ = 0.85 × (5.14) = 4.37 in.

The average stress in prestressing steel, (f_{ps}) $= 270\left(1 - 0.28 \times \frac{5.14}{29.2}\right) = 256.69$ ksi

Step 8.4.3. Nominal Flexural Resistance

LRFD Eq. 5.7.3.2.2-1

Nominal flexural resistance, M_n

$$= A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_s\left(d_s - \frac{a}{2}\right) - A`_s f`_s\left(d`_s - \frac{a}{2}\right)$$

$$= 4.34 \times 256.69 \times \left(29.2 - \frac{4.37}{2}\right) + 0 - 0$$

$$= 30095.64 \text{ kip} - \text{in.} = 2507.97 \text{ kip} - \text{ft}$$

Deck reinforcement can be included in nominal flexural resistance calculation if amounts are known. In most cases, the deck reinforcement constribution is very small and can be omitted.

Step 8.4.4. Factored Flexural Resistance

Factored flexural resistance, M_r = ϕM_n LRFD Eq.5.7.3.2.1-1

Resistance factor, ϕ = 1.00 LRFD Art. 5.5.4.2.1,

for a tension-controlled section

M_r (2507.97 kip – ft) > M_u (1878.76 kip – ft)….. **O.K.** LRFD Eq. 5.7.3.2.1-1

Step 8.4.5. Maximum Reinforcement LRFD Art. 5.7.3.3.1 and Art. 5.5.4.2.1

The check for maximum reinforcement limits was discontinued by AASHTO in 2005. Adequate ductility is ensured with a tension-controlled design of the beam (See **Step 8.4.2**). When beam flexural resistance is not tension-controlled, the resistance factor for the Strength Limits State I is decreased.

Step 8.4.6. Minimum Reinforcement LRFD Art. 5.7.3.3.2

At any section, the amount of prestressed and nonprestressed tensile reinforcement should be adequate to develop a factored flexural resistance, M_r, equal to the lesser of:

- *1.2 times the cracking strength determined on the basis of elastic stress distribution and the modulus of rupture, and,*
- *1.33 times the factored moment required by the applicable strength load combination.*

Check at midspan:

$$M_{cr} = \gamma_3 \left[(\gamma_1 f_r + \gamma_2 f_{cpe})S_c - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right]$$ LRFD Eq. 5.7.3.3.2-1

where,

γ_1 = flexural cracking variability factor =1.6 LRFD Art. 5.7.3.3.2

γ_2 = prestress variability factor =1.1, for bonded tendons LRFD Art. 5.7.3.3.2,

γ_3 = ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement = 1.0, for prestressed concrete structures. LRFD Art. 5.7.3.3.2,

f_r = concrete modulus of rupture $= 0.24\sqrt{f'_c}$ LRFD Art. 5.4.2.6

$= 0.24\sqrt{6.8} = 0.626$ ksi

f_{cpe} = compressive stress in concrete due to effective prestress (after allowance for all losses) at the extreme fiber of the section where tensile stress is generated by externally applied loads

$$= \frac{\eta F_i}{A_b} + \frac{\eta F_i e_{pg}}{S_b} = \frac{0.8 \times 878.75}{467} + \frac{0.8 \times 878.75 \times 7.60}{2360} = 3.770 \text{ ksi}$$

M_{dnc} = total unfactored dead load moment acting on the monolithic or noncomposite section

$= M_g + M_D + M_{cd}$ = (171.49 kip-ft + 265.34 kip-ft + 13.42 kip-ft) × 12 in./ft

= 5,403 kip – in.

S_c = S_{bc} = section modulus for the extreme fiber of the composite section where tensile stress is generated by externally applied loads

= 5,207 in.^{3}

S_{nc} = S_b = section modulus for the extreme fiber of the monolithic or noncomposite section where tensile stress is generated by externally applied loads

= 2,360 in.^{3}

$$M_{cr} = 1.0 \times \left[(1.6 \times 0.626 + 1.1 \times 3.770) \times 5207 - 5403 \times \left(\frac{5207}{2360} - 1\right)\right]$$

$$= 20290.82 \text{ kip} - \text{in.} = 1690.90 \text{ kip} - \text{ft}$$

$1.2M_{cr}$ = 2029.08 kip-ft LRFD Art. 5.7.3.3.2

$1.33M_u$ = 2498.75 kip-ft

Min ($1.2M_{cr}$; $1.33M_u$) =$1.2M_{cr}$

M_r (= 2507.97 kip – ft) ≥ $1.2M_{cr}$ (= 2029.08 kip-ft) **O.K.**

Step 8.5. Lifting Stress Check

*MDOT practice is to check stresses at debonding point and over the supports/lifting points due to the cantilever moment. The check is performed by assuming support/lifting point locations at 3.0 ft from the beam end (**Figure 13**). If the stress conditions are not satisfied, the supports are moved towards the beam end. MDOT procedure does not consider the dynamic effects, and a dead load multiplier of 1 is used.*

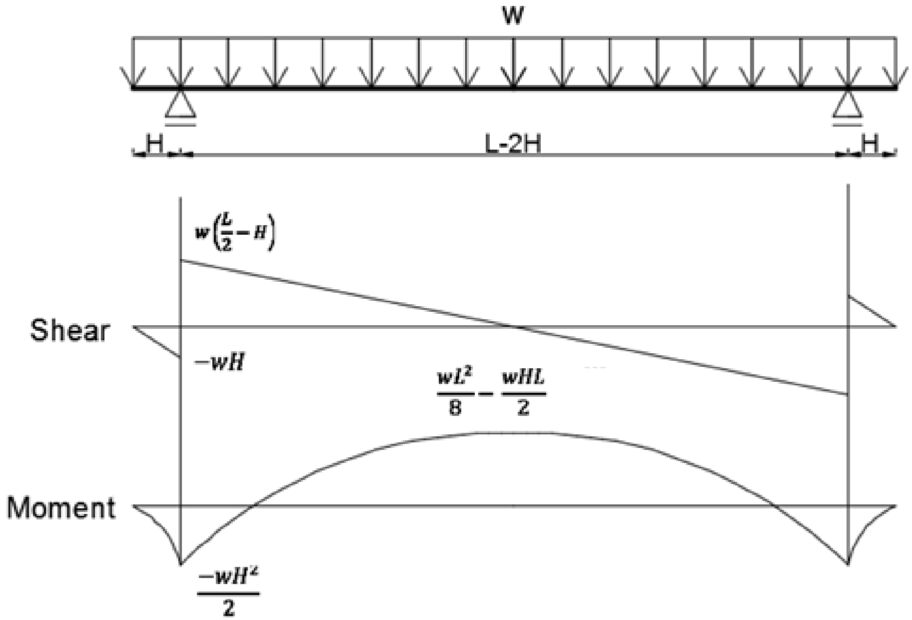


Figure 13. Moment and shear diagrams for lifting stress check

Moment at a distance of x from a support $= \frac{wLx}{2} - \frac{w}{2}(H + x)^2$

Distance between temporary support/lifting points and beam end, H = 3 ft

Stresses over Temporary Support

Moment at temporary support/lifting point located at 3 ft, $M = \frac{-wH^2}{2}$ where $w = w_g$

$$M = \frac{-w_g H^2}{2} = \frac{-0.486 \times 3^2}{2} = -2.19 \text{ kip} - \text{ft}$$

Allowable concrete tensile stress, $\bar{f}_{tt}$ $= 0.24\sqrt{f'_{ct}}$

Allowable concrete compressive stress, $\bar{f}_{ct}$ $= 0.60\ f_{ct}'$

where,

f_{ct}' = concrete compressive strength for lifting stress check

$$= \frac{f'_c + f'_{ci}}{2} = \frac{6.8 + 6.4}{2} = 6.60 \text{ ksi}$$

Hence,

$\bar{f}_{tt} = 0.24 \times \sqrt{6.60}$ = 0.62 ksi

$\bar{f}_{ct} = 0.60 \times 6.60$ = 3.96 ksi

Prestressing strand eccentricity (e_{pg}) at 3 ft = 4.86 in.

Prestressing force at 3 ft, $F_i = 0.217 \times 22 \times 202.50 = 967$ kip

The tensile stresses are calculated and compared to the allowable stresses at top:

$$\frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M}{S_t} \geq (-f_{tt})$$

$$\frac{967}{467} - \frac{967 \times 4.86}{2320} + \frac{-2.19 \times 12}{2320} = 0.034 \text{ ksi} > -0.620 \text{ ksi} \qquad \textbf{O.K.}$$

The compressive stresses are calculated and compared to the allowable stresses at bottom:

$$\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M}{S_b} \leq (f_{ct})$$

$$\frac{967}{467} + \frac{967 \times 4.86}{2360} - \frac{-2.19 \times 12}{2360} = 4.07 \text{ ksi} > 3.96 \text{ ksi} \qquad \textbf{NOT O.K.}$$

Since the stress check is not satisfied, the supports/lifting points need to be moved 1 ft closer to the beam ends.

Moment at temporary support/lifting point located at 2ft, M

$$= \frac{-w_g H^2}{2} = \frac{-0.486 \times 2^2}{2}$$

$$= -0.972 \text{ kip} - \text{ft}$$

Prestressing strand eccentricity (e_{pg}) at 2 ft = 4.86 in.

Prestressing force increases linearly withing the transfer length of 3 ft.

Hence,

Prestressing force at 2 ft, F_i = (0.217 × 22 × 202.50) × (2/3) = 644.49 kip

The tensile stresses are calculated and compared to the allowable stresses at top:

$$\frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M}{S_t} \geq (-f_{tt})$$

$$\frac{644.49}{467} - \frac{644.49 \times 4.86}{2320} + \frac{-0.972 \times 12}{2320} = 0.025 \text{ ksi} > -0.62 \text{ ksi} \qquad \textbf{O.K.}$$

The compressive stresses are calculated and compared to the allowable stresses at bottom:

$$\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M}{S_b} \leq (f_{ct})$$

$$\frac{644.49}{467} + \frac{644.49 \times 4.86}{2360} - \frac{-0.972 \times 12}{2360} = 2.71 \text{ ksi} < 3.96 \text{ ksi} \qquad \textbf{O.K.}$$

The stresses due to positive moment at midspan need to be checked with either 24 strands or 20 strands. The number of strands used for the check depends on the schedule for cutting the top strands (cut strands).

Stresses at Debonding Point

The distance between temporary support/lifting point and debonding point, L_{deb} (**Figure 8**)

= 5.5 – 2.0 = 3.5 ft

Moment at debonding point, M

$$= w_g \frac{L}{2} L_{dra} - \frac{w_g}{2}(H + L_{dra})^2$$

where,

L = L_b

$$M = 0.486 \times \frac{54.625}{2} \times 3.5 - \frac{0.486}{2} \times (2 + 3.5)^2$$

$$= 39.11 \text{ kip} - \text{ft}$$

Prestressing strand eccentricity (e_{pg}) at debonding point (i.e., at 5.5 ft) = 4.98 in.

(refer to **Figure 8** and **Table 7** for additional information)

The tensile stresses are calculated and compared to the allowable stresses at top:

$$\frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M}{S_t} \geq (-f_{tt})$$

$$\frac{967}{467} - \frac{967 \times 4.98}{2320} + \frac{39.11 \times 12}{2320} = 0.20 \text{ ksi} > -0.62 \text{ ksi}$$ **O.K.**

The compressive stresses are calculated and compared to the allowable stresses at bottom:

$$\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M}{S_b} \leq (f_{ct})$$

$$\frac{967}{467} + \frac{967 \times 4.98}{2360} - \frac{39.11 \times 12}{2360} = 3.91 \text{ ksi} < 3.96 \text{ ksi}$$ **O.K.**

Some highway agencies require shear checks at the support and lifting points. However, as per MDOT practice, shear check is omitted.

Step 8.6. Continuity Connection Check

Step 8.6.1. Negative Moment Connection at Strength Limit State

The scope of this example is limited to prestressed concrete beam design. Hence, the continuity connection check is not included. As needed, the following steps can be followed to complete the continuity connection check:

a) Calculate the nominal flexural resistance (M_n) using reinforcement amount and spacing in the cast-in-place deck slab over pier. — LRFD Eq. 5.7.3.2.2-1

b) Select the resistance factor, ϕ — LRFD Art. 5.5.4.2.1

c) Calculate the factored flexural resistance ($M_r = \phi M_n$) — LRFD Eq. 5.7.3.2.1-1

d) Calculate the ultimate moment over the pier (M_u) using Strength I limit state.

e) Check if $M_u < M_r$ — LRFD Eq. 5.7.3.2.1-1

Step 8.6.2. Service State – Compressive Strength Check at Negative Moment Region

In the negative moment region, girder bottom flange is subjected to additional compressive stress proportional to the negative moment at the section. The critical section for compression check is established as the section in the negative moment region where compression due to prestressing is the greatest. Hence, the critical section is at a distance equal to the transfer length measured from the beam end. Analysis is performed by assuming elastic cracked section to check if beam bottom flange compressive strength exceeds $0.6f'_c$.

Step 8.6.3. Crack Control — LRFD Art. 5.7.3.4

Tension reinforcement needs to be distributed to control flexural cracking. — LRFD Art. C5.7.3.4

(a) Calculate the overall thickness of the component, h.

(b) Calculate the thickness of concrete cover measured from extreme tension fiber to center of the flexural reinforcement located closest thereto, d_c.

(c) Calculate $\beta_s = 1 + \frac{d_c}{0.7(h-d_c)}$

(d) Determine the exposure factor, γ_e.

(e) Calculate the tensile stress in the mild steel reinforcement at the service limit state, f_{ss}

(f) Check if $f_{ss} \leq 0.6f_y$. Otherwise, change the amount of steel to satisfy the condition.

(g) Maintain the spacing, s, of mild steel reinforcement in the layer closest to the tension face such that $s \leq \frac{700\,\gamma_c}{\beta_s f_{ss}} - 2d_c$ — LRFD Eq. 5.7.3.4-1

Step 8.6.4. Positive Moment Connection at Strength Limit State

A positive moment may develop at intermediate piers under the effect of prestressing, permanent loads, and creep and shrinkage (if creep and shrinkage is considered in the design). Reinforcement is provided at beam bottom flange at intermediate pierss to resist the factored positive moment (M_u) at negative moment locations. — NCHRP (2004), FHWA (2003)

MDOT currently does not perform this calculation.

Step 8.7. Shear Design

Transverse shear reinforcement is required when $V_u > 0.5\phi(V_c + V_p)$ — LRFD Eq. 5.8.2.4-1

where,

V_u = total factored shear force

V_c = shear strength provided by concrete

V_p = component of the effective prestressing force in the direction of the applied shear

ϕ = resistance factor for shear = 0.9 — LRFD Art. 5.5.4.2.1

Transverse shear is evaluated to identify the length of beam requiring reinforcement.
Even if $V_u \leq 0.5\phi(V_c + V_p)$, shear reinforcement is provided in beams at the maximum spacing.

In this example, transverse shear design procedure is demonstrated on the critical section near the support.

Step 8.7.1. Critical Section for Shear

The critical section near the support is taken as the effective shear depth, d_v, from the internal face of the support. LRFD Art. 5.8.3.2

d_v = effective shear depth (i.e., distance between resultants of tensile and compressive forces) LRFD Art. 5.8.2.9

= $(d_e - a/2) \geq$ greater of $0.9d_e$ or $0.72h_c$

d_e = effective depth from extreme compression fiber to centroid of the tensile force in the tensile reinforcement

a = depth of compression block

h_c = overall depth = 32 in.

For flexural members, the distance between the resultants of the tensile and compressive forces due to flexure can be determined from: LRFD Eq. C5.8.2.9-1

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}}$$

where, M_n = 2,508 kip-ft

$A_{ps} = A_{pst}$ = 4.34 in.^{2}

f_{ps} = 256.69 ksi (from **Step 8.4.2)**

A_s = 0 in^2.

Therefore, d_v = 27 in.

Check if $d_v \geq$ Max $(0.9d_e, 0.72h_c)$ LRFD Art. 5.8.2.9

27 in. $\geq$ Max (26.28 in., 23.04 in.) **O.K.**

Bearing width is not yet established, and conservatively, the distance to the critical section of the shear can be measured from the bearing centerline.

Distance to critical section for shear from bearing centerline, x_{cr}

$= d_v$ = 27 in. = 0.042 L_{ds}

Step 8.7.2. Factored Moment and Shear at Critical Location

In designing continuous for live load (CLL) bridges, the larger value for shear represents the required strength calculated for both simple supported and continuous spans.

As shown in **Table 3**; shear in simple span, $V_{HL\text{-}M}$, governs. Hence, the design is performed accordingly. The following tables show moment and shear at the critical section for shear, $0.042L_{ds}$.

Location	Moment (kip – ft)					
	Simple span					
	M_g	M_D	M_{cd}	M_b	M_{ws}	$M_{HL\text{-}M}$
0.042 L_{ds}	27.69	42.85	1.14	2.51	8.55	106.90

Location	Shear (kip)					
	Simple span					
	V_g	V_D	V_{cd}	V_b	V_{ws}	$V_{HL\text{-}M}$
0.042 L_{ds}	11.83	18.29	0.51	1.07	3.65	68.57

Since simple span controls the shear design, moment and shear due to support settlement is not a concern.

Factored moment at critical section (0.042L_{ds}), M_u

$= 1.25(M_g + M_D + M_{cd} + M_b) + 1.5(M_{ws}) + 1.75(M_{HL\text{-}M})$ LRFD Table 3.4.1.1 (Strength I)

$= 1.25(27.69 + 42.85 + 1.14 + 2.51) + 1.5(8.55) + 1.75(106.90)$

$= 292.64$ kip-ft

Factored shear at critical section (0.042L_{ds}), V_u

$= 1.25(V_g + V_D + V_{cd} + V_b) + 1.5(V_{ws}) + 1.75(V_{HL\text{-}M})$ LRFD Table 3.4.1.1 (Strength I)

$= 1.25(11.83 + 18.29 + 0.51 + 1.07) + 1.5(3.65) + 1.75(68.57)$

$= 165.10$ kip

Step 8.7.3. Shear Strength Provided by Concrete

Concrete contribution to nominal shear resistance, V_c LRFD Eq. 5.8.3.3-3

$$= 0.0316\beta\sqrt{f'_c}b_v d_v$$

where,

β = factor indicating the ability of diagonally cracked concrete to transmit tension and shear

b_v = effective web width taken as the minimum web width within the depth d_v

The following calculations are performed for evaluating β.

Calculation of Strain in Flexural Reinforcement

Strain in flexural reinforcement, ε_s

$$= \frac{\left|\frac{M_u}{d_v}\right| + 0.5N_u + \left|V_u - V_p\right| - A_{ps}f_{po}}{(E_sA_s + E_pA_{ps})}$$ LRFD Eq. 5.8.3.4.2-4

where,

N_u = applied factored normal force at critical section = 0 kip

V_p = component of the effective prestressing force in the direction of the applied shear, = 0 kip

A_{ps} = area of prestressing strands on the flexural tension side of the member =18 × 0.217= 3.91 in.^{2}

f_{po} = parameter taken as modulus of elasticity of prestressing strands multiplied by the locked-in difference in strain between the prestressing strands and the surrounding concrete

For prestressed members, f_{po} can be assumed as $0.7f_{pu}$ LRFD Art. 5.8.3.4.2

$= 0.7 \times 270 = 189$ ksi

Within the transfer length, f_{po} shall be increased linearly from zero at the location where the bond between the strands and concrete commences to its full value at the end of the transfer length. LRFD Art. 5.8.3.4.2

In this example,

d_v + the distance from beam end to the bearing centerline

= 27 + 8.75 in. = 35.75 in. < transfer length of 36 in.

Hence,

$f_{po} = 189 \times (35.75/36) = 187.69$ ksi LRFD Art. 5.8.3.4.2

$|M_u| > |V_u - V_p|\, d_v$ LRFD Art. 5.8.3.4.2

$|M_u| = 292.64 \times 12 = 3511.68$ kip-in

$||V_u - V_p||\, d_v = |165.10 – 0| \times 27 = 4457.70$ kip-in

$|M_u| < |V_u - V_p|\, d_v$ → **NOT O.K.**

Hence,

$M_u = 4457.70$ kip-in = 371.48 kip-ft

$$\varepsilon_s = \frac{\frac{|371.48 \times 12|}{27} + 0 + |165.10 - 0| - 3.91 \times 187.69}{(0 + 28500 \times 3.91)} = -0.004 < 0$$

If ε_s calculated from Eq. 5.8.3.4.2-4 is negative, it can be taken as zero or recalculated with the denominator of Eq. 5.8.3.4.2-4 replaced by ($E_sA_s + E_pA_{ps} + E_cA_c$). However, ε_s should not be taken as less than $–0.40 \times 10^{–3}$. LRFD Art.5.8.3.4.2

where,

A_c = area of concrete on the flexural side of the member (**Figure 14**) LRFD Fig. 5.8.3.4.2.2

= 291.81 in.^{2}

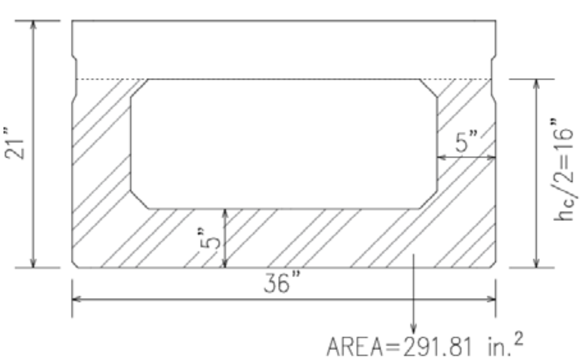


Figure 14. Area of the concrete on the flexural tension side

The flexural tension side of the member shall be taken as the half-depth containing the flexural tension zone. LRFD Fig. 5.8.3.4.2-1

$$\varepsilon_s = \frac{\frac{|371.48 \times 12|}{27} + 0 + |165.10 - 0| - 3.91 \times 187.69}{(0 + 28500 \times 3.91 + 291.81 \times 4749)}$$

$$= -0.00027 > -0.40 \times 10^{-3} \quad \textbf{O.K.}$$

β and θ Calculation

Assume that the section contains at least the minimum transverse reinforcement.

β = factor indicating ability of diagonally cracked concrete to transmit tension and shear

$= \frac{4.8}{1 + 750\varepsilon_s}$ LRFD Eq.5.8.3.4.2-1

$= \frac{4.8}{1 + 750 \times (-2.7 \times 10^{-4})} = 6.02$

θ = angle of inclination of diagonal compressive stress

$= 29 + 3500\varepsilon_s = 28.06°$ LRFD Eq. 5.8.3.4.2-3

MDOT procedure is to iterate for β and θ since the AASHTO LRFD 7th edition (2016) equations are not yet implemented in the BDS.

Shear is carried by the webs.

Total web thickness, $t_{ws} = b_v = 10$ in.

Shear strength provided by concrete, V_c

$= 0.0316\beta\sqrt{f'_c}b_v d_v$ LRFD Eq. 5.8.3.3-3

$= 0.0316 \times 6.02 \times \sqrt{6.8} \times 10 \times 27 = 133.94$ kip

Step 8.7.4. Shear Reinforcement Requirements

Check if the following condition is satisfied; LRFD Eq. 5.8.2.4-1

$V_u > 0.5\phi(V_c + V_p)$

$\phi = 0.9$ for normal weight concrete LRFD Art. 5.5.4.2.1

165.10 kip > 0.5 × 0.9 × (133.94 + 0) = 60.27 kip **O.K.**

Therefore, the transverse shear reinforcement is required.

Area of Shear Reinforcement

$$\frac{V_u}{\phi} \leq V_n = V_c + V_s + V_p$$ LRFD Eq. 5.8.3.3-1

Strength required from shear reinforcement,

$V_s = \frac{V_u}{\phi} - V_c - V_p = \frac{165.10}{0.9} - 133.94 - 0 = 49.50$ kip

Shear strength provided by reinforcement; LRFD Eq. 5.8.3.3-4

$$V_s = \frac{A_v f_{yh} d_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$$

where,

A_v = area of shear reinforcement within a distance s

s = spacing of shear reinforcements

f_{yh} = specified yield strength of shear reinforcement = 60 ksi

α = angle of inclination of shear reinforcement to longitudinal axis

= 90° (vertical shear reinforcement)

Area of shear reinforcement for a spacing s

$$A_v = \frac{V_s s}{f_{yh} d_v \cot\theta} = \frac{49.50\times s}{60\times 27\times \cot 28.06°} = 0.016(s)\ \text{in.}^2$$

Therefore, s = A_v /0.016

Select, #4 - 2 leg stirrups. Thus, A_v = 0.4 in.^{2}

Spacing, s = (0.4) / (0.016) = 25 in.

Shear reinforcement spacing provided in **Table 9** is calculated by considering d_v and associated parameters at the respective sections.

Spacing of Shear Reinforcement

Maximum allowable spacing of shear reinforcement shall be checked. LRFD Art. 5.8.2.7

Check if the concrete shear stress limitation is satisfied; LRFD Eq. 5.8.2.7-1

$v_u < 0.125f'_c$

$$v_u = \frac{|V_u - \phi V_p|}{\phi t_{ws} d_v} = \frac{|165.10 - 0.9\times 0|}{0.9\times 10\times 27} = 0.68\ \text{ksi}$$ LRFD Eq. 5.8.2.9-1

$0.125 \times f'_c = 0.125 \times 6.8 = 0.85$ ksi LRFD Eq. 5.8.2.7-1

0.68 ksi < 0.85 ksi

Then, LRFD Eq. 5.8.2.7-2

$s_{max} \leq \min(0.8d_v, 24\ \text{in.})$

$s_{max} \leq \min(21.6\ \text{in.}, 24\ \text{in.}) = 21.6$ in.

Use #4 – 2 leg stirrups at 12 in. spacing

Shear resistance provided by shear reinforcement, V_s LRFD Eq.5.8.3.3-4

$$= \frac{A_v f_{yh} d_v \cot\theta}{s} = \frac{0.40\times 60\times 27\times \cot 28.06°}{12} = 101.30\ \text{kip}$$

A larger spacing of shear reinforcement could have been specified. However, minimum interface shear reinforcement requirements will necessitate additional steel. (See **Step 8.8**)

Check the adequacy of the section with shear reinforcement;

$$\frac{V_u}{\phi} \le V_n = V_c + V_s + V_p$$ LRFD Eq. 5.8.3.3-1

$$V_n = 133.94 + 101.30 + 0 = 235.24 \text{ kip}$$

$$\frac{165.10}{0.9} = 183.44 \text{ kip} \le 235.24 \text{ kip}$$ **O.K.**

Minimum Reinforcement Requirement

The area of shear reinforcement should be more than

$$0.0316\sqrt{f'_c}\frac{t_w s}{f_{yh}} = 0.0316\sqrt{6.8}\frac{(10)(12)}{60} = 0.16 \text{ in.}^2 < A_v \text{ provided}$$ LRFD Eq. 5.8.2.5-1

O.K.

Step 8.7.5. Maximum Nominal Shear Resistance

In order to ensure that shear reinforcement yields before web crushing, an upper limit of V_n is defined.

Since $V_n = 0.25f'_c t_w d_v + V_p$ and $V_n = V_c + V_s + V_p$ LRFD Eq. 5.8.3.3-2 LRFD Eq. 5.8.3.3-1

$V_c + V_s \le 0.25f'_c t_w d_v$

$V_c + V_s = 133.94 + 101.30 = 235.24$ kip

$0.25f'_c\, t_{ws}\, d_v = 0.25 \times 6.8 \times 10 \times 27 = 459.00$ kip

$V_c + V_s \le 0.25f'_c\, t_w\, d_v$ **O.K.**

The procedure shown in **Step 8.7** is repeated at the end of Step 8.8 to calculate the changing stirrup spacing along the span.

Step 8.8. Interface Shear Transfer

Factored Horizontal Shear LRFD Art. 5.8.4

$$V_{hi} = \frac{V_u}{d_v}$$ LRFD Eq. C5.8.4.2-7

where,

V_{hi} = horizontal factored shear force per unit length of the beam at the strength limit state

V_u = factored shear force at a specified section due to superimposed loads

d_v = distance between tensile and compressive force resultants = 27 in. = $0.042L_{ds}$

Factored shear at critical section ($0.042L_{ds}$), V_u

$= 1.25(V_g + V_D + V_{cd} + V_b) + 1.5(V_{ws}) + 1.75(V_{HL\text{-}M})$ LRFD Table 3.4.1.1 (Strength I)

$= 1.25(11.83 + 18.29 + 0.51 + 1.07) + 1.5(3.65) + 1.75(68.57)$

$= 165.10$ kip

Therefore, the applied horizontal shear, $V_{hi} = \frac{V_u}{d_v} = \frac{165.10}{27} = 6.11 \frac{\text{kip}}{\text{in}}$ LRFD Eq. C5.8.4.2-7

Required Nominal Resistance

$$V_{ni} = \frac{V_{hi}}{\phi} = \frac{6.11}{0.9} = 6.79 \frac{\text{kip}}{\text{in}}$$ LRFD Eq. 5.8.4.1-1

Required Interface Shear Reinforcement

The nominal shear resistance at the interface, V_{ni}

$$= cA_{cv} + \mu[A_{vf}\, f_{yh} + P_c]$$ LRFD Eq. 5.8.4.1-3

where,

c = cohesion factor

= 0.28 (for an intentionally roughened surface) LRFD Art.5.8.4.3

μ = coefficient of friction = 1.0 LRFD Art. 5.8.4.3

b_{tf} = top flange width = 36 in.

A_{cv} = area of concrete section resisting interface shear

$= b_{tf} \times 1$ in.= 36 in.^{2} (for 1 in. longer section along the span)

A_{vf} = area of shear reinforcement crossing the shear plane

P_c = permanent net compressive force normal to the shear plane

It is conservative to neglect P_c. Hence, $P_c = 0$ kip LRFD C5.8.4.1

Solving for A_{vf}

$$V_{ni} = cA_{cv} + \mu\,[A_{vf}\, f_{yh} + P_c]$$

$$6.79 = 0.28 \times 36 + 1.0\,(A_{vf} \times 60 + 0) \rightarrow A_{vf} = -0.055 \text{ in.}^2 < 0$$ LRFD Eq. 5.8.4.1-3

Hence, the resistance provided by cohesion is greater than the stress developed under the applied force, and the minimum required interface reinforcement is provided.

Minimum Interface Shear Reinforcement

Minimum $A_{vf} \geq (0.05\, A_{cv})/f_{yh}$ LRFD Eq. 5.8.4.4-1

Vertical shear reinforcement of #4 – 2 leg stirrup (i.e., A_v = 0.4 in.^{2}) at 12 in. spacing is provided from the beam extending into the deck.

Therefore, $A_{vf} = A_v \times 12/s$ = 0.40 in.^{2}/ft

$0.05\, A_{cv}/f_{yh} = (0.05 \times b_{tf} \times 1)\,/f_{yh} = 0.05(36 \times 1)/60 = 0.03$ in.^{2}/in. = 0.36 in.^{2}/ft

$A_{vf} > 0.05\, A_{cv}/f_{yh}$ **O.K.**

Maximum Nominal Shear Resistance

$$V_{ni} \leq \min\,(K_1\, f'_c\, A_{cv};\; K_2\, A_{cv})$$ LRFD Eq.5.8.4.1-4 and Eq. 5.8.4.1-5

where,

K_1 = fraction of concrete strength available to resist interface shear

= 0.3 (for a cast-in-place concrete slab) LRFD 5.8.4.3

K_2 = limiting interface shear resistance LRFD 5.8.4.3

=1.8 ksi (for a cast-in-place concrete slab with normal weight concrete)

$K_1\, f'_c\, A_{cv} = 0.3 \times 4 \times 36 = 43.2$ kip/in.

$K_2\, A_{cv} = 1.8 \times 36 = 64.8$ kip/in.

$V_{ni,\,provided} = cA_{cv} + \mu[(A_{vf}/12)f_{yh} + P_c]$; where, A_{vf} = 0.40 in.^{2}/ft LRFD Eq. 5.8.4.1-3

$= 0.28 \times 36 + 1.0 \times [(0.40/12) \times 60 + 0) = 12.08$ kip/in.

12.08 kip/in. < Min (43.2 kip/in.; 64.8 kip/in.) **O.K.**

The spacing of shear reinforcement at $0.1L_{ds}$ along the half beam length are given in **Table 9** and standard shear details are presented in **Appendix F**.

Table 9. The Spacing of Shear Reinforcement

Location	V_u/ϕ (kip)	β	V_c (kip)	V_p (kip)	V_s (kip)	s (in.)	s_{max} (in.)	$s_{provided}$ (in.)
Critical section for shear	183.44	6.02	135.60	0.00	47.84	22.13	21.88	12
Transfer location	183.38	6.04	136.05	0.00	47.32	22.68	21.88	12
$0.10 \times L_{ds}$	167.93	5.62	126.76	0.00	41.17	29.13	21.88	12
$0.20 \times L_{ds}$	141.37	5.02	113.07	0.00	28.30	42.19	21.88	18
$0.30 \times L_{ds}$	115.34	4.84	107.77	0.00	7.57	155.83	21.61	18
$0.40 \times L_{ds}$	89.74	3.51	78.15	0.00	11.59	101.05	21.61	18
$0.50 \times L_{ds}$ (Midspan)	64.68	3.33	74.23	0.00	0.00	NR*	21.61	18

* NR: Not Required

s_{max} is calculated using LRFD Eq. 5.8.2.7-1 and LRFD Eq. 5.8.2.7-2.

Step 8.9. Minimum Longitudinal Reinforcement Requirement

Longitudinal reinforcement should be proportioned such that the following condition is satified at each section:

$$A_{ps}f_{ps} + A_sf_y \geq \frac{M_u}{d_v\phi_f} + 0.5\frac{N_u}{\phi_c} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta \qquad \text{LRFD Eq. 5.8.3.5-1}$$

where,

A_s = area of nonprestressed tension reinforcement

f_y = specified minimum yield strength of reinforcing steel

A_{ps} = area of prestressing steel at the tension side of the section

f_{ps} = average stress in prestressing steel at the location for which the nominal resistance is required

M_u = factored moment at the section corresponding to the factored shear force

N_u = applied factored axial force

V_u = factored shear force at section

V_s = shear resistance provided by shear reinforcement ($V_s < V_u/\phi$)

V_p = component of the effective prestressing force in the direction of the applied shear

d_v = effective shear depth

ϕ = resistance factor as appropriate for moment, shear, and axial resistance.

θ = angle of inclination of diagonal compressive stresses

Required Reinforcement at Face of Bearing

For simple supports with $M_u = 0$ kip-ft and $N_u = 0$ kip, the longitudinal reinforcement on the flexural tension side of the beam at the inside edge of the bearing shall satisfy the following:

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\phi} - 0.5V_s - V_p\right)\cot\theta$$ LRFD Eq. 5.8.3.5-2

When the bearing size is not yet established, the above equation can be applied at the bearing centerline for conservative results. Consequently, the assumed failure crack for this analysis radiates from the centerline of the bearing (i.e., 8.75 in. from the end of the beam).

Factored shear force at the bearing centerline, V_u LRFD Table 3.4.1.1 (Strength I)

= 175.48 kip (**Table 5**)

Shear resistance provided by shear reinforcement at 12 in. from the end of the beam is calculated as shown below:

$$V_s = \frac{A_v f_{yh} d_v \cot\theta}{s} = \frac{0.40 \times 60 \times 27 \times \cot 28.06}{12} = 101.30 \text{ kip}$$

where

$\theta = 28.06^{o}$ at the centerline of the bearing

In determining the tensile force that the reinforcement is expected to resist at the inside edge of the bearing area, the values of V_u, V_s, V_p, and θ, calculated for the section d_v from the face of the support may be used. LRFD C5.8.3.5

$$\left(\frac{V_u}{\phi} - 0.5V_s - V_p\right)\cot\theta = \left(\frac{175.48}{0.9} - 0.5 \times 101.30 - 0\right)\cot 28.06°$$
$$= 270.76 \text{ kip}$$

The assumed crack plane crosses the centroid of the group of 18 strands at a distance of 19.14 in. (i.e., 8.75 + 5.54 in. × cot 28.06^{o}) from the beam end.

The transfer length is 36 in. from the end of the beam (60 × strand diameter). LRFD 5.11.4.1

The available prestress from 18 strands at 19.14 in. is a fraction of the effective prestress, f_{pe} of 162 ksi (i.e., 0.8 × 0.75f_{pu}).

$$A_{ps} f_{ps} + A_s f_y = 18 \times 0.217 \times 162 \times \left(\frac{19.14}{36}\right) + 0$$ LRFD Eq.5.8.3.5-2
$$= 336.42 \text{ kip} > 270.76 \text{ kip} \quad \textbf{O.K.}$$

If the bearing width is 9 in., the failure crack will extend from the edge of the bearing, and the assumed crack plane crosses the centroid of the 18 strand group at a distance of 23.64 in. (i.e., 8.75 + 9/2 + 5.54 in. × cot28.06^{o}) from the beam end.

$$A_{ps} f_{ps} + A_s f_y = 18 \times 0.217 \times 162 \times \left(\frac{23.64}{36}\right) + 0$$ LRFD Eq.5.8.3.5-2
$$= 415.52 \text{ kip} > 270.76 \text{ kip} \quad \textbf{O.K.}$$

Step 8.10. Anchorage Zone Reinforcement LRFD Art. 5.10.10.1

Design of the anchorage zone reinforcement is based on the force in the strands just prior to transfer.

P_{pi} = force in the strands prior to transfer

$= 22 \times 0.217 \times 202.50 = 967$ kip

The bursting resistance, $P_r = f_s A_s$ LRFD Eq. 5.10.10.1 - 1

where,

A_s = total area of vertical reinforcement located within a distance of h/4 from the end of the beam

f_s = stress in steel ≤ 20 ksi

The bursting resistance, $P_r \geq 0.04P_{pi}$ LRFD Art. 5.10.10.1

$$P_r \geq 0.04\ (967) = 38.68 \text{ kip}$$

Solving for the required area of steel, $A_{s,req} = P_r/f_s$

$= 38.68/20 = 1.93$ in.^{2}

At least 1.93 in.^{2} of vertical transverse reinforcement is required within a distance of

$h/4 = 21/4 = 5.25$ in.

Starting at 2.5 in. from the beam end use two #4 – 2 leg stirrups at 2.5 in. spacing. BDG 7.11.01

$A_s = 2 \times 2 \times 0.20$ in.^{2} = 1.2 in.^{2}

$A_s < A_{sreq}$ **NOT O.K.**

In most cases, it is difficult to satisfy the anchorage zone requirement. Therefore, until more clarity is brought to the anchorage zone design (such as the use of strut and tie models), designer should be aware that the time of prestress release is most critical. Specifically, end zone reinforcement that is less than the required steel area is consistently used in actual production without objectionable cracking at the member end.

PCI (2011) Section 8.3.1.6.

Confinement Reinforcement

Reinforcement is provided to confine the prestressing steel in the bottom flange to a distance of 1.5h from the beam end. The reinforcement may not be less than #3 deformed bars with spacing not exceeding 6 in. The reinforcement should be a shape that will confine the strands. LRFD Art. 5.10.10.2

For the box beam, this confining steel shall be provided for a distance of $1.5h = 1.5 \times 21 = 31.5$ in.

Step 8.11. Deflection and Camber

Please note that camber sign convention is positive when deflections are negative.

Long-term deflection may be taken as the instantaneous deflection multiplied by a factor 4.0, if the instantaneous deflection is based on gross moment of inertia of the beam. LRFD Art. 5.7.3.6.2

However, a factor 4.0 is not appropriate for this type of precast construction. It is recommended that the designer follow the guidelines of the owner agency for which the bridge is being designed or follow a rigorous, time-dependent analysis. PCI (2011) Ch. 9.4

This example demonstrates MDOT deflection calculation practices.

Step 8.11.1. Deflection due to Static Loads

Deflection is calculated using the modulus of elasticity of concrete and the gross cross-section properties of the noncomposite precast beam.

Camber due to prestressing force at transfer, Δ_p

$$\Delta_p = \frac{F_i(0.098e_{pgm} + 0.027e_{pge})L_b^2}{E_{ci}I_b}$$

Libby (1977) Chapter 6-3

where,

F_i = total prestressing force after transfer = 1,055 kip

e_{pgm} = eccentricity of prestresssing force at midspan = 4.98 in.

e_{pge} = eccentricity of prestresssing force at beam end = 4.86 in.

L_b = beam length at transfer = 54.625 ft

L_{ds} = beam length at erection = 53.167 ft

E_{ci} = modulus of elasticity of beam at transfer = 4,655 ksi

E_{cb} = modulus of elasticity of beam in service = 4,749 ksi

I_b = beam moment of inertia = 24,600 in.^{4}

$$\Delta_p = \frac{1055 \times (0.098 \times 4.98 + 0.027 \times 4.86) \times (54.625 \times 12)^2}{4655 \times 24600}$$

MDOT(2002)

$$= 2.45 \text{ in.}\uparrow$$

Deflection due to beam weight at transfer, Δ_b

$$\Delta_b = \frac{0.104M_{gr}L_b^2}{E_{ci}I_b}$$

MDOT(2002)

where, M_{gr} = moment due to beam weight = 181.27 kip-ft
= 2175.24 kip-in.

$$\Delta_b = \frac{0.104 \times 2175.24 \times (54.625 \times 12)^2}{4655 \times 24600} = 0.85 \text{ in.}\downarrow$$

Resultant camber at transfer, $\Delta_{int} = \Delta_p + (-\Delta_b)$

$= 2.45 - 0.85 = 1.6$ in. ↑

Long term camber due to prestress, Δ_{Lp}

$\Delta_{Lp} = \Delta_p [1.9 + 0.6 (I_b/I_c)]$

where, I_c = composite section moment of intertia

$= 103{,}611$ in.^{4}

$\Delta_{Lp} = 2.45 [1.9 + 0.6 \times (24600/103611)]$

$= 5.00$ in. ↑

Long term camber due to beam weight, Δ_{Lb}

$\Delta_{Lb} = \Delta_b [2.1 + 0.7 (I_b/I_c)]$

$= 0.85 [2.1 + 0.7 \times (24600/103611)]$

$= 1.93$ in. ↓

Net ultimate beam camber $= \Delta_{Lp} + (-\Delta_{Lb})$

$= 5.00 - 1.93 = 3.07$ in. ↑

Long term beam deflection due to slab and haunch weight (ult), Δ_s

$$\Delta_s = \frac{5wL_b^4}{384E_{cb}I_b}\left[1 + \alpha_s C_u \frac{I_b}{I_c}\right]$$

where, $\alpha_s = 0.60$

$C_u = 1.8$

$E_{cb} = 4{,}749$ ksi

w = slab and haunch weight = 0.752 kip/ft

$$\Delta_s = \frac{5 \times \frac{0.752}{12} \times (54.625 \times 12)^4}{384 \times 4749 \times 24600}\left[1 + 0.60 \times 1.8 \times \frac{24600}{103611}\right]$$

$= 1.62$ in. ↓

Deflection due to forms and reinforcement weight, Δ_{fr}

$$\Delta_{fr} = \frac{5wL_b^4}{384E_{cb}I_b}$$

where, w = forms and reinforcement weight = 20 × (75/12) = 125 lb/ft

Weight of forms and reinforcement is assumed to be 20 lb/ft^2

Beam spacing = 75 in.

$$\Delta_{fr} = \frac{5 \times \frac{0.125}{12} \times (54.625 \times 12)^4}{384 \times 4749 \times 24600}$$

$= 0.21$ in. ↓

Deflection due to barrier and future wearing surface weight, Δ_{bfws}

$$\Delta_{bfws} = 2.3 \times \frac{5wL_b^4}{384E_{cb}I_c}$$

where,

w = $w_{bar} + w_{ws}$ = 0.044 + 0.150 = 0.194 kip/ft

w_{bar} = barrier weight = 0.044 kip/ft

w_{ws} = wearing surface weight = 0.150 kip/ft

$$\Delta_{bfws} = 2.3 \times \frac{5 \times \frac{0.194}{12} \times (54.625 \times 12)^4}{384 \times 4749 \times 24600}$$

= 0.77 in. ↓

Step 8.11.2. Deflection due to Live Load and Impact

LRFD Art. 2.5.2.6.2

Live load deflection limit = L_{ds}/800

= 53.167 × 12/800 = 0.80 in.

If owner requires the optional live load criteria, the deflection is the greater of:

LRFD Art. 2.5.2.6.2
LRFD Art. 3.6.1.3.2

- That resulting from the design truck plus impact, Δ_{LT}, or
- That resulting from 25% of the design truck plus impact, Δ_{LT}, taken together with the design lane load, Δ_{LL}.

The dynamic load allowance must be included in the calculation of live load deflection.

LRFD Art. 2.5.2.6.2

All the beams are assumed to deflect equally due to the applied live load. Therefore, the distribution factor for deflection, DFD

LRFD Art. 2.5.2.6.2

= Number of lanes/Number of beams

= 6/12 = 0.5 lanes/beam

Deflection = $\frac{\text{COEFF}}{I_c}\left(\frac{E_s}{E_c}\right)\left(\frac{\text{No.of Design Lanes}}{\text{No.of Beams}}\right)$(Multiple Presence Factor)

MDOT (2002)

COEFF for lane load and truck load are derived for steel beams with $E_s = 29 \times 10^6$ psi. Hence, when the deflection is calculated using the above equation, $E_s = 29 \times 10^6$ psi should be used.

For lane load, COEFF = $0.000496L^4$ where, L = Span in ft.

MDOT (2002)

For truck load, COEFF = $0.0894 (L^3 - 555L + 4780)$ OR

= $0.0794(L^3 - 368L + 2400)$

Multiple presence factor of 0.65 is used for the bridge with 6 design lanes.

LRFD
Table 3.6.1.1.2-1

Deflection due to Lane Load

COEFF = $0.000496L^4$ = $0.000496(53.167)^4$ = 3963

Deflection, $\Delta_{LL} = \frac{3963}{103611}\left(\frac{29 \times 10^6}{4.749 \times 10^6}\right)\left(\frac{6}{12}\right)0.65 = 0.076$ in.

Deflection due to Design Truck and Impact

COEFF = $0.0894(L^3 – 555L + 4780) = 0.0894(53.167^3 – 555 \times 53.167 + 4780)$

= 11225

OR

= $0.0794 (L^3 – 368L + 2400) = 0.0794 (53.167^3 – 368 \times 53.167 + 2400)$

= 10570

With impact, COEFF = 11225 × 1.33 = 14929

Deflection, $\Delta_{LT} = \frac{14929}{103611}\left(\frac{29 \times 10^6}{4.749 \times 10^6}\right)\left(\frac{6}{12}\right)0.65 = 0.286$ in.

Live Load Deflection

Live load deflection = Max (Δ_{LT}, $0.25\Delta_{LT} + \Delta_{LL}$)

= Max (0.286, 0.25 × 0.286 + 0.076) = 0.286 in.

0.286 in. < L_{ds}/800 = 0.80 in. **O.K.**

Step 8.12. Bottom Flange Shortening During Stress Transfer

$$\text{Bottom flange shortening} = \frac{F_i L}{A_b E_{ci}} + \frac{8\Delta_{int}}{L}\left(\frac{I_b}{S_b}\right)$$

where,

- F_i = total prestressing force after transfer = 1,055 kip
- L = beam length = 54.625 ft
- A_b = area of beam = 467 in.^{2}
- E_{ci} = modulus of elasticity of beam at transfer = 4,655 ksi
- Δ_{int} = camber at transfer = 1.6 in.
- I_b = moment of inertia = 24,600 in.^{4}
- S_b = section modulus for bottom fiber = 2,360 in.^{3}

$$\text{Bottom flange shortening} = \frac{1055 \times (54.625 \times 12)}{467 \times 4655} + \frac{8 \times 1.6}{54.625 \times 12}\left(\frac{24600}{2360}\right) = 0.52 \text{ in.}$$

APPENDIX A

UNFACTORED MOMENT AND SHEAR IN INTERIOR AND EXTERIOR BEAMS

Table A-1. Unfactored Moment in Interior Beam (kip-ft)

Location	At release	For service and strength limit state checks											
		Noncomposite section		Composite section									
				Simple span					Continuous span+				
						HL – 93 Mod						HL – 93 Mod	
	Beam, M_{gr}	Beam, M_g	Deck and haunch, M_D	Barrier, M_b	Future wearing surface, M_{ws}	Truck load with impact, M_{LT}[1]	60 kip axle load with impact, M_{LA}[2]	Lane load, M_{LL}[3]	Settlement, M_s	Barrier, M_b	Future wearing surface, M_{ws}	Truck load with impact, M_{LT}[4]	Lane load, M_{LL}[5]
Beam end	0.00												
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Transfer location*	109.11	81.09	77.81	11.74	14.67	170.80	160.93	54.05	0.00	11.36	14.21	-16.43	35.61
0.05× L_{ds}	175.27	147.26	141.43	21.32	26.65	234.23	221.16	74.33	-0.01	15.91	19.89	-22.93	49.70
0.10 × L_{ds}	307.17	279.15	267.85	40.40	50.50	440.64	419.13	140.84	-0.02	29.48	36.85	-45.87	92.36
0.15 × L_{ds}	423.48	395.46	379.46	57.23	71.54	619.24	593.92	199.52	-0.13	40.86	51.07	-68.80	127.99
0.20 × L_{ds}	524.28	496.26	476.18	71.82	89.78	770.01	745.53	250.38	-0.24	49.99	62.48	-91.74	156.60
0.25 × L_{ds}	609.58	581.55	558.03	84.17	105.21	892.35	872.80	293.41	-0.72	56.87	71.09	-114.67	178.17
0.30 × L_{ds}	679.36	651.34	624.99	94.27	117.84	986.87	977.81	328.62	-1.20	61.51	76.89	-137.60	192.71
0.35 × L_{ds}	733.64	705.62	677.07	102.12	127.66	1058.83	1059.65	356.01	-2.64	63.91	79.88	-160.54	200.21
0.40 × L_{ds}	772.41	744.39	714.27	107.74	134.67	1113.47	1118.30	375.57	-4.08	64.06	80.07	-183.48	200.69
0.45 × L_{ds}	795.67	767.65	736.59	111.10	138.88	1139.68	1152.60	387.30	-7.24	61.97	77.46	-206.41	194.13
0.50 × L_{ds} (Midspan)	803.43	775.40	744.03	112.23	140.28	1137.80	1164.54	391.11	-10.39	57.63	72.04	-229.34	180.54
0.55 × L_{ds}	795.67	767.65	736.59	111.10	138.88	1139.68	1152.60	387.30	-15.78	51.05	63.81	-252.28	159.92
0.60 × L_{ds}	772.41	744.39	714.27	107.74	134.67	1113.47	1118.30	375.57	-21.16	42.22	52.78	-275.21	132.27
0.65 × L_{ds}	733.64	705.62	677.07	102.12	127.66	1058.83	1059.65	356.01	-28.53	31.15	38.94	-298.15	97.59
0.70 × L_{ds}	679.36	651.34	624.99	94.27	117.84	986.87	977.81	328.62	-35.90	17.83	22.29	-321.08	55.87
0.75 × L_{ds}	609.58	581.55	558.03	84.17	105.21	892.35	872.80	293.41	-44.05	2.27	2.84	-344.02	7.12
0.80 × L_{ds}	524.28	496.26	476.18	71.82	89.78	770.01	745.53	250.38	-52.20	-15.53	-19.41	-366.95	-48.66
0.85 × L_{ds}	423.48	395.46	379.46	57.23	71.54	619.24	593.92	199.52	-59.31	-35.58	-44.47	-391.07	-111.47
0.90 × L_{ds}	307.17	279.15	267.85	40.40	50.50	440.64	419.13	140.84	-66.42	-57.87	-72.34	-546.40	-181.31
0.95 × L_{ds}	175.27	147.26	141.43	21.32	26.65	234.23	221.16	74.33	-72.50	-82.41	-103.01	-712.65	-258.18
Transfer location*	109.11	81.09	77.50	11.74	14.67	170.80	160.93	54.05	-75.12	-101.60	-126.99	-838.40	-318.30
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-78.57	-109.19	-136.49	-888.17	-342.09
Span 2 - 0	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-79.98	-114.23	-142.78	-920.23	-360.45

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

+ 60 kip axle load is not included in continuous span analysis.

1. M_{LT} = 1.2 × 1.33 × 0.581 × HS-20
2. M_{LA} = 1.2 × 1.33 × 0.581 × 60
3. M_{LL} = 1.2 × 0.581 × 0.64
4. M_{LT} = 1.2 × 1.33 × 0.581 × 0.9 × HS-20
5. M_{LL} = 1.2 × 0.581 × 0.9 × 0.64

Table A-2. Unfactored Shear in Interior Beam (kip)

Location	At release	For service and strength limit state checks											
		Noncomposite section		Composite section									
				Simple span					Continuous span[+]				
						HL – 93 Mod						HL – 93 Mod	
	Beam, V_{gr}	Beam, V_g	Deck and haunch, V_D	Barrier, V_b	Future wearing surface, V_{ws}	Truck load with impact, V_{LT}[1]	60 kip axle load with impact, V_{LA}[2]	Lane load, V_{LL}[3]	Settlement, V_s	Barrier, V_b	Future wearing surface, V_{ws}	Truck load with impact, V_{LT}[4]	Lane load, V_{LL}[5]
Beam end	37.70												
CL of bearing	37.03	37.03	35.54	5.36	6.70	75.18	70.50	23.66	0.94	4.06	5.07	-6.94	16.12
Transfer location*	35.04	35.04	33.63	5.07	6.34	72.04	67.89	22.00	0.94	3.68	4.59	-6.94	14.59
0.05× L_{ds}	33.32	33.32	32.00	4.82	6.03	70.86	66.90	21.36	0.94	3.53	4.42	-6.94	13.99
0.10 × L_{ds}	29.63	29.63	28.43	4.29	5.36	66.65	63.40	19.17	0.94	2.98	3.73	-6.94	11.86
0.15 × L_{ds}	25.92	25.92	24.88	3.75	4.69	62.44	59.89	17.10	0.94	2.45	3.06	-6.94	9.73
0.20 × L_{ds}	22.22	22.22	21.32	3.22	4.02	58.15	56.31	15.15	0.94	1.91	2.39	-9.61	7.60
0.25 × L_{ds}	18.52	18.52	17.77	2.68	3.35	53.94	52.81	13.31	0.94	1.38	1.72	-15.23	5.47
0.30 × L_{ds}	14.81	14.81	14.21	2.14	2.68	49.74	49.30	11.60	0.94	0.84	1.05	-20.74	3.34
0.35 × L_{ds}	11.11	11.11	10.66	1.61	2.01	45.53	45.79	10.00	0.94	0.30	0.38	-26.10	1.21
0.40 × L_{ds}	7.41	7.41	7.11	1.07	1.34	41.24	42.22	8.52	0.94	-0.23	-0.29	-31.54	-0.92
0.45 × L_{ds}	3.70	3.70	3.55	0.54	0.67	37.03	38.71	7.16	0.94	-0.77	-0.96	-36.78	-3.05
0.50 × L_{ds} (Midspan)	0.00	0.00	0.00	0.00	0.00	32.82	35.21	5.92	0.94	-1.30	-1.63	-41.61	-5.18
0.55 × L_{ds}	-3.70	-3.70	-3.55	-0.54	-0.67	-37.03	-38.71	-7.16	0.94	-1.84	-2.30	-46.15	-7.31
0.60 × L_{ds}	-7.41	-7.41	-7.11	-1.07	-1.34	-41.24	-42.22	-8.52	0.94	-2.38	-2.97	-50.38	-9.44
0.65 × L_{ds}	-11.11	-11.11	-10.66	-1.61	-2.01	-45.53	-45.79	-10.00	0.94	-2.91	-3.64	-54.40	-11.57
0.70 × L_{ds}	-14.81	-14.81	-14.21	-2.14	-2.68	-49.74	-49.30	-11.60	0.94	-3.45	-4.31	-58.03	-13.70
0.75 × L_{ds}	-18.52	-18.52	-17.77	-2.68	-3.35	-53.94	-52.81	-13.31	0.94	-3.98	-4.98	-61.36	-15.83
0.80 × L_{ds}	-22.22	-22.22	-21.32	-3.22	-4.02	-58.15	-56.31	-15.15	0.94	-4.52	-5.65	-64.40	-17.96
0.85 × L_{ds}	-25.92	-25.92	-24.88	-3.75	-4.69	-62.44	-59.89	-17.10	0.94	-5.06	-6.32	-67.19	-20.09
0/90 × L_{ds}	-29.63	-29.63	-28.43	-4.29	-5.36	-66.65	-63.40	-19.17	0.94	-5.59	-6.99	-69.62	-22.22
0/95 × L_{ds}	-33.32	-33.32	-32.00	-4.82	-6.03	-70.86	-66.90	-21.36	0.94	-6.13	-7.66	-71.75	-24.35
Transfer location*	-35.04	-35.04	-33.63	-5.07	-6.34	-72.04	-67.89	-22.00	0.94	-6.51	-8.14	-73.07	-25.88
CL of bearing	-37.03	-37.03	-35.54	-5.36	-6.70	-75.18	-70.50	-23.66	0.94	-6.66	-8.33	-73.59	-26.48
Span 2 - 0	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.94	-6.76	-8.45	-73.92	-26.94

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

[+] 60 kip axle load is not included in continuous span analysis.

1. $V_{LT} = 1.2 \times 1.33 \times 0.736 \times$ HS-20
2. $V_{LA} = 1.2 \times 1.33 \times 0.736 \times 60$
3. $V_{LL} = 1.2 \times 0.736 \times 0.64$
4. $V_{LT} = 1.2 \times 1.33 \times 0.736 \times 0.9 \times$ HS-20
5. $V_{LL} = 1.2 \times 0.736 \times 0.9 \times 0.64$

Table A-3. Unfactored Moment in Exterior Beam (kip-ft)

Location	At release	For service and strength limit state checks												
		Noncomposite section		Composite section										
				Simple span					Continuous span[+]					
						HL – 93 Mod						HL – 93 Mod		
	Beam, M_{gr}	Beam, M_g	Deck and haunch, M_D	Barrier, M_b	Future wearing surface, M_{ws}	Truck load with impact, M_{LT}[1]	60 kip axle load with impact, M_{LA}[2]	Lane load, M_{LL}[3]	Settlement, M_s	Barrier, M_b	Future wearing surface, M_{ws}	Truck load with impact, M_{LT}[4]	Lane load, M_{LL}[5]
Beam end	0.00												
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Transfer location*	109.11	81.09	77.50	11.74	14.67	209.90	197.77	66.42	0.00	11.36	14.21	-20.31	44.08
0.05× L_{ds}	175.27	147.26	140.80	21.32	26.65	286.47	273.42	91.89	-0.01	15.86	19.83	-28.35	61.52
0.10 × L_{ds}	307.17	279.15	266.79	40.40	50.50	539.97	518.18	174.12	-0.02	29.48	36.85	-56.71	114.35
0.15 × L_{ds}	423.48	395.46	377.95	57.23	71.54	760.42	734.28	246.67	-0.13	40.86	51.07	-85.06	158.47
0.20 × L_{ds}	524.28	496.26	474.29	71.82	89.78	947.86	921.72	309.54	-0.24	49.99	62.48	-113.42	193.89
0.25 × L_{ds}	609.58	581.55	555.81	84.17	105.21	1102.29	1079.08	362.74	-0.72	56.87	71.09	-141.77	220.59
0.30 × L_{ds}	679.36	651.34	622.51	94.27	117.84	1219.11	1208.90	406.28	-1.20	61.51	76.89	-170.12	238.59
0.35 × L_{ds}	733.64	705.62	674.38	102.12	127.66	1300.85	1310.08	440.14	-2.64	63.91	79.88	-198.48	247.88
0.40 × L_{ds}	772.41	744.39	711.44	107.74	134.67	1370.66	1382.59	464.32	-4.08	64.06	80.07	-226.84	248.47
0.45 × L_{ds}	795.67	767.65	733.67	111.10	138.88	1400.45	1425.00	478.82	-7.24	61.97	77.46	-255.19	240.36
0.50 × L_{ds} (Midspan)	803.43	775.40	741.08	112.23	140.28	1398.43	1440.13	483.66	-10.39	57.63	72.04	-283.54	223.52
0.55 × L_{ds}	795.67	767.65	733.67	111.10	138.88	1400.45	1425.00	478.82	-15.78	51.05	63.81	-311.89	197.00
0.60 × L_{ds}	772.41	744.39	711.44	107.74	134.67	1370.66	1382.59	464.32	-21.16	42.22	52.78	-340.25	163.77
0.65 × L_{ds}	733.64	705.62	674.38	102.12	127.66	1300.85	1310.08	440.14	-28.53	31.15	38.94	-368.59	120.83
0.70 × L_{ds}	679.36	651.34	622.51	94.27	117.84	1219.11	1208.90	406.28	-35.90	17.83	22.29	-396.95	69.17
0.75 × L_{ds}	609.58	581.55	555.81	84.17	105.21	1102.29	1079.08	362.74	-44.05	2.27	2.84	-425.31	8.82
0.80 × L_{ds}	524.28	496.26	474.29	71.82	89.78	947.86	921.72	309.54	-52.20	-15.53	-19.41	-453.66	-60.25
0.85 × L_{ds}	423.48	395.46	377.95	57.23	71.54	760.42	734.28	246.67	-59.31	-35.58	-44.47	-483.48	-138.00
0/90 × L_{ds}	307.17	279.15	266.79	40.40	50.50	539.97	518.18	174.12	-66.42	-57.87	-72.34	-675.52	-224.48
0/95 × L_{ds}	175.27	147.26	140.80	21.32	26.65	286.47	273.42	91.89	-72.50	-82.41	-103.01	-881.06	-319.65
Transfer location*	109.11	81.09	77.50	11.74	14.67	209.90	197.77	66.42	-75.12	-109.19	-136.49	-942.59	-349.11
CL of bearing	28.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-78.57	-109.19	-136.49	-1098.05	-423.53
Span 2 - 0	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-79.98	-114.23	-142.78	-1138.01	-443.06

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

[+] 60 kip axle load is not included in continuous span analysis.

1. $M_{LT} = 1.2 \times 1.33 \times 0.714 \times$ HS-20
2. $M_{LA} = 1.2 \times 1.33 \times 0.714 \times 60$
3. $M_{LL} = 1.2 \times 0.714 \times 0.64$
4. $M_{LT} = 1.2 \times 1.33 \times 0.714 \times 0.9 \times$ HS-20
5. $M_{LL} = 1.2 \times 0.714 \times 0.9 \times 0.64$

Table A-4. Unfactored Shear in Exterior Beam (kip)

Location	At release	For service and strength limit state checks											
		Noncomposite section		Composite section									
				Simple span					Continuous span+				
						HL – 93 Mod						HL – 93 Mod	
	Beam, V_{gr}	Beam, V_g	Deck and haunch, V_D	Barrier, V_b	Future wearing surface, V_{ws}	Truck load with impact, V_{LT}[1]	60 kip axle load with impact, V_{LA}[2]	Lane load, V_{LL}[3]	Settlement, V_s	Barrier, V_b	Future wearing surface, V_{ws}	Truck load with impact, V_{LT}[4]	Lane load, V_{LL}[5]
Beam end	37.70												
CL of bearing	37.03	37.03	35.39	5.36	6.70	72.36	68.71	23.10	0.94	4.06	5.07	-6.78	15.73
Transfer location*	35.04	35.04	33.49	5.07	6.34	69.88	65.86	21.48	0.94	3.68	4.59	-6.78	14.24
0.05× L_{ds}	33.32	33.32	31.86	4.82	6.03	68.41	65.30	20.85	0.94	3.52	4.40	-6.78	13.66
0.10 × L_{ds}	29.63	29.63	28.32	4.29	5.36	64.47	61.88	18.71	0.94	2.98	3.73	-6.78	11.57
0.15 × L_{ds}	25.92	25.92	24.78	3.75	4.69	60.53	58.45	16.69	0.94	2.45	3.06	-6.78	9.49
0.20 × L_{ds}	22.22	22.22	21.24	3.22	4.02	56.59	54.96	14.78	0.94	1.91	2.39	-9.38	7.42
0.25 × L_{ds}	18.52	18.52	17.70	2.68	3.35	52.64	51.54	12.99	0.94	1.38	1.72	-14.87	5.34
0.30 × L_{ds}	14.81	14.81	14.16	2.14	2.68	47.72	48.12	11.32	0.94	0.84	1.05	-20.24	3.26
0.35 × L_{ds}	11.11	11.11	10.62	1.61	2.01	43.78	44.69	9.76	0.94	0.30	0.38	-25.48	1.18
0.40 × L_{ds}	7.41	7.41	7.08	1.07	1.34	39.83	41.20	8.32	0.94	-0.23	-0.29	-30.78	-0.90
0.45 × L_{ds}	3.70	3.70	3.54	0.54	0.67	35.89	37.78	6.99	0.94	-0.77	-0.96	-35.90	-2.98
0.50 × L_{ds} (Midspan)	0.00	0.00	0.00	0.00	0.00	31.95	34.36	5.77	0.94	-1.30	-1.63	-40.62	-5.06
0.55 × L_{ds}	-3.70	-3.70	-3.54	-0.54	-0.67	-35.89	-37.78	-6.99	0.94	-1.84	-2.30	-45.04	-7.14
0.60 × L_{ds}	-7.41	-7.41	-7.08	-1.07	-1.34	-39.83	-41.20	-8.32	0.94	-2.38	-2.97	-49.19	-9.21
0.65 × L_{ds}	-11.11	-11.11	-10.62	-1.61	-2.01	-43.78	-44.69	-9.76	0.94	-2.91	-3.64	-53.11	-11.29
0.70 × L_{ds}	-14.81	-14.81	-14.16	-2.14	-2.68	-47.72	-48.12	-11.32	0.94	-3.45	-4.31	-56.65	-13.37
0.75 × L_{ds}	-18.52	-18.52	-17.70	-2.68	-3.35	-52.64	-51.54	-12.99	0.94	-3.98	-4.98	-59.90	-15.45
0.80 × L_{ds}	-22.22	-22.22	-21.24	-3.22	-4.02	-56.59	-54.96	-14.78	0.94	-4.52	-5.65	-62.86	-17.54
0.85 × L_{ds}	-25.92	-25.92	-24.78	-3.75	-4.69	-60.53	-58.45	-16.69	0.94	-5.06	-6.32	-65.59	-19.61
0/90 × L_{ds}	-29.63	-29.63	-28.32	-4.29	-5.36	-64.47	-61.88	-18.71	0.94	-5.59	-6.99	-67.96	-21.69
0/95 × L_{ds}	-33.32	-33.32	-31.86	-4.82	-6.03	-68.41	-65.30	-20.85	0.94	-6.13	-7.66	-70.04	-23.77
Transfer location*	-35.04	-35.04	-33.49	-5.07	-6.34	-69.88	-65.86	-21.48	0.94	-6.51	-8.14	-70.55	-24.37
CL of bearing	-37.03	-37.03	-35.39	-5.36	-6.70	-72.36	-68.71	-23.10	0.94	-6.66	-8.33	-71.83	-25.85
Span 2 - 0	-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.94	-6.76	-8.45	-72.12	-26.22

L_{ds} – Design span of 83.75 ft

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

+ 60 kip axle load is not included in continuous span analysis.

1. V_{LT} = 1.2 × 1.33 × 0.714 × HS-20
2. V_{LA} = 1.2 × 1.33 × 0.714 × 60
3. V_{LL} = 1.2 × 0.714 × 0.64
4. V_{LT} = 1.2 × 1.33 × 0.714 × 0.9 × HS-20
5. V_{LL} = 1.2 × 0.714 × 0.9 × 0.64

APPENDIX B

PRESTRESSING DESIGN OF BULB-TEE BEAM USING MAGNEL DIAGRAM

STEP B-1. SECTION PROPERTIES

Noncomposite Section

Distance from the centroid to upper limit of kern, $k_t = -S_b/A_b$ = -9.11 in.

Distance from the centroid to lower limit of kern, $k_b = S_t/A_b$ = 9.31 in.

Definition: The kern is the region on the cross-section in which a compressive force can be applied without generating any tensile stress within the cross-section.

Kern is calculated using an upper and lower limit (k_t, k_b) with respect to the centroid of the beam cross-section (**Figure B-1**). In this example, the positive axis is pointing downward towards the bottom fiber from the centroid.

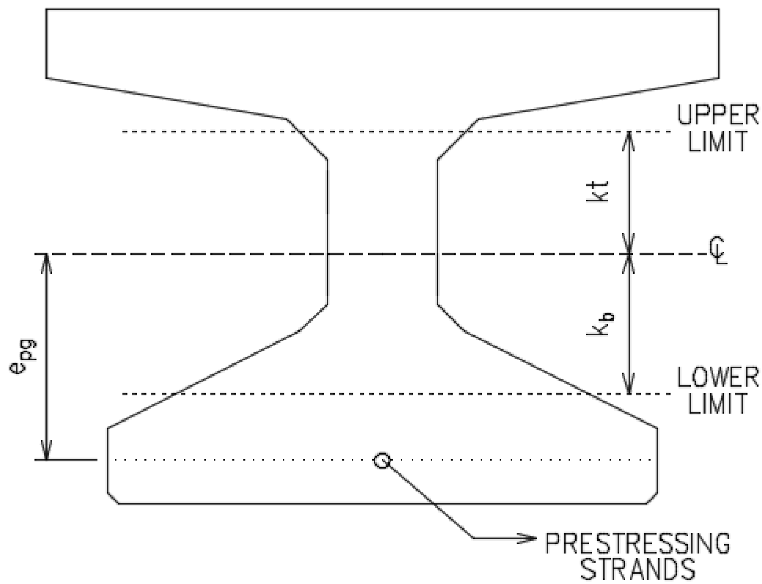


Figure B-1. Upper and lower limits of a kern in a prestressed beam

Composite Section

Interior beam:

Distance from centroid to upper limit of the central kern, $k_t = -S_{bc}/A_c$ = -8.22 in.

Distance from centroid to lower limit of the central kern, $k_b = S_{tc}/A_c$ = 12.75 in.

Exterior beam:

Distance from centroid to upper limit of the central kern, $k_t = -S_{bc}/A_c$ = -8.24 in.

Distance from centroid to lower limit of the central kern, $k_b = S_{tc}/A_c$ = 12.73 in.

STEP B-2. PRESTRESSING STRAND DESIGN

Step B.2.1. Stress Conditions at Midspan

Stresses at release and in service need to be maintained at or below the respective stress limits.

Sign Convention: Tensile stress is designated to be negative.
The following conditions and stress limits are considered:

At release:
Noncomposite section top and bottom fibers stresses under a prestressing force and the beam's self-weight on the overall beam length should be below the allowable tension and compression stress limits.

Condition 1: Beam top fiber tensile stress check at release

$$\frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M_{gr}}{S_t} \geq \left(-\bar{f}_{ti}\right)$$ LRFD Table 5.9.4.1.2-1

$$-\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_t} - \frac{M_{gr}}{S_t} \leq \bar{f}_{ti}$$

where,

F_i = prestressing force at release (kip)
A_b = area of beam cross-section (in.^{2})
e_{pg} = eccentricity of strands with respect to girder centroid (in.)
S_t = section modulus for top fiber (in.^{3})
M_{gr} = moment due to beam weight at release (kip-ft)
$\bar{f}_{ti}$ = allowable concrete tensile stress at release (ksi)

Condition 2: Beam bottom fiber compression stress check at release

$$\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M_{gr}}{S_b} \leq \bar{f}_{ci}$$ LRFD Art. 5.9.4.1.1

where,

S_b = section modulus of noncomposite beam for bottom fiber (in.^{3})
$\bar{f}_{ci}$ = allowable concrete compressive stress at release (ksi)

In service:

Conditions are:

- Moment due to beam, cast-in-place deck, and haunch weight on noncomposite section.
- Moment due to non-structural elements (barrier and future wearing surface) weight and live load on composite section.

Condition 3: Beam top fiber compression stress check under effective prestress and permanent loads

$$\frac{\eta F_i}{A_b} - \frac{\eta F_i e_{pg}}{S_t} + \frac{(M_g + M_D)}{S_t} + \frac{(M_b + M_{ws})}{S_{tc}} \leq \bar{f}_{cpl}$$

LRFD Table 5.9.4.2.1-1

where,

η = ratio of effective stress after losses to prestressing steel stress prior to transfer

M_g = moment due to beam weight (kip-in)

M_D = moment due to deck and haunch weight (kip-in)

M_b = moment due to barrier weight (kip-in)

M_{ws} = moment due to future wearing surface (kip-in)

S_{tc} = section modulus of composite beam for top fiber (in.^{3})

$\bar{f}_{cpl}$ = allowable compressive stress for concrete subjected to effective prestress and permanent loads (ksi)

Condition 4: Beam top fiber compression stress check under effective prestress, permanent loads, and transient loads

LRFD Table 5.9.4.2.1-1

$$\frac{\eta F_i}{A_b} - \frac{\eta F_i e_{pg}}{S_t} + \frac{(M_g + M_D)}{S_t} + \frac{(M_b + M_{ws} + M_{LT} + M_{LL})}{S_{tc}} \leq \bar{f}_{ctl}$$

where,

$\bar{f}_{ctl}$ = allowable compressive stress for concrete subjected to effective prestress, permanent loads, and transient loads (ksi)

M_{LT} = moment due to design truck load (kip-in)

M_{LL} = moment due to design lane load (kip-in)

Condition 5: Beam bottom fiber tension stress check under effective prestress, permanent loads, and transient loads

LRFD Table 5.9.4.2.2-1

$$\frac{\eta F_i}{A_b} + \frac{\eta F_i e_{pg}}{S_b} - \frac{(M_g + M_D)}{S_b} - \frac{(M_b + M_{ws} + 0.8M_{LT} + 0.8M_{LL})}{S_{bc}} \geq (-\bar{f}_{ts})$$

$$-\frac{\eta F_i}{A_b} - \frac{\eta F_i e_{pg}}{S_b} + \frac{(M_g + M_D)}{S_b} + \frac{(M_b + M_{ws} + 0.8M_{LT} + 0.8M_{LL})}{S_{bc}} \leq \bar{f}_{ts}$$

where,

S_{bc} = section of the composite beam modulus for bottom fiber (in.^{3})

$\bar{f}_{ts}$ = allowable tensile stress for concrete with bonded steel and subjected to not worse than moderate corrosion condition (ksi)

To determine the required prestressing force at midspan, Service III limit state is used. For required prestressing force and associated eccentricity calculations and midspan stress checks, the five inequality conditions given above are merged graphically to demonstrate the iterative approach of the design.

In this process, the five conditions are rearranged to characterize five inequalities representing the relationship between eccentricity (e_{pg}) and initial prestressing force (F_i = $f_{pi}A_{ps}N$; where A_{ps} is the area of a prestressing strand and N is the number of strands).

In addition to five conditions, a sixth condition is imposed by limiting eccentricity of the prestressing strands that are to be bounded by a concrete cover. In this example, a 2 in. concrete cover is specified.

Please note the sign convention where tensile stresses are negative (-):

Condition 1:

$$e_{pg} \leq k_b + \left[\frac{1}{F_i}\right](M_{gr} + \bar{f}_{ti}S_t)$$

Condition 2:

$$e_{pg} \leq k_t + \left[\frac{1}{F_i}\right](M_{gr} + \bar{f}_{ci}S_b)$$

Condition 3:

$$e_{pg} \geq k_b + \left[\frac{1}{\eta F_i}\right][(M_g + M_D) + \frac{(M_b + M_{ws})S_t}{S_{tc}} - \bar{f}_{cpl}S_t]$$

Condition 4:

$$e_{pg} \geq k_b + \left[\frac{1}{\eta F_i}\right][(M_g + M_D) + \frac{(M_b + M_{ws} + M_{LT} + M_{LL})S_t}{S_{tc}} - \bar{f}_{ctl}S_t]$$

Condition 5:

$$e_{pg} \geq k_t + \left[\frac{1}{\eta F_i}\right][(M_g + M_D) + \frac{(M_b + M_{ws} + 0.8M_{LT} + 0.8M_{LL})S_b}{S_{bc}} - \bar{f}_{ts}S_b]$$

Condition 6:

$$e_{pg} \leq y_b - 2\text{ in.}$$

y_b = distance from the centroid to the extreme bottom fiber of the noncomposite precast beam (in.)

The six inequalities are illustrated graphically in **Figure B-2**. The hatched area represents the feasibility domain of the prestressing force (F_i) and strand eccentricity (e_{pg}) combinations that do not violate all five stress limits and the maximum allowable eccentricity limit.

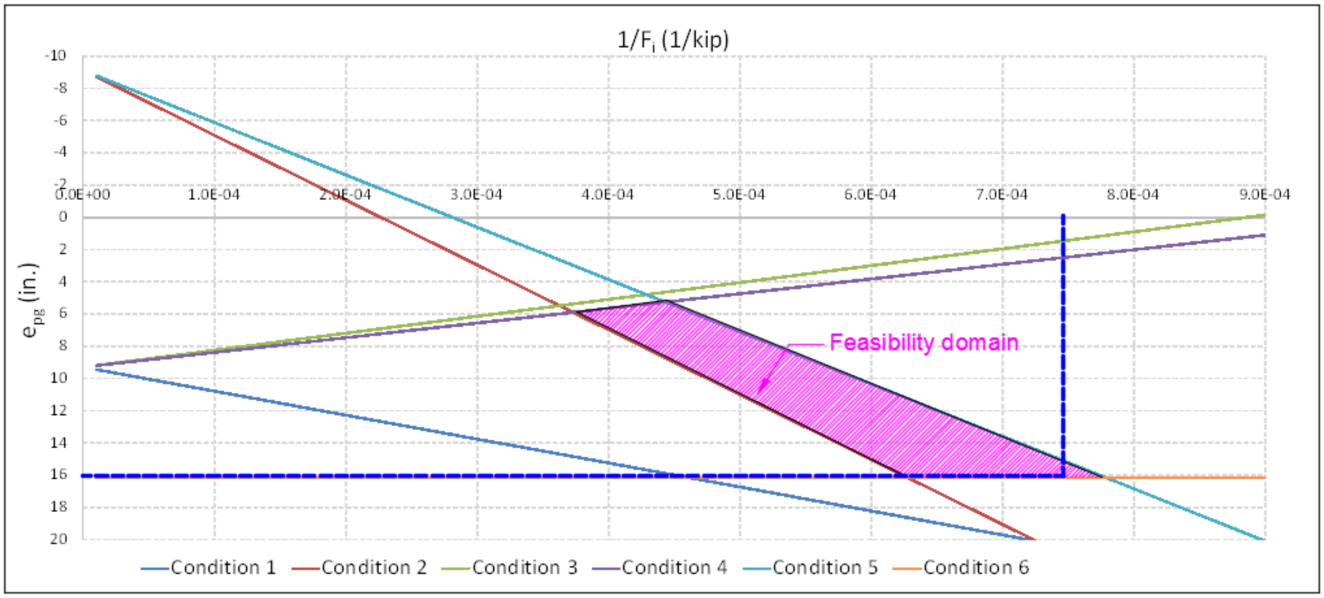


Figure B-2. Feasibility domain of e_{pg} and F_i at midspan

Step B.2.2. Eccentricity at Midspan

The strand center of gravity location, measured from the bottom of the beam, is 5-15% of the beam depth. As an initial assumption, location is set to 6% of the beam depth. Therefore,

$$e_{pg} = y_b - 0.06 \times h = 16 \text{ in.}$$

With e_{pg} = 16 in. on y-axis, $1/F_i$ located on x-axis and inside the feasibility domain is obtained from **Figure B-2**.

$1/F_i$	$= 7.5 \times 10^{-4}$ kips^{-1}
Hence, F_i	= 1,333 kips
Required number of strands, N	$= F_i/(A_{ps}f_{pi}) = 30.3$
Select, N	= 32
Prestressing force at release, F_i	$= f_{pi}A_{ps}N$
	= (202.50 ksi)(0.217 in.^{2})(32)
	= 1,406 kips

Step B.2.3. Feasibility Check at Midspan

A check is required to confirm that F_i and e_{pg} remain in the feasibility domain.

As an example,

Prestressing force at midspan of the beam, F_i = 1,406 kips

Hence, $1/F_i = 1/1406 \text{ kip} = 7.11\times10^{-4}$ kips^{-1}

As per the strand arrangement shown in Figure 13 of bulb-tee beam design example, e_{pg} = 15.14 in.

As shown in **Figure B-3**, the point defined by $1/F_i = 7.11 \times 10^{-4}$ kips^{-1} and e_{pg} = 15.14 in. for service conditions is located within the feasibility domain. Hence, the strand arrangement at midspan satisfies all the stress limits considered in **Step B.2.1**.

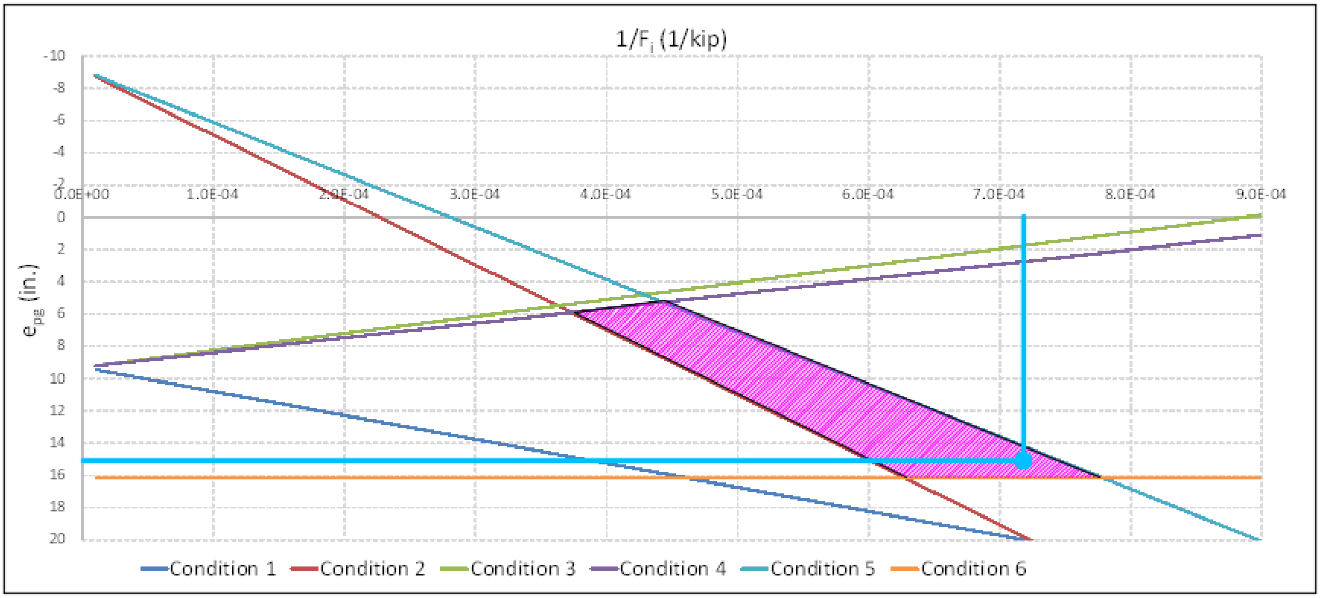


Figure B-3. Strand arrangement check with feasibility domain

Step B.2.4. Limiting Kern along Beam Length

As discussed earlier, upper and lower limits of kern along the beam length define the required number of prestressing strands and their arrangement needed to satisfy the tension and compression stress limits along the beam length.

The process requires representing the controlling inequality condition derived from five stress inequalities, defined in **Step B.2.1**, without the effect of beam weight and moments. As an example, the stress inequality for Condition 1 (top fiber stresses at release) is reformulated in the steps shown below:

Equation from **Step B.2.1**

$$e_{pg} \leq k_b + \left[\frac{1}{F_i}\right](M_{gr} + \bar{f}_{ti}S_t)$$

With the effect of beam self-weight excluded, i.e., $M_{gr} = 0$

$$e_{pg} \leq k_b + \frac{\bar{f}_{ti}S_t}{F_i}; \text{ where, } S_t = k_b A_b, \text{ and}$$

$$e_{pg} \leq k_b + \frac{\bar{f}_{ti}k_b A_b}{F_i}$$

Defining stress at the beam centroid under initial prestressing force, $f_{gi} = F_i/A_b$

$$e_{pg} \leq k_b(1 + \frac{\bar{f}_{ti}}{f_{gi}})$$

Following a similar procedure, all five inequalities are written in the following format:

Condition 1:	Condition 2:	Condition 3:	Condition 4:	Condition 5:
$e_{pg} \leq k_b(1+\frac{\bar{f}_{ti}}{f_{gi}})$	$e_{pg} \leq k_t(1-\frac{\bar{f}_{ci}}{f_{gi}})$	$e_{pg} \geq k_b\left(1-\frac{\bar{f}_{cpl}}{f_g}\right)$	$e_{pg} \geq k_b\left(1-\frac{\bar{f}_{ctl}}{f_g}\right)$	$e_{pg} \geq k_t\left(1+\frac{\bar{f}_{ts}}{f_g}\right)$

where, f_g = stress at the beam centroid due to effective prestressing force $= (F_i\eta)/A_b$

For the given beam section and initial prestressing,

$f_g = (F_i\eta)/A_b = 1{,}280.8$ psi
$f_{gi} = F_i/A_b = 1{,}601.7$ psi

In the first two conditions, f_{ti} and f_{ci}, are used for defining the lower limit of kern.

$$k`_b = \min\left[k_b\left(1+\frac{\bar{f}_{ti}}{f_{gi}}\right), k_t\left(1+\frac{\bar{f}_{ci}}{f_{gi}}\right)\right]$$

$$k`_b = \min[12.8 \text{ in.}, 12.4 \text{ in.}] = 12.4 \text{ in.}$$

Hence, the first condition governs the lower limit of kern.

The other three conditions with f_{ctl}, f_{cpl}, and f_{ts} are used to define the upper limit of the kern.

$$k`_t = \max\left[k_b\left(1-\frac{\bar{f}_{cpl}}{f_g}\right), k_b\left(1-\frac{\bar{f}_{ctl}}{f_g}\right), k_t\left(1-\frac{\bar{f}_{ts}}{f_g}\right)\right]$$

$$k`_t = \max[-15.2 \text{ in.}, -23.4 \text{ in.}, -12.8 \text{ in.}] = -12.8 \text{ in.}$$

Hence, the fifth condition governs the upper limit of the kern.

Figure B-4 shows the upper and lower limits of the kern (k_t and k_b).
Figure B-5 shows the upper and lower limits of the kern based on the stress limits (k_t' and k_b').

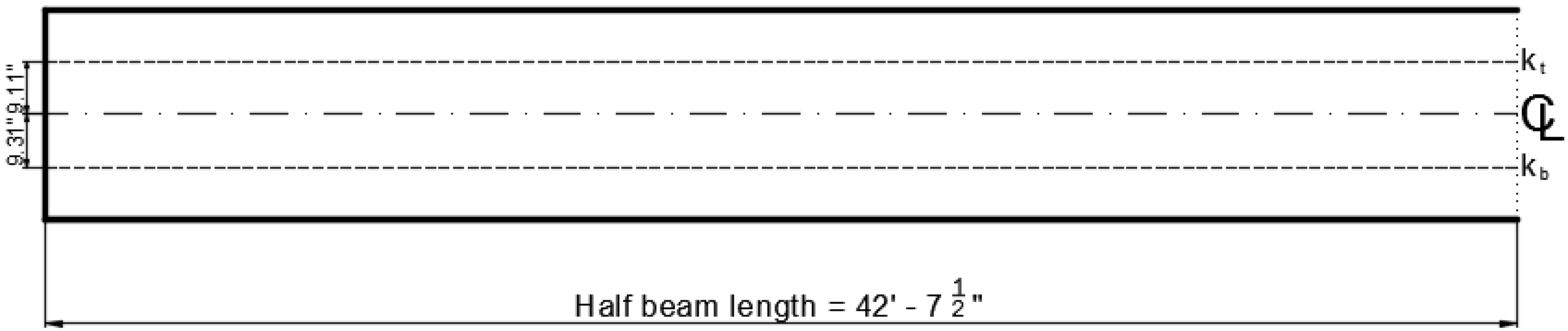


Figure B-4. The upper and lower limits of the kern (k_t and k_b)

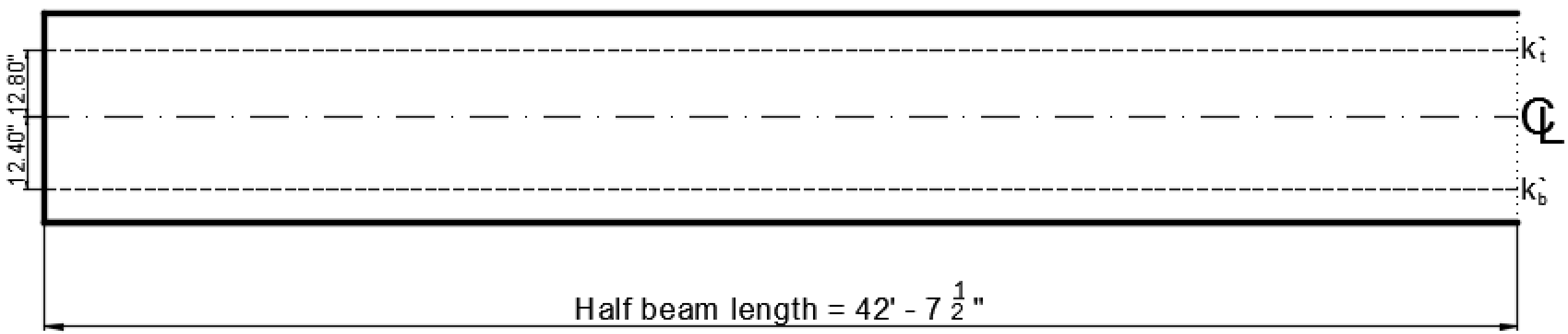


Figure B-5. The upper and lower limits of the kern based on the stress limits (k_t' and k_b')

Step B.2.5. Upper and Lower Bound Eccentricity along the Beam Length

The governing conditions that define the lower and upper limits of the kern are described in **Step B.2.4**.

In this calculation step, decreasing moment along the beam length from midspan to beam end will define the upper and lower eccentricity limits.

Upper limit of the kern (e_{pgu}) is calculated with the 5th condition

$$e_{pgu} = k`_t + \left[\frac{1}{\eta F_i}\right]\left(M_g + M_D\right) + \frac{(M_b + M_{ws} + 0.8M_{LT} + 0.8M_{LL})S_b}{S_{bc}}$$

Lower limit of the kern (e_{pgl}) is calculated with the 2nd condition

$$e_{pgl} = k`_b + \left[\frac{M_{gr}}{F_i}\right]$$

Using the above two equations, e_{pgu} and e_{pgl} are calculated along the beam length and shown in **Table B-1**. **Figure B-6** demonstrates e_{pgu} and e_{pgl} variation along the span. The hatched area represents the feasibility region for the center of gravity of prestressing strands without violating the stress limits at transfer and in service.

Table B-1. Upper and Lower Bounds of Eccentricity along the Half Span

Location	**e_{pgu} (in.)**	**e_{pgl} (in.)**
Beam end		12.40
CL of bearing	-12.81	12.42
0.05 × L_{ds}	-8.45	13.96
0.10 × L_{ds}	-4.55	15.13
0.15 × L_{ds}	-1.12	16.15
0.20 × L_{ds}	1.84	17.04
0.25 × L_{ds}	4.33	17.79
0.30 × L_{ds}	6.35	18.41
0.35 × L_{ds}	7.91*	18.89
0.40 × L_{ds}	9.05*	19.23
0.45 × L_{ds}	9.73*	19.44
0.50 × L_{ds} (Midspan)	9.96*	19.51

* The moment due to axle load controls the design.

Note: Strand centroid is designed to be inside the lower limit of the kern.

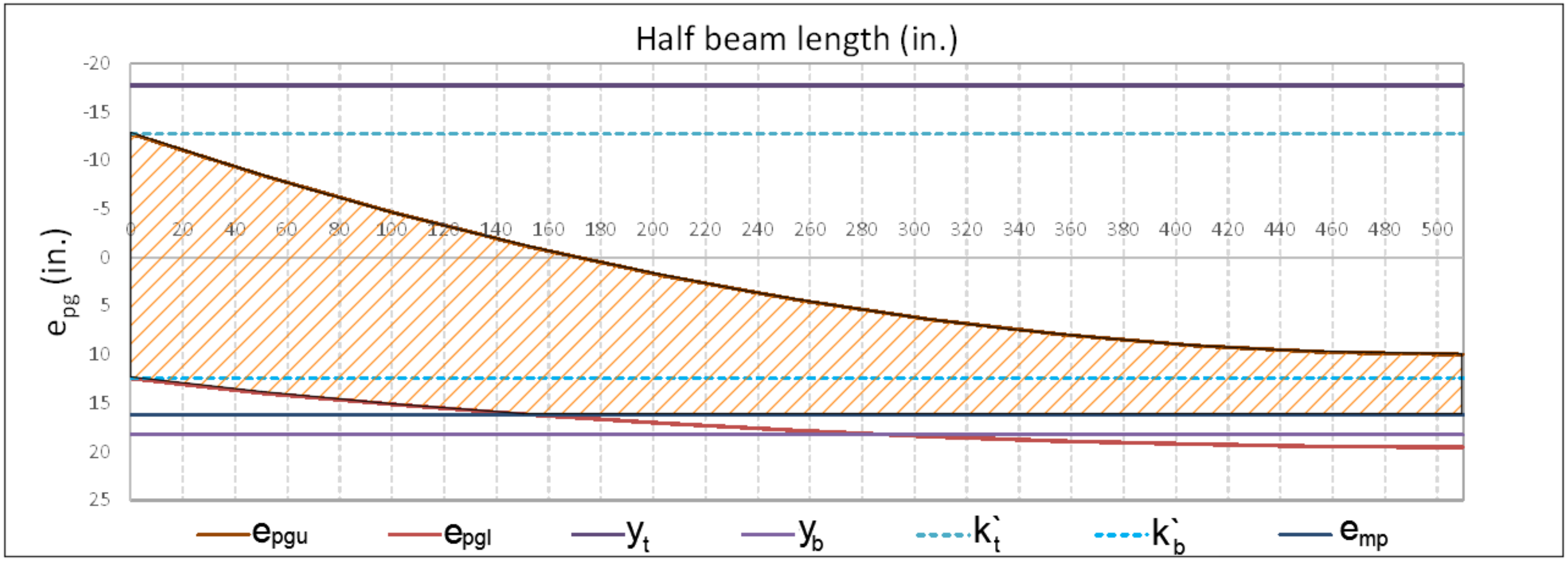


Figure B-6. Upper and lower limits of kern along the half beam length

Step B.2.6. Strand Arrangement

The strand arrangement is defined so that F_i remains between the upper and lower limits of the kern.

Figure B-6 shows strand arrangement at beam end and midspan cross sections.

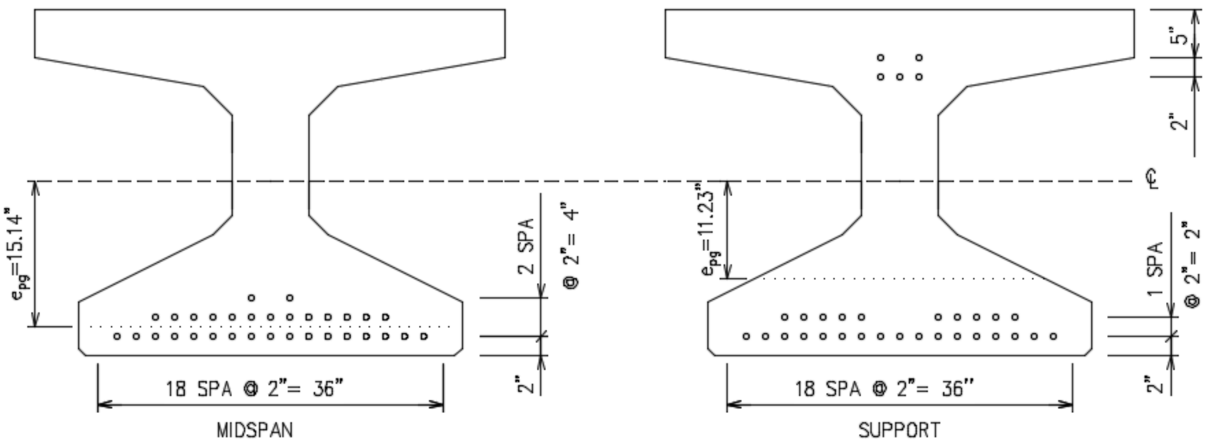


Figure B-7. Strand arrangement at midspan and over the supports

Step B.2.7. Strand Profile

The longitudinal strand profile along the span is designed. **Figure B-8** shows the strand profile along half span of the beam. In this example, to assure the satisfaction of stress limits along the beam length, draping point is located at $0.4L_{ds}$. Typically, the draping point is placed at $0.4L_{ds}$. This position of draping is often validated from the relationship between standard section depth and span. The objective of draping or harping is to maintain the eccentricity within upper and lower bounds of the kern along the span shown in **Figure B-6**. The green line in **Figure B-9** represents the eccentricity (e_{pg}) along the half beam length.

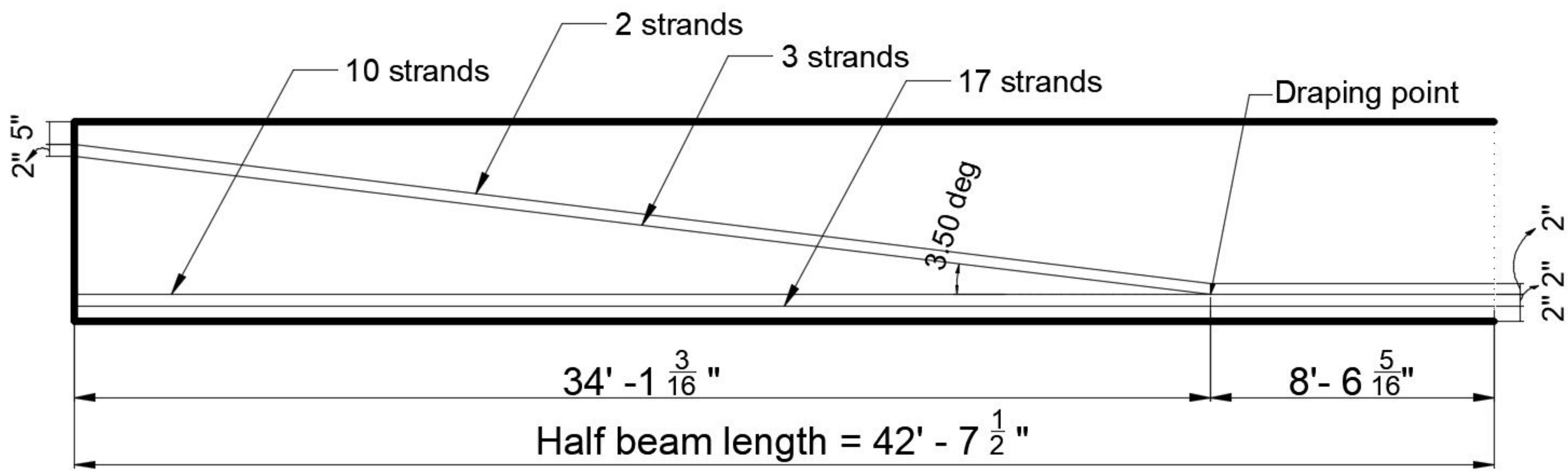


Figure B-8. Longitudinal strand profile along half beam length

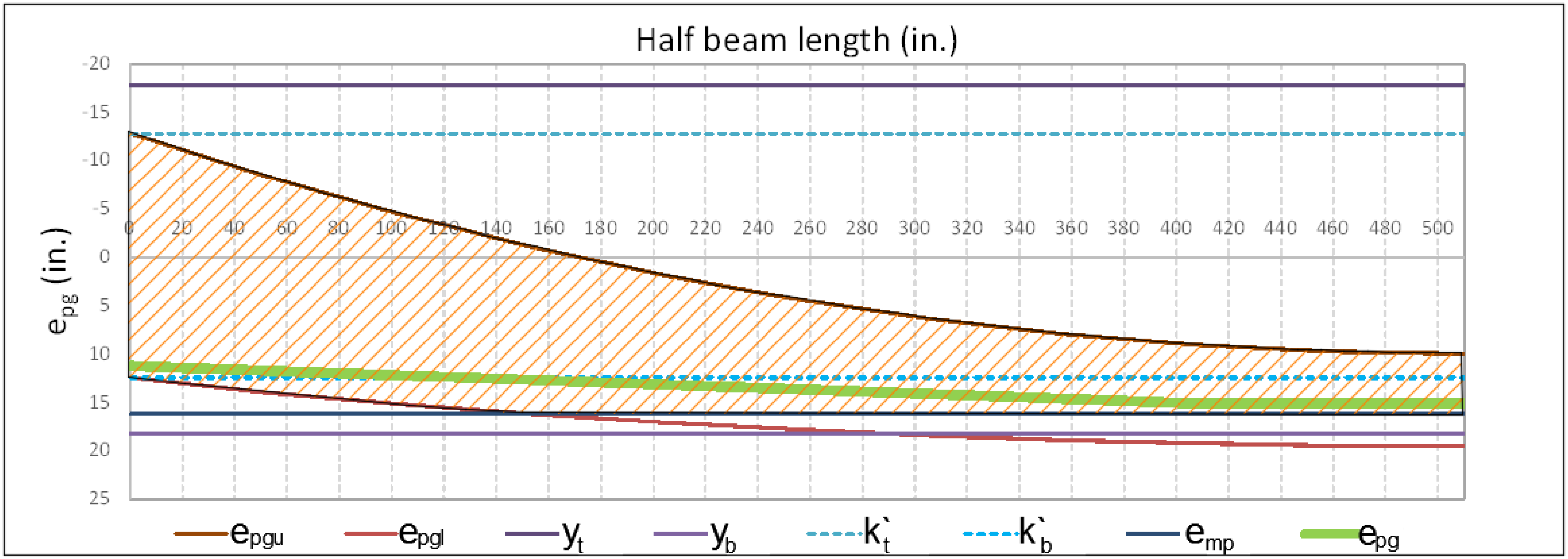


Figure B-9. Position of strand center of gravity (e_{pg}) along the beam length

APPENDIX C

TIME DEPENDENT REFINED CALCULATIONS OF PRESTRESSING LOSSES

Time-Dependent Losses between Prestress Transfer and Deck Placement

The construction schedule is assumed as follows:

	Concrete age in days	Section
At transfer	t_i=0.75	Noncomposite
At deck placement	t_d=56	Noncomposite
In service	t_f = 20,000	Composite

Total loss between prestress transfer and deck placement is the summation of prestress losses from shrinkage and creep of concrete, and relaxation of prestressing strands.

Losses from Concrete Shrinkage

LRFD Art. 5.9.5.4.2a

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}$$ LRFD Eq. 5.9.5.4.2a-1

where,

Δf_{pSR} = prestress loss from shrinkage of concrete between time of transfer and deck placement, ksi

ε_{bid} = concrete shrinkage strain between the time of transfer and deck placement, in./in.

$\varepsilon_{bid} = k_{vs}\, k_{hs}\, k_f\, k_{td}\, 0.48 \times 10^{-3}$ LRFD Eq. 5.4.2.3.3-1

E_p = Modulus of elasticity of prestressing strands, ksi

K_{id} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for time period between prestress transfer and deck placement

k_{vs} = the factor for the effect of volume-to-surface ratio (V/S) of the beam

= 1.45 – 0.13 V/S = 1.45 - 0.13 × 4.16 = 0.91 < 1.0 LRFD Eq. 5.4.2.3.2-2

→ k_{vs}=1.0 $k_{VS} \geq 1.0$

H = relative humidity = 75% (Michigan climate) LRFD Fig.5.4.2.3.3-1

k_{hs} = humidity factor for shrinkage = 2 - 0.014H LRFD Eq.5.4.2.3.3-2

= 2-0.014 × 75 = 0.95

k_f = factor for the effect of concrete strength LRFD Eq. 5.4.2.3.2-4

$$= \frac{5}{1+f'_{ci}} = \frac{5}{1+6.3} = 0.685$$

k_{td} = time development factor at deck placement for shrinkage

$$= \frac{t}{61-4f'_{ci}+t}$$ LRFD Eq. 5.4.2.3.2-5

t = maturity of concrete = t_d - t_i = 56 - 0.75 = 55.25 days

$$k_{td} = \frac{t}{61-4f'_{ci}+t} = \frac{55.25}{61-4\times 6.3+55.25} = 0.607 = k_{tdd}$$

k_{tdd} will be used in $\Psi_b(t_d, t_i)$ formulation to calculate creep of concrete between transfer and deck placement.

$$\varepsilon_{bid} = 1.0 \times 0.95 \times 0.685 \times 0.607 \times 0.48 \times 10^{-3}$$

$$= 1.90 \times 10^{-4} \text{ in./in.}$$

$$K_{id} = \frac{1}{1 + \frac{E_p}{E_{ci}}\frac{A_{ps}}{A_g}\left(1 + \frac{A_g e_{pg}^2}{I_g}\right)[1 + 0.7\Psi_b(t_f, t_i)]}$$ LRFD Eq. 5.9.5.4.2a-2

$\Psi_b(t_f, t_i)$ = girder creep coefficient at final time from loading introduced at transfer

$=1.9\ k_{vs}\ k_{hc}\ k_f\ k_{td}\ t_i^{-0.118}$ LRFD Eq. 5.4.2.3.2-1

k_{hc} = humidity factor for creep =1.56 - 0.008H LRFD Eq. 5.4.2.3.2-3

= 1.56 - 0.008 × 75 = 0.96

k_{td} = time development factor at deck placement for creep

$= \frac{t}{61 - 4f'_{ci} + t}$ LRFD Eq. 5.4.2.3.2-5

t = maturity of concrete = t_f - t_i= 20,000 - 0.75

= 19,999.25 days

$$k_{td} = \frac{t}{61 - 4f'_{ci} + t} = \frac{19999.25}{61 - 4 \times 6.3 + 19999.25} = 0.998 = k_{tdf}$$

k_{tdf} will be used in $\Psi_b(t_f, t_d)$ formulation to calculate creep of concrete between deck placement and in-service.

$\Psi_b(t_f, t_i) = 1.9 \times 1.0 \times 0.96 \times 0.685 \times 0.998 \times 0.75^{-0.118} = 1.29$ LRFD Eq. 5.4.2.3.2-1

$$K_{id} = \frac{1}{1 + \frac{28500}{4631} \times \frac{0.217 \times 32}{878.30} \times \left(1 + \frac{878.30 \times 15.14^2}{145592}\right) \times [1 + 0.7 \times 1.29]}$$ LRFD Eq. 5.9.5.4.2a-2

$= 0.819$

The prestress loss from shrinkage of concrete between transfer and deck placement

$$\Delta f_{pSR} = \varepsilon_{bid} E_p K_{id}$$ LRFD Eq. 5.9.5.4.2a-1

$$\Delta f_{pSR} = 1.90 \times 10^{-4} \times 28500 \times 0.819 = 4.43 \text{ ksi}$$

Loss due to Concrete Creep LRFD Art. 5.9.5.4.2b

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp} \Psi_b(t_d, t_i) K_{id}$$ LRFD Eq. 5.9.5.4.2b-1

where,

Δf_{pCR}, = prestress loss from creep of beam concrete between transfer and deck placement

$\Psi_b(t_d, t_i)$ = girder creep coefficient at time of deck placement due to loading introduced at transfer

$=1.9\ k_{vs}\ k_{hc}\ k_f\ k_{tdd}\ t_i^{-0.118}$ LRFD Eq. 5.4.2.3.2-1

$=1.9 \times 1.0 \times 0.96 \times 0.685 \times 0.607 \times 0.75^{-0.118} = 0.785$

$$\Delta f_{pCR} = \frac{E_p}{E_{ci}} f_{cgp}\Psi_b(t_d, t_i)K_{id} = \frac{28500}{4631} \times 2.52 \times 0.785 \times 0.819$$ LRFD Eq. 5.9.5.4.2b-1

$= 9.97$ ksi

Losses from Strand Relaxation LRFD Art. 5.9.5.4.2c

$$\Delta f_{pR1} = \frac{f_{pt}}{K_L}\left(\frac{f_{pt}}{f_{py}} - 0.55\right)$$ LRFD Eq. 5.9.5.4.2c-1

where,

Δf_{pR1} = prestress loss from relaxation of strands between transfer and deck placement

f_{pt} = stress in prestressing strands immediately after transfer ≥ $0.55f_{py}$

= f_{pi} - Δf_{pES} = 202.50 – 15.51 = 186.99 ksi

K_L= 30 for low relaxation strands and 7 for other prestressing steel, unless more accurate manufacture's data is available

$$\Delta f_{pR1} = \frac{f_{pt}}{K_L}\left(\frac{f_{pt}}{f_{py}} - 0.55\right) = \frac{186.99}{30}\left(\frac{186.99}{243} - 0.55\right) = 1.37 \text{ ksi}$$

For low-relaxation strands, the relaxation loss, Δf_{pR1} = 1.2 ksi LRFD Art. 5.9.5.4.2c

Time Dependent Losses after Deck Placement

The total time-dependent losses between deck placement and in-service is the summation of prestress losses from shrinkage and creep of beam concrete, and the relaxation of strands.

Losses due to Concrete Shrinkage LRFD Art. 5.9.5.4.3a

$$\Delta f_{pSD} = \varepsilon_{bdf}E_pK_{df}$$ LRFD Eq. 5.9.5.4.2a-1

where,

Δf_{pSD} = losses from shrinkage of beam concrete between deck placement and in-service

ε_{bdf} = shrinkage strain of beam between deck placement and in-service

$= \varepsilon_{bif} - \varepsilon_{bid}$

ε_{bif} = total shrinkage strain of beam concrete between transfer and in-service

$= k_{vs}k_{hs}k_f k_{tdf} 0.48 \times 10^{-3}$ LRFD Eq. 5.4.2.3.3-1

$= 1.0 \times 0.95 \times 0.685 \times 0.998 \times 0.48 \times 10^{-3} = 3.12 \times 10^{-4}$ in./in.

Therefore, $\varepsilon_{bdf} = \varepsilon_{bif} - \varepsilon_{bid} = 3.12 \times 10^{-4} - 1.90 \times 10^{-4} = 1.22 \times 10^{-4}$ in./in.

K_{df} = transformed section coefficient that accounts for time-dependent interaction between concrete and bonded steel in the section being considered for the time between deck placement and in-service.

$$K_{df} = \frac{1}{1 + \frac{E_p}{E_{ci}} \frac{A_{ps}}{A_c}\left(1 + \frac{A_c e_{pc}^2}{I_c}\right)[1 + 0.7\Psi_b(t_f, t_i)]}$$ LRFD Eq. 5.9.5.4.3a-2

e_{pc} = eccentricity of prestressing force with respect to centroid of composite section

= e_{pg} + (y_{bc} - y_b) = 15.14 + (28.57 - 18.20) = 25.51 in.

A_c = area of section from gross composite concrete section properties of the girder and the deck, and the deck-to-girder modular ratio

= 1,563 in.^{2}

I_c = moment of inertia of gross composite concrete section calculated using material properties at service

= 367,259 in.^{4}

$$K_{df} = \frac{1}{1 + \frac{28500}{4631} \times \frac{0.217 \times 32}{1563} \times \left(1 + \frac{1563 \times 25.51^2}{367259}\right) \times [1 + 0.7 \times 1.29]}$$

$= 0.836$

Prestress loss from shrinkage of concrete between deck placement and in-service

$$\Delta f_{pSD} = \varepsilon_{bdf} E_p K_{df} = 1.22 \times 10^{-4} \times 28500 \times 0.836 = 2.91 \text{ ksi}$$

Losses due to Concrete Creep LRFD Art. 5.9.5.4.3b

$$\Delta f_{pCD} = \frac{E_p}{E_{ci}} f_{cgp}[\Psi_b(t_f, t_i) - \Psi_b(t_d, t_i)]K_{df} + \frac{E_p}{E_c} \Delta f_{cd} \Psi_b(t_f, t_d) K_{df}$$ LRFD Eq. 5.9.5.4.3b-1

where,

Δf_{pCD} = prestress loss from creep of beam concrete between deck placement and in-service

E_c = modulus of elasticity for beam in service = 5,250 ksi

Δf_{cd} = change in concrete stress at centroid of prestressing strands from long-term losses between transfer and deck placement, combined with deck weight and superimposed loads

$$= -(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}) \frac{A_{ps}}{A_b}\left(1 + \frac{A_b e_{pg}^2}{I_b}\right) - \left(\frac{M_D e_{pg}}{I_b} + \frac{(M_b + M_{ws}) e_{pc}}{I_c}\right)$$

$$= -(4.43 + 9.97 + 1.37) \frac{0.217 \times 32}{878.30}\left(1 + \frac{878.30 \times 15.14^2}{145592}\right) - \left(\frac{744.03 \times 12 \times 15.14}{145592} + \frac{(112.23 + 140.28) \times 12 \times 25.51}{367259}\right)$$

$= -1.436$ ksi

$\Psi_b(t_f, t_d)$ = girder creep coefficient at service due to loading at deck placement

$= 1.9\ k_{vs}\ k_{hc}\ k_f\ k_{tdf}\ t_d^{-0.118}$ LRFD Eq. 5.4.2.3.2-1

k_{td} = time development factor at service for creep = $\frac{t}{61 - 4f'_{ci} + t}$ LRFD Eq. 5.4.2.3.2-5

t = maturity of concrete

$= t_f - t_d$ = 20,000 – 56 = 19,944 days

$$k_{dt} = \frac{t}{61 - 4f'_{ci} + t} = \frac{19944}{61 - 4 \times 6.3 + 19944} = 0.998$$

$$\Psi_b(t_f, t_d) = 1.9 \times 1.0 \times 0.96 \times 0.685 \times 0.998 \times 56^{-0.118} = 0.775$$

The prestress loss due to creep of concrete between deck placement and service

$$\Delta f_{pCD} = \frac{28500}{4631} \times 2.52 \times [1.29 - 0.785] \times 0.836 + \frac{28500}{4906} \times (-1.436) \times 0.775 \times 0.836$$
$$= 1.14\ \text{ksi}$$

Losses due to Strand Relaxation LRFD Art. 5.9.5.4.3c

The prestress loss due to relaxation of prestressing strands in composite section between time of deck placement and service, Δf_{pR2}

$$\Delta f_{pR2} = \Delta f_{pR1} = 1.37\ \text{ksi}$$ LRFD Eq. 5.9.5.4.3c-1

Total losses at midspan

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pLT}$$ LRFD Eq. 5.9.5.1-1

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2}$$
$$= 15.51 + 4.43 + 9.97 + 1.37 + 2.91 + 1.14 + 1.37$$
$$= 36.70\ \text{ksi}$$

Ratio of effective prestress after losses to stress prior to transfer, η

$$= \frac{f_{pi} - \Delta f_{pT}}{f_{pi}} = \frac{202.50 - 36.70}{202.50} = 0.82$$

The losses calculated with the refined analysis are 18%, and sufficiently close to the lump sum estimate of 20%.

APPENDIX D

UNFACTORED MOMENT AND SHEAR IN INTERIOR BOX BEAMS

Table D-1. Unfactored Moment in Interior Beam (kip-ft)

Location	At release	For service and strength limit state checks												
		Noncomposite section			Composite section									
					Simple span					Continuous span+				
							HL – 93 Mod						HL – 93 Mod	
	Beam, M_{gr}	Beam, M_g	Deck and haunch, M_D	Concrete Diaphragm, M_{cd}	Barrier, M_b	Future wearing surface, M_{ws}	Truck load with impact, M_{LT}^1	60 kip axle load with impact, M_{LA}^2	Lane load, M_{LL}^3	Settlement, M_s	Barrier, M_b	Future wearing surface, M_{ws}	Truck load with impact, M_{LT}^4	Lane load, M_{LL}^5
Span 1										-88.37	-14.33	-48.84	-291.81	-104.42
Beam end	0.00									-87.57	-14.09	-48.03	-289.70	-102.70
CL of bearing	9.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-84.44	-13.17	-44.91	-281.53	-96.02
Transfer location*	37.40	27.96	43.27	1.15	2.53	8.63	87.73	90.82	17.07	-74.69	-10.43	-35.55	-256.07	-76.00
0.05× L_{ds}	42.00	32.41	50.15	1.34	2.93	10.00	102.69	106.31	23.37	-73.03	-9.97	-33.99	-251.73	-72.68
0.10 × L_{ds}	71.37	61.82	95.66	2.68	5.60	19.08	205.33	212.58	37.77	-61.62	-7.11	-24.24	-221.93	-51.82
0.15 × L_{ds}	96.90	87.53	135.44	4.03	7.92	27.02	279.71	295.30	53.40	-50.21	-4.54	-15.46	-192.14	-33.06
0.20 × L_{ds}	119.32	109.68	169.70	5.37	9.93	33.85	354.01	377.95	67.15	-38.80	-2.29	-7.80	-178.21	-16.67
0.25 × L_{ds}	138.20	128.61	199.01	6.71	11.64	39.70	400.03	437.04	78.58	-27.39	-0.34	-1.17	-166.16	-2.50
0.30 × L_{ds}	153.57	144.16	223.06	8.05	13.05	44.49	446.00	496.08	88.14	-15.98	1.29	4.40	-154.11	9.41
0.35 × L_{ds}	165.74	156.03	241.43	9.40	14.13	48.16	471.95	531.53	95.37	-4.57	2.61	8.89	-143.56	19.01
0.40 × L_{ds}	174.22	164.70	254.85	10.74	14.91	50.83	497.88	566.95	100.73	6.93	3.63	12.37	-134.87	26.45
0.45 × L_{ds}	179.34	169.88	262.87	12.08	15.38	52.43	499.62	578.78	103.76	18.25	4.32	14.71	-126.19	31.46
0.50 × L_{ds} (Midspan)	181.27	171.49	265.34	13.42	15.53	52.93	501.35	590.58	104.93	29.66	4.72	16.10	-117.50	34.42
0.55 × L_{ds}	179.34	169.88	262.87	12.08	15.38	52.43	499.62	578.78	103.76	41.07	4.78	16.30	-119.49	34.86
0.60 × L_{ds}	174.22	164.70	254.85	10.74	14.91	50.83	497.88	566.95	100.73	52.48	4.56	15.55	-121.48	33.24
0.65 × L_{ds}	165.74	156.03	241.43	9.40	14.13	48.16	471.95	531.53	95.37	63.89	4.01	13.66	-123.46	29.20
0.70 × L_{ds}	153.57	144.16	223.06	8.05	13.05	44.49	446.00	496.08	88.14	75.30	3.16	10.76	-129.41	23.00
0.75 × L_{ds}	138.20	128.61	199.01	6.71	11.64	39.70	400.03	437.04	78.58	86.71	1.99	6.78	-138.56	14.49
0.80 × L_{ds}	119.32	109.68	169.70	5.37	9.93	33.85	354.01	377.95	67.15	98.12	0.51	1.73	-147.72	3.71
0.85 × L_{ds}	96.90	87.53	135.44	4.03	7.92	27.02	279.71	295.30	53.40	109.53	-1.27	-4.34	-158.87	-9.28
0/90 × L_{ds}	71.37	61.82	95.66	2.68	5.60	19.08	205.33	212.58	37.77	120.94	-3.38	-11.52	-186.97	-24.64
0/95 × L_{ds}	42.00	32.41	50.15	1.34	2.93	10.00	102.69	106.31	23.37	132.35	-5.78	-19.69	-215.07	-42.11
Transfer location*	37.40	27.96	43.27	1.15	2.53	8.63	87.73	90.82	17.07	134.01	-6.16	-21.02	-219.16	-44.93
CL of bearing	9.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	143.76	-8.51	-29.02	-243.17	-62.05
Beam end	0.00									146.89	-9.30	-31.71	-250.88	-67.79
Span 3										147.69	-9.50	-32.40	-252.91	-69.27

L_{ds} – Design span

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

+ 60 kip axle load is not included in continuous span analysis.

1. M_{LT} = 1.2 × 1.33 × 0.464 × HS-20
2. M_{LA} = 1.2 ×1.33 × 0.464 × 60
3. M_{LL} = 1.2 × 0.464 × 0.64
4. M_{LT} = 1.2 × 1.33 × 0.464 × 0.9 × HS-20
5. M_{LL} = 1.2 × 0.464 × 0.9 × 0.64

Table D-2. Unfactored Shear in Interior Beam (kip)

Location	At release	For service and strength limit state checks												
		Noncomposite section			Composite section									
					Simple span					Continuous span[+]				
							HL – 93 Mod						HL – 93 Mod	
	Beam, V_{gr}	Beam, V_g	Deck and haunch, V_D	Concrete Diaphragm, V_{cd}	Barrier, V_b	Future wearing surface, V_{ws}	Truck load with impact, V_{LT}[1]	60 kip axle load with impact, V_{LA}[2]	Lane load, V_{LL}[3]	Settlement, V_s	Barrier, V_b	Future wearing surface, V_{ws}	Truck load with impact, V_{LT}[4]	Lane load, V_{LL}[5]
Span 1										4.29	1.30	4.42	54.16	12.66
Beam end	13.27									4.29	1.29	4.40	53.96	12.58
CL of bearing	12.92	12.92	19.99	0.51	1.17	4.00	58.82	59.46	12.68	4.29	1.26	4.29	53.15	12.27
Transfer location*	11.82	11.82	18.28	0.51	1.07	3.65	55.79	56.93	11.62	4.29	1.16	3.95	50.64	11.29
0.05× L_{ds}	11.63	11.63	17.99	0.51	1.05	3.59	55.27	56.49	11.45	4.29	1.14	3.89	50.21	11.12
0.10 × L_{ds}	10.34	10.34	15.99	0.51	0.94	3.19	51.70	53.52	10.27	4.29	1.02	3.49	47.26	9.98
0.15 × L_{ds}	9.04	9.04	13.99	0.51	0.82	2.79	48.13	50.55	9.16	4.29	0.91	3.09	44.31	8.84
0.20 × L_{ds}	7.75	7.75	11.99	0.51	0.70	2.39	44.56	47.57	8.12	4.29	0.79	2.69	41.44	7.70
0.25 × L_{ds}	6.46	6.46	10.00	0.51	0.59	1.99	40.99	44.60	7.13	4.29	0.67	2.29	38.57	6.56
0.30 × L_{ds}	5.17	5.17	8.00	0.51	0.47	1.60	37.43	41.63	6.21	4.29	0.56	1.89	35.70	5.42
0.35 × L_{ds}	3.88	3.88	6.00	0.51	0.35	1.20	33.86	38.65	5.36	4.29	0.44	1.50	32.90	4.28
0.40 × L_{ds}	2.58	2.58	4.00	0.51	0.23	0.80	30.29	35.68	4.56	4.29	0.32	1.10	30.18	3.14
0.45 × L_{ds}	1.29	1.29	2.00	0.51	0.12	0.40	26.83	32.71	3.84	4.29	0.21	0.70	27.46	2.00
0.50 × L_{ds} (Midspan)	0.00	0.00	0.00	0.51	0.00	0.00	23.36	29.73	3.17	4.29	0.09	0.30	24.74	0.86
0.55 × L_{ds}	-1.29	-1.29	-2.00	-0.51	-0.12	-0.40	-26.83	-32.71	-3.84	4.29	-0.03	-0.10	22.49	0.29
0.60 × L_{ds}	-2.58	-2.58	-4.00	-0.51	-0.23	-0.80	-30.29	-35.68	-4.56	4.29	-0.15	-0.50	20.25	-1.43
0.65 × L_{ds}	-3.88	-3.88	-6.00	-0.51	-0.35	-1.20	-33.86	-38.65	-5.36	4.29	-0.26	-0.90	18.00	-2.57
0.70 × L_{ds}	-5.17	-5.17	-8.00	-0.51	-0.47	-1.60	-37.43	-41.63	-6.21	4.29	-0.38	-1.30	16.03	-3.71
0.75 × L_{ds}	-6.46	-6.46	-10.00	-0.51	-0.59	-1.99	-40.99	-44.60	-7.13	4.29	-0.50	-1.70	14.28	-4.85
0.80 × L_{ds}	-7.75	-7.75	-11.99	-0.51	-0.70	-2.39	-44.56	-47.57	-8.12	4.29	-0.61	-2.09	12.52	-5.99
0.85 × L_{ds}	-9.04	-9.04	-13.99	-0.51	-0.82	-2.79	-48.13	-50.55	-9.16	4.29	-0.73	-2.49	10.84	-7.13
0/90 × L_{ds}	-10.34	-10.34	-15.99	-0.51	-0.94	-3.19	-51.70	-53.52	-10.27	4.29	-0.85	-2.89	9.76	-8.27
0/95 × L_{ds}	-11.63	-11.63	-17.99	-0.51	-1.05	-3.59	-55.27	-56.49	-11.45	4.29	-0.97	-3.29	8.67	-9.41
Transfer location*	-11.82	-11.82	-18.28	-0.51	-1.07	-3.65	-55.79	-56.93	-11.62	4.29	-0.98	-3.35	8.52	-9.58
CL of bearing	-12.92	-12.92	-19.99	-0.51	-1.17	-4.00	-58.82	-59.46	-12.68	4.29	-1.08	-3.69	7.59	-10.56
Beam end	-13.27									4.29	-1.11	-3.80	7.29	-10.87
Span 3										4.29	-1.12	-3.83	7.22	-10.95

L_{ds} – Design span

* Transfer location is at 36 in. (= 60 × Strand diameter) from beam end.

[+] 60 kip axle load is not included in continuous span analysis.

1. V_{LT} = 1.2 × 1.33 × 0.621 × HS-20
2. V_{LA} = 1.2 ×1.33 × 0.621 × 60
3. V_{LL} = 1.2 × 0.621 × 0.64
4. V_{LT} = 1.2 × 1.33 × 0.621 × 0.9 × HS-20
5. V_{LL} = 1.2 × 0.621× 0.9 × 0.64

APPENDIX E

PRESTRESSING DESIGN OF BOX BEAM USING MAGNEL DIAGRAM

STEP E-1. SECTION PROPERTIES

Noncomposite Section

Distance from the centroid to upper limit of kern, $k_t = -S_b/A_b$ = -5.05 in.

Distance from the centroid to lower limit of kern, $k_b = S_t/A_b$ = 4.97 in.

Definition: The kern is the region on the cross-section in which a compressive force can be applied without generating any tensile stress within the cross-section.

Kern is calculated using an upper and lower limit (k_t, k_b) with respect to the centroid of the beam cross-section (**Figure E-1**). In this example, the positive axis is pointing downward towards the bottom fiber from the centroid.

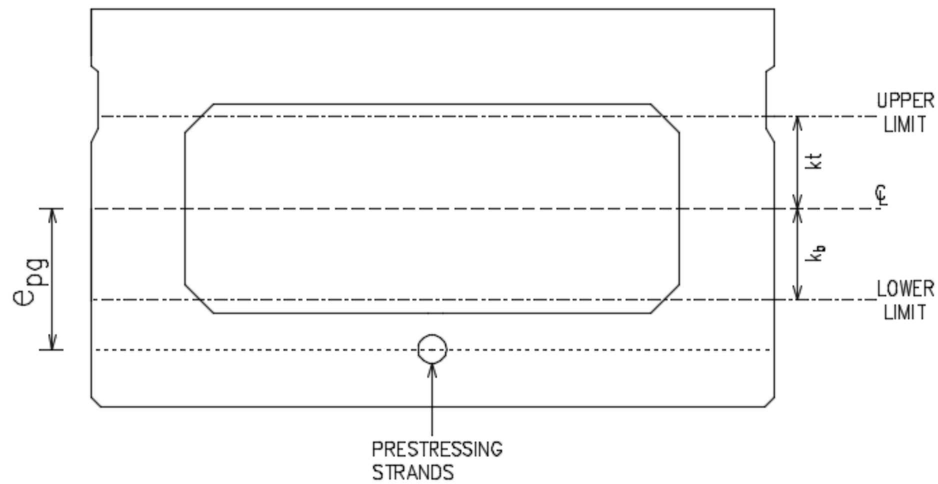


Figure E-1. Upper and lower limits of a kern in a prestressed beam

Composite Section

Interior beam:

Distance from centroid to upper limit of the central kern, $k_t = -S_{bc}/A_c$ = -4.76 in.

Distance from centroid to lower limit of the central kern, $k_b = S_{tc}/A_c$ = 7.83 in.

STEP E-2. PRESTRESSING STRAND DESIGN

Step E.2.1. Stress Conditions at Midspan

Stresses at release and in-service need to be maintained at or below the respective stress limits.

Sign Convention: Tensile stress is designated to be negative.
The following conditions and stress limits are considered:

At release:
Noncomposite section top and bottom fiber stresses under a prestressing force and the beam's self-weight on the overall beam length should be below the allowable tension and compression stress limits.

Condition 1: Beam top fiber tensile stress check at release

$$\frac{F_i}{A_b} - \frac{F_i e_{pg}}{S_t} + \frac{M_{gr}}{S_t} \geq \left(-\bar{f}_{ti}\right)$$

LRFD Table 5.9.4.1.2-1

$$-\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_t} - \frac{M_{gr}}{S_t} \leq \bar{f}_{ti}$$

where,

F_i = prestressing force at release (kip)
A_b = area of beam cross-section (in.^{2})
e_{pg} = eccentricity of strands with respect to girder centroid (in.)
S_t = section modulus for top fiber (in.^{3})
M_{gr} = moment due to beam weight at release (kip-ft)
$\bar{f}_{ti}$ = allowable concrete tensile stress at release (ksi)

Condition 2: Beam bottom fiber compression stress check at release

$$\frac{F_i}{A_b} + \frac{F_i e_{pg}}{S_b} - \frac{M_{gr}}{S_b} \leq \bar{f}_{ci}$$

LRFD Art. 5.9.4.1.1

where,

S_b = section modulus of noncomposite beam for bottom fiber (in.^{3})
$\bar{f}_{ci}$ = allowable concrete compressive stress at release (ksi)

In-service:

Conditions are:

- Moment due to beam, cast-in-place deck, and haunch weight on noncomposite section.
- Moment due to non-structural elements (barrier and future wearing surface) weight and live load on composite section.

Condition 3: Beam top fiber compression stress check under effective prestress and permanent loads

$$\frac{\eta F_i}{A_b} - \frac{\eta F_i e_{pg}}{S_t} + \frac{(M_g + M_D + M_{cd})}{S_t} + \frac{(M_b + M_{ws})}{S_{tc}} \leq \bar{f}_{cpl}$$

LRFD Table 5.9.4.2.1-1

where,

η = ratio of effective stress after losses to prestressing steel stress prior to transfer

M_g = moment due to beam weight (kip-in)

M_D = moment due to deck and haunch weight (kip-in)

M_{CD} = moment due to concrete diaphragm weight (kip-in)

M_b = moment due to barrier weight (kip-in)

M_{ws} = moment due to future wearing surface (kip-in)

S_{tc} = section modulus of composite beam for top fiber (in.^{3})

$\bar{f}_{cpl}$ = Allowable compressive stress for concrete subjected to effective prestress and permanent loads (ksi)

Condition 4: Beam top fiber compression stress check under effective prestress, permanent loads, and transient loads

LRFD Table 5.9.4.2.1-1

$$\frac{\eta F_i}{A_b} - \frac{\eta F_i e_{pg}}{S_t} + \frac{(M_g + M_D + M_{cd})}{S_t} + \frac{(M_b + M_{ws} + M_{LT} + M_{LL})}{S_{tc}} \leq \bar{f}_{ctl}$$

where,

$\bar{f}_{ctl}$ = allowable compressive stress for concrete subjected to effective prestress, permanent loads, and transient loads (ksi)

M_{LT} = moment due to truck load (kip-in)

M_{LL} = moment due to lane load (kip-in)

Condition 5: Beam bottom fiber tension stress check under effective prestress, permanent loads, and transient loads

LRFD Table 5.9.4.2.2-1

$$\frac{\eta F_i}{A_b} + \frac{\eta F_i e_{pg}}{S_b} - \frac{(M_g + M_D + M_{cd})}{S_b} - \frac{(M_b + M_{ws} + 0.8M_{LT} + 0.8M_{LL})}{S_{bc}} \geq (-\bar{f}_{ts})$$

$$-\frac{\eta F_i}{A_b} - \frac{\eta F_i e_{pg}}{S_b} + \frac{(M_g + M_D + M_{cd})}{S_b} + \frac{(M_b + M_{ws} + 0.8M_{LT} + 0.8M_{LL})}{S_{bc}} \leq \bar{f}_{ts}$$

where,

S_{bc} = Section modulus of the composite beam for bottom fiber (in.^{3})

$\bar{f}_{ts}$ = Allowable tensile stress for concrete with bonded steel and subjected to not worse than moderate corrosion condition (ksi)

To determine the required prestressing force at midspan, Service III limit state is used. For required prestressing force and associated eccentricity calculations and midspan stress checks, the five inequality conditions given above are merged graphically to demonstrate the iterative approach of the design.

In this process, the five conditions are rearranged to characterize five inequalities representing the relationship between eccentricity (e_{pg}) and initial prestressing force ($F_i = f_{pi}A_{ps}N$; where A_{ps} is the area of a prestressing strand and N is the number of strands).

In addition to five conditions, a sixth condition is imposed by limiting eccentricity of the prestressing strands that are to be bounded by a concrete cover. In this example, a 2 in. concrete cover is specified.

Please note the sign convention where tensile stresses are negative (-);

Condition 1:

$$e_{pg} \le k_b + \left[\frac{1}{F_i}\right](M_{gr} + \bar{f}_{ti}S_t)$$

Condition 2:

$$e_{pg} \le k_t + \left[\frac{1}{F_i}\right](M_{gr} + \bar{f}_{ci}S_b)$$

Condition 3:

$$e_{pg} \ge k_b + \left[\frac{1}{\eta F_i}\right]\left[\left(M_g + M_D + M_{cd}\right) + \frac{(M_b + M_{ws})S_t}{S_{tc}} - \bar{f}_{cpl}S_t\right]$$

Condition 4:

$$e_{pg} \ge k_b + \left[\frac{1}{\eta F_i}\right]\left[\left(M_g + M_D + M_{cd}\right) + \frac{(M_b + M_{ws} + M_{LT} + M_{LL})S_t}{S_{tc}} - \bar{f}_{ctl}S_t\right]$$

Condition 5:

$$e_{pg} \ge k_t + \left[\frac{1}{\eta F_i}\right]\left[\left(M_g + M_D + M_{cd}\right) + \frac{(M_b + M_{ws} + 0.8M_{LT} + 0.8M_{LL})S_b}{S_{bc}} - \bar{f}_{ts}S_b\right]$$

Condition 6:

$$e_{pg} \le y_b - 2 \text{ in.}$$

y_b = distance from the centroid to the extreme bottom fiber of the noncomposite precast beam (in.)

The six inequalities are illustrated graphically in **Figure E-2**. The hatched area represents the feasibility domain of the prestressing force (F_i) and strand eccentricity (e_{pg}) combinations that do not violate all five stress limits and the maximum allowable eccentricity limit.

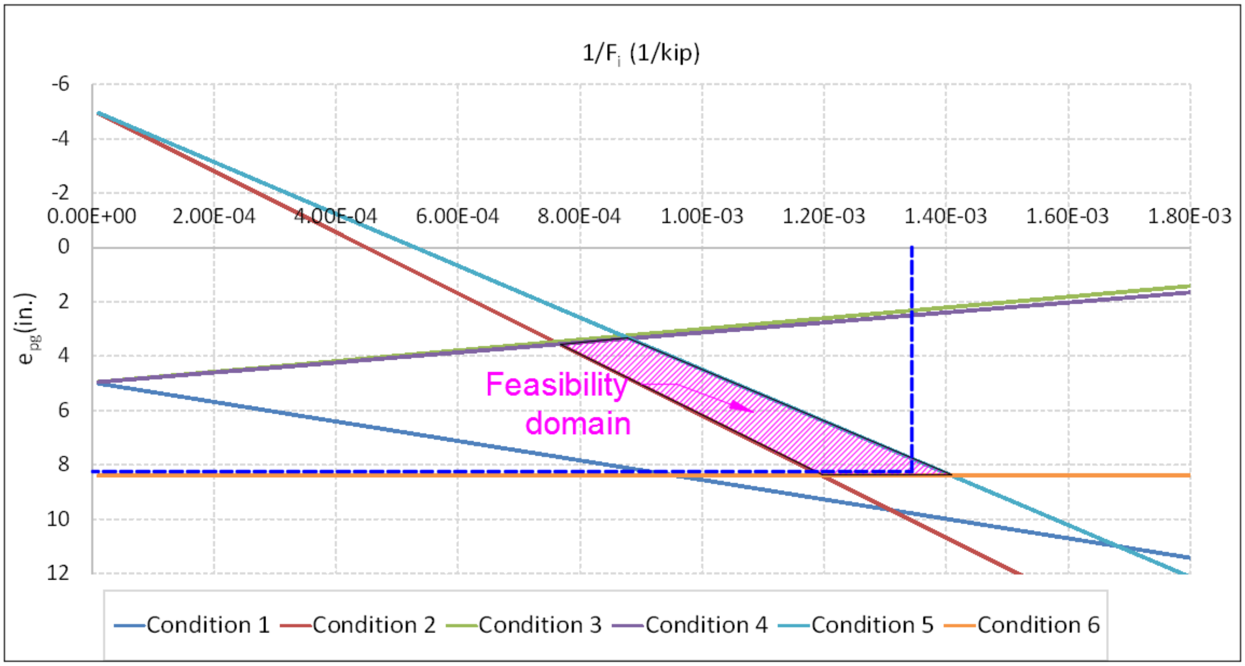


Figure E-2. Feasibility domain of e_{pg} and F_i at midspan

Step E.2.2. Eccentricity at Midspan

The strand center of gravity location, measured from the bottom of the beam, is 5% to 15% of the beam depth. As an initial assumption, location is set to 10% of beam depth. Therefore,

$$e_{pg} = y_b - 0.10 \times h = 10.40 - 0.10 \times 21 = 8.3 \text{ in.}$$

With e_{pg} = 8.3 in. on the y-axis, $1/F_i$ located on the x-axis and inside the feasibility domain is obtained from **Figure E-2**.

$1/F_i$ $= 1.35 \times 10^{-3}$ kips^{-1}

Hence, F_i = 741 kip

Required number of strands, N $= F_i/(A_{ps}f_{pi}) = 741/(0.217 \times 202.5) = 16.86$

Select, N = 20

Prestressing force at release, F_i $= f_{pi}A_{ps}N$

$= (202.5 \text{ ksi})(0.217 \text{ in.}^2)(20)$

$= 878.75$ kips

Step E.2.3. Feasibility Check at Midspan

A check is required to confirm that F_i and e_{pg} remain in the feasibility domain.

As an example,

Prestressing force at midspan of the beam, F_i = 878.75 kips

Hence, $1/F_i = 1/878.75 \text{ kip} = 1.14 \times 10^{-3}$ kips^{-1}

As per the strand arrangement shown in Figure 7 of box-beam design example, e_{pg} = 7.60 in.

As shown in **Figure E-3**, the point defined by $1/F_i = 1.14\times10^{-3}$ $kips^{-1}$ and e_{pg} = 7.60 in. for service condition is located within the feasibility domain. Hence, the strand arrangement at midspan satisfies all the stress limits considered in **Step E.2.1**.

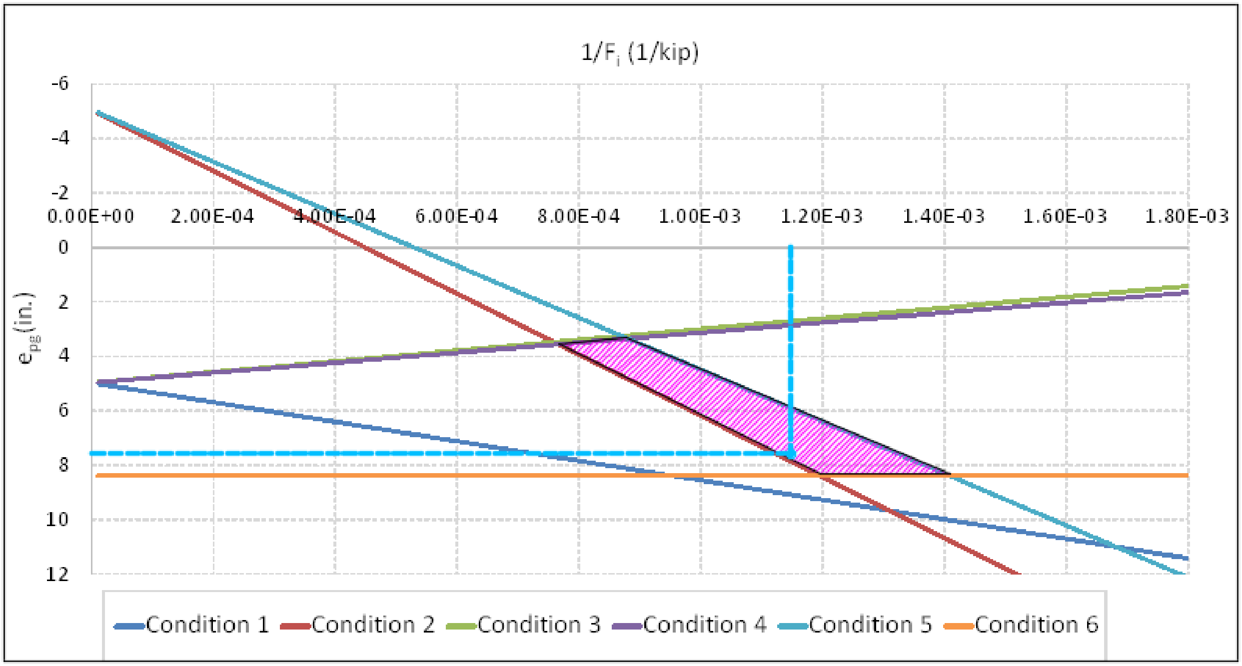


Figure E-3. Strand arrangement check with feasibility domain

Step E.2.4. Limiting Kern along Beam Length

As discussed earlier, upper and lower limits of kern along the beam length define the required number of prestressing strands and their arrangement needed to satisfy the tension and compression stress limits along the beam length.

The process requires representing the controlling inequality condition derived from five stress inequalities defined in **Step E.2.1**. without the effect of beam weight and moments. As an example, the stress inequality for Condition 1 (top fiber stresses at release) is reformulated in the steps shown below:

Equation from **Step E.2.1**

$$e_{pg} \leq k_b + \left[\frac{1}{F_i}\right](M_{gr} + \bar{f}_{ti}S_t)$$

With the effect of beam self-weight excluded, i.e., $M_{gr} = 0$

$$e_{pg} \leq k_b + \frac{\bar{f}_{ti}S_t}{F_i};\text{ where, } S_t = k_bA_b\text{, and}$$

$$e_{pg} \leq k_b + \frac{\bar{f}_{ti}k_bA_b}{F_i}$$

Defining stress at the beam centroid under initial prestressing force, $f_{gi} = F_i/A_b$

$$e_{pg} \leq k_b(1 + \frac{\bar{f}_{ti}}{f_{gi}})$$

Following a similar procedure, all five inequalities are written in the following format:

Condition 1:	Condition 2:	Condition 3:	Condition 4:	Condition 5:
$e_{pg} \leq k_b(1 + \frac{\bar{f}_{ti}}{f_{gi}})$	$e_{pg} \leq k_t(1 - \frac{\bar{f}_{ci}}{f_{gi}})$	$e_{pg} \geq k_b\left(1 - \frac{\bar{f}_{cpl}}{f_g}\right)$	$e_{pg} \geq k_b\left(1 - \frac{\bar{f}_{ctl}}{f_g}\right)$	$e_{pg} \geq k_t\left(1 + \frac{\bar{f}_{ts}}{f_g}\right)$

where, f_g = stress at the beam centroid due to effective prestressing force = $(\eta F_i)/A_b$

For the given beam section and initial prestressing,

$$f_g = (F_i\eta)/A_b = 1{,}506 \text{ psi}$$
$$f_{gi} = F_i/A_b = 1{,}882 \text{ psi}$$

In the first two conditions, f_{ti} and f_{ci}, are used for defining the lower limit of kern.

$$k`_b = \min\left[k_b\left(1 + \frac{\bar{f}_{ti}}{f_{gi}}\right), k_t\left(1 + \frac{\bar{f}_{ci}}{f_{gi}}\right)\right]$$

$$k`_b = \min[6.6 \text{ in.}, 5.3 \text{ in.}] = 5.3 \text{ in.}$$

Hence, the second condition governs the lower limit of kern.

The other three conditions with f_{ctl}, f_{cpl}, and f_{ts} are used to define the upper limit of the kern.

$$k`_t = \max\left[k_b\left(1 - \frac{\bar{f}_{cpl}}{f_g}\right), k_b\left(1 - \frac{\bar{f}_{ctl}}{f_g}\right), k_t\left(1 - \frac{\bar{f}_{ts}}{f_g}\right)\right]$$

$$k`_t = \max[-5.1 \text{ in.}, -8.5 \text{ in.}, -6.7 \text{ in}] = -5.1 \text{ in.}$$

Hence, the third condition governs the upper limit of the kern.

Step E.2.5. Upper and Lower Bound Eccentricity along the Beam Length

The governing conditions that define the lower and upper limits of the kern are described in **Step E.2.4**.

In this calculation step, decreasing moment along the beam length from midspan to beam end will define the upper and lower eccentricity limits.

Upper limit of the kern is calculated with the 3rd condition:

$$e_{pgu} = k`_b + \left[\frac{1}{\eta F_i}\right](M_g + M_D + M_{cd}) + \frac{(M_b + M_{ws})S_t}{S_{tc}}$$

Lower limit of the kern is calculated with the 2nd condition:

$$e_{pgl} = k`_t + \left[\frac{M_{gr}}{F_i}\right]$$

Using the above two equations, e_{pgu} and e_{pgl} are calculated along the span and illustrated in **Table E-1**. **Figure E-4** demonstrates e_{pgu} and e_{pgl} variation along the span. The hatched area represents the feasibility region for the center of gravity of prestressing strands without violating the stress limits at transfer and in service.

Table E-1. Upper and Lower Bounds of Eccentricity along the Half Span

Location	e_{pgu} (in.)	e_{pgl} (in.)
Beam end		5.30
CL of bearing	-5.10	5.43
0.05 × L_{ds}	-3.61	5.87
0.10 × L_{ds}	-2.25	6.27
0.15 × L_{ds}	-1.06	6.62
0.20 × L_{ds}	-0.04	6.93
0.25 × L_{ds}	0.84	7.19
0.30 × L_{ds}	1.57	7.40
0.35 × L_{ds}	2.13	7.56
0.40 × L_{ds}	2.55	7.68
0.45 × L_{ds}	2.81	7.70
0.50 × L_{ds} (Midspan)	2.90	7.78

Note: Strand centroid is designed to be inside the lower limit of the kern.

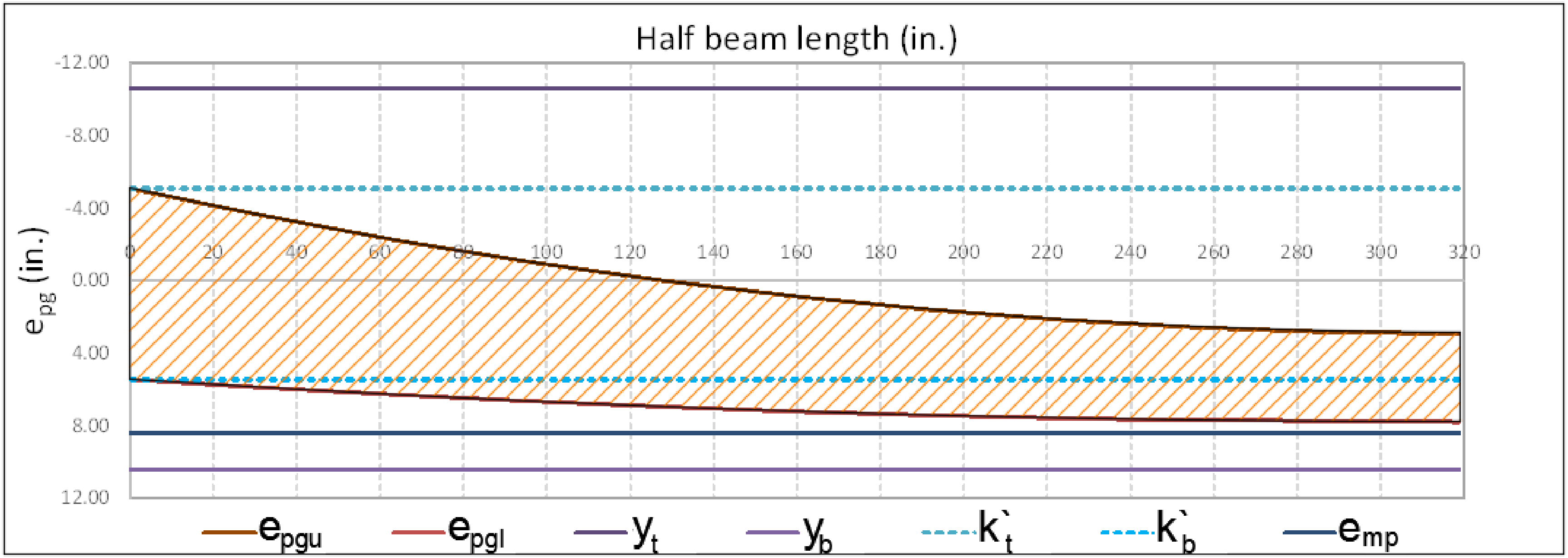


Figure E-4. Upper and lower limits of kern along the half beam length

Step E.2.5. Strand Arrangement

The strand arrangement is defined after a trial and error process so that F_i remains between the upper and lower limits of the kern.

Figure E-5 shows strand arrangement at midspan and beam end cross sections.

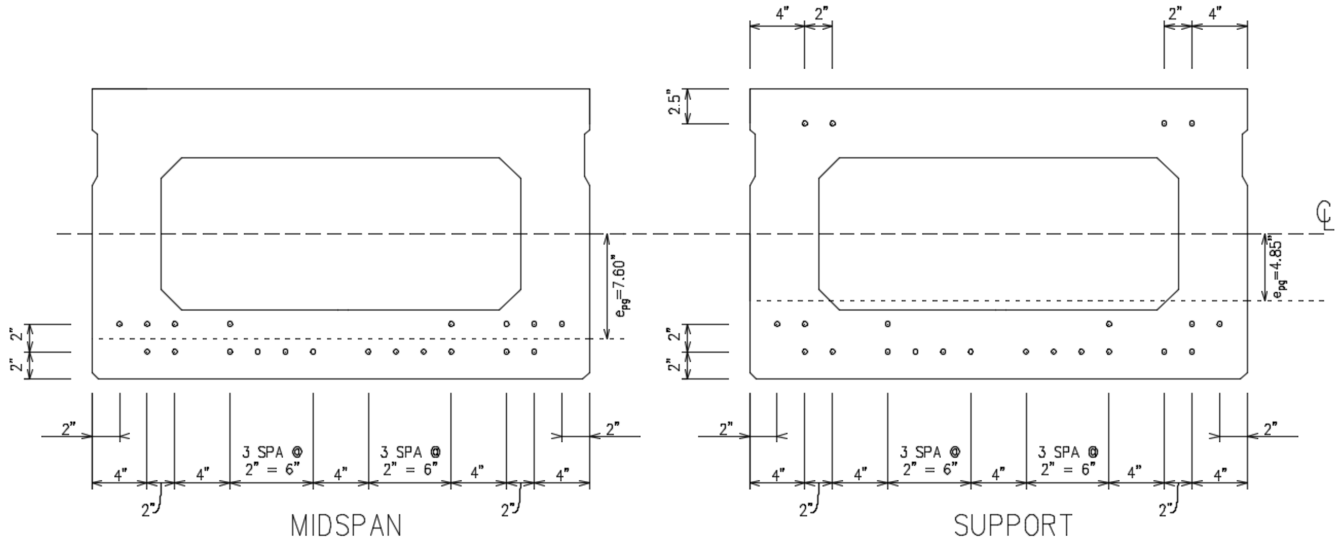


Figure E-5. Strand arrangement at midspan and over the supports

Step E.2.6. Strand Profile

The longitudinal strand profile along the beam length is designed. **Figure E-6** and **Figure E-7** show the strand profile along half length of the beam at release and in service, respectively. In this example, to satisfy the stress limits along the beam length, debonding point at the bottom of the beam is located at $0.1L_b$ from beam end while top strands (a.k.a. cut strands) are bonded up to $0.25L_b$ from beam end.

The objective of debonding is to maintain the eccentricity within the lower and upper bounds of the limit kern along the beam length. Adding cut strands at top of the beam for controlling the eccentricity by changing the total prestressing force and eccentricity complicates the design procedure. To assure the satisfaction of stress limits along the beam length, based on the designed strand profile, the upper and lower bounds of eccentricity should be recalculated at many sections along the span.

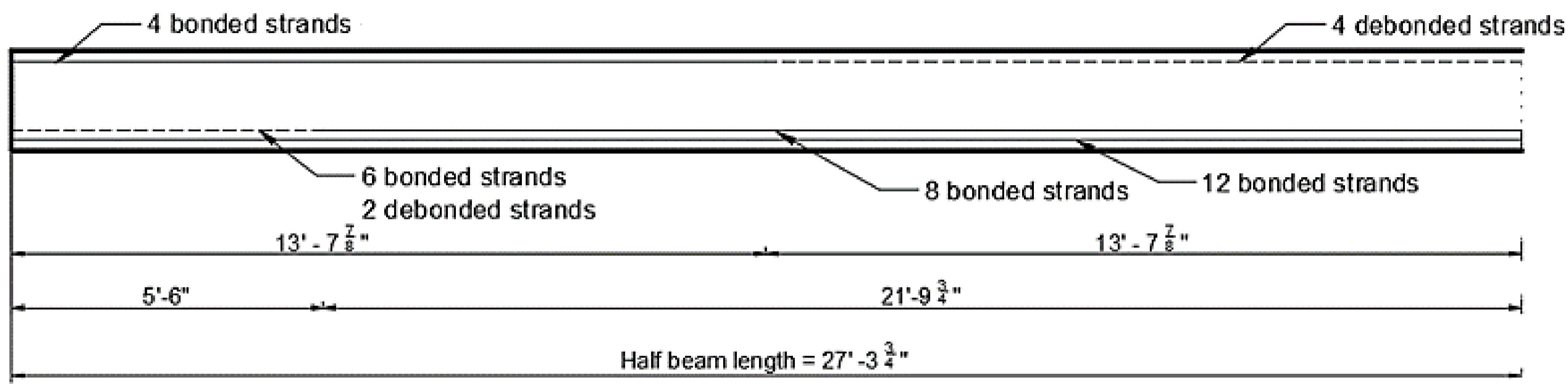


Figure E-6. Longitudinal strand profile along half length of the beam at release

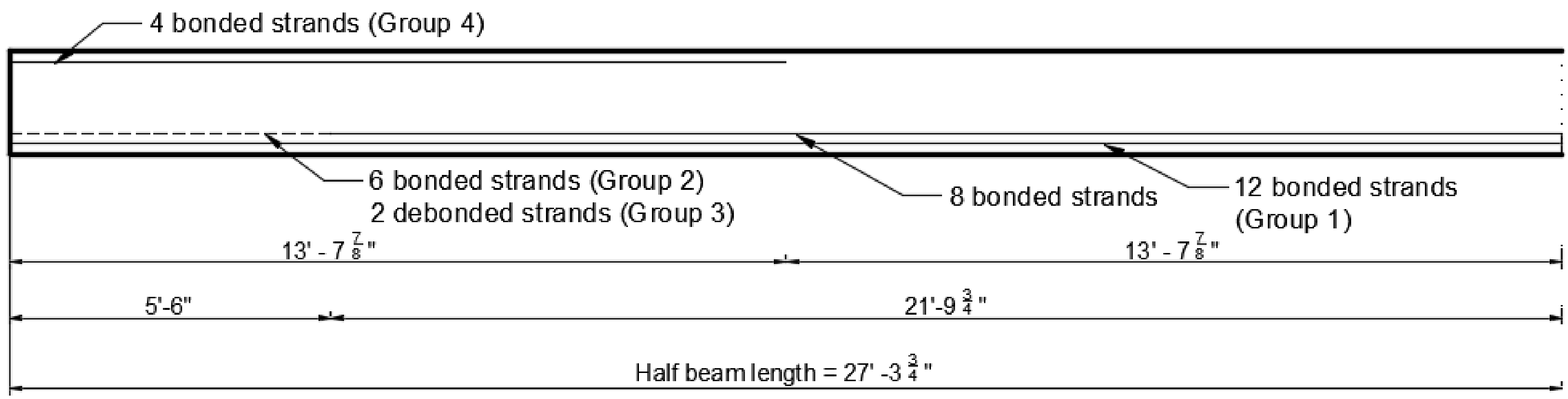
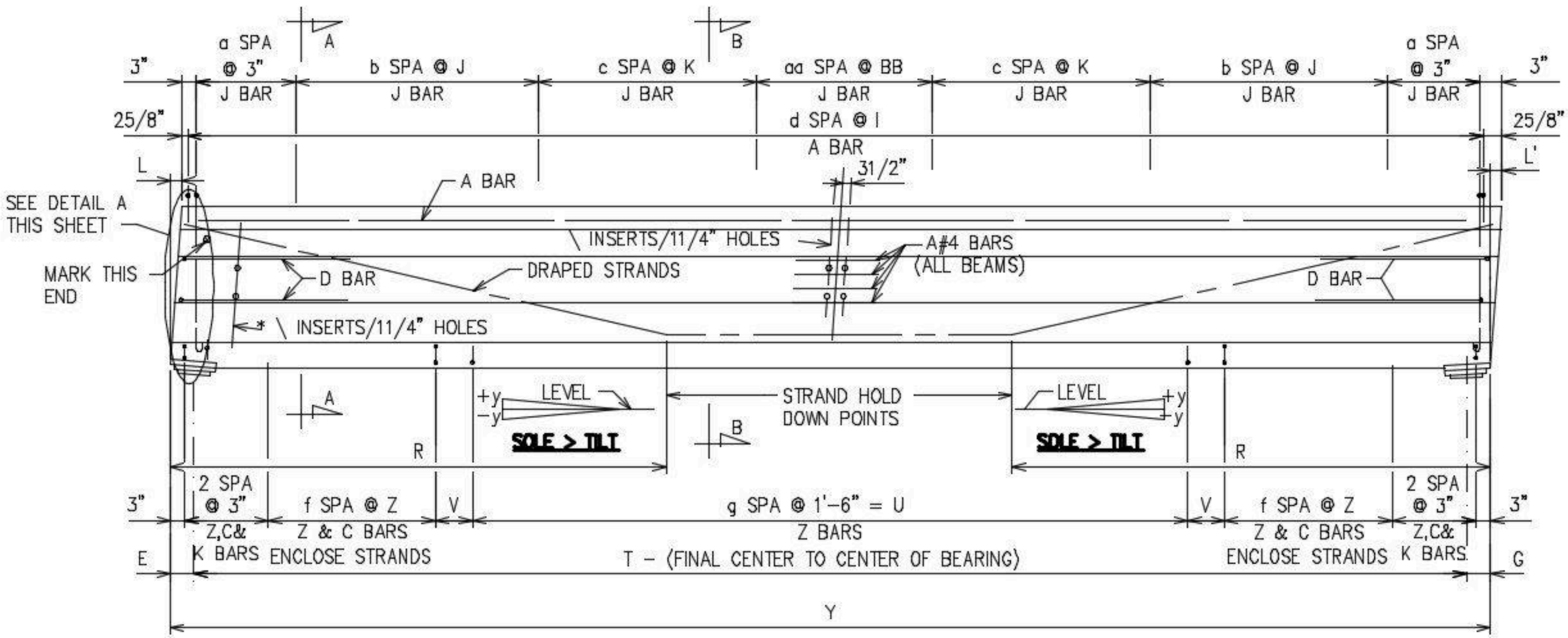


Figure E-7. Longitudinal strand profile along half length of the beam in service

APPENDIX F

STANDARD SHEAR DETAILS

BEAM DIMENSIONS		
SPAN	1	2
NO.REQ.	3	3
a	2	2
b	12	12
c	16	16
J	8.5"	8.5"
K	2'–0"	2'–0"
aa	2	2
BB	1'–4.5"	1'–4.5"



BEAM ELEVATION

Figure F-1. Standard shear details of an interior bulb-tee beam

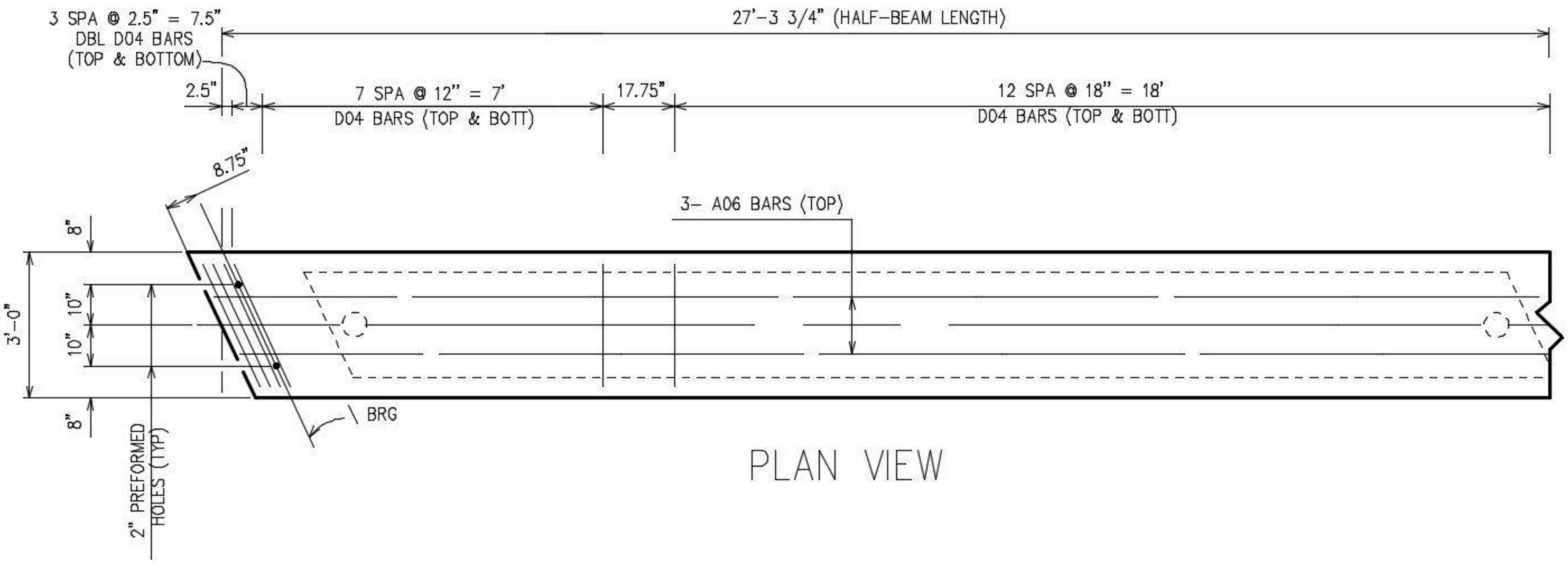
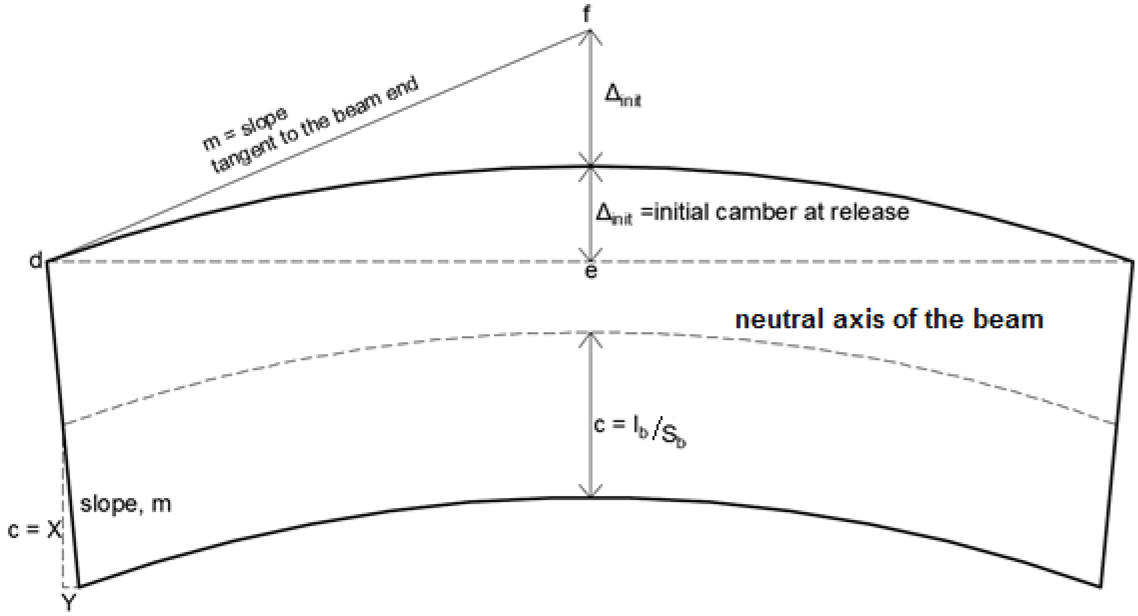
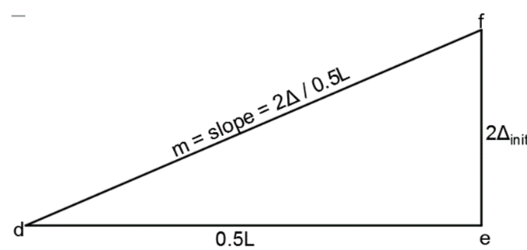


Figure F-2. Standard shear details of an interior box beam

APPENDIX G

DERIVATION FOR BOTTOM FLANGE SHORTENING CALCULATION





$$\text{slope, m} = 2\frac{\Delta_{\text{init}}}{0.5\text{L}} = \frac{\text{Y}}{\text{X}} = \frac{\text{Y}}{\frac{\text{I}_\text{b}}{\text{S}_\text{b}}}$$

$$\text{Y} = 2\left(\frac{\Delta_{\text{init}}}{0.5\text{L}}\right)\left(\frac{\text{I}_\text{b}}{\text{S}_\text{b}}\right)$$

Total bottom flange shortening at release;

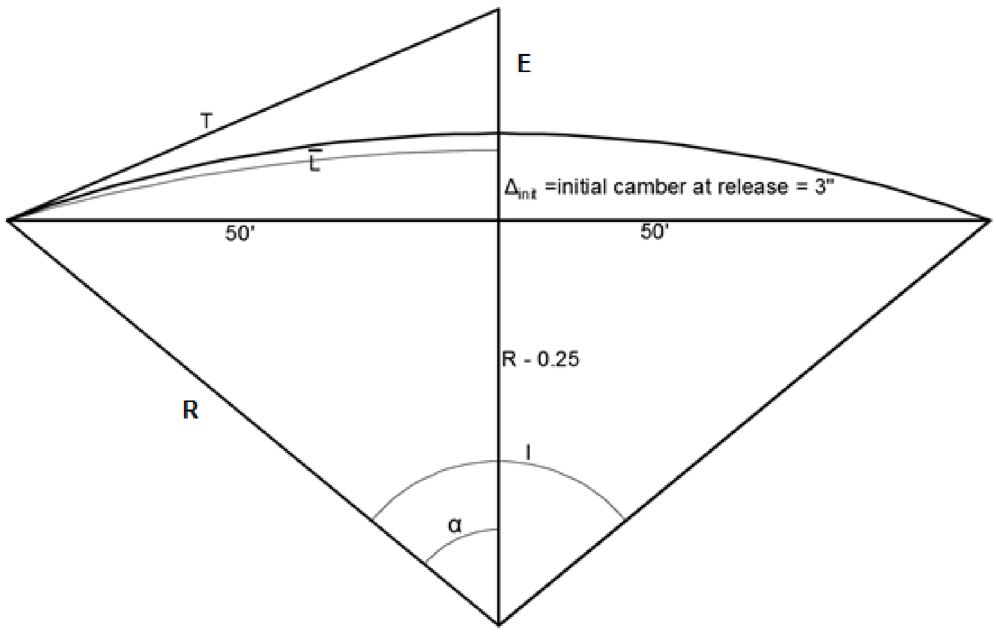
$$= \frac{\text{PL}}{\text{AE}} + 2\text{Y} = \frac{\text{PL}}{\text{AE}} + 2\left(\frac{2\ \Delta_{\text{init}}}{0.5\text{L}}\right)\left(\frac{\text{I}_\text{b}}{\text{S}_\text{b}}\right)$$

$$= \frac{\text{PL}}{\text{AE}} + \frac{8\Delta_{\text{init}}}{\text{L}}\left(\frac{\text{I}_\text{b}}{\text{S}_\text{b}}\right)$$

Note that 2Y in the above equation represents the deformation at both ends of the beam.

Assume: 100 ft span with 3 in. initial camber.

Show that $E = \Delta_{init}$



Solve for radius;

$$R^2 = 50^2 + (R - 0.25)^2 = 50^2 + R^2 - 0.5R + 0.0625$$

Since 0.0625 is negligible, $0.5R = 50^2$

$$R = 5000 \text{ ft}$$

$$\alpha_{rad} = \sin^{-1}\left(\frac{50}{5000}\right) = 0.01 \text{ rad}$$

Length of arc = $\hat{L} = R\alpha_{rad} = 5000\ (0.01) = 50$ ft

Since the angle between T and R is 90 deg,

Length of tangent = $T = R\tan\alpha = 5000\tan(0.01) = 50.002 \cong 50$ ft

Therefore, $T = \hat{L}$

$$E = T\left(\tan\frac{I}{4}\right) = T\left(\tan\frac{\alpha}{2}\right) = 50\tan\left(\frac{0.01}{2}\right) = 0.25 \text{ in.}$$

$$E = \Delta_{init} = 0.25 \text{ in.}$$

APPENDIX H

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