DESIGN OF A HAMMERHEAD PIER AND FOUNDATIONS

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May 10, 2023

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ACKNOWLEDGEMENTS

This project is funded by the Michigan Department of Transportation. It is carried out by the Michigan Department of Transportation Center of Excellence for Structural Durability at Western Michigan University. We, the authors, would like to acknowledge Mr. Juan Alcantar as the project manager and the advisory panel members for their assistance in finding relevant information, reviewing the drafts, and responding to our questions in a timely manner to successfully complete this example. We appreciate the contribution of Ms. Kanchani Basnayake and Mr. Harsha Amunugama for the development of this example.

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Section 7 Hammerhead Pier with Spread Footing
Step 7.1 Preliminary Dimensions

Description

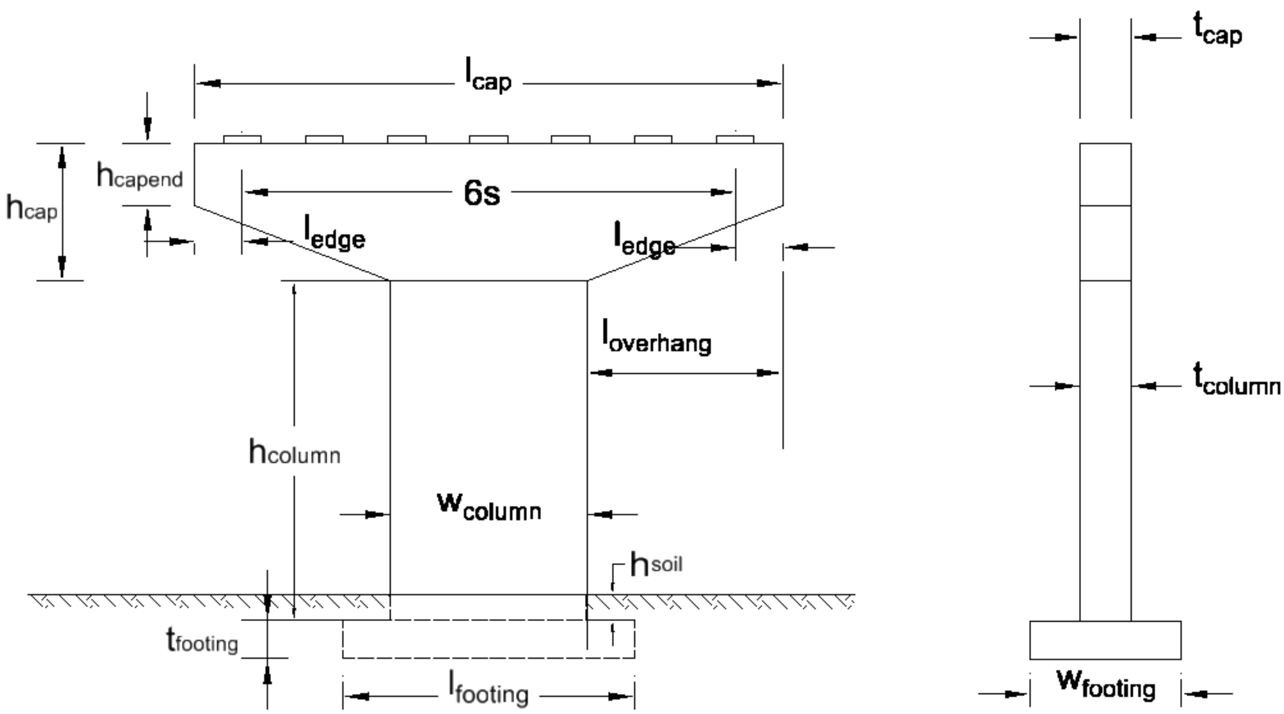
This section illustrates the design of a hammerhead pier supported by a spread footing for an interstate freeway bridge. The design is implemented in accordance with the Michigan Department of Transportation (MDOT) policies published as of 09/30/2022. This design follows the requirements of the 9th Edition of the AASHTO LRFD Bridge Design Specification, as modified and supplemented by the Bridge Design Manual (BDM), Bridge Design Guides (BDG), and 2020 Standard Specifications for Construction (SSFC). Certain material and design parameters are selected to be in compliance with MDOT practice reflected in the Bridge Design System (BDS), the MDOT legacy software.

The pier is designed for the superstructure described in the *Two-Span Continuous Bridge Steel Plate Girder Design Example* developed by Attanayake et al. (2021). Refer to Section 2 of the *Design of Highway Bridge Abutments and Foundations Example* developed by Attanayake and Hu (2023) for the design criteria, bridge information, material properties, and soil types and properties.

These examples are available at https://mdotjboss.state.mi.us/SpecProv/trainingmaterials.htm#2108560.

The preliminary dimensions are selected based on site-specific conditions, highway agency standards, and past experience.

The following figure shows the pier geometry and dimensional variables:



The preliminary dimensions selected for this example are given below.

Pier cap length $l_{cap} := W_{deck} = 63.75\,ft$

Pier cap end height $h_{capend} := 5ft$

Pier cap height $h_{cap} := 11ft$

Pier cap thickness $t_{cap} := 4ft$

Length of the overhang $l_{overhang} := 21.25ft$

Column width $w_{column} := 21.25ft$

Column thickness $t_{column} := 4ft$

Column height $h_{column} := 14ft$

A spread footing is selected for this pier.

Footing length $l_{footing} := 32.25ft$

Footing thickness $t_{footing} := 3ft$

Footing width $w_{footing} := 18ft$

Depth of soil above the footing $h_{soil} := 3ft$

Note: The depth from the ground level to the bottom of the footing needs to be maintained at a minimum of 4 ft for frost depth. Typically, a one-foot deep soil profile is maintained with normal grading when the pier is at a median. The depth of the soil may change to 2 to 3 ft based on the pavement profile when the pier is closer to the pavement.

Girder spacing $S = 9.719\,ft$

Distance from the exterior girder to the edge of the pier cap

$$l_{edge} := \frac{l_{cap} - S \cdot (N_{beams} - 1)}{2} = 2.719\,ft$$

Concrete Cover Requirements for Reinforcing Steel

Unless otherwise shown on the plans, the minimum concrete clear cover for reinforcement shall satisfy the following requirements: **BDM 8.02.N**

Concrete cast against earth: 3 in.

For all other cases unless shown on plans: 2 in.

The following concrete cover is used since it is greater than the required minimum. **BDG 5.16.01, 5.18.01, 5.22.01**

Cover for the footing $Cover_{ft} := 4in$

Since the concrete cover requirements for pier caps and columns are not provided in the BDM and BDG, the following dimensions are taken from the MDOT Sample Bridge Plans.

Cover for the pier cap $Cover_{cap} := 3.5in$

Cover for the pier columns $Cover_{col} := 4in$

Step 7.2 Application of Dead Load

Description

This step describes the application of the dead load on the hammerhead pier.

Dead Load Girder Reactions

The superstructure dead load reactions per bearing are taken from the *Steel Plate Girder Design Example*. All the beam seats are assumed to be at the same elevation.

When calculating superstructure loads on the substructure, 75% of the barrier dead load should be applied with the fascia beam load. The remaining 25% of the barrier load should be applied with the first interior girder load.

BDM 7.01.04.J

Note: The exterior and interior girder shear values presented in the *Steel Plate Girder Design Example* (Table 12 and 13) were calculated by equally distributing the barrier loads to all the girders. Therefore, the girder reactions over the pier due to barrier loads need to be recalculated as shown below.

<u>Exterior Girders</u>

Table 12 of the Steel Plate Girder Design Example

Reaction due to the weight of structural components and non-structural attachments (DC), including the stay-in-place formwork but excluding barrier weight

$$R_{DCEx\_noBarrier} := 161.4kip$$

Reaction due to 75% of the barrier weight (DB) on the exterior girder $R_{DCEx\_barrier} := 44kip$

Total exterior girder reaction due to DC $R_{DCEx} := R_{DCEx\_noBarrier} + R_{DCEx\_barrier} = 205.4 \cdot kip$

Reaction due to the weight of the future wearing surface (DW) $R_{DWEx} := 26.6kip$

<u>First Interior Girder</u>

Table 13 of the Steel Plate Girder Design Example

Reaction due to the weight of structural components and non-structural attachments (DC), including the stay-in-place formwork but excluding barrier weight

$$R_{DC1stIn\_noBarrier} := 190.4kip$$

Reaction due to 25% of the barrier weight (DB) on the first interior girder $R_{DC1stIn\_barrier} := 14.5kip$

Total first interior girder reaction due to DC $R_{DC1stIn} := R_{DC1stIn\_noBarrier} + R_{DC1stIn\_barrier} = 204.9 \cdot kip$

Weight of the future wearing surface (DW) $R_{DWIn} := 26.4kip$

<u>Other Interior Girders</u>

Table 13 of the Steel Plate Girder Design Example

Reaction due to the weight of structural components and non-structural attachments (DC), including the stay-in-place formwork but excluding barrier weight

$$R_{DCIn} := 190.4kip$$

Dead Load Calculation

Dead load of superstructure

Weight of structural components and non-structural attachments (DC)

$$DC_{Sup} := 2 \cdot R_{DCEx} + 2 \cdot R_{DC1stIn} + \left(N_{beams} - 4\right) \cdot R_{DCIn}$$

$$DC_{Sup} = 1.392 \times 10^{3} \cdot kip$$

Weight of future wearing surface (DW) $DW_{Sup} := 2 \cdot R_{DWEx} + \left(N_{beams} - 2\right) \cdot R_{DWIn} = 185.2 \cdot kip$

Pier cap weight $DC_{cap} := W_c \cdot t_{cap} \cdot \left[2 \cdot \left(\frac{h_{capend} + h_{cap}}{2} \right) \cdot l_{overhang} + h_{cap} \cdot w_{column} \right] = 344.25 \cdot kip$

Pier column weight $DC_{column} := W_c \cdot t_{column} \cdot h_{column} \cdot w_{column} = 178.5 \cdot kip$

Pier footing weight $DC_{footing} := W_c \cdot w_{footing} \cdot t_{footing} \cdot l_{footing} = 261.225 \cdot kip$

Step 7.3 Application of Live Load

Description

This step describes the application of the live load on the hammerhead pier.

Girder Reactions Due to Live Load on a Single Lane

MDOT uses a modified version of the HL-93 loading in the AASHTO LRFD Bridge Design Specifications. The combination of a single design truck load, a single 60-kip load (axle load), or a two design truck load for continuous spans, and a design lane load are multiplied by a factor of 1.2 to designate the design loading as HL-93 Mod.

Factor for HL-93 Mod	$f_{HL93Mod} := 1.2$	**BDM 7.01.04-A**
Dynamic Load Allowance	$IM := 0.33$	**LRFD Table 3.6.2.1-1**

Several software programs are available for designers to calculate the maximum live load effects by developing 3D bridge models and simulating live load positions along and across the lanes. However, this example demonstrates a commonly used, easy-to-implement, approach for the same purpose. The process includes the following steps:

1. Develop a single line girder model representing girder cross-sections, effective deck cross-section, composite and noncomposite segments of the girder, and boundary conditions.
2. Apply relevant truck and lane loads as independent loads to calculate the maximum reaction per lane at the girder support over the pier. For example, refer to Table A-2 and A-4 in the *Steel Plate Girder Design Example* for loads and the format of results.
3. Multiply selected support reactions with applicable factors. For example, the support reaction due to truck load is multiplied by the impact factor. When the support reactions are due to the truck pair for continuous spans, both reactions due to truck and lane loads are multiplied by a factor of 0.9 to account for the 10% reduction specified in the AASHTO LRFD Bridge Design Specifications.
4. Calculate an equivalent pair of wheel loads, P_{wheel}, that will result in the same support reactions developed by the truck load on a single lane.
5. Calculate an equivalent 10-ft long line load, W_{lane}, that will result in the same support reactions developed by the lane load applied on a single lane.
6. Apply these P_{wheel} and W_{lane} loads on the bridge deck to generate girder end reactions that will ultimately result in the maximum force effects in the pier cap, columns, and footing.

Since the load distribution factors in the LRFD Specifications are not used in this process to calculate girder end reactions, a multiple presence factor is applied to the truck and lane loads depending on the number of design lanes considered in the analysis.

$$MPF(lane) := \begin{cases} 1.2 & \text{if } lane = 1 \\ 1.0 & \text{if } lane = 2 \\ 0.85 & \text{if } lane = 3 \\ 0.65 & \text{otherwise} \end{cases}$$

LRFD Table 3.6.1.1.2-1

The *Steel Plate Girder Design Example* presents unfactored girder reactions for truck and lane loads. The following three live load cases are used in the *Steel Plate Girder Design Example* to determine the design forces and moments:

Case 1: design truck + design lane, **LRFD 3.6.1.3.1**
Case 2: a single 60-kip axle load + design lane, and
Case 3: 90% of two design trucks spaced a minimum of 50-ft apart + 90% of design lane.

Case 1 is the governing case for girder reactions over the pier. Case 1 reactions given in Appendix A of the *Steel Plate Girder Design Example* on a per-lane basis do not include the factors for HL-93 Mod and the dynamic load allowance.

Table A-2 and A-4 in the *Steel Plate Girder Design Example* present the exterior and interior girder reactions per lane. As shown in Table A-2 and A-4, the exterior girder reactions are slightly greater than the interior girder reactions. For this design, exterior girder reactions are used to calculate P_{wheel} and W_{lane} loads.

Since the load Case 1 (i.e. the design truck + lane load combination) produces the maximum reactions over the pier, the following loads are selected.

Maximum reaction at the girder supports over the pier due to the design truck load

$$V_{Truck} := 67.6kip$$

Table A-2 of the Steel Plate Girder Design Example

Maximum reaction at the girder supports over the pier due to the design lane load

$$V_{Lane} := 40.5kip$$

Table A-2 of the Steel Plate Girder Design Example

The unfactored concentrated load representing the girder reaction per wheel line for pier cap and column design

$$P_{wheel} := \frac{V_{Truck}}{2} \cdot f_{HL93Mod} \cdot (1 + IM) = 53.945 \cdot kip$$

Because of the damping effect of soil, only the static effects of the design truck or tandem are considered for the design of footings. Hence, the unfactored concentrated load per wheel line is calculated by excluding the dynamic load allowance as shown below.

LRFD 3.6.2.1

$$P_{wheel\_ft} := \frac{V_{Truck}}{2} \cdot f_{HL93Mod} = 40.56 \cdot kip$$

Next, the unfactored uniformly distributed load representing girder reactions over the pier due to the design lane load is calculated. This load is transversely distributed over a 10 ft wide strip. The dynamic load allowance is not applied to this load.

LRFD 3.6.2.1

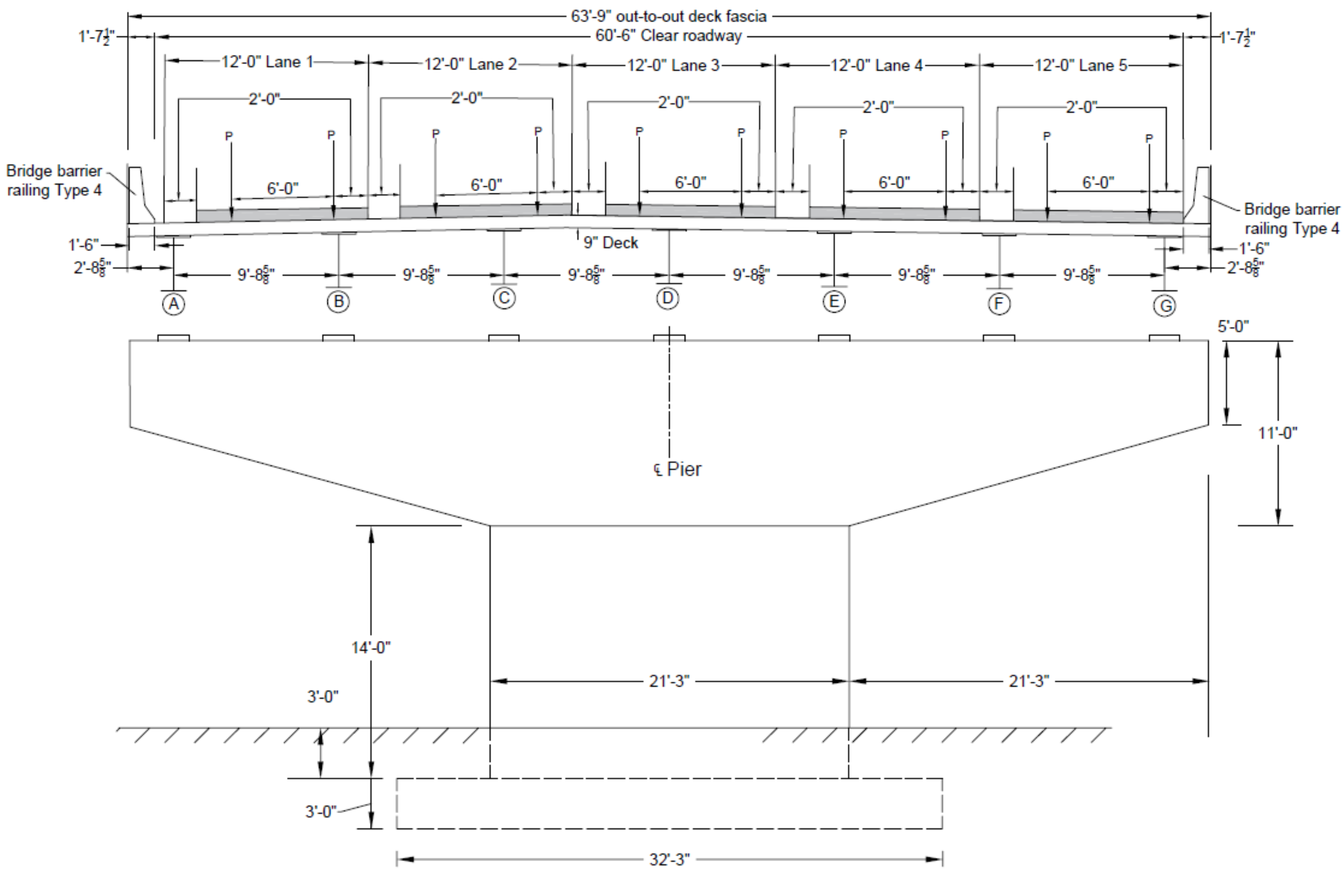
$$W_{lane} := \frac{V_{Lane} \cdot f_{HL93Mod}}{10ft} = 4.86 \cdot \frac{kip}{ft}$$

Critical Live Load Positions and Girder Reactions for Pier Cap and Column Design

This superstructure can accommodate a maximum of five (5) 12-ft wide design lanes. Therefore, the maximum live load effects on the pier cap, column, and footing are determined by considering a combination of one, two, three, four, or five loaded lanes. Since the width of the lane load is 10 ft and the axle is 6 ft, these loads are placed across the 12-ft wide lane to develop the girder end reactions that ultimately result in the maximum force effects on the pier cap, column, and footing.

LRFD 3.6.1

The following figure illustrates the controlling truck and lane load positions when all five lanes are loaded. The Lane 5 load is placed closer to the barrier to develop the maximum moment in the overhang portion of the pier cap.



The next step is to compute the reactions due to the above loads at each of the seven bearing locations. The reactions are calculated by assuming that the deck is pinned at the interior girder locations while being continuous over the exterior girders.

Only Lane 5 loaded

$$R_{G5} := \frac{P_{wheel} \cdot (8.8125ft + 2.2125ft) + W_{lane} \cdot 10ft \cdot 5.8125ft}{S} = 90.261 \cdot kip$$

$$R_{F5} := P_{wheel} \cdot 2 + W_{lane} \cdot 10ft - R_{G5} = 66.228 \cdot kip$$

$R_{A5} := 0$ $\quad R_{B5} := 0$ $\quad R_{C5} := 0$ $\quad R_{D5} := 0$ $\quad R_{E5} := 0$

Only Lane 4 loaded

$$R_{F4} := \frac{P_{wheel} \cdot (6.53125ft + 0.53125ft) + W_{lane} \cdot 8.53125ft \cdot (0.5 \cdot 8.53125ft)}{S} = 57.399 \cdot kip$$

$$R_{D4} := \frac{W_{lane} \cdot 1.46875ft \cdot (0.5 \cdot 1.46875ft)}{S} = 0.539 \cdot kip$$

$$R_{E4} := P_{wheel} \cdot 2 + W_{lane} \cdot 10ft - R_{F4} - R_{D4} = 98.551 \cdot kip$$

$R_{A4} := 0$ $\quad R_{B4} := 0$ $\quad R_{C4} := 0$ $\quad R_{G4} := 0$

Only Lane 3 loaded

$$R_{E3} := \frac{P_{wheel} \cdot (4.25ft) + W_{lane} \cdot 6.25ft \cdot (0.5 \cdot 6.25ft)}{S} = 33.357 \cdot kip$$

$$R_{C3} := \frac{P_{wheel} \cdot (1.75ft) + W_{lane} \cdot 3.75ft \cdot (0.5 \cdot 3.75ft)}{S} = 13.23 \cdot kip$$

$$R_{D3} := P_{wheel} \cdot 2 + W_{lane} \cdot 10ft - R_{E3} - R_{C3} = 109.903 \cdot kip$$

$R_{A3} := 0$ $\quad R_{B3} := 0$ $\quad R_{F3} := 0$ $\quad R_{G3} := 0$

Only Lane 2 loaded

$$R_{D2} := \frac{P_{wheel} \cdot (1.96875ft) + W_{lane} \cdot 3.96875ft \cdot (0.5 \cdot 3.96875ft)}{S} = 14.866 \cdot kip$$

$$R_{B2} := \frac{P_{wheel} \cdot (4.03125ft) + W_{lane} \cdot 6.03125ft \cdot (0.5 \cdot 6.03125ft)}{S} = 31.471 \cdot kip$$

$$R_{C2} := P_{wheel} \cdot 2 + W_{lane} \cdot 10ft - R_{B2} - R_{D2} = 110.153 \cdot kip$$

$R_{A2} := 0$ $\quad R_{E2} := 0$ $\quad R_{F2} := 0$ $\quad R_{G2} := 0$

Only Lane 1 loaded

$$R_{A1} := \frac{P_{wheel} \cdot (0.3125ft + 6.3125ft) + W_{lane} \cdot 8.3125ft \cdot (0.5 \cdot 8.3125ft)}{S} = 54.049 \cdot kip$$

$$R_{C1} := \frac{W_{lane} \cdot 1.6875ft \cdot (0.5 \cdot 1.6875ft)}{S} = 0.712 \cdot kip$$

$$R_{B1} := P_{wheel} \cdot 2 + W_{lane} \cdot 10ft - R_{A1} - R_{C1} = 101.728 \cdot kip$$

$R_{D1} := 0$ $\quad R_{E1} := 0$ $\quad R_{F1} := 0$ $\quad R_{G1} := 0$

Unfactored Live Load Girder Reactions under Different Load Cases

The following are the calculations for live load girder reactions with Lane 5 loaded, Lanes 4 and 5 loaded, Lanes 3 to 5 loaded, Lanes 2 to 5 loaded, and all 5 lanes loaded cases:

Only Lane 5 loaded

$R_{A\_1L} := R_{A5} \cdot MPF(1) = 0$ $\quad R_{B\_1L} := R_{B5} \cdot MPF(1) = 0$ $\quad R_{C\_1L} := R_{C5} \cdot MPF(1) = 0$

$R_{D\_1L} := R_{D5} \cdot MPF(1) = 0$ $\quad R_{E\_1L} := R_{E5} \cdot MPF(1) = 0$ $\quad R_{F\_1L} := R_{F5} \cdot MPF(1) = 79.474 \cdot kip$

$R_{G\_1L} := R_{G5} \cdot MPF(1) = 108.314 \cdot kip$

Lanes 4 and 5 loaded

$R_{A\_2L} := (R_{A4} + R_{A5}) \cdot MPF(2) = 0$

$R_{B\_2L} := (R_{B4} + R_{B5}) \cdot MPF(2) = 0$

$R_{C\_2L} := (R_{C4} + R_{C5}) \cdot MPF(2) = 0$

$R_{D\_2L} := (R_{D4} + R_{D5}) \cdot MPF(2) = 0.539 \cdot kip$

$R_{E\_2L} := (R_{E4} + R_{E5}) \cdot MPF(2) = 98.551 \cdot kip$

$R_{F\_2L} := (R_{F4} + R_{F5}) \cdot MPF(2) = 123.627 \cdot kip$

$R_{G\_2L} := (R_{G4} + R_{G5}) \cdot MPF(2) = 90.261 \cdot kip$

Lanes 3 to 5 loaded

$R_{A\_3L} := (R_{A3} + R_{A4} + R_{A5}) \cdot MPF(3) = 0$

$R_{B\_3L} := (R_{B3} + R_{B4} + R_{B5}) \cdot MPF(3) = 0$

$R_{C\_3L} := (R_{C3} + R_{C4} + R_{C5}) \cdot MPF(3) = 11.245 \cdot kip$

$R_{D\_3L} := (R_{D3} + R_{D4} + R_{D5}) \cdot MPF(3) = 93.876 \cdot kip$

$R_{E\_3L} := (R_{E3} + R_{E4} + R_{E5}) \cdot MPF(3) = 112.122 \cdot kip$

$R_{F\_3L} := (R_{F3} + R_{F4} + R_{F5}) \cdot MPF(3) = 105.083 \cdot kip$

$R_{G\_3L} := (R_{G3} + R_{G4} + R_{G5}) \cdot MPF(3) = 76.722 \cdot kip$

Lanes 2 to 5 loaded

$R_{A\_4L} := (R_{A2} + R_{A3} + R_{A4} + R_{A5}) \cdot MPF(4) = 0$

$R_{B\_4L} := (R_{B2} + R_{B3} + R_{B4} + R_{B5}) \cdot MPF(4) = 20.456 \cdot kip$

$R_{C\_4L} := (R_{C2} + R_{C3} + R_{C4} + R_{C5}) \cdot MPF(4) = 80.198 \cdot kip$

$R_{D\_4L} := (R_{D2} + R_{D3} + R_{D4} + R_{D5}) \cdot MPF(4) = 81.45 \cdot kip$

$R_{E\_4L} := (R_{E2} + R_{E3} + R_{E4} + R_{E5}) \cdot MPF(4) = 85.74 \cdot kip$

$R_{F\_4L} := (R_{F2} + R_{F3} + R_{F4} + R_{F5}) \cdot MPF(4) = 80.358 \cdot kip$

$R_{G\_4L} := (R_{G2} + R_{G3} + R_{G4} + R_{G5}) \cdot MPF(4) = 58.67 \cdot kip$

All 5 lanes loaded

$R_{A\_5L} := (R_{A1} + R_{A2} + R_{A3} + R_{A4} + R_{A5}) \cdot MPF(5) = 35.132 \cdot kip$

$R_{B\_5L} := (R_{B1} + R_{B2} + R_{B3} + R_{B4} + R_{B5}) \cdot MPF(5) = 86.58 \cdot kip$

$R_{C\_5L} := (R_{C1} + R_{C2} + R_{C3} + R_{C4} + R_{C5}) \cdot MPF(5) = 80.661 \cdot kip$

$R_{D\_5L} := (R_{D1} + R_{D2} + R_{D3} + R_{D4} + R_{D5}) \cdot MPF(5) = 81.45 \cdot kip$

$R_{E\_5L} := (R_{E1} + R_{E2} + R_{E3} + R_{E4} + R_{E5}) \cdot MPF(5) = 85.74 \cdot kip$

$R_{F\_5L} := (R_{F1} + R_{F2} + R_{F3} + R_{F4} + R_{F5}) \cdot MPF(5) = 80.358 \cdot kip$

$R_{G\_5L} := (R_{G1} + R_{G2} + R_{G3} + R_{G4} + R_{G5}) \cdot MPF(5) = 58.67 \cdot kip$

Girder Reactions Due to Live Load for Footing Design

Because of the damping effect of soil, the dynamic impact is excluded when the live load effects are calculated for the design of foundations. Calculations below include an equivalent pair of wheel loads, P_{wheel_ft}, that will result in the same support reactions developed by a truck load on a single lane without the dynamic amplification. As described below, this wheel load is applied on predefined lanes to calculate girder end reactions.

Only Lane 5 loaded

$$R_{G5\_ft} := \frac{P_{wheel\_ft} \cdot (8.8125ft + 2.2125ft) + W_{lane} \cdot 10ft \cdot 5.8125ft}{S} = 75.078 \cdot kip$$

$$R_{F5\_ft} := P_{wheel\_ft} \cdot 2 + W_{lane} \cdot 10ft - R_{G5\_ft} = 54.642 \cdot kip$$

$R_{A5\_ft} := 0$ $R_{B5\_ft} := 0$ $R_{C5\_ft} := 0$ $R_{D5\_ft} := 0$ $R_{E5\_ft} := 0$

Only Lane 4 loaded

$$R_{F4\_ft} := \frac{P_{wheel\_ft} \cdot (6.53125ft + 0.53125ft) + W_{lane} \cdot 8.53125ft \cdot (0.5 \cdot 8.53125ft)}{S} = 47.672 \cdot kip$$

$$R_{D4\_ft} := \frac{W_{lane} \cdot 1.46875ft \cdot (0.5 \cdot 1.46875ft)}{S} = 0.539 \cdot kip$$

$$R_{E4\_ft} := P_{wheel\_ft} \cdot 2 + W_{lane} \cdot 10ft - R_{F4\_ft} - R_{D4\_ft} = 81.508 \cdot kip$$

$R_{A4\_ft} := 0$ $R_{B4\_ft} := 0$ $R_{C4\_ft} := 0$ $R_{G4\_ft} := 0$

Only Lane 3 loaded

$$R_{E3\_ft} := \frac{P_{wheel\_ft} \cdot (4.25ft) + W_{lane} \cdot 6.25ft \cdot (0.5 \cdot 6.25ft)}{S} = 27.504 \cdot kip$$

$$R_{C3\_ft} := \frac{P_{wheel\_ft} \cdot (1.75ft) + W_{lane} \cdot 3.75ft \cdot (0.5 \cdot 3.75ft)}{S} = 10.819 \cdot kip$$

$$R_{D3\_ft} := P_{wheel\_ft} \cdot 2 + W_{lane} \cdot 10ft - R_{E3\_ft} - R_{C3\_ft} = 91.397 \cdot kip$$

$R_{A3\_ft} := 0$ $R_{B3\_ft} := 0$ $R_{F3\_ft} := 0$ $R_{G3\_ft} := 0$

Only Lane 2 loaded

$$R_{D2\_ft} := \frac{P_{wheel\_ft} \cdot (1.96875ft) + W_{lane} \cdot 3.96875ft \cdot (0.5 \cdot 3.96875ft)}{S} = 12.155 \cdot kip$$

$$R_{B2\_ft} := \frac{P_{wheel\_ft} \cdot (4.03125ft) + W_{lane} \cdot 6.03125ft \cdot (0.5 \cdot 6.03125ft)}{S} = 25.919 \cdot kip$$

$$R_{C2\_ft} := P_{wheel\_ft} \cdot 2 + W_{lane} \cdot 10ft - R_{B2\_ft} - R_{D2\_ft} = 91.646 \cdot kip$$

$R_{A2\_ft} := 0$ $R_{E2\_ft} := 0$ $R_{F2\_ft} := 0$ $R_{G2\_ft} := 0$

Only Lane 1 loaded

$$R_{A1\_ft} := \frac{P_{wheel\_ft} \cdot (0.3125ft + 6.3125ft) + W_{lane} \cdot 8.3125ft \cdot (0.5 \cdot 8.3125ft)}{S} = 44.925 \cdot kip$$

$$R_{C1\_ft} := \frac{W_{lane} \cdot 1.6875ft \cdot (0.5 \cdot 1.6875ft)}{S} = 0.712 \cdot kip$$

$$R_{B1\_ft} := P_{wheel\_ft} \cdot 2 + W_{lane} \cdot 10ft - R_{A1\_ft} - R_{C1\_ft} = 84.083 \cdot kip$$

$R_{D1\_ft} := 0$ $\quad R_{E1\_ft} := 0$ $\quad R_{F1\_ft} := 0$ $\quad R_{G1\_ft} := 0$

The design live load should be placed to generate the maximum soil bearing pressure. The greatest eccentricity and loads to maximize the soil bearing pressure are developed when the live load is on all 5 lanes.

Girder end reactions due to live load on all five lanes

$$R_{AFt\_5L} := \left(R_{A1\_ft} + R_{A2\_ft} + R_{A3\_ft} + R_{A4\_ft} + R_{A5\_ft}\right) \cdot MPF(5) = 29.201 \cdot kip$$

$$R_{BFt\_5L} := \left(R_{B1\_ft} + R_{B2\_ft} + R_{B3\_ft} + R_{B4\_ft} + R_{B5\_ft}\right) \cdot MPF(5) = 71.501 \cdot kip$$

$$R_{CFt\_5L} := \left(R_{C1\_ft} + R_{C2\_ft} + R_{C3\_ft} + R_{C4\_ft} + R_{C5\_ft}\right) \cdot MPF(5) = 67.066 \cdot kip$$

$$R_{DFt\_5L} := \left(R_{D1\_ft} + R_{D2\_ft} + R_{D3\_ft} + R_{D4\_ft} + R_{D5\_ft}\right) \cdot MPF(5) = 67.659 \cdot kip$$

$$R_{EFt\_5L} := \left(R_{E1\_ft} + R_{E2\_ft} + R_{E3\_ft} + R_{E4\_ft} + R_{E5\_ft}\right) \cdot MPF(5) = 70.858 \cdot kip$$

$$R_{FFt\_5L} := \left(R_{F1\_ft} + R_{F2\_ft} + R_{F3\_ft} + R_{F4\_ft} + R_{F5\_ft}\right) \cdot MPF(5) = 66.505 \cdot kip$$

$$R_{GFt\_5L} := \left(R_{G1\_ft} + R_{G2\_ft} + R_{G3\_ft} + R_{G4\_ft} + R_{G5\_ft}\right) \cdot MPF(5) = 48.801 \cdot kip$$

The total unfactored live load at the footing when all 5 lanes are loaded

$$R_{LLFooting} := R_{AFt\_5L} + R_{BFt\_5L} + R_{CFt\_5L} + R_{DFt\_5L} + R_{EFt\_5L} + R_{FFt\_5L} + R_{GFt\_5L} = 421.59 \cdot kip$$

Step 7.4 Application of Other Loads

Description

This step describes the application of braking force, wind load, temperature load, earth load, and vehicle collision load. Other loads, such as ice load and centrifugal force, are not applicable for this example. For illustrative purposes, the calculation of ice load and centrifugal force are presented in Appendix 5.B and 5.C.

Page Content

Braking Force

Since the abutments have expansion bearings, the fixed bearings at the pier resist the braking force along the longitudinal direction of the bridge.

The braking force (BR) shall be taken as the greater of: **LRFD 3.6.4**

- 25% of the axle weight of the design truck / tandem
- 5% of the design truck / tandem weight plus lane load.

The braking force is applied on all design lanes assuming that the bridge carries traffic in one direction.

Braking force per lane due to 25% of the axle weight of the design truck / tandem

$$BR_1 := 25\% \cdot (32kip + 32kip + 8kip) = 18 \cdot kip$$

Braking force per lane due to 5% of the design truck / tandem weight plus lane load

$$BR_2 := 5\% \cdot \left(72kip + 0.64 \frac{kip}{ft} \cdot 2L_{span}\right) = 10 \cdot kip$$

Note: The MDOT practice, as reflected in BDS, is to take only 5% of the design truck plus lane load as the breaking force. In addition, the HL-93 modification factor is not included in the braking force calculation. This example follows MDOT practice.

Braking force selected for the design $BRK := BR_2 = 10 \cdot kip$

Next, calculate the braking force considering 1 to 5 loaded lanes.

Braking force due to 1 loaded lane $BRK_{1L} := BRK \cdot MPF(1) = 12 \cdot kip$

Braking force due to 2 loaded lanes $BRK_{2L} := 2BRK \cdot MPF(2) = 20 \cdot kip$

Braking force due to 3 loaded lanes $BRK_{3L} := 3BRK \cdot MPF(3) = 25.5 \cdot kip$

Braking force due to 4 loaded lanes $BRK_{4L} := 4BRK \cdot MPF(4) = 26 \cdot kip$

Braking force due to 5 loaded lanes $BRK_{5L} := 5BRK \cdot MPF(5) = 32.5 \cdot kip$

The braking force is assumed to be equally shared by the bearings at the pier.

The braking force shall be assumed to act horizontally at a distance of 6 ft above the roadway surface. **LRFD 3.6.4**

Note: The MDOT practice is to apply the horizontal component of the breaking force at the bearings. The impact of the eccentricity of the load with respect to the bearing elevation is not considered.

Wind Load

Since the expansion bearings are located over the abutments, the fixed bearings at the pier resist the longitudinal component of the wind load acting on the superstructure.

Wind Load on Superstructure

LRFD 3.8.1.1, 3.8.1.2

To calculate the wind load acting on the superstructure, the total depth from the top of the barrier to the bottom of the girder is considered. Then, the wind exposure area is calculated by multiplying the tributary length for a specific direction and the superstructure depth. Finally, the wind load is calculated by multiplying the wind pressure and the wind exposure area.

Since the expansion bearings at the abutment are restrained in the transverse direction, the tributary length for the transverse direction wind load on the pier with fixed bearings is equal to one-half of each adjacent span. Because of the expansion bearings at the abutments, the entire bridge length is selected as the tributary length for the longitudinal direction.

Total depth of the superstructure $D_{total} := h_{Railing} + t_{Deck} + t_{Haunch} + d_{Girder} = 7.083\ ft$

Tributary length for the transverse wind load on superstructure $L_{WindT} := L_{span} = 100\ ft$ One half of each adjacent span

Tributary length for the longitudinal wind load on superstructure $L_{WindL} := 2L_{span} = 200\ ft$ Length of entire superstructure

Effective area for the transverse wind load on superstructure $A_{WSuperT} := D_{total} \cdot L_{WindT} = 708.333\ ft^2$

Effective area for the longitudinal wind load on superstructure $A_{WSuperL} := D_{total} \cdot L_{WindL} = 1.417 \times 10^3\ ft^2$

Basic wind speed (mph) for Strength III load combination $V_{wStrIII} := 115$ **LRFD Figure 3.8.1.1.2-1**

Basic wind speed (mph) for Strength V load combination $V_{wStrV} := 80$ **LRFD Table 3.8.1.1.2-1**

Basic wind speed (mph) for Service I load combination $V_{wSerI} := 70$ **LRFD Table 3.8.1.1.2-1**

Gust effect factor $Gust := 1$ **LRFD Table 3.8.1.2.1-1, no sound barrier**

Drag coefficient, superstructure $C_{DSup} := 1.3$ **LRFD Table 3.8.1.2.1-2**

Superstructure height (ft) when the height is less than 33 ft $Z := 33$ **LRFD 3.8.1.2.1**

Wind exposure category for the site B

Pressure exposure and elevation coefficient for Strength III and Service IV load combinations

$$K_{ZSup} := \frac{\left(2.5 \cdot \ln\left(\frac{Z}{0.9832}\right) + 6.87\right)^2}{345.6} = 0.709$$

LRFD Eq. 3.8.1.2.1-2

The wind pressure acting on the superstructure is calculated for different load combinations. **LRFD Eq. 3.8.1.2.1-1**

Wind pressure on the superstructure (ksf), Strength III $P_{ZSup.StrIII} := 2.56 \cdot 10^{-6} \cdot K_{ZSup} \cdot V_{wStrIII}^2 \cdot Gust \cdot C_{DSup} = 0.031$

Wind pressure on the superstructure (ksf), Strength V $P_{ZSup.StrV} := 2.56 \cdot 10^{-6} \cdot V_{wStrV}^2 \cdot Gust \cdot C_{DSup} = 0.021$

Wind pressure on the superstructure (ksf), Service I $P_{ZSup.SerI} := 2.56 \cdot 10^{-6} \cdot V_{wSerI}^2 \cdot Gust \cdot C_{DSup} = 0.016$

The superstructure wind load acting on the pier depends on the wind attack angle which is measured from a line perpendicular to the longitudinal axis of the bridge. **LRFD 3.8.1.2.2**

Since the span length and height of the bridge are less than 150 ft and 33 ft respectively, the following wind load components are used: **LRFD 3.8.1.2.3a**

- Transverse: 100 percent of the wind load calculated based on the wind direction perpendicular to the longitudinal axis of the bridge
- Longitudinal: 25 percent of the transverse load.

Only the pier has fixed bearings. Therefore, the longitudinal component of the wind load on the superstructure is equally shared by the bearings at the pier.

Wind load at each bearing due to the transverse wind loads on the superstructure, Strength III

$$WS_{TStrIII} := \frac{P_{ZSup.StrIII} \cdot ksf \cdot A_{WSuperT}}{N_{beams}} = 3.158 \cdot kip$$

Wind load at each bearing due to the transverse wind loads on the superstructure, Strength V

$$WS_{TStrV} := \frac{P_{ZSup.StrV} \cdot ksf \cdot A_{WSuperT}}{N_{beams}} = 2.155 \cdot kip$$

Wind load at each bearing due to the transverse wind loads on the superstructure, Service I

$$WS_{TSerI} := \frac{P_{ZSup.SerI} \cdot ksf \cdot A_{WSuperT}}{N_{beams}} = 1.65 \cdot kip$$

Wind load at each bearing due to the longitudinal wind loads on the superstructure, Strength III

$$WS_{LStrIII} := WS_{TStrIII} \cdot \frac{A_{WSuperL}}{A_{WSuperT}} \cdot 0.25 = 1.579 \cdot kip$$

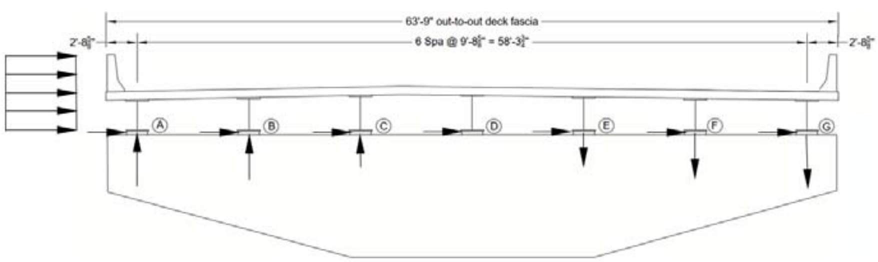
Wind load at each bearing due to the longitudinal wind loads on the superstructure, Strength V

$$WS_{LStrV} := WS_{TStrV} \cdot \frac{A_{WSuperL}}{A_{WSuperT}} \cdot 0.25 = 1.078 \cdot kip$$

Wind load at each bearing due to the longitudinal wind loads on the superstructure, Service I

$$WS_{LSerI} := WS_{TSerI} \cdot \frac{A_{WSuperL}}{A_{WSuperT}} \cdot 0.25 = 0.825 \cdot kip$$

The transverse load acting on the superstructure also applies a moment to the pier cap. This moment acts about the transverse centerline of the pier cap and induces vertical loads at the bearings, as illustrated in the following figure.



The following calculations show the moments about the longitudinal axis of the bridge due to transverse wind loads on the superstructure:

Strength III

$$M_{TStrIII} := P_{ZSup.StrIII} \cdot ksf \cdot A_{WSuperT} \cdot \frac{D_{total}}{2} = 78.285 \cdot kip \cdot ft$$

Strength V

$$M_{TStrV} := P_{ZSup.StrV} \cdot ksf \cdot A_{WSuperT} \cdot \frac{D_{total}}{2} = 53.433 \cdot kip \cdot ft$$

Service I

$$M_{TSerI} := P_{ZSup.SerI} \cdot ksf \cdot A_{WSuperT} \cdot \frac{D_{total}}{2} = 40.91 \cdot kip \cdot ft$$

Moment of inertia of the girders (as a group)

$$I_{girders} := 2 \cdot (3S)^2 + 2(2S)^2 + 2 \cdot S^2 = 2.645 \times 10^3\ ft^2$$

The magnitude of the vertical forces acting on the bearings is calculated below.

Vertical forces at bearings A and G, Strength III

$$R_{WS\_AGStrIII} := \frac{M_{TStrIII} \cdot (3S)}{I_{girders}} = 0.863 \cdot kip$$

Vertical forces at bearings A and G, Strength V $R_{WS\_AGStrV} := \dfrac{M_{TStrV}\cdot(3S)}{I_{girders}} = 0.589\cdot kip$

Vertical forces at bearings A and G, Service I $R_{WS\_AGSerI} := \dfrac{M_{TSerI}\cdot(3S)}{I_{girders}} = 0.451\cdot kip$

Vertical forces at bearings B and F, Strength III $R_{WS\_BFStrIII} := \dfrac{M_{TStrIII}\cdot(2S)}{I_{girders}} = 0.575\cdot kip$

Vertical forces at bearings B and F, Strength V $R_{WS\_BFStrV} := \dfrac{M_{TStrV}\cdot(2S)}{I_{girders}} = 0.393\cdot kip$

Vertical forces at bearings B and F, Service I $R_{WS\_BFSerI} := \dfrac{M_{TSerI}\cdot(2S)}{I_{girders}} = 0.301\cdot kip$

Vertical forces at bearings C and E, Strength III $R_{WS\_CEStrIII} := \dfrac{M_{TStrIII}\cdot(S)}{I_{girders}} = 0.288\cdot kip$

Vertical forces at bearings C and E, Strength V $R_{WS\_CEStrV} := \dfrac{M_{TStrV}\cdot(S)}{I_{girders}} = 0.196\cdot kip$

Vertical forces at bearings C and E, Service I $R_{WS\_CESerI} := \dfrac{M_{TSerI}\cdot(S)}{I_{girders}} = 0.15\cdot kip$

Note: The MDOT practice is to equally distribute the horizontal component of the transverse wind load to the bearings and neglect the effect of eccentricity. The above calculation is for illustrative purposes only. The vertical forces induced at the bearings by the eccentric transverse wind load are not considered in the design.

Vertical Wind Load

The vertical upward wind load is calculated as 0.02 ksf times the width of the deck for the Strength III load combination. This line load is applied at the windward quarter of the deck width. **LRFD 3.8.2**

Note: Since the MDOT practice is not to consider the vertical wind load, it is excluded from the analysis and design presented in this example .

Wind Load on the Substructure

Drag coefficient for the substructure $C_{DSub} := 1.6$ **LRFD Table 3.8.1.2.1-2**

The wind pressure acting on the substructure is calculated for different load combinations. **LRFD Eq. 3.8.1.2.1-1**

Wind pressure on the substructure (ksf), Strength III $P_{ZSub.StrIII} := 2.56\cdot 10^{-6}\cdot K_{ZSup}\cdot V_{wStrIII}^{\ 2}\cdot Gust\cdot C_{DSub} = 0.038$

Wind pressure on the substructure (ksf), Strength V $P_{ZSub.StrV} := 2.56\cdot 10^{-6}\cdot V_{wStrV}^{\ 2}\cdot Gust\cdot C_{DSub} = 0.026$

Wind pressure on the substructure (ksf), Service I $P_{ZSub.SerI} := 2.56\cdot 10^{-6}\cdot V_{wSerI}^{\ 2}\cdot Gust\cdot C_{DSub} = 0.02$

For simplicity, apply the same pressure along the transverse and longitudinal directions. The transverse and longitudinal wind forces calculated from the wind pressure acting on the corresponding exposed areas are to be applied simultaneously. These loads shall also act simultaneously with the superstructure wind loads.

LRFD 3.8.1.2.3b

Height difference between the middle and end of the pier cap $h_d := h_{cap} - h_{capend} = 6\,ft$

Exposed area of the pier cap in the longitudinal direction $A_{capL} := l_{cap} \cdot h_{cap} - 2\frac{1}{2} l_{overhang} \cdot h_d = 573.75\,ft^2$

Exposed area of the pier cap in the transverse direction $A_{capT} := t_{cap} \cdot h_{cap} = 44\,ft^2$

The column height exposed to wind is the distance from the ground to the bottom of the cap.

Exposed area of the pier column in the longitudinal direction $A_{colL} := w_{column} \cdot (h_{column} - h_{soil}) = 233.75\,ft^2$

Exposed area of the pier column in the transverse direction $A_{colT} := t_{column} \cdot (h_{column} - h_{soil}) = 44\,ft^2$

Longitudinal Component of the Substructure Wind Load Acting on the Pier, WS_{SubL}

Strength III $WS_{SubL.StrIII} := P_{ZSub.StrIII} \cdot ksf \cdot (A_{capL} + A_{colL}) = 31.014 \cdot kip$

Strength V $WS_{SubL.StrV} := P_{ZSub.StrV} \cdot ksf \cdot (A_{capL} + A_{colL}) = 21.168 \cdot kip$

Service I $WS_{SubL.SerI} := P_{ZSub.SerI} \cdot ksf \cdot (A_{capL} + A_{colL}) = 16.207 \cdot kip$

Transverse Component of the Substructure Wind Load Acting on the Pier, WS_{SubT}

Strength III $WS_{SubT.StrIII} := P_{ZSub.StrIII} \cdot ksf \cdot (A_{capT} + A_{colT}) = 3.38 \cdot kip$

Strength V $WS_{SubT.StrV} := P_{ZSub.StrV} \cdot ksf \cdot (A_{capT} + A_{colT}) = 2.307 \cdot kip$

Service I $WS_{SubT.SerI} := P_{ZSub.SerI} \cdot ksf \cdot (A_{capT} + A_{colT}) = 1.766 \cdot kip$

These loads are applied at the centroid of the loaded area for each direction. The distances from the bottom of the column to the points of application of the resultant force are required to calculate the moment due to wind loads.

Distance from the base of the column to the point of application of the longitudinal wind load on the substructure

$$H_{WSSubL} := \frac{l_{cap} \cdot h_{cap} \cdot \left(h_{column} + \frac{h_{cap}}{2}\right) - (l_{overhang} \cdot h_d) \cdot \left(h_{column} + \frac{1}{3} h_d\right) + A_{colL} \cdot \left(\frac{h_{column} - h_{soil}}{2} + h_{soil}\right)}{A_{capL} + A_{colL}}$$

$$H_{WSSubL} = 16.868\,ft$$

Distance from the bottom of the column to the point of application of the transverse wind load on the substructure

$$H_{WSSubT} := \frac{A_{capT} \cdot \left(h_{column} + \frac{h_{cap}}{2}\right) + A_{colT} \cdot \left(\frac{h_{column} - h_{soil}}{2} + h_{soil}\right)}{A_{capT} + A_{colT}} = 14\,ft$$

Wind Load on Live Load

Since the individual span length and height of this girder bridge are less than 150 ft and 33 ft respectively, the following wind load components acting on the live load are used: **LRFD 3.8.1.3**

- 0.10 klf, transverse
- 0.04 klf, longitudinal.

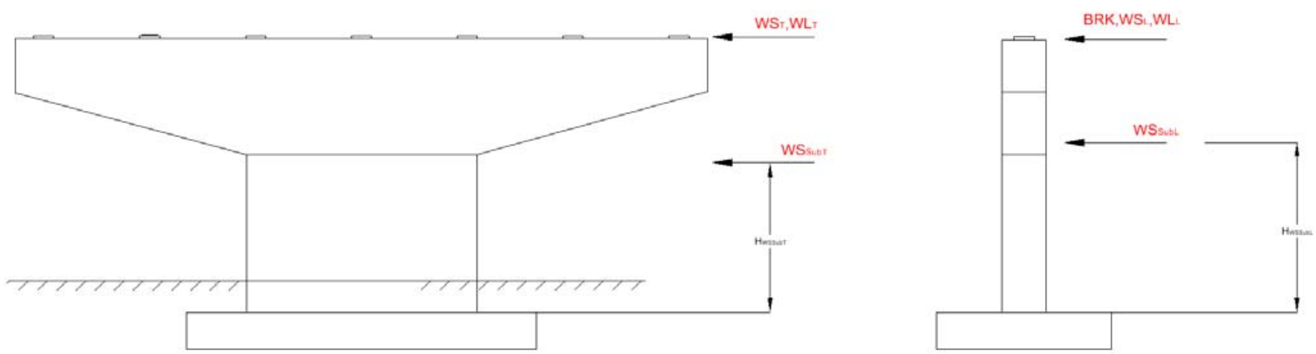
The transverse and longitudinal components of the load acting on each bearing are:

$$WL_{TBearing} := \frac{0.1\frac{kip}{ft} \cdot L_{WindT}}{N_{beams}} = 1.429 \cdot kip \qquad WL_{LBearing} := \frac{0.04\frac{kip}{ft} \cdot L_{WindL}}{N_{beams}} = 1.143 \cdot kip$$

The wind load on live load acts at 6 ft above the roadway. **LRFD 3.8.1.3**

Note: The MDOT practice does not consider the eccentricity of the wind load acting on the live load. Only the horizontal force is distributed to the bearings.

The following figure shows the braking force and the wind load applied on the pier in the transverse and longitudinal direction of the bridge.



Temperature Load

Since this bridge has two equal spans and expansion bearings over the abutments, the center of movement in the longitudinal direction is located at the pier. Therefore, the bearing pads at the pier do not deform when the superstructure deforms due to change in temperature. As a result, the pier is not subjected to transverse forces.

Vertical Earth Load

Vertical earth load on the footing $\quad EV_{Ft} := \gamma_s \cdot h_{soil} \cdot \left(w_{footing} \cdot l_{footing} - t_{column} \cdot w_{column}\right) = 178.38 \cdot kip$

Vehicle Collision Load

The draft language for incorporating AASHTO LRFD vehicle collision force is being reviewed by the bridge committee. Once approved, the AASHTO LRFD vehicle collision force shall be accounted for in the design of all new bridges, bridge replacements, and pier replacements.

MDOT's preference is to locate the pier outside of the clear zone as defined in Section 7.01.11 of the MDOT Road Design Manual. After the draft language is approved, the updated BDM will describe the the preference for accounting for the vehicle collision force when the pier cannot be located outside of the clear zone.

The pier design described in this example does not consider the vehicle collision force assuming that the pier is located outside of the clear zone defined in Section 7.01.11 of the MDOT Road Design Manual.

Step 7.5 Combined Load Effects

Description

This step presents the procedure of combining all load effects and calculating the total factored forces and moments acting at the pier cap, column, and footing.

Page Content

Strength I, Strength III, Strength V and Service I limit states are considered for the analysis and design of the pier.

LRFD 3.4.1

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75LS + 0.5TU

Strength III = 1.25DC + 1.5DW + 1.5EH + 1.35EV + 1.0WS + 0.5TU

Strength V = 1.25DC + 1.5DW + 1.35LL + 1.35BR + 1.0WS + 1.0WL + 1.5EH + 1.35EV + 1.35LS + 0.5TU

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

BR = vehicular braking force
DC = dead load of structural components and nonstructural attachments
DW = dead load of the future wearing surface and utilities
EH = horizontal earth pressure load
EV = vertical pressure from the earth fill
LL = vehicular live load
LS = live load surcharge
WL = wind on live load
WS = wind load on structure
TU = force effect due to uniform temperature

Limit states that are not shown here either do not control or are not applicable.

Note: These load combinations should include the maximum and minimum load factors; only the maximum factors are shown for clarity.

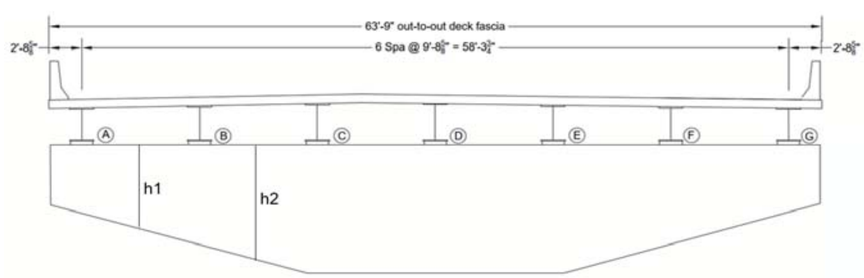
Pier Cap Load Effects

In this example, the pier cap is designed using both the strut-and-tie method (STM) and the traditional method for illustrative purposes. Therefore, the load effects required for the STM and traditional method are calculated.

Load Effects for the Strut-and-Tie Method

LRFD 5.8.2

For the STM, the pier cap self-weight is applied at each bearing location as a concentrated load based on the tributary width of the segment. For example, Girder A reaction includes the weight of the pier cap section located between the end of the cap and the midpoint between girders A and B. Similarly, Girder B reaction includes the weight of the pier cap section located between the midpoints of girders A - B and B - C.



Height of the pier cap at the midpoint between girders A and B

$$h1 := h_{capend} + \frac{h_d}{l_{overhang}} \cdot \left(l_{edge} + \frac{S}{2} \right) = 7.14\ ft$$

Height of the pier cap at the midpoint between girders B and C

$$h2 := h_{capend} + \frac{h_d}{l_{overhang}} \cdot \left(l_{edge} + \frac{3S}{2} \right) = 9.884\ ft$$

Tributary weight of the pier cap on Girder A $Cap_{DC\_A} := W_c \cdot t_{cap} \cdot \left(\frac{h_{capend} + h1}{2}\right) \cdot \left(l_{edge} + \frac{S}{2}\right) = 27.599 \cdot kip$

Tributary weight of the pier cap on Girder B $Cap_{DC\_B} := W_c \cdot t_{cap} \cdot \left(\frac{h1 + h2}{2}\right) \cdot S = 49.634 \cdot kip$

Tributary weight of the pier cap on Girder C

$$Cap_{DC\_C} := W_c \cdot t_{cap} \cdot \left[\frac{h2 + h_{cap}}{2} \cdot \left(l_{overhang} - l_{edge} - 1.5S\right) + \left(2.5S + l_{edge} - l_{overhang}\right) \cdot h_{cap}\right] = 62.82 \cdot kip$$

Tributary weight of the pier cap on Girder D $Cap_{DC\_D} := W_c \cdot t_{cap} \cdot S \cdot h_{cap} = 64.144 \cdot kip$

Due to symmetry, the tributary weights of the pier cap on Girder E, F, and G are equal to Girder C, B, and A, respectively.

$Cap_{DC\_E} := Cap_{DC\_C} = 62.82 \cdot kip$ $Cap_{DC\_F} := Cap_{DC\_B} = 49.634 \cdot kip$ $Cap_{DC\_G} := Cap_{DC\_A} = 27.599 \cdot kip$

Strength I is the controlling limit state for the application of the STM.

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75LS + 0.5TU

Lane 5 loaded

$$R_{uA\_1L} := 1.25 \cdot \left(R_{DCEx} + Cap_{DC\_A}\right) + 1.5 \cdot R_{DWEx} + 1.75R_{A\_1L} = 331.149 \cdot kip$$

$$R_{uB\_1L} := 1.25 \cdot \left(R_{DC1stIn} + Cap_{DC\_B}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{B\_1L} = 357.768 \cdot kip$$

$$R_{uC\_1L} := 1.25 \cdot \left(R_{DCIn} + Cap_{DC\_C}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{C\_1L} = 356.125 \cdot kip$$

$$R_{uD\_1L} := 1.25 \cdot \left(R_{DCIn} + Cap_{DC\_D}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{D\_1L} = 357.78 \cdot kip$$

$$R_{uE\_1L} := 1.25 \cdot \left(R_{DCIn} + Cap_{DC\_E}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{E\_1L} = 356.125 \cdot kip$$

$$R_{uF\_1L} := 1.25 \cdot \left(R_{DC1stIn} + Cap_{DC\_F}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{F\_1L} = 496.847 \cdot kip$$

$$R_{uG\_1L} := 1.25 \cdot \left(R_{DCEx} + Cap_{DC\_G}\right) + 1.5 \cdot R_{DWEx} + 1.75R_{G\_1L} = 520.698 \cdot kip$$

Lanes 4 and 5 loaded

$$R_{uA\_2L} := 1.25 \cdot \left(R_{DCEx} + Cap_{DC\_A}\right) + 1.5 \cdot R_{DWEx} + 1.75R_{A\_2L} = 331.149 \cdot kip$$

$$R_{uB\_2L} := 1.25 \cdot \left(R_{DC1stIn} + Cap_{DC\_B}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{B\_2L} = 357.768 \cdot kip$$

$$R_{uC\_2L} := 1.25 \cdot \left(R_{DCIn} + Cap_{DC\_C}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{C\_2L} = 356.125 \cdot kip$$

$$R_{uD\_2L} := 1.25 \cdot \left(R_{DCIn} + Cap_{DC\_D}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{D\_2L} = 358.724 \cdot kip$$

$$R_{uE\_2L} := 1.25 \cdot \left(R_{DCIn} + Cap_{DC\_E}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{E\_2L} = 528.59 \cdot kip$$

$$R_{uF\_2L} := 1.25 \cdot \left(R_{DC1stIn} + Cap_{DC\_F}\right) + 1.5 \cdot R_{DWIn} + 1.75R_{F\_2L} = 574.115 \cdot kip$$

$$R_{uG\_2L} := 1.25 \cdot \left(R_{DCEx} + Cap_{DC\_G}\right) + 1.5 \cdot R_{DWEx} + 1.75R_{G\_2L} = 489.106 \cdot kip$$

Lanes 3 to 5 loaded

$$R_{uA\_3L} := 1.25\cdot\left(R_{DCEx} + Cap_{DC\_A}\right) + 1.5\cdot R_{DWEx} + 1.75R_{A\_3L} = 331.149\cdot kip$$

$$R_{uB\_3L} := 1.25\cdot\left(R_{DC1stIn} + Cap_{DC\_B}\right) + 1.5\cdot R_{DWIn} + 1.75R_{B\_3L} = 357.768\cdot kip$$

$$R_{uC\_3L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_C}\right) + 1.5\cdot R_{DWIn} + 1.75R_{C\_3L} = 375.804\cdot kip$$

$$R_{uD\_3L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_D}\right) + 1.5\cdot R_{DWIn} + 1.75R_{D\_3L} = 522.063\cdot kip$$

$$R_{uE\_3L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_E}\right) + 1.5\cdot R_{DWIn} + 1.75R_{E\_3L} = 552.338\cdot kip$$

$$R_{uF\_3L} := 1.25\cdot\left(R_{DC1stIn} + Cap_{DC\_F}\right) + 1.5\cdot R_{DWIn} + 1.75R_{F\_3L} = 541.663\cdot kip$$

$$R_{uG\_3L} := 1.25\cdot\left(R_{DCEx} + Cap_{DC\_G}\right) + 1.5\cdot R_{DWEx} + 1.75R_{G\_3L} = 465.413\cdot kip$$

Lanes 2 to 5 loaded

$$R_{uA\_4L} := 1.25\cdot\left(R_{DCEx} + Cap_{DC\_A}\right) + 1.5\cdot R_{DWEx} + 1.75R_{A\_4L} = 331.149\cdot kip$$

$$R_{uB\_4L} := 1.25\cdot\left(R_{DC1stIn} + Cap_{DC\_B}\right) + 1.5\cdot R_{DWIn} + 1.75R_{B\_4L} = 393.566\cdot kip$$

$$R_{uC\_4L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_C}\right) + 1.5\cdot R_{DWIn} + 1.75R_{C\_4L} = 496.472\cdot kip$$

$$R_{uD\_4L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_D}\right) + 1.5\cdot R_{DWIn} + 1.75R_{D\_4L} = 500.318\cdot kip$$

$$R_{uE\_4L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_E}\right) + 1.5\cdot R_{DWIn} + 1.75R_{E\_4L} = 506.171\cdot kip$$

$$R_{uF\_4L} := 1.25\cdot\left(R_{DC1stIn} + Cap_{DC\_F}\right) + 1.5\cdot R_{DWIn} + 1.75R_{F\_4L} = 498.394\cdot kip$$

$$R_{uG\_4L} := 1.25\cdot\left(R_{DCEx} + Cap_{DC\_G}\right) + 1.5\cdot R_{DWEx} + 1.75R_{G\_4L} = 433.821\cdot kip$$

All 5 lanes loaded

$$R_{uA\_5L} := 1.25\cdot\left(R_{DCEx} + Cap_{DC\_A}\right) + 1.5\cdot R_{DWEx} + 1.75R_{A\_5L} = 392.63\cdot kip$$

$$R_{uB\_5L} := 1.25\cdot\left(R_{DC1stIn} + Cap_{DC\_B}\right) + 1.5\cdot R_{DWIn} + 1.75R_{B\_5L} = 509.282\cdot kip$$

$$R_{uC\_5L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_C}\right) + 1.5\cdot R_{DWIn} + 1.75R_{C\_5L} = 497.282\cdot kip$$

$$R_{uD\_5L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_D}\right) + 1.5\cdot R_{DWIn} + 1.75R_{D\_5L} = 500.318\cdot kip$$

$$R_{uE\_5L} := 1.25\cdot\left(R_{DCIn} + Cap_{DC\_E}\right) + 1.5\cdot R_{DWIn} + 1.75R_{E\_5L} = 506.171\cdot kip$$

$$R_{uF\_5L} := 1.25\cdot\left(R_{DC1stIn} + Cap_{DC\_F}\right) + 1.5\cdot R_{DWIn} + 1.75R_{F\_5L} = 498.394\cdot kip$$

$$R_{uG\_5L} := 1.25\cdot\left(R_{DCEx} + Cap_{DC\_G}\right) + 1.5\cdot R_{DWEx} + 1.75R_{G\_5L} = 433.821\cdot kip$$

Factored Girder Reactions for the Application of the Strut-and-Tie Method (kip)

	Lane 5 loaded	Lanes 4 and 5 loaded	Lanes 3 to 5 loaded	Lanes 2 to 5 loaded	All 5 lanes loaded
Girder A	331.15	331.15	331.15	331.15	392.63
Girder B	357.77	357.77	357.77	393.57	509.28
Girder C	356.13	356.13	375.80	496.47	497.28
Girder D	357.78	358.72	522.06	500.32	500.32
Girder E	356.13	528.59	552.34	506.17	506.17
Girder F	496.85	574.12	541.66	498.39	498.39
Girder G	520.70	489.11	465.41	433.82	433.82

Load Effects for the Traditional Method

Strength I is the controlling limit state for the design of the pier cap. Service I is the controlling serviceability limit state. The critical design location is at 21.25 ft from the end of the cap. The reactions at the two outermost bearings (F and G) and the pier cap overhang self-weight develop the critical moments and shear at this critical section. By examining the girder reactions under different live load cases, it is determined that the controlling live load effects may be developed under the Lane 5 loaded case or the Lanes 4 and 5 loaded case.

Self-weight of the pier cap overhang
$$DC_{CapOverhang} := \frac{h_{capend} + h_{cap}}{2} \cdot l_{overhang} \cdot t_{cap} \cdot W_c = 102 \cdot kip$$

Moment arm of self-weight of the pier cap overhang to the critical section

$$Arm_{DCOverhang} := \frac{l_{overhang} \cdot h_{capend} \cdot \frac{1}{2} \cdot l_{overhang} + \frac{1}{2} \cdot h_d \cdot l_{overhang} \cdot \frac{1}{3} \cdot l_{overhang}}{l_{overhang} \cdot h_{capend} + \frac{1}{2} h_d \cdot l_{overhang}} = 9.297\ ft$$

Distance from Girder G to the critical section
$$Arm_{G\_cap} := l_{overhang} - l_{edge} = 18.531\ ft$$

Distance from Girder F to the critical section
$$Arm_{F\_cap} := l_{overhang} - l_{edge} - S = 8.813\ ft$$

Strength I

Factored shear force at the critical section under the Lane 5 loaded case

$$V_{u\_1LStrI} := 1.25 \cdot \left(DC_{CapOverhang} + R_{DCEx} + R_{DC1stIn}\right) + 1.5 \cdot \left(R_{DWEx} + R_{DWIn}\right) + 1.75\left(R_{F\_1L} + R_{G\_1L}\right)$$

$$V_{u\_1LStrI} = 1.049 \times 10^3 \cdot kip$$

Factored shear force at the critical section under the Lanes 4 and 5 loaded case

$$V_{u\_2LStrI} := 1.25\cdot\left(DC_{CapOverhang} + R_{DCEx} + R_{DC1stIn}\right) + 1.5\cdot\left(R_{DWEx} + R_{DWIn}\right) + 1.75\left(R_{F\_2L} + R_{G\_2L}\right)$$

$$V_{u\_2LStrI} = 1.094\times10^{3}\cdot kip$$

Controlling shear force at the critical section $V_{u\_StrI} := \max\left(V_{u\_1LStrI}, V_{u\_2LStrI}\right) = 1.094\times10^{3}\cdot kip$

Factored moment at the critical section under the Lane 5 loaded case

$$M_{u\_1LStrI} := 1.25\cdot DC_{CapOverhang}\cdot Arm_{DCOverhang} + \left(1.25\cdot R_{DCEx} + 1.5\cdot R_{DWEx} + 1.75\cdot R_{G\_1L}\right)\cdot Arm_{G\_cap} \cdots + \left(1.25\cdot R_{DC1stIn} + 1.5\cdot R_{DWIn} + 1.75\cdot R_{F\_1L}\right)\cdot Arm_{F\_cap}$$

$$M_{u\_1LStrI} = 1.403\times10^{4}\cdot kip\cdot ft$$

Factored moment at the critical location under the Lanes 4 and 5 loaded case

$$M_{u\_2LStrI} := 1.25\cdot DC_{CapOverhang}\cdot Arm_{DCOverhang} + \left(1.25\cdot R_{DCEx} + 1.5\cdot R_{DWEx} + 1.75\cdot R_{G\_2L}\right)\cdot Arm_{G\_cap} \cdots + \left(1.25\cdot R_{DC1stIn} + 1.5\cdot R_{DWIn} + 1.75\cdot R_{F\_2L}\right)\cdot Arm_{F\_cap}$$

$$M_{u\_2LStrI} = 1.412\times10^{4}\cdot kip\cdot ft$$

Controlling moment at the critical section $M_{u\_StrI} := \max\left(M_{u\_1LStrI}, M_{u\_2LStrI}\right) = 1.412\times10^{4}\cdot kip\cdot ft$

Service I

Factored shear force at the critical section under the Lane 5 loaded case

$$V_{u\_1LSerI} := \left(DC_{CapOverhang} + R_{DCEx} + R_{DC1stIn}\right) + \left(R_{DWEx} + R_{DWIn}\right) + \left(R_{F\_1L} + R_{G\_1L}\right)$$

$$V_{u\_1LSerI} = 753.088\cdot kip$$

Factored shear force at the critical section under the Lanes 4 and 5 loaded case

$$V_{u\_2LSerI} := \left(DC_{CapOverhang} + R_{DCEx} + R_{DC1stIn}\right) + \left(R_{DWEx} + R_{DWIn}\right) + \left(R_{F\_2L} + R_{G\_2L}\right)$$

$$V_{u\_2LSerI} = 779.189\cdot kip$$

Controlling shear force at the critical section $V_{u\_SerI} := \max\left(V_{u\_1LSerI}, V_{u\_2LSerI}\right) = 779.189\cdot kip$

Factored moment at the critical section under the Lane 5 loaded case

$$M_{u\_1LSerI} := DC_{CapOverhang}\cdot Arm_{DCOverhang} + \left(R_{DCEx} + R_{DWEx} + R_{G\_1L}\right)\cdot Arm_{G\_cap} \cdots + \left(R_{DC1stIn} + R_{DWIn} + R_{F\_1L}\right)\cdot Arm_{F\_cap}$$

$$M_{u\_1LSerI} = 9.993\times10^{3}\cdot kip\cdot ft$$

Factored moment at the critical section under the Lanes 4 and 5 loaded case

$$M_{u\_2LSerI} := DC_{CapOverhang}\cdot Arm_{DCOverhang} + \left(R_{DCEx} + R_{DWEx} + R_{G\_2L}\right)\cdot Arm_{G\_cap} \cdots + \left(R_{DC1stIn} + R_{DWIn} + R_{F\_2L}\right)\cdot Arm_{F\_cap}$$

$$M_{u\_2LSerI} = 1.005\times10^{4}\cdot kip\cdot ft$$

Controlling moment at the critical section $M_{u\_SerI} := \max\left(M_{u\_1LSerI}, M_{u\_2LSerI}\right) = 1.005\times10^{4}\cdot kip\cdot ft$

Pier Column Load Effects

Strength V is the controlling limit state for the design of the pier column under biaxial bending with an axial load. The critical section for the design is located at the column - footing connection. The critical load effects for the Strength V limit state are achieved by minimizing the axial effects while maximizing the transverse and longitudinal moments. This is accomplished by excluding the future wearing surface load and using minimum load factors for the dead loads. Since the live load placements indicate that the Lane 5 loaded case or Lanes 4 and 5 loaded case could develop the critical design forces and moments, Strength V combinations with respect to those two lane positions are evaluated.

Strength V = 0.9DC + 1.35LL + 1.35BR + 1.0WS + 1.0WL

To calculate the moments acting at the critical section of the column, the moment arms of various loads are calculated as shown below:

Moment arm of the vertical load at the Girder E bearing $Arm_E := S = 9.719\,ft$

Moment arm of the vertical load at the Girder F bearing $Arm_F := 2S = 19.438\,ft$

Moment arm of the vertical load at the Girder G bearing $Arm_G := 3S = 29.156\,ft$

The braking force, the wind load on the superstructure, and the wind load acting on the live load are all applied at the bearings as horizontal loads.

Distance from the column base to the top of the pier cap $Arm_{col} := h_{cap} + h_{column} = 25\,ft$

Axial Force and Moment at the Base of the Pier Column

Next, the factored axial forces and moments at the base of the pier column are calculated for the Lane 5 loaded case and the Lanes 4 and 5 loaded case.

The Lane 5 Loaded Case

Factored axial load

$$N_{uColStrV\_1L} := 0.9\cdot\left(DC_{Sup} + DC_{cap} + DC_{column}\right) + 1.35\cdot\left(R_{F\_1L} + R_{G\_1L}\right)$$

$$N_{uColStrV\_1L} = 1.977\times 10^{3}\cdot kip$$

Factored moment about the transverse axis of the pier column

$$\begin{aligned} M_{uTColStrV\_1L} := {} & 1.35\cdot\left(R_{F\_1L}\cdot Arm_F + R_{G\_1L}\cdot Arm_G\right) \ldots \\ & + 1.0\cdot\left(N_{beams}\cdot WS_{TStrV}\cdot Arm_{col} + WS_{SubT.StrV}\cdot H_{WSSubT}\right) \ldots \\ & + 1.0\cdot\left(N_{beams}\cdot WL_{TBearing}\cdot Arm_{col}\right) \end{aligned}$$

$$M_{uTColStrV\_1L} = 7.008\times 10^{3}\cdot kip\cdot ft$$

Factored moment about the longitudinal axis of the pier column

$$\begin{aligned} M_{uLColStrV\_1L} := {} & 1.35\cdot\left(BRK_{1L}\cdot Arm_{col}\right) \ldots \\ & + 1.0\cdot\left(N_{beams}\cdot WS_{LStrV}\cdot Arm_{col} + WS_{SubL.StrV}\cdot H_{WSSubL}\right) \ldots \\ & + 1.0\cdot\left(N_{beams}\cdot WL_{LBearing}\cdot Arm_{col}\right) \end{aligned}$$

$$M_{uLColStrV\_1L} = 1.151\times 10^{3}\cdot kip\cdot ft$$

The Lanes 4 and 5 Loaded Case

Factored axial load

$$N_{uColStrV\_2L} := 0.9\cdot(DC_{Sup} + DC_{cap} + DC_{column}) \cdots + 1.35\cdot(R_{D\_2L} + R_{E\_2L} + R_{F\_2L} + R_{G\_2L})$$

$$N_{uColStrV\_2L} = 2.146 \times 10^{3}\cdot kip$$

Factored moment about the transverse axis of the pier column

$$M_{uTColStrV\_2L} := 1.35\cdot(R_{E\_2L}\cdot Arm_{E} + R_{F\_2L}\cdot Arm_{F} + R_{G\_2L}\cdot Arm_{G}) \cdots + 1.0\cdot(N_{beams}\cdot WS_{TStrV}\cdot Arm_{col} + WS_{SubT.StrV}\cdot H_{WSSubT}) \cdots + 1.0\cdot(N_{beams}\cdot WL_{TBearing}\cdot Arm_{col})$$

$$M_{uTColStrV\_2L} = 8.749 \times 10^{3}\cdot kip\cdot ft$$

Factored moment about the longitudinal axis of the pier column

$$M_{uLColStrV\_2L} := 1.35\cdot(BRK_{2L}\cdot Arm_{col}) \cdots + 1.0\cdot(N_{beams}\cdot WS_{LStrV}\cdot Arm_{col} + WS_{SubL.StrV}\cdot H_{WSSubL}) \cdots + 1.0\cdot(N_{beams}\cdot WL_{LBearing}\cdot Arm_{col})$$

$$M_{uLColStrV\_2L} = 1.421 \times 10^{3}\cdot kip\cdot ft$$

Shear Forces at the Base of the Pier Column

Since the Strength III or Strength V limit state could develop the controlling shear forces at the base of the pier column, shear forces due to both limit states are calculated.

Strength III = 1.25DC + 1.5DW + 1.0WS

Strength V = 1.25DC + 1.5DW + 1.35BR + 1.0WS + 1.0WL

The shear parallel to the longitudinal axis of the pier (transverse shear) and the shear parallel to the transverse direction of the pier (longitudinal shear) are calculated as shown below.

Strength III

Factored transverse shear force

$$V_{uTColStrIII} := 1.0\cdot(N_{beams}\cdot WS_{TStrIII} + WS_{SubT.StrIII})$$

$$V_{uTColStrIII} = 25.484\cdot kip$$

Factored longitudinal shear force

$$V_{uLColStrIII} := N_{beams}\cdot WS_{LStrIII} + WS_{SubL.StrIII}$$

$$V_{uLColStrIII} = 42.066\cdot kip$$

Strength V

Factored transverse shear force

$$V_{uTColStrV} := 1.0\cdot(N_{beams}\cdot WS_{TStrV} + WS_{SubT.StrV} + N_{beams}\cdot WL_{TBearing})$$

$$V_{uTColStrV} = 27.394\cdot kip$$

Factored longitudinal shear force

$$V_{uLColStrV} := 1.35\cdot BRK_{5L} + 1.0\cdot(N_{beams}\cdot WS_{LStrV} + WS_{SubL.StrV} + N_{beams}\cdot WL_{LBearing})$$

$$V_{uLColStrV} = 80.587\cdot kip$$

The controlling transverse shear force $V_{uTCol} := \max\left(V_{uTColStrIII}, V_{uTColStrV}\right) = 27.394 \cdot kip$

The controlling longitudinal shear force $V_{uLCol} := \max\left(V_{uLColStrIII}, V_{uLColStrV}\right) = 80.587 \cdot kip$

Pier Footing Load Effects

The bearing pressure distribution depends on the rigidity of the footing and the soil type and condition. The pier footings are usually rigid, and the assumption q = (P/A) +/- (Mc/I) is valid. For an accurate calculation of bearing pressure distribution, the footing may be analyzed as a beam on an elastic foundation.

The live load on all five lanes develop the critical load effects for the footing design.

Moment arm of Girder A and G reactions to the center of footing $Arm_{AG} := 3S = 29.156\,ft$

Moment arm of Girder B and F reactions to the center of footing $Arm_{BF} := 2S = 19.438\,ft$

Moment arm of Girder C and E reactions to the center of footing $Arm_{CE} := S = 9.719\,ft$

For convenience, define the longitudinal axis of the footing as x-axis and the transverse axis as y-axis.

Strength I

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75LS + 0.5TU

Factored vertical force

$$F_{VFtStrI} := 1.25 \cdot \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + 1.5DW_{Sup} \dots + 1.75R_{LLFooting} + 1.35 \cdot EV_{Ft}$$

$$F_{VFtStrI} = 3.976 \times 10^{3} \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge $V_{TFtStrI} := 0$

Factored shear force parallel to the longitudinal axis of the bridge $V_{LFtStrI} := 1.75 \cdot BRK_{5L} = 56.875 \cdot kip$

Factored moment about the longitudinal axis of the footing $M_{XFtStrI} := 1.75 \cdot BRK_{5L} \cdot \left(Arm_{col} + t_{footing}\right) = 1.593 \times 10^{3} \cdot kip \cdot ft$

Factored moment about the transverse axis of the footing

$$M_{YFtStrI} := 1.75 \cdot \left[\left(R_{GFt\_5L} - R_{AFt\_5L}\right) \cdot Arm_{AG} + \left(R_{FFt\_5L} - R_{BFt\_5L}\right) \cdot Arm_{BF} \dots + \left(R_{EFt\_5L} - R_{CFt\_5L}\right) \cdot Arm_{CE}\right] = 894.546 \cdot kip \cdot ft$$

Note: DC, DW, and EV are symmetrically placed loads over the footing. Only the eccentrically placed traffic loading contributes to the moment about the transverse axis of the footing.

Strength III

Strength III = 1.25DC + 1.5DW + 1.5EH + 1.35EV + 1.0WS + 0.5TU

Factored vertical force

$$F_{VFtStrIII} := 1.25 \cdot \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + 1.5DW_{Sup} \dots + 1.35 \cdot EV_{Ft}$$

$$F_{VFtStrIII} = 3.238 \times 10^{3} \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrIII} := N_{beams} \cdot WS_{TStrIII} + WS_{SubT.StrIII} = 25.484 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrIII} := N_{beams} \cdot WS_{LStrIII} + WS_{SubL.StrIII} = 42.066 \cdot kip$$

Factored moment about the longitudinal axis of the footing

$$M_{XFtStrIII} := N_{beams} \cdot WS_{LStrIII} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubL.StrIII} \cdot \left(H_{WSSubL} + t_{footing}\right)$$

$$M_{XFtStrIII} = 925.652 \cdot kip \cdot ft$$

Factored moment about the transverse axis of the footing

$$M_{YFtStrIII} := N_{beams} \cdot WS_{TStrIII} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubT.StrIII} \cdot \left(H_{WSSubT} + t_{footing}\right)$$

$$M_{YFtStrIII} = 676.372 \cdot kip \cdot ft$$

Strength V

Strength V = 1.25DC + 1.5DW + 1.35LL + 1.35BR + 1.0WS + 1.0WL + 1.5EH + 1.35EV + 1.35 LS + 0.5TU

Factored vertical force

$$F_{VFtStrV} := 1.25 \cdot \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + 1.5DW_{Sup} \cdots \\ + 1.35 \cdot R_{LLFooting} + 1.35 \cdot EV_{Ft}$$

$$F_{VFtStrV} = 3.807 \times 10^{3} \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrV} := N_{beams} \cdot \left(WS_{TStrV} + WL_{TBearing}\right) + WS_{SubT.StrV} = 27.394 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrV} := 1.35 \cdot BRK_{5L} + N_{beams} \cdot \left(WS_{LStrV} + WL_{LBearing}\right) + WS_{SubL.StrV} = 80.587 \cdot kip$$

Factored moment about the longitudinal axis of the footing

$$M_{XFtStrV} := 1.35 \cdot BRK_{5L} \cdot \left(Arm_{col} + t_{footing}\right) + N_{beams} \cdot WS_{LStrV} \cdot \left(Arm_{col} + t_{footing}\right) \cdots \\ + N_{beams} \cdot WL_{LBearing} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubL.StrV} \cdot \left(H_{WSSubL} + t_{footing}\right)$$

$$M_{XFtStrV} = 2.084 \times 10^{3} \cdot kip \cdot ft$$

Factored moment about the transverse axis of the footing

$$M_{YFtStrV} := 1.35 \left[\left(R_{GFt\_5L} - R_{AFt\_5L}\right) \cdot Arm_{AG} + \left(R_{FFt\_5L} - R_{BFt\_5L}\right) \cdot Arm_{BF} \cdots \\ + \left(R_{EFt\_5L} - R_{CFt\_5L}\right) \cdot Arm_{CE}\right] \cdots \\ + N_{beams} \cdot WS_{TStrV} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubT.StrV} \cdot \left(H_{WSSubT} + t_{footing}\right) \cdots \\ + N_{beams} \cdot WL_{TBearing} \cdot \left(Arm_{col} + t_{footing}\right)$$

$$M_{YFtStrV} = 1.432 \times 10^{3} \cdot kip \cdot ft$$

Service I

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

Factored vertical force

$$F_{VFtSerI} := \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + DW_{Sup} \cdots + R_{LLFooting} + EV_{Ft}$$

$$F_{VFtSerI} = 2.961 \times 10^{3} \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtSerI} := N_{beams} \cdot \left(WS_{TSerI} + WL_{TBearing}\right) + WS_{SubT.SerI} = 23.317 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtSerI} := BRK_{5L} + N_{beams} \cdot \left(WS_{LSerI} + WL_{LBearing}\right) + WS_{SubL.SerI} = 62.482 \cdot kip$$

Factored moment about the longitudinal axis of the footing

$$M_{XFtSerI} := BRK_{5L} \cdot \left(Arm_{col} + t_{footing}\right) + N_{beams} \cdot WS_{LSerI} \cdot \left(Arm_{col} + t_{footing}\right) \cdots + N_{beams} \cdot WL_{LBearing} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubL.SerI} \cdot \left(H_{WSSubL} + t_{footing}\right)$$

$$M_{XFtSerI} = 1.618 \times 10^{3} \cdot kip \cdot ft$$

Factored moment about the transverse axis of the footing

$$M_{YFtSerI} := \left(R_{GFt\_5L} - R_{AFt\_5L}\right) \cdot Arm_{AG} + \left(R_{FFt\_5L} - R_{BFt\_5L}\right) \cdot Arm_{BF} \cdots + \left(R_{EFt\_5L} - R_{CFt\_5L}\right) \cdot Arm_{CE} + N_{beams} \cdot WS_{TStrV} \cdot \left(Arm_{col} + t_{footing}\right) \cdots + WS_{SubT.StrV} \cdot \left(H_{WSSubT} + t_{footing}\right) + N_{beams} \cdot WL_{TBearing} \cdot \left(Arm_{col} + t_{footing}\right)$$

$$M_{YFtSerI} = 1.253 \times 10^{3} \cdot kip \cdot ft$$

Step 7.6 Pier Cap Design: Strut-and-Tie Method

Description

This step presents the pier cap design using the strut-and-tie method.

Page	Content

Geometry and Member Forces of the Strut-and-Tie Model

When a structural member meets the definition of a deep component, the LRFD Bridge Design Specifications recommend, but do not mandate, that the STM be used to determine force effects and the required amount of reinforcing steel. The STM accounts for nonlinear strain distribution, nonuniform shear distribution, and the mechanical interaction of V_u, T_u and M_u.

LRFD 5.8.2.1

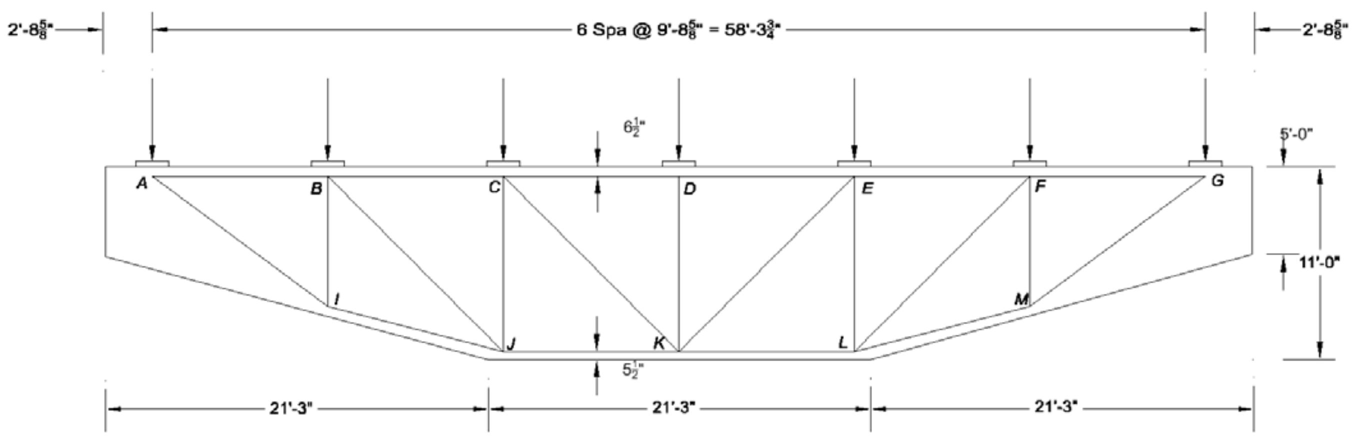
A few key considerations in strut-and-tie modeling are as follows:

LRFD 5.8.2.2

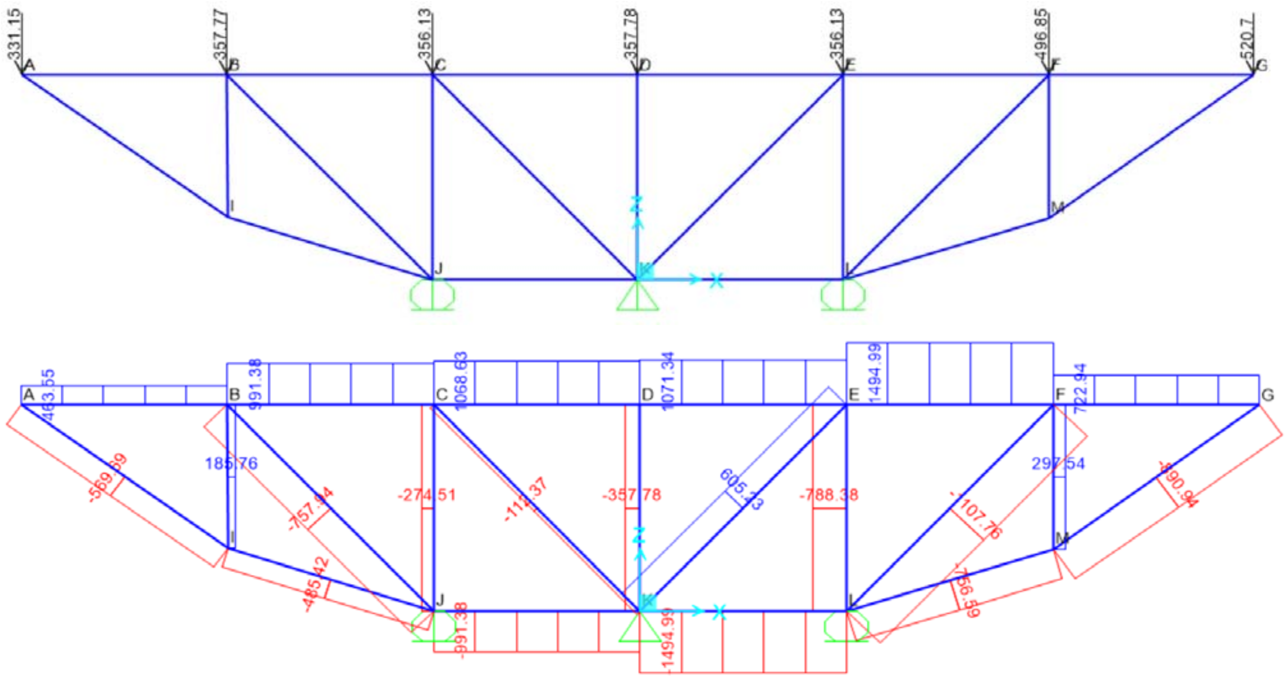
1. The truss must be in external and internal equilibrium.
2. A tie must be located at the centroid of the reinforcement that carries the tie force.
3. The angle between a strut and a tie entering the same node must be greater than 25°.
4. Reasonable and conservative assumptions and simplifications must be made when necessary.
5. In general, a model with fewest and shortest ties is the most efficient.
6. When using strut-and-tie modeling, design iterations may be necessary to determine the geometry of the model.

The strut-and-tie model of the pier cap is shown in the following figure. In this model, it is assumed that the top tie is at 6.5 in. from the top of the pier cap, and the bottom strut is at 5.5 in. from the bottom of the pier cap.

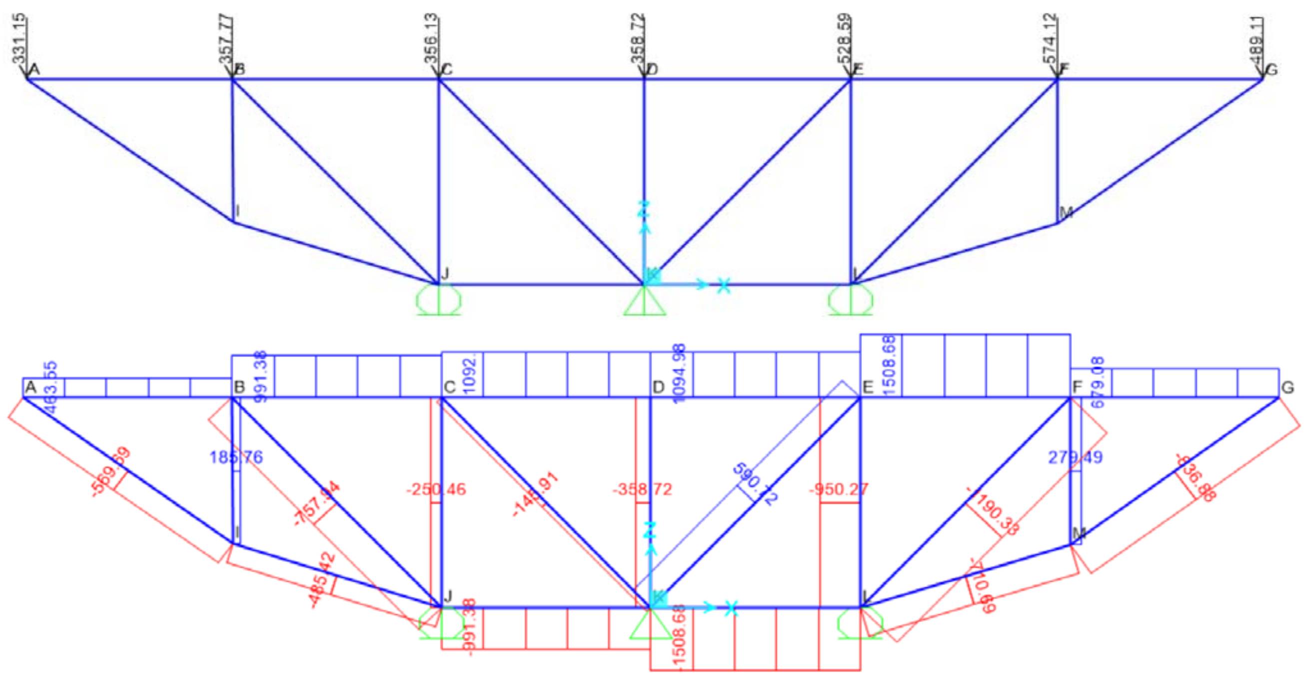
$\text{centroid}_{top} := 6.5\text{in}$ $\qquad$ $\text{centroid}_{bot} := 5.5\text{in}$



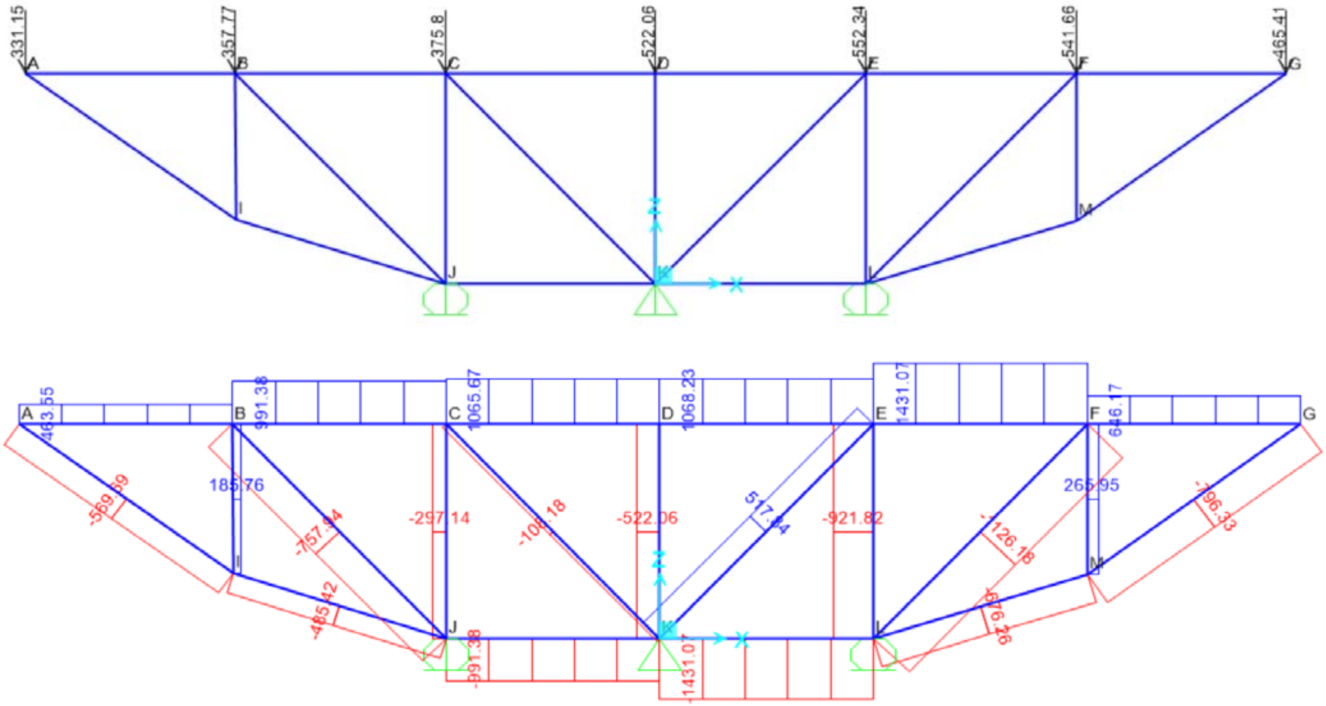
The strut and tie (i.e. truss member) forces were calculated for the Strength I limit state with different live load cases using a structural analysis software. Step 7.5 presents the calculation of girder reactions under different live load cases. The loads and the corresponding truss member forces are shown in the following figures. The truss members with red and blue axial force labels represent struts and ties, respectively.



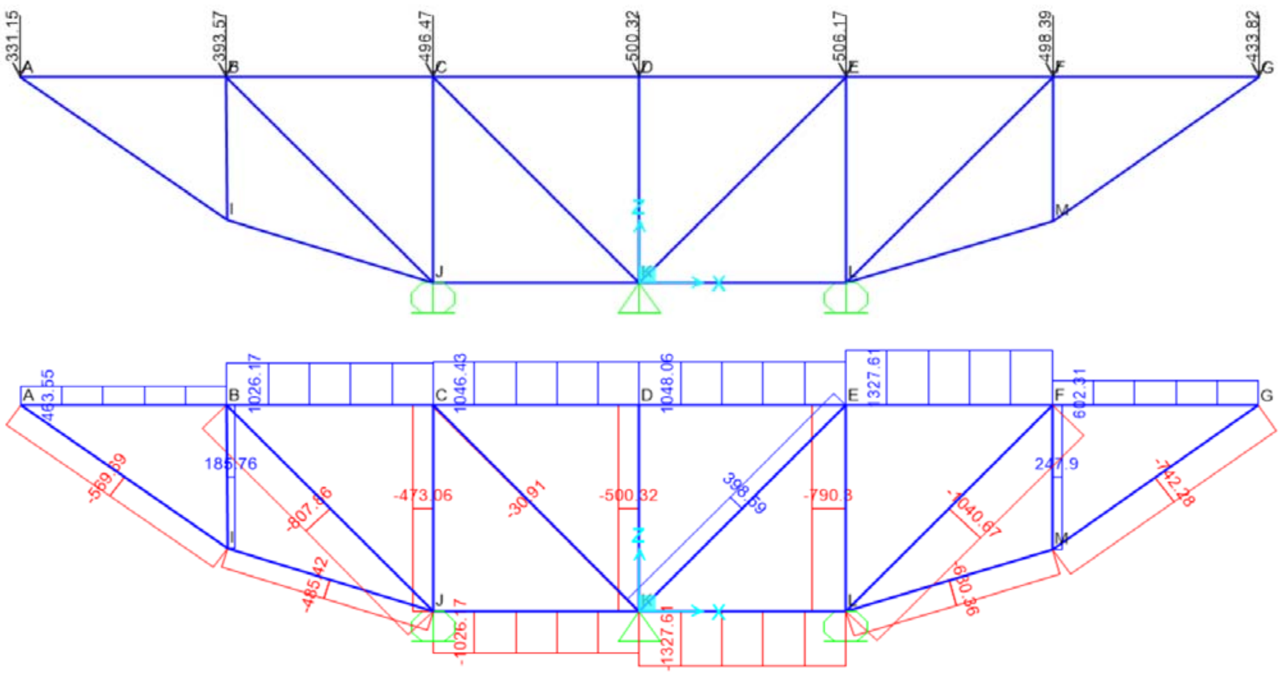
Loads and member forces under the Lane 5 loaded case



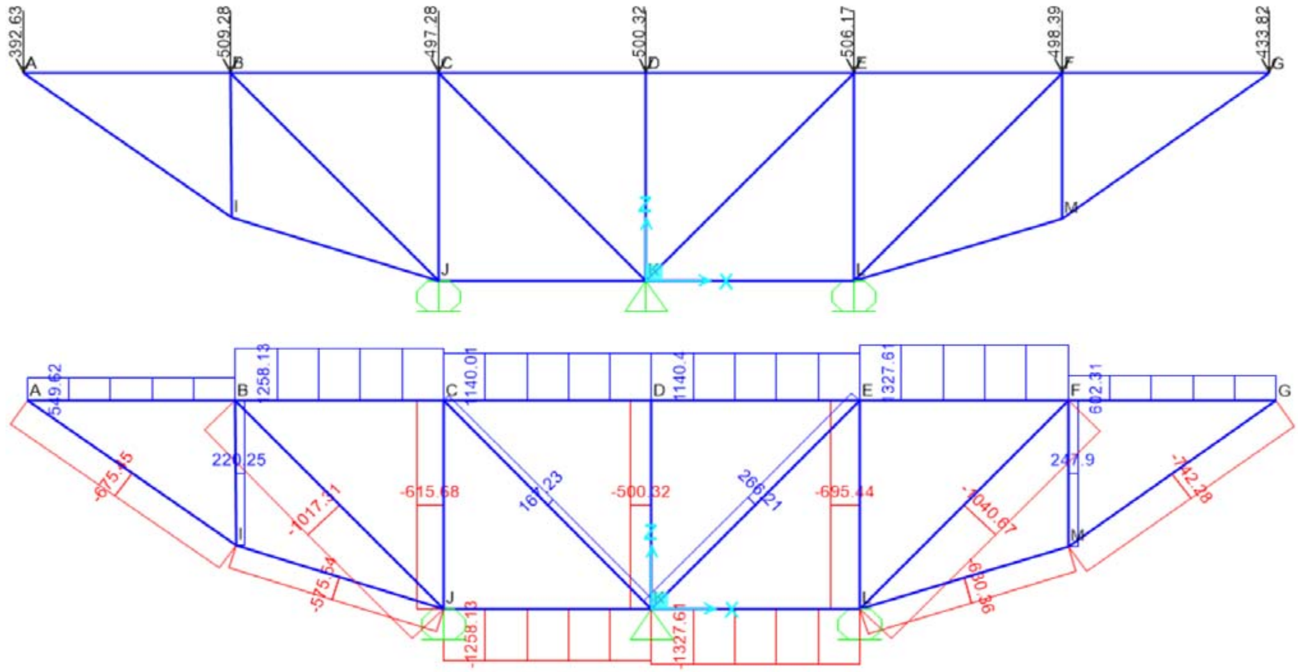
Loads and member forces under the Lanes 4 and 5 loaded case



Loads and member forces under the Lanes 3 to 5 loaded case



Loads and member forces under the Lanes 2 to 5 loaded case



Loads and member forces under the all 5 lanes loaded case

Summary Table of Truss Member Forces

The following table summarizes the member forces (in kips) of the truss under different live load cases. Positive and negative values represent tension and compression, respectively. The maximum tension or compression force in each member is highlighted in yellow.

Member	Lane 5 loaded	Lanes 4 and 5 loaded	Lanes 3 to 5 loaded	Lanes 2 to 5 loaded	All five lanes loaded
AB	463.55	463.55	463.55	463.55	549.62
BC	991.38	991.38	991.38	1026.17	1258.13
CD	1068.63	1092	1065.67	1046.43	1140.01
DE	1071.34	1094.98	1068.32	1048.06	1140.4
EF	1494.99	1508.68	1431.07	1327.61	1327.61
FG	722.94	679.08	646.17	602.31	602.31
AI	-569.69	-569.69	-569.69	-569.69	-675.45
BI	185.76	185.76	185.76	185.76	220.25
BJ	-757.94	-757.94	-757.94	-807.86	-1017.31
CJ	-274.51	-250.46	-297.14	-473.06	-615.68
CK	-112.37	-145.91	-108.18	-30.91	167.23
DK	-357.78	-358.72	-522.06	-500.32	-500.32
EK	605.23	590.72	517.84	398.59	266.21
EL	-788.38	-950.27	-921.82	-790.3	-695.44
FL	-1107.76	-1190.33	-1126.18	-1040.67	-1040.67
FM	297.54	279.49	265.95	247.9	247.9
GM	-890.94	-836.88	-796.33	-742.28	-742.28
IJ	-485.42	-485.42	-485.42	-485.42	-575.54
JK	-991.38	-991.38	-991.38	-1026.17	-1258.13
KL	-1494.99	-1508.68	-1431.07	-1327.61	-1327.61
LM	-756.59	-710.69	-676.26	-680.36	-680.36

As described in Step 7.3 and 7.4, the loads are applied on the bridge superstructure to develop the maximum moment and shear in the pier cap segment DG. Once the design details are developed for this segment, the same details are used for the segment AD due to symmetry. Hence, the strut and tie forces and the forces at the nodes located in the pier cap segment DG are considered for the design. The following designs are described in this step:

- Bearing size check at nodes F and G
- Tension tie reinforcement design for ties EF and FG
- Stirrup design using the forces in the vertical tie FM, the only vertical tie in the pier cap segment DG.

Additionally, the tension tie anchorage check, the crack control reinforcement design, and the skin reinforcement design are performed.

Bearing Size Check

The nodes are characterized based on the strut and tie interaction at a node. **LRFD 5.8.2.2**

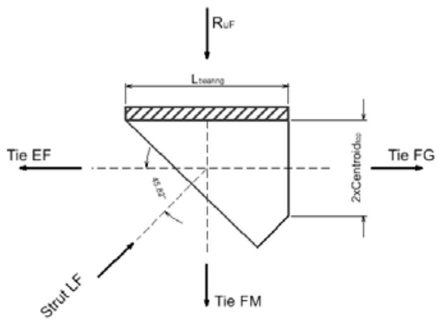
CCC: Nodes where only struts intersect (e.g. nodes J and L)
CCT: Nodes where a tie intersects the node from only one direction (e.g. nodes A, G, I, and M)
CTT: Nodes where ties intersect in two different directions (e.g. nodes B, C, D, E, F, and K)

The nominal resistance (P_n) at the bearing node face is calculated based on the limiting compressive stress and the effective area beneath the bearing device. **LRFD 5.8.2.5**

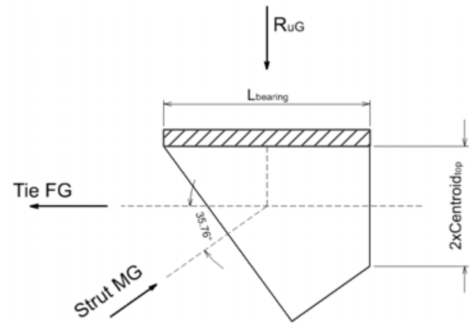
By examining girder end reactions, the maximum reactions at nodes F and G are identified for the following:

- CTT node: the Lanes 4 and 5 loaded case developed the maximum load at node F.
- CCT node: the Lane 5 loaded case developed the maximum load at node G.

$R_{uF\_2L} = 574.115 \cdot kip$ $\quad R_{uG\_1L} = 520.698 \cdot kip$



Geometry and Forces of Node F



Geometry and Forces of Node G

The required bearing area for node F (CTT) is calculated as follows:

Confinement modification factor, conservatively taken as 1.0 $\quad m := 1$ **LRFD 5.8.2.5.3a**

Resistance factor for bearing on concrete $\quad \phi_{bearing} := 0.7$ **LRFD 5.5.4.2**

Concrete efficiency factor for bearing face, assuming crack control reinforcement is present $\quad v_{CTT} := 0.85 - \frac{f_c}{20ksi} = 0.7$ **LRFD Table 5.8.2.5.3a-1**

Required bearing area for node F $\quad BrArea_{req\_F} := \frac{R_{uF\_2L} - 1.25Cap_{DC\_F}}{\phi_{bearing} \cdot m \cdot v_{CTT} \cdot f_c} = 348.349 \cdot in^2$ **LRFD 5.8.2.5**

Bearing plate size $\quad L_{bearing} := 22in \quad W_{bearing} := 19in$

Bearing plate area $\quad BrArea_{Provided} := L_{bearing} \cdot W_{bearing} = 418 \cdot in^2$

Check the adequacy of the bearing area provided by the plate at F $\quad Check := if(BrArea_{Provided} > BrArea_{req\_F}, "OK", "Not OK") = "OK"$

The required bearing area for node G (CCT) is calculated as follows:

Concrete efficiency factor for bearing face, assuming crack control reinforcement is present $\quad v_{CCT} := 0.7$ **LRFD Table 5.8.2.5.3a-1**

Required bearing area for node G $\quad BrArea_{req\_G} := \frac{R_{uG\_1L} - 1.25Cap_{DC\_G}}{\phi_{bearing} \cdot m \cdot v_{CCT} \cdot f_c} = 330.748 \cdot in^2$ **LRFD 5.8.2.5**

Check the adequacy of the bearing area provided by the plate at G $\quad Check := if(BrArea_{Provided} > BrArea_{req\_G}, "OK", "Not OK") = "OK"$

Tension Tie Reinforcement Design

Tie EF Design

As per the forces in the summary table, the Lanes 4 and 5 loaded case generates the maximum tension in the tie. The required area of tension tie reinforcement at the top of the pier cap and between girders E and F is calculated.

The maximum force in the tie $P_{uEF} := 1508.68kip$

Resistance factor for tension members $\phi_{tension} := 0.9$ **LRFD 5.5.4.2**

Required reinforcing steel area $A_{st\_EF} := \dfrac{P_{uEF}}{\phi_{tension} \cdot f_y} = 27.939 \cdot in^2$ **LRFD Eq. 5.8.2.4-1**

For the top reinforcement, try two rows of 9 No. 11 bars spaced at 5 in.

Select the reinforcing steel bar size $bar_{EF} := 11$

Select the number of reinforcing steel bars $n_{barEF} := 18$

Select the reinforcing steel bar spacing $S_{barEF} := 5in$

Nominal diameter of a bar $d_{barEF} := Dia(bar_{EF}) = 1.41 \cdot in$

Cross-section area of a bar $A_{barEF} := Area(bar_{EF}) = 1.56 \cdot in^2$

Total reinforcing steel area provided $A_{sProvided\_EF} := n_{barEF} \cdot A_{barEF} = 28.08 \cdot in^2$

Check the adequacy of tie EF $Check := if(A_{sProvided\_EF} > A_{st\_EF}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Tie FG Design

As per the forces in the summary table, the Lane 5 loaded case generates the maximum tension in the tie. The required area of tension tie reinforcement at the top of the pier cap and between girders F and G is calculated.

The maximum force in the tie $P_{uFG} := 722.94kip$

Required reinforcing steel area $A_{st\_FG} := \dfrac{P_{uFG}}{\phi_{tension} \cdot f_y} = 13.388 \cdot in^2$ **LRFD Eq. 5.8.2.4-1**

For the top reinforcement, try one row of 9 No. 11 bars spaced at 5 in.

Select the reinforcing steel bar size $bar_{FG} := 11$

Select the number of reinforcing steel bars $n_{barFG} := 9$

Select the reinforcing steel bar spacing $S_{barFG} := 5in$

Nominal diameter of a bar $d_{barFG} := Dia(bar_{FG}) = 1.41 \cdot in$

Cross-section area of a bar $A_{barFG} := Area(bar_{FG}) = 1.56 \cdot in^2$

Total reinforcing steel area provided $A_{sProvided\_FG} := n_{barFG} \cdot A_{barFG} = 14.04 \cdot in^2$

Check the adequacy of tie FG $Check := if(A_{sProvided\_FG} > A_{st\_FG}, "OK", "Not OK") = "OK"$

Stirrup Design

The vertical tension tie FM is designed to resist the factored tension force. Tie FM is the only vertical tie located in the pier cap segment DG. As per the forces in the summary table, the Lane 5 loaded case generates the maximum tension in the tie. This tension force is resisted by the stirrups provided within the specific tension tie region (i.e. the width of the tension tie).

The maximum force in the tie $P_{uFM} := 297.54kip$

Required reinforcing steel area $A_{st\_FM} := \dfrac{P_{uFM}}{\phi_{tension} \cdot f_y} = 5.51 \cdot in^2$ **LRFD Eq. 5.8.2.4-1**

Try No. 5 bars as double-stirrups (i.e. with four legs). $bar := 5$ $leg := 4$

Cross-section area of a bar $A_{bar} := Area(bar) = 0.31 \cdot in^2$

Total stirrup area provided $A_{st} := leg \cdot A_{bar} = 1.24 \cdot in^2$

Required number of double-stirrups $n_{stirrup} := \dfrac{A_{st\_FM}}{A_{st}} = 4.444$

These stirrups need to be distributed over a length defined by the midpoint between Girder E and F and the midpoint between Girder F and G.

Length of the region $L_{stirrup} := S = 9.719\ ft$

Required stirrup spacing $s_{stirrup} := \dfrac{L_{stirrup}}{n_{stirrup}} = 26.246 \cdot in$

Crack Control Reinforcement **LRFD 5.8.2.6**

The pier cap is required to have an orthogonal grid of reinforcement to control the width of cracks. The maximum spacing of these reinforcements is limited to the smaller of d/4 and 12 in. Since the pier cap depth is 11 ft and the depth at the end of the overhang is 5 ft, 12 in. spacing controls.

Pier cap width $b_w := t_{cap} = 4\ ft$

Required spacing of vertical reinforcement for crack control $s_v := \dfrac{A_{st}}{0.003 \cdot b_w} = 8.611 \cdot in$ **LRFD Eq. 5.8.2.6-1**

Required stirrup spacing $s_{stirRequired} := \min(s_{stirrup}, s_v) = 8.611 \cdot in$

Check if the required spacing satisfies the maximum limit $Check := if(s_{stirRequired} < 20in, "OK", "Not OK") = "OK"$

Select a stirrup spacing $s_{stir} := 8in$

Use pairs of No. 5 double-legged stirrups at 8 in. spacing in the pier cap.

Bottom Strut Check

Smeared nodes are the interior nodes that are not bounded by a bearing plate. Since all the nodes in the bottom struts are smeared nodes, the evaluation of concrete stresses is unnecessary. **LRFD C5.8.2.2**

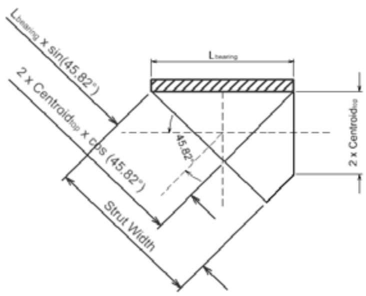
Diagonal Strut Check

The strut LF carries the largest diagonal compressive force. As per the forces in the summary table, the Lanes 4 and 5 loaded case generates the maximum compression in the strut.

The maximum force in the strut $\quad P_{uLF} := 1190.33\text{kip}$

Strut LF is connected to Node F. Ties EF, FM and FG are also connected to the same node.

Angle between LF and EF $\quad \alpha_s := \operatorname{atan}\left(\dfrac{h_{cap} - \text{centroid}_{top} - \text{centroid}_{bot}}{S}\right) = 45.817 \cdot \text{deg}$



Width of the strut $\quad w_{LF} := L_{bearing} \cdot \sin(\alpha_s) + 2 \cdot \text{centroid}_{top} \cdot \cos(\alpha_s) = 24.837 \cdot \text{in}$

Thickness of the strut $\quad t_{LF} := t_{cap} = 48 \cdot \text{in}$

Effective cross-section area of the strut $\quad A_{cn\_LF} := t_{LF} \cdot w_{LF} = 1.192 \times 10^3 \cdot \text{in}^2$

Node F is a CTT node. The surface where Strut LF meets the node is a strut-to-node interface.

Resistance factor for the strut $\quad \phi_{strut} := 0.7 \quad$ **LRFD 5.5.4.2**

Factored resistance of the strut $\quad P_{r\_LF} := \phi_{strut} \cdot m \cdot v_{CTT} \cdot f_c \cdot A_{cn\_LF} = 1752 \cdot \text{kip}$

Check the adequacy of the strut $\quad \text{Check} := \text{if}\left(P_{r\_LF} > P_{uLF}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Tension Tie Anchorage Check

Tension ties shall be anchored in the nodal regions. **LRFD 5.8.2.4.2**

The longitudinal bars at the top of the pier cap must be developed at the inner edge of the bearing at Node G.

First, calculate the available embedment length to develop the bars beyond the edge of the bearing.

Available development length $\quad l_{d\_available} := l_{edge} + \dfrac{L_{bearing}}{2} - \text{Cover}_{cap} = 40.125 \cdot \text{in}$

The longitudinal bar size provided at the top of the pier cap $\quad \text{bar} := 11$

Diameter of the bar $\quad d_{bar} := \text{Dia}(\text{bar}) = 1.41 \cdot \text{in}$

Required development length for the straight epoxy-coated bars with spacing less than 6 in. $\quad l_{d\_required} := 116\text{in} \quad$ **BDG 7.14.02**

Check if $l_{d\_available} > l_{d\_required}$ $\quad$ $Check := if(l_{d\_available} > l_{d\_required}, "OK", "Not OK") = "Not OK"$

Since an adequate length is not available to develop the bars, evaluate the possibility of using hooked bars to provide the required development length.

The basic development length for a 90 degree hooked bar

$$l_{hb} := 38 \cdot \frac{d_{bar}}{\sqrt{\frac{f_c}{ksi}}} = 30.934 \cdot in$$

LRFD Eq. 5.10.8.2.4a-2

Reinforcement confinement factor $\quad$ $\lambda_{rc} := 0.8$

Coating factor for epoxy coated bars $\quad$ $\lambda_{cw} := 1.2$

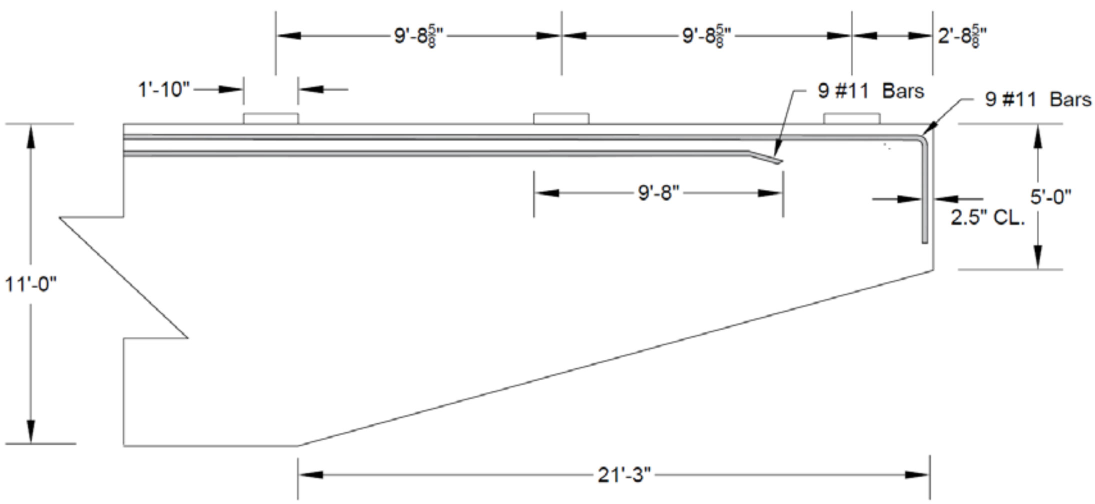
Excess reinforcement factor $\quad$ $\lambda_{er} := \frac{A_{st\_EF}}{A_{sProvided\_EF}} = 0.995$

Factor for normal weight concrete $\quad$ $\lambda := 1$

Required development length $\quad$ $l_{dh} := l_{hb} \cdot \frac{(\lambda_{rc} \cdot \lambda_{cw} \cdot \lambda_{er})}{\lambda} = 29.547 \cdot in$ $\quad$ **LRFD Eq. 5.10.8.2.4a-1**

Check the adequacy of the development length $\quad$ $Check := if(l_{d\_available} > l_{dh}, "OK", "Not OK") = "OK"$

The flexural reinforcement on the top of the pier cap is shown below. The bars at the top layer are hooked. The bars at the 2[nd] layer can be terminated after providing the required development length beyond the inside edge of the bearing at Girder F.



Crack Control Reinforcement Design

LRFD 5.8.2.6

The pier cap is required to have an orthogonal grid of reinforcement to control the width of cracks. The area of crack control reinforcement in each direction should be equal to or greater than 0.003 times the width of the member and the spacing of the reinforcement in the respective direction.

The maximum spacing of these reinforcements is limited to the smaller of d/4 and 12 in. Since the pier cap depth is 11 ft and the depth at the end of the overhang is 5 ft, 12 in. spacing controls.

Horizontal Reinforcement

Select a trial bar size $bar := 7$

Cross-section area of a bar $A_{bar} := Area(bar) = 0.6 \cdot in^2$

Select the number of bars $n_{bar} := 4$

Select a vertical spacing between the bars $s_h := 12in$

Check the adequacy of the crack control reinforcement

$$Check := if\left(n_{bar} \cdot \frac{A_{bar}}{t_{cap} \cdot s_h} > 0.003, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

LRFD Eq. 5.8.2.6-2

Vertical Reinforcement

Two double-legged stirrups made of No. 5 bars were selected. The horizontal spacing of the stirrups is 8 in. The adequacy of stirrups to control horizontal crack width needs to be checked.

Selected bar size $bar := 5$

Number of legs in a stirrup $leg = 4$

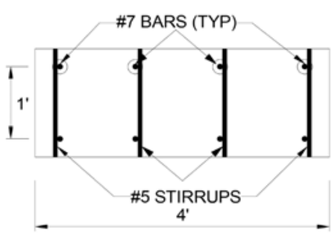
Cross-section area of a bar $A_{bar} := Area(bar) = 0.31 \cdot in^2$

Horizontal spacing of stirrups $s_{stir} = 8 \cdot in$

Check the adequacy of the crack control reinforcement

$$Check := if\left(leg \cdot \frac{A_{bar}}{t_{cap} \cdot s_{stir}} > 0.003, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

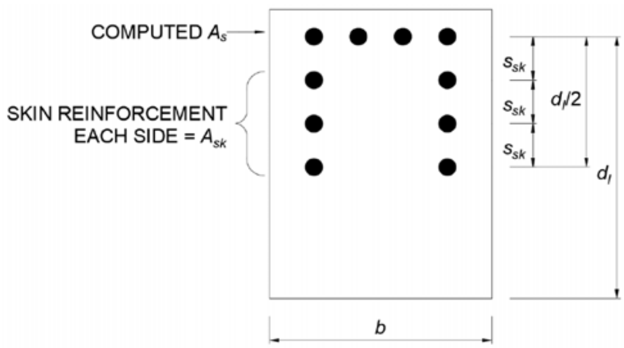
LRFD Eq. 5.8.2.6-1



Skin Reinforcement Design

LRFD 5.6.7

Concrete flexural members with depths exceeding 3 ft have a tendency to develop excessively wide cracks in the upper parts of their tension zones. To reduce the width of these cracks, it is necessary to provide additional longitudinal reinforcing steel in the zone of flexural tension near the vertical side faces of their web. This additional steel, which is referred to as the longitudinal skin reinforcement, must be uniformly distributed along both side faces for a distance equal to d/2 closer to the flexural reinforcing steel, as shown below.



Distance from the extreme compression fiber to the centroid of the extreme tension steel $\quad d_l := h_{cap} - Cover_{cap} = 128.5 \cdot in$

The maximum spacing of skin reinforcement $\quad s_{minSkin} := \min\left(\frac{d_l}{6}, 12in\right) = 12 \cdot in$

Required area of skin reinforcement on each side face of the pier cap $\quad A_{sk1} := 0.012 \cdot (d_l - 30in) \cdot \frac{in}{ft} = 1.182 \cdot \frac{in^2}{ft}$ **LRFD Eq. 5.6.7-3**

One fourth of the required flexural tensile reinforcement $\quad A_{sk2} := \frac{1}{4} \cdot \frac{A_{st\_EF}}{0.5 d_l} = 1.305 \cdot \frac{in^2}{ft}$

The required area of skin reinforcement on each side face of the pier cap, not to exceed one fourth of the flexural tensile reinforcement $\quad A_{sk\_required} := \min(A_{sk1}, A_{sk2}) = 1.182 \cdot \frac{in^2}{ft}$

The skin reinforcement shall be uniformly distributed along both side faces of the pier cap for a distance of $d_l/2$ closer to the flexural tension reinforcement, which is located at the top of the pier cap. Since No.7 bars at 12 in. spacing were selected as crack control horizontal reinforcing bars, it is necessary to check if they are adequate to act as the skin reinforcement.

Selected bar size for each side face $\quad bar := 7$

Selected reinforcing steel bar spacing $\quad s_h = 12 \cdot in$

Cross-section area of a reinforcing bar $\quad A_{bar} := Area(bar) = 0.6 \cdot in^2$

Check the adequacy of crack control reinforcing bars as the skin reinforcement

$$Check := if\left(A_{bar} \cdot \frac{12in}{s_h \cdot ft} > A_{sk\_required}, \text{"OK"}, \text{"Not OK"}\right) = \text{"Not OK"}$$

The crack control reinforcing bars selected for the pier cap are not sufficient to fulfil the skin reinforcement requirement.

Add one more No. 7 bar between the two horizontal No. 7 crack control bars on each side of the pier cap.

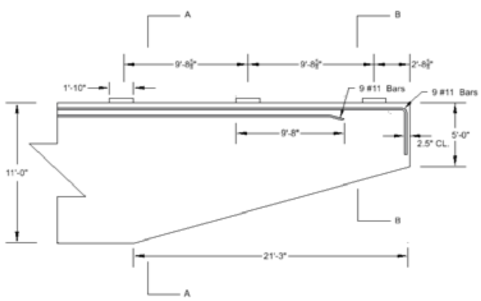
Spacing of the skin reinforcing bars $\quad s_{sk} := 6in$

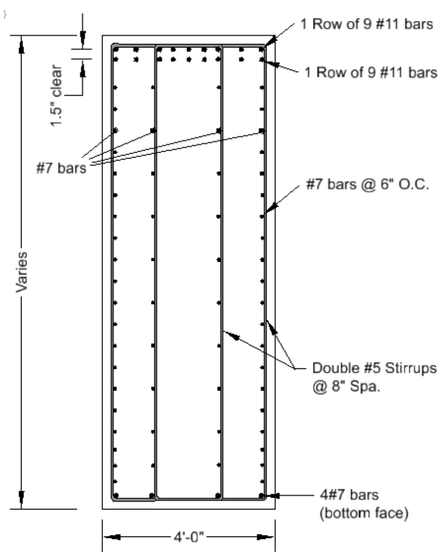
Check the adequacy of the skin reinforcement

$$Check := if\left(A_{bar}\cdot\frac{12in}{s_{sk}\cdot ft} > A_{sk\_required}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

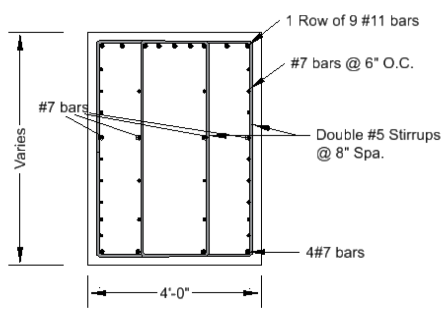
Although the skin reinforcement is only required for a distance of $d_l/2$ nearest the flexural tension reinforcement, a common practice is to distribute them to the entire depth of the section.

The typical pier cap cross-sections are shown below.





Section A-A



Section B-B

Note: Certain details are not shown in this drawing for clarity of the main reinforcement. Refer to MDOT Bridge Design Guides for additional details.

Step 7.7 Pier Cap Design: Traditional Method

Description

This step presents the pier cap design using the traditional method.

Page Content

Regardless of the member dimensions, the traditional sectional design method is based on the following assumptions:

- The longitudinal strains vary linearly along the depth of the member.
- The shear distribution remains uniform over the depth of the member.

The traditional method requires separate designs for V_u and M_u at different locations along the member.

Strength I is the controlling limit state for the pier cap design. The Service I limit state is used to check the crack width control requirements. The critical design section is located at 21.25 ft from the end of the cap. The reactions at the two outermost bearings (Girders F and G) and the self-weight of the overhang contribute to the critical moments and forces at the section. Step 7.5 presents the controlling shear forces and moments at the critical section.

Design for Flexure

LRFD 5.6.3.2

As a trial, consider the following for the top reinforcement:

- 1st row with 9 No. 11 bars spaced at 5 in.
- 2nd row with 5 No. 11 and 4 No. 10 bars.

$bar1 := 11$ $\quad n_{bar1} := 14$ $\quad bar2 := 10$ $\quad n_{bar2} := 4$

Nominal diameter of a No. 11 reinforcing bar $\quad d_{bar1} := Dia(bar1) = 1.41 \cdot in$

Cross-section area of a No. 11 reinforcing bar $\quad A_{bar1} := Area(bar1) = 1.56 \cdot in^2$

Nominal diameter of a No. 10 reinforcing bar $\quad d_{bar2} := Dia(bar2) = 1.27 \cdot in$

Cross-section area of a No. 10 reinforcing bar $\quad A_{bar2} := Area(bar2) = 1.27 \cdot in^2$

Total area of reinforcing steel provided as the top reinforcement $\quad A_{sProvided\_cap} := n_{bar1} \cdot A_{bar1} + n_{bar2} \cdot A_{bar2} = 26.92 \cdot in^2$

The Strength I limit state moment at the critical section

$M_{u\_StrI} = 1.412 \times 10^4 \cdot kip \cdot ft$ $\quad$ From Step 7.5

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses.

Effective depth $\quad d_e := h_{cap} - Cover_{cap} = 128.5 \cdot in$

Resistance factor for flexure $\quad \phi_f := 0.9$ $\quad$ **LRFD 5.5.4.2**

Width of the compression face of the member $\quad b := t_{cap} = 4\,ft$

Stress block factor $\quad \beta_1 := \min\left[\max\left[0.85 - 0.05 \cdot \left(\frac{f_c - 4ksi}{ksi}\right), 0.65\right], 0.85\right] = 0.85$ $\quad$ **LRFD 5.6.2.2**

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption $A_s := 1in^2$

Given $M_{u\_StrI} = \phi_f \cdot A_s \cdot f_y \cdot \left[d_e - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]$ **LRFD 5.6.3.2**

Required area of steel $A_{sRequired\_cap} := Find(A_s) = 25.681 \cdot in^2$

Check if $A_{sProvided} > A_{sRequired}$ $Check := if(A_{sProvided\_cap} > A_{sRequired\_cap}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Moment capacity of the section with the provided steel

$$M_{CapacityCap} := \phi_f \cdot A_{sProvided\_cap} \cdot f_y \cdot \left[d_e - \frac{1}{2} \cdot \left(\frac{A_{sProvided\_cap} \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]$$

$$M_{CapacityCap} = 1.477 \times 10^4 \cdot kip \cdot ft$$

Distance from the extreme compression fiber to the neutral axis

$$c := \frac{A_{sProvided\_cap} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 15.52 \cdot in$$

Check the validity of assumption, $f_s = f_y$ $Check := if\left(\frac{c}{d_e} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor $\gamma_1 := 1.6$ For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement $\gamma_3 := 0.67$ For ASTM A615 Grade 60 reinforcement

Section modulus $S_c := \frac{1}{6} \cdot b \cdot h_{cap}^2 = 1.394 \times 10^5 \cdot in^3$

Cracking moment $M_{cr} := \gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c = 5.176 \times 10^3 \cdot kip \cdot ft$

1.33 times the factored moment demand $1.33 \cdot M_{u\_StrI} = 1.878 \times 10^4 \cdot kip \cdot ft$

The factored moment to satisfy the minimum reinforcement requirement $M_{req} := min(1.33 M_{u\_StrI}, M_{cr}) = 5.176 \times 10^3 \cdot kip \cdot ft$

Check the adequacy of the section capacity $Check := if(M_{CapacityCap} > M_{req}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. Checking for crack control assures that the actual stress in the reinforcement does not exceed the service limit state stress.

Spacing requirement for the mild steel reinforcement in the layer closest to the tension face

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

LRFD Eq. 5.6.7-1

Exposure factor for the Class 1 exposure condition

$\gamma_e := 1.00$

Distance from extreme tension fiber to the center of the closest flexural reinforcement

$d_c := Cover_{cap} = 3.5 \cdot in$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face

$$\beta_s := 1 + \frac{d_c}{0.7\left(h_{cap} - d_c\right)} = 1.039$$

The position of the cross-section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis.

Assumed distance from the extreme compression fiber to the neutral axis

$x := 6 \cdot in$

Given
$$\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_c} \cdot A_{sProvided\_cap} \cdot \left(d_e - x\right)$$

Position of the neutral axis

$x_{na} := Find(x) = 29.764 \cdot in$

Tensile force in the reinforcing steel due to service limit state moment

$$T_s := \frac{M_{u\_SerI}}{d_e - \frac{x_{na}}{3}} = 1 \times 10^3 \cdot kip$$

Stress in the reinforcing steel due to service limit state moment

$$f_{ss1} := \frac{T_s}{A_{sProvided\_cap}} = 37.773 \cdot ksi$$

f_{ss} (not to exceed $0.6f_y$)

$f_{ss} := \min\left(f_{ss1}, 0.6f_y\right) = 36 \cdot ksi$

Required reinforcement bar spacing

$$s_{barRequred} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 11.716 \cdot in$$

Spacing of the steel reinforcement bars

$$s_{bar} := \frac{\left(b - 2 \cdot Cover_{cap}\right)}{8} = 5.125 \cdot in$$

Check if the spacing provided < the required spacing

$Check := if\left(s_{bar} < s_{barRequred}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Shrinkage and Temperature Reinforcement Requirement

LRFD 5.10.6

The following calculations check the adequacy of the flexural reinforcing steel to control shrinkage and temperature stresses in the pier cap.

For bars, the area of reinforcement per-foot (A_s), on each face and in each direction, shall satisfy

$$A_S \geq \frac{1.3bh}{2(b+h)f_y}$$

provided that

$$0.11in^2 \leq A_S \leq 0.6in^2$$

The following calculation evaluates the above limits to identify the minimum area of shrinkage and temperature reinforcement needed for the pier cap.

Minimum area of shrinkage and temperature reinforcement

$$A_{shrink.temp} := \min\left[\left[\begin{array}{c}\left(0.60\,\dfrac{in^2}{ft}\right)\\ \max\left[\left[\begin{array}{c}\left(0.11\,\dfrac{in^2}{ft}\right)\\ \left[\dfrac{1.3\cdot h_{cap}\cdot t_{cap}\cdot\dfrac{kip}{in\cdot ft}}{2\left(h_{cap}+t_{cap}\right)\cdot f_y}\right]\end{array}\right]\right]\end{array}\right]\right] = 0.381\cdot\frac{in^2}{ft}$$

Check if the provided area of steel > the required area of shrinkage and temperature steel

$$Check := if\left[\left(A_{sProvided\_cap} > A_{shrink.temp}\cdot t_{cap}\right), \text{"OK"}, \text{"Not OK"}\right] = \text{"OK"}$$

Design for Shear

LRFD 5.7.3.4.1

A simplified design procedure can be used since the section is not subjected to an axial tension and contains at least the minimum amount of transverse reinforcement.

Maximum factored shear force at the critical section $V_{u\_StrI} = 1.094\times 10^3\cdot kip$ From Step 7.5

Effective width of the section $b_v := b = 48\cdot in$

Depth of the equivalent rectangular stress block $a := \dfrac{A_{sProvided\_cap}\cdot f_y}{0.85\cdot f_c\cdot b} = 13.196\cdot in$

Effective shear depth $d_v := \max\left(d_e - \dfrac{a}{2}, 0.9\cdot d_e, 0.72\cdot h_{cap}\right) = 121.902\cdot in$ **LRFD 5.7.2.8**

Factor indicating the ability of diagonally cracked concrete to transmit tension and shear $\beta := 2$

Angle of inclination of diagonal compressive stresses $\theta := 45$

Nominal shear resistance of concrete $V_c := 0.0316\cdot\beta\cdot\sqrt{f_c\cdot ksi}\cdot b\cdot d_v = 640.5\cdot kip$ **LRFD Eq. 5.7.3.3-3**

Resistance factor for shear (for normal weight concrete) $\phi_v := 0.9$ **LRFD 5.5.4.2**

Shear stress on the concrete $v_u := \dfrac{V_{u\_StrI}}{\phi_v\cdot b_v\cdot d_v} = 0.208\cdot ksi$ **LRFD Eq. 5.7.2.8-1**

$$Check := if\left(v_u < 0.125\cdot f_c, \text{"Max. spacing = 24 in."}, \text{"Max. spacing = 12 in."}\right) = \text{"Max. spacing = 24 in."}$$

The maximum spacing of the transverse reinforcement shall not exceed 24 in. **LRFD 5.7.2.6**

Select trial stirrup size and number of legs $bar := 5$ $leg := 4$

Select stirrup spacing $s := 8in$

Cross-section area of one leg of a stirrup $A_{bar} := \text{Area}(bar) = 0.31 \cdot in^2$

Total stirrup area $A_v := leg \cdot A_{bar} = 1.24 \cdot in^2$

Check minimum transverse reinforcement requirement

$$0.0316 \cdot \beta \cdot \frac{\sqrt{f_c \cdot ksi} \cdot b \cdot s}{f_y} = 0.701 \cdot in^2$$

$$Check := if\left(A_v > 0.0316 \cdot \beta \cdot \frac{\sqrt{f_c \cdot ksi} \cdot b \cdot s}{f_y}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Shear resistance provided by stirrups

$$V_s := \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s} = 699.905 \cdot kip$$ **LRFD Eq. 5.7.3.3-4**

The nominal shear resistance, V_n, at the critical section is calculated as follows:

$$V_{n1} := V_c + V_s = 1.34 \times 10^3 \cdot kip$$

$$V_{n2} := 0.25 f_c \cdot b \cdot d_v = 4.388 \times 10^3 \cdot kip$$ **LRFD Eq. 5.7.3.3-2**

$$V_n := \min\left(V_{n1}, V_{n2}\right) = 1.34 \times 10^3 \cdot kip$$

Factored shear resistance $V_r := \phi_v \cdot V_n = 1.206 \times 10^3 \cdot kip$

Check if the factored shear resistance > the factored shear force

$$Check := if\left(V_r > V_{u\_StrI}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Step 7.8 Pier Column Design

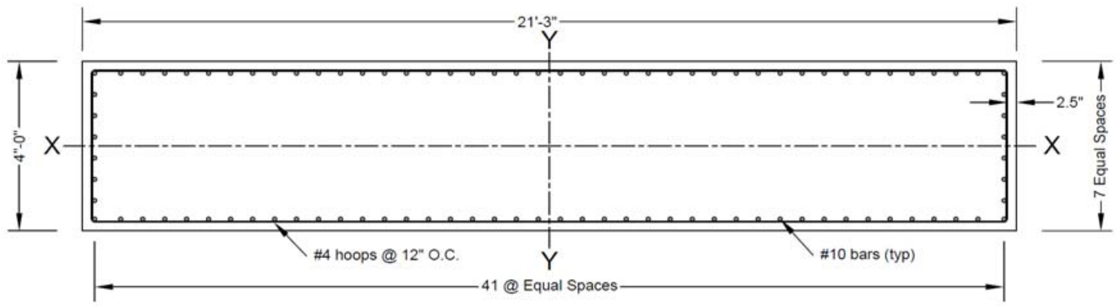
Description

This step presents the column design.

Page **Content**

Preliminary Design

Assumed section dimensions and reinforcement details are shown in the following figure:



Design for Axial Load and Biaxial Bending

The selected column reinforcing steel, shown in the above figure, is checked against the maximum and minimum requirements. **LRFD 5.6.4.2**

Number of bars and bar size $N_{bars} := 96$ $\quad bar := 10$

Cross-section area of a bar $A_{bar} := Area(bar) = 1.27 \cdot in^2$

Total area of reinforcing steel $A_{s\_col} := N_{bars} \cdot A_{bar} = 121.92 \cdot in^2$

Gross area of the pier column $A_{g\_col} := t_{column} \cdot w_{column} = 1.224 \times 10^4 \cdot in^2$

$$\frac{A_{s\_col}}{A_{g\_col}} = 9.961 \times 10^{-3}$$

Check the maximum reinforcing steel limit $Check := if\left(\frac{A_{s\_col}}{A_{g\_col}} \le 0.08, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$ **LRFD Eq. 5.6.4.2-1**

Check the minimum reinforcing steel limit $Check := if\left(\frac{A_{s\_col}}{A_{g\_col}} \ge 0.135 \cdot \frac{f_c}{f_y}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$ **LRFD Eq. 5.6.4.2-3**

The slenderness ratio about each axis of the column is calculated below.

The unbraced lengths used for the slenderness ratio about each axis is the full height of the pier, which is the height from the top of the footing to the top of the pier cap. Because of the expansion bearings at the abutment, the pier is not restrained against sway in the longitudinal direction of the bridge. Hence, the effective length factor in that direction, K_x, is taken as 2.1. The effective length factor in the transverse direction of the bridge, K_y, is taken as 1.0 since the sway of the pier in that direction is prevented by the bridge superstructure.

Column moment of inertia about x-axis $I_{xx} := \frac{w_{column} \cdot t_{column}^3}{12} = 2.35 \times 10^6 \cdot in^4$

Column moment of inertia about y-axis $I_{yy} := \frac{t_{column} \cdot w_{column}^3}{12} = 6.633 \times 10^7 \cdot in^4$

Radius of gyration about x-axis $r_{xx} := \sqrt{\frac{I_{xx}}{A_{g\_col}}} = 13.856 \cdot in$

Radius of gyration about y-axis $r_{yy} := \sqrt{\frac{I_{yy}}{A_{g\_col}}} = 73.612 \cdot in$

Effective length factor about the x-x axis $K_x := 2.1$

Effective length factor about the y-y axis $K_y := 1.0$

Length of the pier $L_u := h_{column} + h_{cap} = 25\,ft$

Slenderness ratios about x- and y- axes $\frac{K_x \cdot L_u}{r_{xx}} = 45.466$ $\frac{K_y \cdot L_u}{r_{yy}} = 4.075$

Check the slenderness ratio about x-axis $Check := if\left(\frac{K_x \cdot L_u}{r_{xx}} < 100, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$ **LRFD 5.6.4.3**

Check the slenderness ratio about y-axis $Check := if\left(\frac{K_y \cdot L_u}{r_{yy}} < 100, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

The slenderness effects may not be considered when the slenderness ratio of an unbraced member is less than 22. **LRFD 5.6.4.3**

To calculate the moment magnification factor for the moment about the x-axis, the column flexural stiffness (EI) about x-axis needs to be defined. The calculation process requires defining (a) the ratio of maximum factored permanent load moments to the maximum factored total load moment, (b) the moment of inertia of the gross concrete section about the centroidal axis, and (c) the moment of inertia of longitudinal reinforcement about the centroidal axis.

For this pier, the force effects contributing to the moment about the x-axis are the braking force and wind loads acting on the structure and live load. Since none of these are permanent loads, the ratio of the maximum factored permanent load moments to the maximum factored total moment is zero..

Ratio of the maximum factored permanent load moments to the maximum factored total moment $\beta_d := 0$

Number of equal spacings provided between reinforcing steel bars in the y-direction

$SP_y := 7$

Spacing of the reinforcing bars in the y-direction $spa_y := \frac{t_{column} - 2Cover_{col}}{SP_y} = 5.714 \cdot in$

Moment of inertia of longitudinal steel about the x-axis

$$I_{sx} := \frac{\pi \cdot d_{bar}{}^4}{64} \cdot N_{bars} + 2 \cdot 42A_{bar} \cdot \left(\frac{7spa_y}{2}\right)^2 + 4 \cdot A_{bar} \cdot \left(\frac{5spa_y}{2}\right)^2 + 4 \cdot A_{bar} \cdot \left(\frac{3spa_y}{2}\right)^2 + 4 \cdot A_{bar} \cdot \left(\frac{spa_y}{2}\right)^2$$

$$I_{sx} = 4.414 \times 10^4 \cdot in^4$$

The column flexural stiffness is the maximum of the following two values:

$$EI_1 := \frac{\frac{E_c \cdot I_{xx}}{5} + E_s \cdot I_{sx}}{1 + \beta_d} = 2.984 \times 10^9 \cdot kip \cdot in^2$$

LRFD Eq. 5.6.4.3-1

$$EI_2 := \frac{\frac{E_c \cdot I_{xx}}{2.5}}{\left(1 + \beta_d\right)} = 3.408 \times 10^9 \cdot kip \cdot in^2$$

LRFD Eq. 5.6.4.3-2

$$EI := \max\left(EI_1, EI_2\right) = 3.408 \times 10^9 \cdot kip \cdot in^2$$

Stiffness reduction factor for concrete members $\phi_K := 0.75$ **LRFD 4.5.3.2.2a**

The moment magnification factor is calculated as follows. As stated in Step 7.5, the Lane 5 loaded case and the Lanes 4 and 5 loaded case under the Strength V limit state are the critical load cases for the axial load and biaxial bending design of the column. Therefore, the moment magnification factors for these two load cases are calculated.

Euler buckling load

$$P_e := \frac{\pi^2 \cdot EI}{\left(K_x \cdot L_u\right)^2} = 8.475 \times 10^4 \cdot kip$$

LRFD Eq. 4.5.3.2.2b-5

Moment magnification factor for Lane 5 loaded case

$$\delta_{s\_1L} := \frac{1}{1 - \left(\frac{N_{uColStrV\_1L}}{\phi_K \cdot P_e}\right)} = 1.032$$

LRFD Eq. 4.5.3.2.2b-4

Moment magnification factor for Lanes 4 and 5 loaded case

$$\delta_{s\_2L} := \frac{1}{1 - \left(\frac{N_{uColStrV\_2L}}{\phi_K \cdot P_e}\right)} = 1.035$$

LRFD Eq. 4.5.3.2.2b-4

The forces and moments acting at the base of the column are calculated in Step 7.5. The forces and moments from Lane 5 and Lanes 4 and 5 loaded cases are used to evaluate the adequacy of the column capacity.

Lane 5 loaded case

$$P_{u\_1L} := N_{uColStrV\_1L} = 1.977 \times 10^3 \cdot kip$$

$$M_{uy\_1L} := M_{uTColStrV\_1L} = 7.008 \times 10^3 \ ft \cdot kip$$

$$M_{ux\_1L} := M_{uLColStrV\_1L} \cdot \delta_{s\_1L} = 1.188 \times 10^3 \ ft \cdot kip$$

Lanes 4 and 5 loaded case

$$P_{u\_2L} := N_{uColStrV\_2L} = 2.146 \times 10^3 \cdot kip$$

$$M_{uy\_2L} := M_{uTColStrV\_2L} = 8.749 \times 10^3 \ ft \cdot kip$$

$$M_{ux\_2L} := M_{uLColStrV\_2L} \cdot \delta_{s\_2L} = 1.47 \times 10^3 \ ft \cdot kip$$

Resistance factor for compression $\phi_{axial} := 0.75$ **LRFD 5.5.4.2**

Check if the factored axial load is greater or less than $0.1\phi f_c' A_g$ to select the appropriate equation for proportioning the member subjected to biaxial flexure and compression. **LRFD 5.6.4.5**

$$0.1 \cdot \phi_{axial} \cdot f_c \cdot A_{g\_col} = 2.754 \times 10^3 \cdot kip$$

$$Check := if\left(P_{u\_1L} < 0.1 \cdot \phi_{axial} \cdot f_c \cdot A_{g\_col}, \text{"Use Eq. 5.6.4.5-3"}, \text{"Use eq. 5.6.4.5-1"}\right) = \text{"Use Eq. 5.6.4.5-3"}$$

$$Check := if\left(P_{u\_2L} < 0.1 \cdot \phi_{axial} \cdot f_c \cdot A_{g\_col}, \text{"Use Eq. 5.6.4.5-3"}, \text{"Use eq. 5.6.4.5-1"}\right) = \text{"Use Eq. 5.6.4.5-3"}$$

Note: Instead of using M_{rx} and M_{ry} (with the AASHTO LRFD Eq. 5.6.4.5-3), following typical industry practice, the factored resultant flexural resistance, M_r, of the column is used in the approximate calculation procedure described below.

$M_r := 43990 kip \cdot ft$ Calculated using a commercial software

$$\frac{\sqrt{M_{ux\_1L}^2 + M_{uy\_1L}^2}}{M_r} = 0.162 \qquad \frac{\sqrt{M_{ux\_2L}^2 + M_{uy\_2L}^2}}{M_r} = 0.202$$

$$Check := if\left(\frac{\sqrt{M_{ux\_1L}^2 + M_{uy\_1L}^2}}{M_r} \le 1, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$ **LRFD Eq. 5.6.4.5-3**

$$Check := if\left(\frac{\sqrt{M_{ux\_2L}^2 + M_{uy\_2L}^2}}{M_r} \le 1, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$ **LRFD Eq. 5.6.4.5-3**

Although the column has a fairly large excess flexural capacity, an optimal column size is not considered for the following reasons:

(1) In this design example, the requirements of the pier cap dictate the column dimensions (a reduction in the column width will increase the moment in the pier cap).

(2) A short and squat column, such as the one in this example, generally has a relatively large excess capacity even when only minimally reinforced.

Design for Shear

The maximum factored shear forces parallel to the longitudinal and transverse axes of the column are presented in Step 7.5.

Factored shear parallel to the longitudinal axis of the column $V_{uTCol} = 27.394 \cdot kip$

Factored shear parallel to the transverse axis of the column $V_{uLCol} = 80.587 \cdot kip$

For simplicity, shear designs are carried out independently for longitudinal and transverse directions using the maximum shear force in each direction.

Since the column is not subjected to axial tension and contains at least the minimum amount of the transverse reinforcement, the simplified procedure is used. **LRFD 5.7.3.4.1**

Shear Parallel to the Transverse Axis of the Column

Maximum factored shear force $V_{uLCol} = 80.587 \cdot kip$

Effective width of the section $b_v := w_{column} = 21.25\ ft$

Effective shear depth, conservatively taken as 0.72h $d_v := 0.72 \cdot t_{column} = 34.56 \cdot in$ **LRFD 5.7.2.8**

Factor indicating ability of diagonally cracked concrete to transmit tension and shear $\beta := 2$

Nominal shear resistance of concrete $V_c := 0.0316 \cdot \beta \cdot \lambda \cdot \sqrt{f_c \cdot ksi} \cdot b_v \cdot d_v = 964.7 \cdot kip$ **LRFD Eq. 5.7.3.3-3**

Resistance factor for shear $\phi_v = 0.9$ **LRFD 5.5.4.2**

Check if the transverse reinforcement is required **LRFD 5.7.2.3**

$Check := if\left(V_{uTCol} < 0.5\phi_v \cdot V_c, \text{"Shear reinforcement NOT required"}, \text{"Shear reinforcement required"}\right)$

$Check = \text{"Shear reinforcement NOT required"}$

Shear Parallel to the Longitudinal Axis of the Column

Maximum factored shear force $V_{uTCol} = 27.394 \cdot kip$

Effective width of the section $b_v := t_{column} = 4\ ft$

Effective shear depth, conservatively taken as 0.72h $d_v := 0.72 \cdot w_{column} = 183.6 \cdot in$ **LRFD 5.7.2.8**

Factor indicating ability of diagonally cracked concrete to transmit tension and shear $\beta := 2$

Nominal shear resistance of concrete $V_c := 0.0316 \cdot \beta \cdot \lambda \cdot \sqrt{f_c \cdot ksi} \cdot b_v \cdot d_v = 964.7 \cdot kip$ **LRFD Eq. 5.7.3.3-3**

Resistance factor for shear $\phi_v = 0.9$ **LRFD 5.5.4.2**

Check if the transverse reinforcement is required **LRFD 5.7.2.3**

$Check := if\left(V_{uTCol} < 0.5\phi_v \cdot V_c, \text{"Shear reinforcement NOT required"}, \text{"Shear reinforcment required"}\right)$

$Check = \text{"Shear reinforcement NOT required"}$

Although the transverse reinforcement is not required for shear resistance, transverse confinement steel in the form of hoops, ties, or spirals is required for compression members.

Note: MDOT uses No. 4 as the minimum bar size to avoid damages during shipping and handling. **BDM 7.04.01 G**

The spacing of ties along the vertical axis of the column with single bars or bundles of No. 9 bars or smaller shall not exceed the lesser of the least dimension of the member or 12.0 in. Since the column has No. 10 single bars as the vertical reinforcement, select a spacing of 12 in. for the transverse confinement steel. **LRFD 5.10.4.3**

Use No. 4 bars as hoops at a spacing of 12 in. on center.

Step 7.9 Geotechnical Design of the Footing

Description

This step presents the geotechnical design of a spread footing considering the following strength and serviceability limit states:

1. bearing resistance – strength limit state
2. settlement – service limit state
3. sliding resistance – strength limit state
4. load eccentricity (overturning) – strength limit state.

LRFD 10.6.1.1

Step 7.10 presents the structural design of the footing.

Page Content

Bearing Resistance Check

For eccentrically loaded footing, the use of a reduced effective area is allowed for bearing resistance or settlement calculation. The point of load application shall be at the centroid of the reduced area. **LRFD 10.6.1.3**

Note: As a practice, the average pressure and the values at the toe and heel under different load cases and limit states are provided to the MDOT Geotechnical Services Section for verification.

This example presents the LRFD and MDOT methods.

Strength I

Factored vertical force under dead load (DL)

$$F_{VFtDL} := 1.25 \cdot \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) \cdots + 1.5DW_{Sup} + 1.35 \cdot EV_{Ft}$$

$$F_{VFtDL} = 3.238 \times 10^{3} \cdot kip$$

Factored vertical force with live load $F_{VFtStrI} = 3.976 \times 10^{3} \cdot kip$ From Step 7.5

Factored moment about the longitudinal axis of the footing $M_{XFtStrI} = 1.593 \times 10^{3} \cdot kip \cdot ft$ From Step 7.5

Factored moment about the transverse axis of the footing $M_{YFtStrI} = 894.546 \cdot kip \cdot ft$ From Step 7.5

Eccentricity in the footing width direction $e_B := \dfrac{M_{XFtStrI}}{F_{VFtStrI}} = 0.401\ ft$

Eccentricity in the footing length direction $e_L := \dfrac{M_{YFtStrI}}{F_{VFtStrI}} = 0.225\ ft$

<u>LRFD Method</u>

Effective footing width $B_{eff} := w_{footing} - 2 \cdot e_B = 17.199\ ft$ **LRFD Eq. 10.6.1.3-1**

Effective footing length $L_{eff} := l_{footing} - 2 \cdot e_L = 31.8\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $q_{bearing\_StrI} := \dfrac{F_{VFtStrI}}{B_{eff} \cdot L_{eff}} = 7.27 \cdot ksf$

<u>MDOT Method</u>

Average bearing pressure under DL $q_{bearingDL\_StrI} := \dfrac{F_{VFtDL}}{w_{footing} \cdot l_{footing}} = 5.579 \cdot ksf$

Average bearing pressure $q_{centerStrI} := \dfrac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} = 6.849 \cdot ksf$

Maximum bearing pressure

$$q_{maxStrI} := \frac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} \left(1 + 6 \cdot \frac{e_B}{w_{footing}} + 6 \cdot \frac{e_L}{l_{footing}}\right) = 8.051 \cdot ksf$$

Minimum bearing pressure $$q_{minStrI} := \frac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} \left(1 - 6 \cdot \frac{e_B}{w_{footing}} - 6 \cdot \frac{e_L}{l_{footing}}\right) = 5.648 \cdot ksf$$

Strength III

Factored vertical force $F_{VFtStrIII} = 3.238 \times 10^3 \cdot kip$ From Step 7.5

Factored moment about the longitudinal axis of the footing $M_{XFtStrIII} = 925.652 \cdot kip \cdot ft$ From Step 7.5

Factored moment about the transverse axis of the footing $M_{YFtStrIII} = 676.372 \cdot kip \cdot ft$ From Step 7.5

Eccentricity in the footing width direction $$e_B := \frac{M_{XFtStrIII}}{F_{VFtStrIII}} = 0.286\ ft$$

Eccentricity in the footing length direction $$e_L := \frac{M_{YFtStrIII}}{F_{VFtStrIII}} = 0.209\ ft$$

LRFD Method

Effective footing width $B_{eff} := w_{footing} - 2 \cdot e_B = 17.428\ ft$ **LRFD Eq. 10.6.1.3-1**

Effective footing length $L_{eff} := l_{footing} - 2 \cdot e_L = 31.832\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $$q_{bearing\_StrIII} := \frac{F_{VFtStrIII}}{B_{eff} \cdot L_{eff}} = 5.837 \cdot ksf$$

MDOT Method

Average bearing pressure $$q_{centerStrIII} := \frac{F_{VFtStrIII}}{w_{footing} \cdot l_{footing}} = 5.579 \cdot ksf$$

Maximum bearing pressure $$q_{maxStrIII} := \frac{F_{VFtStrIII}}{w_{footing} \cdot l_{footing}} \left(1 + 6 \cdot \frac{e_B}{w_{footing}} + 6 \cdot \frac{e_L}{l_{footing}}\right) = 6.327 \cdot ksf$$

Minimum bearing pressure $$q_{minStrIII} := \frac{F_{VFtStrIII}}{w_{footing} \cdot l_{footing}} \left(1 - 6 \cdot \frac{e_B}{w_{footing}} - 6 \cdot \frac{e_L}{l_{footing}}\right) = 4.83 \cdot ksf$$

Strength V

Factored vertical force $F_{VFtStrV} = 3.807 \times 10^3 \cdot kip$ From Step 7.5

Factored moment about the longitudinal axis of the footing $M_{XFtStrV} = 2.084 \times 10^3 \cdot kip \cdot ft$ From Step 7.5

Factored moment about the transverse axis of the footing $M_{YFtStrV} = 1.432 \times 10^3 \cdot kip \cdot ft$ From Step 7.5

Eccentricity in the footing width direction $$e_B := \frac{M_{XFtStrV}}{F_{VFtStrV}} = 0.547\ ft$$

Eccentricity in the footing length direction $e_L := \frac{M_{YFtStrV}}{F_{VFtStrV}} = 0.376\ ft$

LRFD Method

Effective footing width $B_{eff} := w_{footing} - 2 \cdot e_B = 16.905\ ft$ **LRFD Eq. 10.6.1.3-1**

Effective footing length $L_{eff} := l_{footing} - 2 \cdot e_L = 31.498\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure $q_{bearing\_StrV} := \frac{F_{VFtStrV}}{B_{eff} \cdot L_{eff}} = 7.15 \cdot ksf$

MDOT Method

Average bearing pressure $q_{centerStrV} := \frac{F_{VFtStrV}}{w_{footing} \cdot l_{footing}} = 6.559 \cdot ksf$

Maximum bearing pressure $q_{maxStrV} := \frac{F_{VFtStrV}}{w_{footing} \cdot l_{footing}} \left(1 + 6 \cdot \frac{e_B}{w_{footing}} + 6 \cdot \frac{e_L}{l_{footing}}\right) = 8.215 \cdot ksf$

Minimum bearing pressure $q_{minStrV} := \frac{F_{VFtStrV}}{w_{footing} \cdot l_{footing}} \left(1 - 6 \cdot \frac{e_B}{w_{footing}} - 6 \cdot \frac{e_L}{l_{footing}}\right) = 4.903 \cdot ksf$

Service I

Factored vertical force under dead load (DL) $F_{VFtDLSerI} := DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing} \cdots + DW_{Sup} + EV_{Ft}$

$F_{VFtDLSerI} = 2.539 \times 10^3 \cdot kip$

Factored vertical force with live load $F_{VFtSerI} = 2.961 \times 10^3 \cdot kip$ From Step 7.5

Factored moment about the longitudinal axis of the footing $M_{XFtSerI} = 1.618 \times 10^3 \cdot kip \cdot ft$ From Step 7.5

Factored moment about the transverse axis of the footing $M_{YFtSerI} = 1.253 \times 10^3 \cdot kip \cdot ft$ From Step 7.5

Eccentricity in the footing width direction $e_B := \frac{M_{XFtStrI}}{F_{VFtStrI}} = 0.401\ ft$

Eccentricity in the footing length direction $e_L := \frac{M_{YFtStrI}}{F_{VFtStrI}} = 0.225\ ft$

LRFD Method

Effective footing width $B_{eff} := w_{footing} - 2 \cdot e_B = 17.199\ ft$ **LRFD Eq. 10.6.1.3-1**

Effective footing length $L_{eff} := l_{footing} - 2 \cdot e_L = 31.8\ ft$ **LRFD Eq. 10.6.1.3-1**

Bearing pressure
$$q_{bearing\_SerI} := \frac{F_{VFtSerI}}{B_{eff} \cdot L_{eff}} = 5.414 \cdot ksf$$

MDOT Method

Footing pressure under DL
$$q_{bearingDL\_SerI} := \frac{F_{VFtDLSerI}}{w_{footing} \cdot l_{footing}} = 4.374 \cdot ksf$$

Average bearing pressure
$$q_{centerSerI} := \frac{F_{VFtSerI}}{w_{footing} \cdot l_{footing}} = 5.101 \cdot ksf$$

Maximum bearing pressure
$$q_{maxSerI} := \frac{F_{VFtSerI}}{w_{footing} \cdot l_{footing}} \left(1 + 6 \cdot \frac{e_B}{w_{footing}} + 6 \cdot \frac{e_L}{l_{footing}}\right) = 5.995 \cdot ksf$$

Minimum bearing pressure
$$q_{minSerI} := \frac{F_{VFtSerI}}{w_{footing} \cdot l_{footing}} \left(1 - 6 \cdot \frac{e_B}{w_{footing}} - 6 \cdot \frac{e_L}{l_{footing}}\right) = 4.206 \cdot ksf$$

Summary

LRFD Method

For the LRFD method, the controlling bearing pressure under strength limit states is
$$q_b := \max\left(q_{bearing\_StrI}, q_{bearing\_StrIII}, q_{bearing\_StrV}\right) = 7.27 \cdot ksf$$

The controlling bearing pressure needs to be checked with the factored bearing resistance of the soil provided by the Geotechnical Services Section.

MDOT Method

The controlling center, maximum, and minimum bearing pressure values under strength limit states are listed below.
$$q_{center} := \max\left(q_{bearing\_StrI}, q_{bearing\_StrIII}, q_{bearing\_StrV}\right) = 7.27 \cdot ksf$$
$$q_{max} := \max\left(q_{maxStrI}, q_{maxStrIII}, q_{maxStrV}\right) = 8.215 \cdot ksf$$
$$q_{min} := \max\left(q_{minStrI}, q_{minStrIII}, q_{minStrV}\right) = 5.648 \cdot ksf$$

A summary of bearing pressure values is shown in the following table:

	Average bearing pressure DL only (psf)	Average bearing pressure (psf)	Bearing pressure max. (psf)	Bearing presssure min. (psf)
Service	4374	5101	5995	4206
Strength	5579	7270	8215	5648
Allowable	Provided by the Geotechnical Services Section			

The Geotechnical Services Section uses these values for the verification of bearing resistance and settlement limits. If the bearing pressure exceeds the bearing strength of the soil, the size of the footing needs to be increased.

BDM 7.03.02G

Settlement Check

The Geotechnical Services Section uses the controlling bearing pressure from the service limit state to check if the total settlement of foundation is less than 1.5 in., the allowable limit. **BDM 7.03.02G 2b**

For the LRFD method, the controlling bearing pressure for settlement analysis is

$$q_{b\_settlement} := q_{bearing\_SerI} = 5.414 \cdot ksf$$

The Geotechnical Services Section uses this controlling bearing pressure to calculate the total settlement of the foundation.

For the MDOT method, the bearing pressures under the service limit state are provided to the Geotechnical Services Section to calculate the settlement.

Note: Besides the total settlement, considerations should be given to prevent the differential settlement between the abutments and pier from exceeding the tolerable differential settlement limit. Differential settlement limits are given in the *Steel Plate Girder Design Example.*

Sliding Resistance Check

Spread footings must be designed to resist lateral loads without sliding. The sliding resistance of a footing on cohesionless soil is a function of the normal force and the interface friction between the foundation and the soil. **LRFD 10.6.3.4**

The Geotechnical Services Section should provide a coefficient of sliding resistance (μ) for a design. MDOT typically uses a sliding resistance coefficient of 0.5 for cast-in-place concrete footings. Consult the Geotechnical Services Section to identify the most suitable coefficient for a specific design.

Coefficient of sliding resistance $\mu := 0.5$

The strength limit states are used for this check. Since the resistance is proportional to the vertical loads, the following conditions are used:

- Minimum load factors are used for all vertical loads.
- Maximum load factors are used for the loads that contribute to horizontal sliding forces.
- Live load is excluded.
- Since DW is the future wearing surface load, it is excluded.

The sliding resistance provided by the passive earth pressure is included in the design. **BDM 7.03.02F**

Passive earth pressure coefficient provided by the Geotechnical Services Section $k_p := 3.3$

Passive earth pressure at the footing base $p_p := k_p \cdot \gamma_s \cdot \left(h_{soil} + t_{footing}\right) = 2.376 \cdot ksf$

Nominal passive resistance of soil $R_{ep} := \frac{1}{2} \cdot p_p \cdot \left(h_{soil} + t_{footing}\right) \cdot l_{footing} = 229.878 \cdot kip$

Resistance factor for passive resistance $\phi_{ep} := 0.5$ **BDM 7.03.02F, LRFD Table 10.5.5.5.2-1**

Resistance factor for shear resistance between soil and foundation $\phi_{\tau} := 0.8$ **BDM 7.03.02F, LRFD Table 10.5.5.5.2-1**

<u>Strength I</u>

Factored shear force parallel to the transverse axis of the footing $V_{LFtStrI} = 56.875 \cdot kip$

Factored shear force parallel to the longitudinal axis of the footing $V_{TFtStrI} = 0 \cdot kip$

Factored sliding force (Demand) $V_{sliding} := \sqrt{V_{LFtStrI}^2 + V_{TFtStrI}^2} = 56.875 \cdot kip$

Minimum vertical load

$$F_{VFtStrIMin} := 0.9 \cdot (DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}) + 1.0 \cdot (EV_{Ft}) = 2.137 \times 10^3 \cdot kip$$

Sliding resistance $V_{resistance} := \phi_T \cdot \mu \cdot F_{VFtStrIMin} = 854.631 \cdot kip$

Check if $V_{resistance} > V_{sliding}$ $Check := if(V_{resistance} > V_{sliding}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Strength III

Factored shear force parallel to the transverse axis of the footing $V_{LFtStrIII} = 42.066 \cdot kip$

Factored shear force parallel to the longitudinal axis of the footing $V_{TFtStrIII} = 25.484 \cdot kip$

Factored sliding force (Demand) $V_{sliding} := \sqrt{V_{LFtStrIII}^2 + V_{TFtStrIII}^2} = 49.183 \cdot kip$

Minimum vertical load

$$F_{VFtStrIIIMin} := 0.9 \cdot (DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}) + 1.0 \cdot (EV_{Ft}) = 2.137 \times 10^3 \cdot kip$$

Sliding resistance (Capacity) $V_{resistance} := \phi_T \cdot \mu \cdot F_{VFtStrIIIMin} = 854.631 \cdot kip$

Check if $V_{resistance} > V_{sliding}$ $Check := if(V_{resistance} > V_{sliding}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Strength V

Factored shear force parallel to the transverse axis of the footing $V_{LFtStrV} = 80.587 \cdot kip$

Factored shear force parallel to the longitudinal axis of the footing $V_{TFtStrV} = 27.394 \cdot kip$

Factored sliding force (Demand) $V_{sliding} := \sqrt{V_{LFtStrV}^2 + V_{TFtStrV}^2} = 85.115 \cdot kip$

Minimum vertical load

$$F_{VFtStrVMin} := 0.9 \cdot (DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}) + 1.0 \cdot (EV_{Ft}) = 2.137 \times 10^3 \cdot kip$$

Sliding resistance (Capacity) $V_{resistance} := \phi_T \cdot \mu \cdot F_{VFtStrVMin} = 854.631 \cdot kip$

Check if $V_{resistance} > V_{sliding}$ $Check := if(V_{resistance} > V_{sliding}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Eccentric Load Limitation (Overturning) Check

The eccentricity of loading at the strength limit state, evaluated based on factored loads, shall not exceed one-sixth of the corresponding dimension measured from the centerline of the footing for stability. **LRFD 10.6.3.3**

The eccentricity in the footing length direction is not of a concern. The following calculations present the evaluation of the eccentricity in the footing width direction:

Strength I

Minimum vertical force

$F_{VFtStrIMin} = 2.137 \times 10^3 \cdot kip$

Moment about the longitudinal axis of the footing

$M_{XFtStrI} = 1.593 \times 10^3 \, ft \cdot kip$

Eccentricity in the footing width direction measured from the centerline

$$e_B := \frac{M_{XFtStrI}}{F_{VFtStrIMin}} = 0.745 \, ft$$

1/6 of footing width

$$\frac{w_{footing}}{6} = 3 \, ft$$

Check if the load eccentricity limitation is satisfied

$$Check := if\left(e_B < \frac{w_{footing}}{6}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Strength III

Minimum vertical force

$F_{VFtStrIIIMin} = 2.137 \times 10^3 \cdot kip$

Moment about the longitudinal axis of the footing

$M_{XFtStrIII} = 925.652 \, ft \cdot kip$

Eccentricity in the footing width direction measured from the centerline

$$e_B := \frac{M_{XFtStrIII}}{F_{VFtStrIIIMin}} = 0.433 \, ft$$

Check if the load eccentricity limitation is satisfied

$$Check := if\left(e_B < \frac{w_{footing}}{6}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Strength V

Minimum vertical force

$F_{VFtStrVMin} = 2.137 \times 10^3 \cdot kip$

Moment about the longitudinal axis of the footing

$M_{XFtStrV} = 2.084 \times 10^3 \, ft \cdot kip$

Eccentricity in the footing width direction measured from the centerline

$$e_B := \frac{M_{XFtStrV}}{F_{VFtStrVMin}} = 0.976 \, ft$$

Check if the load eccentricity limitation is satisfied

$$Check := if\left(e_B < \frac{w_{footing}}{6}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Step 7.10 Structural Design of the Footing

Description

This step presents the structural design of the pier footing.

Page Contents

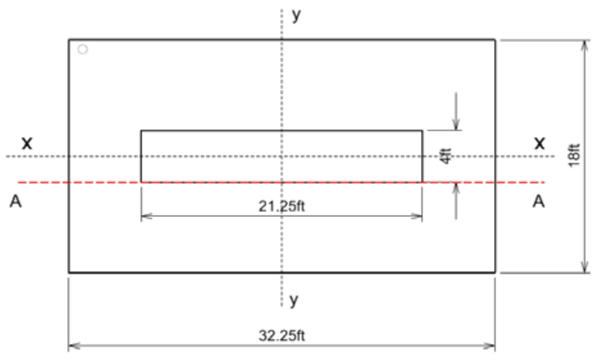
For structural design of an eccentrically loaded foundation, a triangular or trapezoidal bearing pressure distribution shall be used.

LRFD 10.6.5

Design for Flexure

Transverse Reinforcement

The critical section A-A for the design of transverse flexural reinforcement is located at the face of the column, as shown in the following figure.



Distance from the edge of footing to the face of the column

$$l_{col\_x} := \frac{w_{footing} - t_{column}}{2} = 7\,ft$$

Section modulus of the footing about the x-axis

$$S_{XFt} := \frac{1}{6} l_{footing} \cdot w_{footing}^{\;2} = 1.742 \times 10^{3} \cdot ft^{3}$$

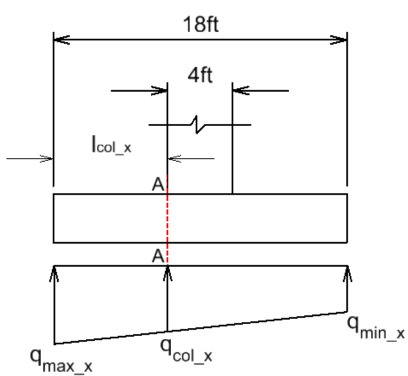
As per the combined load effects presented in Step 7.5, the Strength I limit state is the governing case for flexural design.

Factored vertical force

$$F_{VFtStrI} = 3.976 \times 10^{3} \cdot kip$$

Factored moment about the x-axis

$$M_{XFtStrI} = 1.593 \times 10^{3} \cdot kip \cdot ft$$



Maximum and minimum bearing pressure

$$q_{max\_x} := \frac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} + \frac{M_{XFtStrI}}{S_{XFt}} = 7.764 \cdot ksf$$

$$q_{min\_x} := \frac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} - \frac{M_{XFtStrI}}{S_{XFt}} = 5.935 \cdot ksf$$

Bearing pressure at the critical section $q_{col\_x} := q_{min\_x} + \frac{(q_{max\_x} - q_{min\_x})}{w_{footing}} \cdot (w_{footing} - l_{col\_x}) = 7.053 \cdot ksf$

This example uses a simplified analysis method to determine the maximum moments at the face of the wall by selecting load factors to produce the maximum bearing pressure and minimum resisting loads. This method is conservative and eliminates the need for using multiple combinations.

As shown below, minimum load factors are used for the resisting forces (such as the overburden pressure and footing self-weight) to calculate the maximum moment at the face of the wall.

The moment demand at the critical section on a per-foot basis:

$$M_{ux} := q_{col\_x} \cdot \frac{l_{col\_x}^2}{2} + (q_{max\_x} - q_{col\_x}) \cdot \frac{l_{col\_x}^2}{3} - 0.9 \cdot W_c \cdot t_{footing} \cdot \frac{l_{col\_x}^2}{2} - 1.0 \gamma_s \cdot h_{soil} \cdot \frac{l_{col\_x}^2}{2}$$

$$M_{ux} = 165.665 \cdot \frac{kip \cdot ft}{ft}$$

Flexural Resistance

LRFD 5.6.3.2

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses.

Select a trial bar size $bar := 9$

Nominal diameter of a reinforcing steel bar $d_{bx} := Dia(bar) = 1.128 \cdot in$

Cross-section area of a reinforcing steel bar on the flexural tension side $A_{bar} := Area(bar) = 1 \cdot in^2$

The spacing of the main reinforcing steel bars in walls and slabs shall not be greater than the lesser of 1.5 times the thickness of the member or 18 in. **LRFD 5.10.3.2**

The spacing of shrinkage and temperature reinforcement shall not exceed the following: **LRFD 5.10.6**
12 in. for walls and footings greater than 18 in.
For all other situations, 3 times the component thickness but not less than 18 in.

Note: MDOT limits reinforcement spacing to a maximum of 18 in. in base walls and pier footings adjacent to roadways. **BDG 5.22.01**

Footing thickness $t_{footing} = 3\,ft$

Selected a spacing for reinforcing steel bars $s_{bar} := 8 \cdot in$

Select a 1-ft wide strip for the design.

Area of tension steel in 1-ft wide strip $A_{sProvide\_x} := \frac{A_{bar} \cdot 12in}{s_{bar}} = 1.5 \cdot in^2$

Assume that the moment about the x-axis is greater than the moment about y-axis, and place the reinforcing bars along footing width direction at the bottom of the footing. Then, place the reinforcing bars along footing length direction directly on top of the bars along the width direction. Later, verify this assumption and make necessary adjustments.

Effective depth $d_{ex} := t_{footing} - Cover_{ft} = 32 \cdot in$

Resistance factor for flexure $\phi_f := 0.9$ **LRFD 5.5.4.2**

Width of the compression face of the section $b := 12in$

Stress block factor $\beta_1 = 0.85$

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption $A_s := 1in^2$

Given $M_{ux} \cdot ft = \phi_f \cdot A_s \cdot f_y \cdot \left[d_{ex} - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b} \right) \right]$

Required area of steel $A_{sRequired\_x} := Find(A_s) = 1.194 \cdot in^2$

Check if $A_{sProvided} > A_{sRequired}$ $Check := if(A_{sProvide\_x} > A_{sRequired\_x}, "OK", "Not OK") = "OK"$

Moment capacity of the section with the provided steel

$$M_{Provided\_x} := \phi_f \cdot A_{sProvide\_x} \cdot f_y \cdot \frac{\left[d_{ex} - \frac{1}{2} \cdot \left(\frac{A_{sProvide\_x} \cdot f_y}{0.85 \cdot f_c \cdot b} \right) \right]}{ft}$$

$$M_{Provided\_x} = 206.074 \cdot \frac{kip \cdot ft}{ft}$$

Distance from the extreme compression fiber to the neutral axis

$$c := \frac{A_{sProvide\_x} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 3.46 \cdot in$$

Check the validity of assumption, $f_s = f_y$ $Check\_f_s := if\left(\frac{c}{d_e} < 0.6, "OK", "Not OK" \right) = "OK"$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor $\gamma_1 := 1.6$ For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement $\gamma_3 := 0.67$ For ASTM A615 Grade 60 reinforcement

Section modulus $S_c := \frac{1}{6} \cdot b \cdot t_{footing}^{\;2} = 2.592 \times 10^3 \cdot in^3$

Cracking moment $M_{cr} := \frac{\gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c}{ft} = 96.254 \cdot \frac{kip \cdot ft}{ft}$

1.33 times the factored moment demand $1.33 \cdot M_{ux} = 220.334 \cdot \frac{kip \cdot ft}{ft}$

The factored moment to satisfy the minimum reinforcement requirement $M_{req} := min(1.33M_{ux}, M_{cr}) = 96.254 \cdot \frac{kip \cdot ft}{ft}$

Check the adequacy of section capacity $Check := if(M_{Provided\_x} > M_{req}, "OK", "Not OK") = "OK"$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. Checking for crack control assures that the actual stress in the reinforcement does not exceed the service limit state stress.

The spacing requirement for the mild steel reinforcement in the layer closest to the tension face

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

LRFD Eq. 5.6.7-1

Exposure factor for the Class 1 exposure condition

$$\gamma_e := 1.00$$

Distance from extreme tension fiber to the center of the closest flexural reinforcement

$$d_c := Cover_{ft} = 4 \cdot in$$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face

$$\beta_s := 1 + \frac{d_c}{0.7\left(t_{footing} - d_c\right)} = 1.179$$

The calculation of tensile stress in nonprestressed reinforcement at the service limit state, f_{ss}, requires establishing the neutral axis location and the moment demand at the critical section.

The position of the section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis.

Assumed distance from the extreme compression fiber to the neutral axis

$$x := 5 \cdot in$$

Given

$$\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_c} \cdot A_{sProvide\_x} \cdot \left(d_{ex} - x\right)$$

Position of the neutral axis

$$x_{na} := Find(x) = 7.062 \cdot in$$

Vertical force and moment at the base of the footing under the Service I limit state

$$F_{VFtSerI} = 2.961 \times 10^3 \cdot kip \qquad M_{XFtSerI} = 1.618 \times 10^3 \cdot kip \cdot ft$$

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtSerI}}{w_{footing} \cdot l_{footing}} + \frac{M_{XFtSerI}}{S_{XFt}} = 6.03 \cdot ksf$$

$$q_{min} := \frac{F_{VFtSerI}}{w_{footing} \cdot l_{footing}} - \frac{M_{XFtSerI}}{S_{XFt}} = 4.172 \cdot ksf$$

Bearing pressure at the critical section under the Service I limit state

$$q_{colSerI} := q_{min} + \frac{\left(q_{max} - q_{min}\right)}{w_{footing}} \cdot \left(w_{footing} - l_{col\_x}\right) = 5.307 \cdot ksf$$

The moment at the critical section under the Service I limit state

$$M_{rSerI\_x} := q_{colSerI} \cdot \frac{l_{col\_x}^{\ 2}}{2} + \left(q_{max} - q_{colSerI}\right) \cdot \frac{l_{col\_x}^{\ 2}}{3} - W_c \cdot t_{footing} \cdot \frac{l_{col\_x}^{\ 2}}{2} - \gamma_s \cdot h_{soil} \cdot \frac{l_{col\_x}^{\ 2}}{2}$$

$$M_{rSerI\_x} = 121.98 \cdot \frac{kip \cdot ft}{ft}$$

Tensile force in the reinforcing steel due to the service limit state moment

$$T_s := \frac{M_{rSerI\_x}}{d_{ex} - \frac{x_{na}}{3}} \cdot ft = 49.4 \cdot kip$$

Stress in the reinforcing steel due to the service limit state moment

$$f_{ss1} := \frac{T_s}{A_{sProvide\_x}} = 32.916 \cdot ksi$$

f_{ss} (not to exceed $0.6f_y$)

$$f_{ss} := \min\left(f_{ss1}, 0.6f_y\right) = 32.916 \cdot ksi$$

Required reinforcement spacing

$$s_{barRequired} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 10.044 \cdot in$$

Check if the spacing provided < the required spacing

$$Check := if\left(s_{bar} < s_{barRequired}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Shrinkage and Temperature Reinforcement

The following calculations check the adequacy of the flexural reinforcing steel to control shrinkage and temperature stresses in the footing.

LRFD 5.10.6

Minimum area of shrinkage and temperature reinforcement

$$A_{shrink.temp} := \min\left[\left[\begin{array}{c} \left(0.60 \frac{in^2}{ft}\right) \\ \max\left[\left[\begin{array}{c} \left(0.11 \frac{in^2}{ft}\right) \\ \left[\frac{1.3 \cdot w_{footing} \cdot t_{footing} \cdot \frac{kip}{in \cdot ft}}{2\left(w_{footing} + t_{footing}\right) \cdot f_y}\right] \end{array}\right]\right] \end{array}\right]\right] \cdot ft = 0.334 \cdot in^2$$

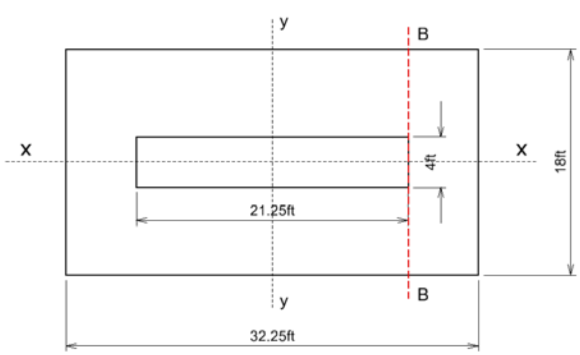
Check if the provided area of steel > the required area of shrinkage and temperature steel

$$Check := if\left(A_{sProvide\_x} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Therefore, the flexural design requires the use of No. 9 bars at 8.0 in. spacing (A_s = 1.5 in.^{2}/ft) as the transverse flexural reinforcement at the bottom of the footing.

Longitudinal Reinforcement

As shown in the following figure, the critical section B-B for the design of longitudinal flexural reinforcement is located at the face of the column:



Distance from the edge of footing to the face of the column

$$l_{col\_y} := \frac{l_{footing} - w_{column}}{2} = 5.5\ \text{ft}$$

Section modulus of the footing about y-axis

$$S_{YFt} := \frac{1}{6} w_{footing} \cdot l_{footing}^{2} = 3.12 \times 10^{3} \cdot \text{ft}^{3}$$

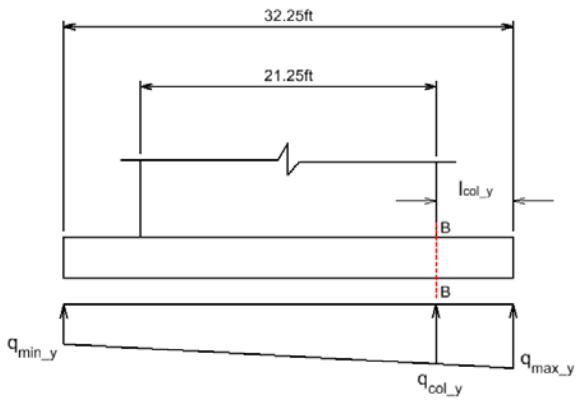
As per the combined load effects presented in Step 7.5, the Strength I limit state is the governing case for flexural design.

Factored vertical force

$$F_{VFtStrI} = 3.976 \times 10^{3} \cdot \text{kip}$$

Factored moment about y-axis

$$M_{YFtStrI} = 894.546 \cdot \text{kip} \cdot \text{ft}$$



Maximum and minimum bearing pressure

$$q_{max\_y} := \frac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} + \frac{M_{YFtStrI}}{S_{YFt}} = 7.136 \cdot \text{ksf}$$

$$q_{min\_y} := \frac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} - \frac{M_{YFtStrI}}{S_{YFt}} = 6.563 \cdot \text{ksf}$$

Bearing pressure at the critical section

$$q_{col\_y} := q_{min\_y} + \frac{\left(q_{max\_y} - q_{min\_y}\right)}{l_{footing}} \cdot \left(l_{footing} - l_{col\_y}\right) = 7.038 \cdot \text{ksf}$$

This example uses a simplified analysis method to determine the maximum moments at the face of the column by selecting load factors to produce the maximum soil pressure and minimum resisting loads. This method is conservative and eliminates the need for using multiple combinations.

As shown below, minimum load factors are used for the resisting forces (such as the overburden pressure and footing self-weight) to calculate the maximum moment at the critical section.

The moment demand at the critical section on a per-foot basis

$$M_{uy} := q_{col\_y} \cdot \frac{l_{col\_y}^{2}}{2} + \left(q_{max\_y} - q_{col\_y}\right) \cdot \frac{l_{col\_y}^{2}}{3} - 0.9 \cdot W_c \cdot t_{footing} \cdot \frac{l_{col\_y}^{2}}{2} - 1.0 \gamma_s \cdot h_{soil} \cdot \frac{l_{col\_y}^{2}}{2}$$

$$M_{uy} = 95.871 \cdot \frac{\text{kip} \cdot \text{ft}}{\text{ft}}$$

$$\text{Check} := \text{if}\left(M_{ux} > M_{uy}, \text{"Assumption is valid"}, \text{"Revise design"}\right) = \text{"Assumption is valid"}$$

Flexural Resistance

LRFD 5.6.3.2

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses.

Select a trial bar size $bar := 7$

Nominal diameter of a reinforcing steel bar $d_{by} := \mathrm{Dia}(bar) = 0.875 \cdot in$

Cross-section area of a reinforcing steel bar on the flexural tension side $A_{bar} := \mathrm{Area}(bar) = 0.6 \cdot in^2$

The spacing of the main reinforcing steel bars in walls and slabs shall not be greater than the lesser of 1.5 times the thickness of the member or 18 in. **LRFD 5.10.3.2**

The spacing of shrinkage and temperature reinforcement shall not exceed the following: **LRFD 5.10.6**

12 in. for walls and footings greater than 18 in.

For all other situations, 3 times the component thickness but not less than 18 in.

Note: MDOT limits reinforcement spacing to a maximum of 18 in. in base walls and pier footings adjacent to roadways. **BDG 5.22.01**

Footing thickness $t_{footing} = 3\,ft$

Select a spacing for reinforcing steel bars $s_{bar} := 8 \cdot in$

Select a 1-ft wide strip for the design.

Area of tension steel provided in a 1-ft wide strip $A_{sProvided\_y} := \dfrac{A_{bar} \cdot 12in}{s_{bar}} = 0.9 \cdot in^2$

Effective depth $d_{ey} := t_{footing} - Cover_{ft} - \dfrac{d_{bx}}{2} - \dfrac{d_{by}}{2} = 30.999 \cdot in$

Resistance factor for flexure $\phi_f := 0.9$ **LRFD 5.5.4.2**

Width of the compression face of the section $b := 12in$

Stress block factor $\beta_1 = 0.85$

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption $A_s := 1in^2$

Given $$M_{uy} \cdot ft = \phi_f \cdot A_s \cdot f_y \cdot \left[d_{ey} - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]$$

Required area of steel $A_{sRequired\_y} := \mathrm{Find}(A_s) = 0.703 \cdot in^2$

Check if $A_{sProvided} > A_{sRequired}$ $Check := \mathrm{if}\left(A_{sProvided\_y} > A_{sRequired\_y}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Moment capacity of the section with the provided steel

$$M_{Provided} := \phi_f \cdot A_{sProvided\_y} \cdot f_y \cdot \frac{\left[d_{ey} - \frac{1}{2} \cdot \left(\frac{A_{sProvided\_y} \cdot f_y}{0.85 \cdot f_c \cdot b}\right)\right]}{ft}$$

$$M_{Provided} = 121.97 \cdot \frac{kip \cdot ft}{ft}$$

Distance from the extreme compression fiber to the neutral axis

$$c := \frac{A_{sProvided\_y} \cdot f_y}{0.85 \cdot f_c \cdot \beta_1 \cdot b} = 2.08 \cdot in$$

Check the validity of assumption, $f_s = f_y$

$$Check\_f_s := if\left(\frac{c}{d_e} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor $\gamma_1 := 1.6$ For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement $\gamma_3 := 0.67$ For ASTM A615 Grade 60 reinforcement

Section modulus

$$S_c := \frac{1}{6} \cdot b \cdot t_{footing}^2 = 2.592 \times 10^3 \cdot in^3$$

Cracking moment

$$M_{cr} := \frac{\gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c}{ft} = 96.254 \cdot \frac{kip \cdot ft}{ft}$$

1.33 times the factored moment demand

$$1.33 \cdot M_{uy} = 127.508 \cdot \frac{kip \cdot ft}{ft}$$

The factored moment to satisfy the minimum reinforcement requirement

$$M_{req} := \min\left(1.33 M_{uy}, M_{cr}\right) = 96.254 \cdot \frac{kip \cdot ft}{ft}$$

Check the adequacy of section capacity

$$Check := if\left(M_{Provided} > M_{req}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. Checking for crack control assures that the actual stress in the reinforcement does not exceed the service limit state stress.

The spacing requirement for the mild steel reinforcement in the layer closest to the tension face

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

LRFD Eq. 5.6.7-1

Exposure factor for the Class 1 exposure condition $\gamma_e := 1.00$

Distance from extreme tension fiber to the center of the closest flexural reinforcement

$$d_c := Cover_{ft} = 4 \cdot in$$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face

$$\beta_s := 1 + \frac{d_c}{0.7\left(t_{footing} - d_c\right)} = 1.179$$

The calculation of tensile stress in nonprestressed reinforcement at the service limit state, f_{ss}, requires establishing the neutral axis location and the moment demand at the critical section.

The position of the cross-section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis.

Assumed distance from the extreme compression fiber to the neutral axis $x := 5 \cdot in$

Given $\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_c} \cdot A_{sProvided\_y} \cdot (d_{ey} - x)$

Position of the neutral axis $x_{na} := Find(x) = 5.528 \cdot in$

Vertical force and moment at the base of the footing under the Service I limit state

$F_{VFtSerI} = 2.961 \times 10^3 \cdot kip$ $M_{YFtSerI} = 1.253 \times 10^3 \cdot kip \cdot ft$ From Step 7.5

Maximum and minimum bearing pressure

$$q_{max} := \frac{F_{VFtSerI}}{w_{footing} \cdot l_{footing}} + \frac{M_{YFtSerI}}{S_{YFt}} = 5.502 \cdot ksf$$

$$q_{min} := \frac{F_{VFtSerI}}{w_{footing} \cdot l_{footing}} - \frac{M_{YFtSerI}}{S_{YFt}} = 4.699 \cdot ksf$$

Bearing pressure at the critical section under the Service I limit state

$$q_{colSerI} := q_{min} + \frac{(q_{max} - q_{min})}{l_{footing}} \cdot (l_{footing} - l_{col\_y}) = 5.365 \cdot ksf$$

The moment at the critical section under the Service I limit state

$$M_{rSerI\_y} := q_{colSerI} \cdot \frac{l_{col\_y}^2}{2} + (q_{max} - q_{colSerI}) \cdot \frac{l_{col\_y}^2}{3} - W_c \cdot t_{footing} \cdot \frac{l_{col\_y}^2}{2} - \gamma_s \cdot h_{soil} \cdot \frac{l_{col\_y}^2}{2}$$

$$M_{rSerI\_y} = 70.279 \cdot \frac{kip \cdot ft}{ft}$$

Tensile force in the reinforcing steel due to the service limit state moment

$$T_s := \frac{M_{rSerI\_y}}{d_{ey} - \frac{x_{na}}{3}} \cdot ft = 28.9 \cdot kip$$

Stress in the reinforcing steel due to service limit state moment

$$f_{ss1} := \frac{T_s}{A_{sProvided\_y}} = 32.14 \cdot ksi$$

f_{ss} (not to exceed $0.6f_y$) $f_{ss} := \min(f_{ss1}, 0.6f_y) = 32.14 \cdot ksi$

Required reinforcement spacing

$$s_{barRequired} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 10.48 \cdot in$$

Check if the spacing provided < the required spacing

$Check := if(s_{bar} < s_{barRequired}, "OK", "Not OK") = "OK"$

Shrinkage and Temperature Reinforcement

Check if the provided area of steel > the required area of shrinkage and temperature steel

$$Check := if\left(A_{sProvided\_y} > A_{shrink.temp}, "OK", "Not OK"\right) = "OK"$$

Therefore, the flexural design requires the use of No. 7 bars at 8.0 in. spacing ($A_s = 0.9$ in.^{2}/ft) as the longitudinal flexural reinforcement at the bottom of the footing.

Design for Shear

One-Way Shear at a Section Parallel to the Transverse Axis of the Footing

The factored shear force at the critical section is computed by calculating the resultant force due to the bearing pressure acting on the footing base area that is outside of the critical section.

Note: Since the transverse and longitudinal load effects are considered independently, bearing pressure distribution along the footing length is uniform. Therefore, a 1-ft wide strip is considered for the design.

Effective width of the section $b = 12 \cdot in$

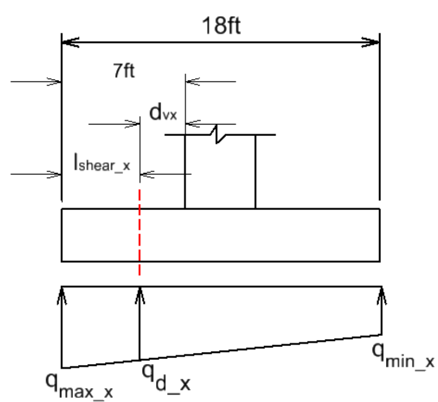
Depth of an equivalent rectangular stress block

$$a := \frac{A_{sProvide\_x} \cdot f_y}{0.85 \cdot f_c \cdot b} = 2.941 \cdot in$$

Effective shear depth

$$d_{vx} := \max\left(d_{ex} - \frac{a}{2}, 0.9 \cdot d_{ex}, 0.72 \cdot t_{footing}\right) = 30.529 \cdot in$$ **LRFD 5.7.2.8**

As shown in the following figure, the critical section for shear is located at a distance d_{vx} from the face of the column:



Distance from end of the footing to the critical section for shear

$$l_{shear\_x} := l_{col\_x} - d_{vx} = 4.456 \cdot ft$$

Bearing stress at the critical section for shear

$$q_{d\_x} := q_{min\_x} + \frac{\left(q_{max\_x} - q_{min\_x}\right)}{w_{footing}} \cdot \left(w_{footing} - l_{shear\_x}\right) = 7.311 \cdot ksf$$

Minimum load factors are used for the resisting forces (such as the overburden pressure and footing self-weight) to calculate the maximum shear at the critical section.

Factored shear demand at the critical section

$$V_{uFt\_x} := \frac{\left(q_{max\_x} + q_{d\_x}\right)}{2} \cdot l_{shear\_x} - 0.9 \cdot W_c \cdot t_{footing} \cdot l_{shear\_x} - 1.0 \cdot \gamma_s \cdot h_{soil} \cdot l_{shear\_x} = 30.178 \cdot \frac{kip}{ft}$$

For a concrete footing, in which the distance from the point of zero shear to the face of the base wall is less than $3d_v$, the simplified procedure for nonprestressed sections can be used. **LRFD 5.7.3.4.1**

$$Check := if\left(l_{col\_x} < 3 \cdot d_{vx}, \text{"Use the simplied method"}, \text{"Do not use the simplifed method"}\right) = \text{"Use the simplied method"}$$

Factor indicating the ability of diagonally cracked concrete to transmit tension and shear $\beta := 2$

Nominal shear resistance of concrete, V_n, is calculated as follows:

$$V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b \cdot d_{vx} = 40.1 \cdot kip$$ **LRFD Eq. 5.7.3.3-3**

$$V_{c2} := 0.25 f_c \cdot b \cdot d_{vx} = 274.765 \cdot kip$$ **LRFD Eq. 5.7.3.3-2**

$$V_n := \min\left(V_{c1}, V_{c2}\right) = 40.103 \cdot kip$$

Resistance factor for shear $\phi_v := 0.9$ **LRFD 5.5.4.2**

Factored shear resistance (Capacity) $V_r := \phi_v \cdot V_n = 36.093 \cdot kip$

Check if the shear capacity > the shear demand $Check := if\left(\frac{V_r}{ft} > V_{uFt\_x}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

One-Way Shear at a Section Parallel to the Longitudinal Axis of the Footing

The factored shear force at the critical section is the resultant force due to the bearing pressure acting on the footing base area located outside the critical section.

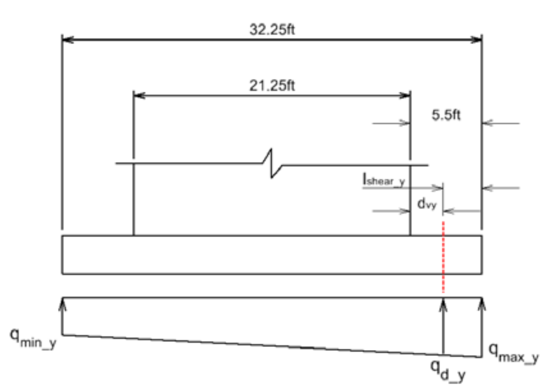
Note: Since the transverse and longitudinal load effects are considered independently, bearing pressure distribution along the footing width is uniform. Therefore, a 1-ft wide strip is considered for the design.

Effective width of the section $b = 12 \cdot in$

Depth of an equivalent rectangular stress block $a := \frac{A_{sProvided\_y} \cdot f_y}{0.85 \cdot f_c \cdot b} = 1.765 \cdot in$

Effective shear depth $d_{vy} := \max\left(d_{ey} - \frac{a}{2}, 0.9 \cdot d_{ey}, 0.72 \cdot t_{footing}\right) = 30.116 \cdot in$ **LRFD 5.7.2.8**

As shown in the following figure, the critical section for shear is located at a distance d_{vy} from the face of the column:



Distance from end of the footing to the critical section $$l_{shear\_y} := l_{col\_y} - d_{vy} = 2.99 \cdot ft$$

Bearing stress at the critical section $$q_{d\_y} := q_{min\_y} + \frac{(q_{max\_y} - q_{min\_y})}{w_{footing}} \cdot (w_{footing} - l_{shear\_y}) = 7.041 \cdot ksf$$

Minimum load factors are used for the resisting forces (such as the overburden pressure and footing self-weight) to calculate the maximum shear at the critical section.

Factored shear demand at the critical section

$$V_{uFt\_y} := \frac{(q_{max\_y} + q_{d\_y})}{2} \cdot l_{shear\_y} - 0.9 \cdot W_c \cdot t_{footing} \cdot l_{shear\_y} - 1.0 \cdot \gamma_s \cdot h_{soil} \cdot l_{shear\_y} = 18.909 \cdot \frac{kip}{ft}$$

For a concrete footing, in which the distance from the point of zero shear to the face of the column is less than $3d_v$, the simplified procedure for nonprestressed sections can be used. **LRFD 5.7.3.4.1**

$$Check := if(l_{col\_x} < 3 \cdot d_{vy}, \text{"Use the simplied method"}, \text{"Do not use the simplifed method"}) = \text{"Use the simplied method"}$$

Factor indicating the ability of diagonally cracked concrete to transmit tension and shear $$\beta := 2$$

The nominal shear resistance of concrete, V_n, is calculated as follows.

$$V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_c \cdot ksi} \cdot b \cdot d_{vy} = 39.6 \cdot kip$$ **LRFD Eq. 5.7.3.3-3**

$$V_{c2} := 0.25 f_c \cdot b \cdot d_{vy} = 271.045 \cdot kip$$ **LRFD Eq. 5.7.3.3-2**

$$V_n := min(V_{c1}, V_{c2}) = 39.56 \cdot kip$$

Resistance factor for shear $$\phi_v := 0.9$$ **LRFD 5.5.4.2**

Factored shear resistance (Capacity) $$V_r := \phi_v \cdot V_n = 35.604 \cdot kip$$

Check if the shear capacity > the shear demand $$Check := if\left(\frac{V_r}{ft} > V_{uFt\_y}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Two-way Shear

Two-way shear (punching shear) in the footing is checked at a critical perimeter around the pier column.

The critical perimeter around the column, b_o, is located at a minimum of $0.5d_v$ from the perimeter of the column. **LRFD 5.12.8.6.3**

An average effective shear depth, d_v, should be used since the two-way shear area includes both the x- and y-directions of the footing.

Average effective shear depth $$d_{v\_avg} := \frac{(d_{vx} + d_{vy})}{2} = 2.527\ ft$$

Critical perimeter $$b_0 := 2 \cdot (w_{column} + d_{v\_avg}) + 2 \cdot (t_{column} + d_{v\_avg}) = 60.608\ ft$$

Ratio of long to short side of the critical perimeter $\beta_c := \frac{w_{column}}{t_{column}} = 5.313$

Nominal shear resistance $V_{n1\_2way} := \left(0.063 + \frac{0.126}{\beta_c}\right) \cdot \sqrt{f_c \cdot ksi} \cdot b_0 \cdot d_{v\_avg} = 3.312 \times 10^3 \cdot kip$

$V_{n2\_2way} := 0.126 \cdot \left(\sqrt{f_c \cdot ksi} \cdot b_0 \cdot d_{v\_avg}\right) = 4.813 \times 10^3 \cdot kip$

LRFD Eq. 5.12.8.6.3-1

$V_{n\_2way} := \min\left(V_{n1\_2way}, V_{n2\_2way}\right) = 3.312 \times 10^3 \cdot kip$

Factored shear resistance (Capacity)) $V_{r\_2way} := \phi_v \cdot V_{n\_2way} = 2.981 \times 10^3 \cdot kip$

To calculate the shear force acting on the critical perimeter, the average bearing pressure is used. The Strength I is the governing limit state.

Average bearing pressure $q_{average} := \frac{F_{VFtStrI}}{w_{footing} \cdot l_{footing}} = 6.849 \cdot \frac{kip}{ft^2}$

Resultant shear force acting on the area outside of the critical perimeter (Demand)

$$V_{u\_2way} := q_{average} \cdot \left[w_{footing} \cdot l_{footing} - \left(w_{column} + d_{v\_avg}\right) \cdot \left(t_{column} + d_{v\_avg}\right)\right] = 2.913 \times 10^3 \cdot kip$$

Check if the factored two-way shear resistance > the demand $Check := \text{if}\left(V_{r\_2way} > V_{u\_2way}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Development Length of Reinforcement

The flexural reinforcing steel must be developed on each side of the critical section for its full development length.

LRFD 5.10.8.1.2

Longitudinal Direction of the Footing

Available development length $l_{dy\_avail} := \frac{l_{footing} - w_{column}}{2} - Cover_{ft} = 62 \cdot in$

Assuming that the bars are at high stress, the required development length for No. 7 bars at 8 in. spacing

$l_{dy.req} := 21in$

BDG 7.14.01

Check if $l_{dy.avail} > l_{dy.req}$ $Check := \text{if}\left(l_{dy\_avail} > l_{dy.req}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Transverse Direction of the Footing

Available development length $l_{dx\_avail} := \frac{w_{footing} - t_{column}}{2} - Cover_{ft} = 80 \cdot in$

Assuming that the bars are at high stress, the required development length for No. 9 bars at 8 in. spacing

$l_{dx.req} := 35in$

BDG 7.14.01

Check if $l_{dx.avail} > l_{dx.req}$ $Check := \text{if}\left(l_{dx\_avail} > l_{dx.req}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Shrinkage and Temperature Reinforcement

This shrinkage and temperature reinforcement requirement for the steel at the bottom of the footing was already checked and the requirements were satisfied.

The reinforcement at the top of the footing should satisfy the shrinkage and temperature reinforcement requirements. **LRFD 5.10.6**

The spacing of shrinkage and temperature reinforcement shall not exceed the following: **LRFD 5.10.6**

12 in. for walls and footings greater than 18 in.

For all other situations, 3 times the component thickness but not less than 18 in.

Note: MDOT limits reinforcement spacing to a maximum of 18 in. for pier footings adjacent to roadways. **BDG 5.22.01**

Select a trial bar size $bar := 6$

Nominal diameter of a reinforcing steel bar $d_{bar} := Dia(bar) = 0.75 \cdot in$

Cross-section area of a reinforcing steel bar $A_{bar} := Area(bar) = 0.44 \cdot in^2$

Select a spacing for reinforcing steel bars $s_{barST} := 12 \cdot in$

Provided horizontal reinforcement area $A_{sProvidedST} := \dfrac{A_{bar} \cdot 12in}{s_{barST}} = 0.44 \cdot in^2$

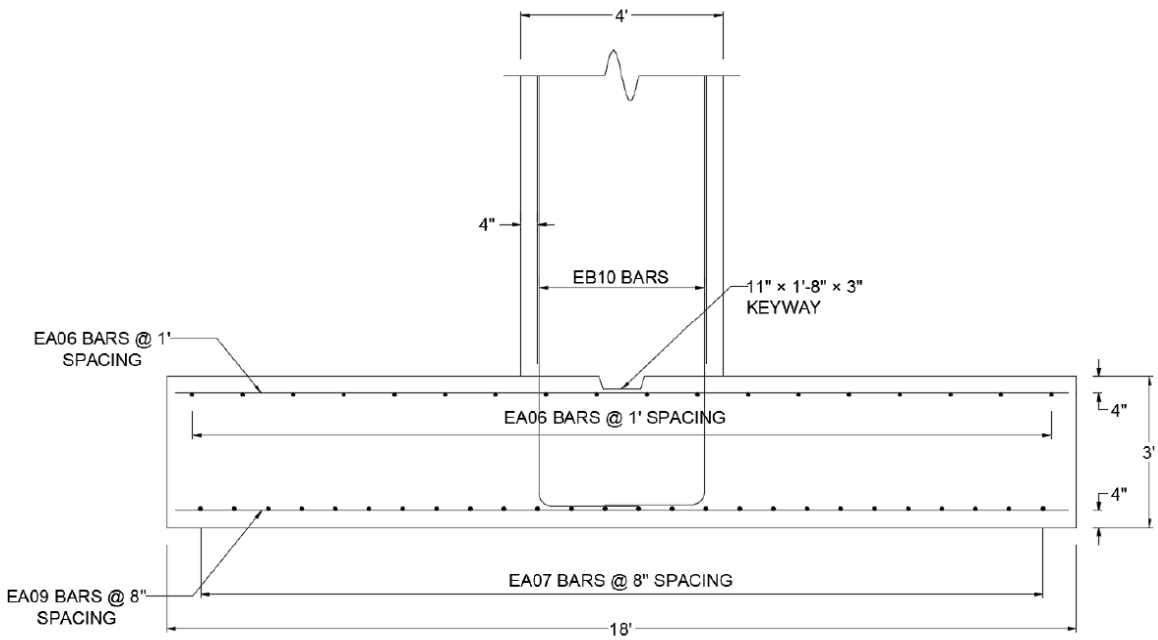
Required shrinkage and temperature steel area in the transverse and longitudinal directions (calculated previously in the flexural design) $A_{shrink.temp} = 0.334 \cdot in^2$

Check if the provided area of steel > the required area of shrinkage and temperature steel $Check := if(A_{sProvidedST} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Therefore, use No. 6 bars at 12.0 in. spacing ($A_s = 0.44$ in.^{2}/ft) as the shrinkage and temperature reinforcement at the top of the footing in both longitudinal and transverse directions.

The footing design presented in this step results in the following details:

- No. 9 bars @ 8.0 in. spacing ($A_s = 1.5$ in.^{2}/ft) as the transverse flexural reinforcement at the bottom of the footing
- No. 7 bars @ 8.0 in. spacing ($A_s = 0.9$ in.^{2}/ft) as the longitudinal flexural reinforcement at the bottom of the footing
- No. 6 bars @ 12.0 in. spacing ($A_s = 0.44$ in.^{2}/ft) as the shrinkage and temperature reinforcement at the top of the footing in both longitudinal and transverse directions.



Note: Certain details are not shown in this drawing for clarity of main reinforcement.
Refer to MDOT Bridge Design Guides for additional details.

Section 8 Hammerhead Pier with Pile Foundation

Step 8.1 Preliminary Dimensions

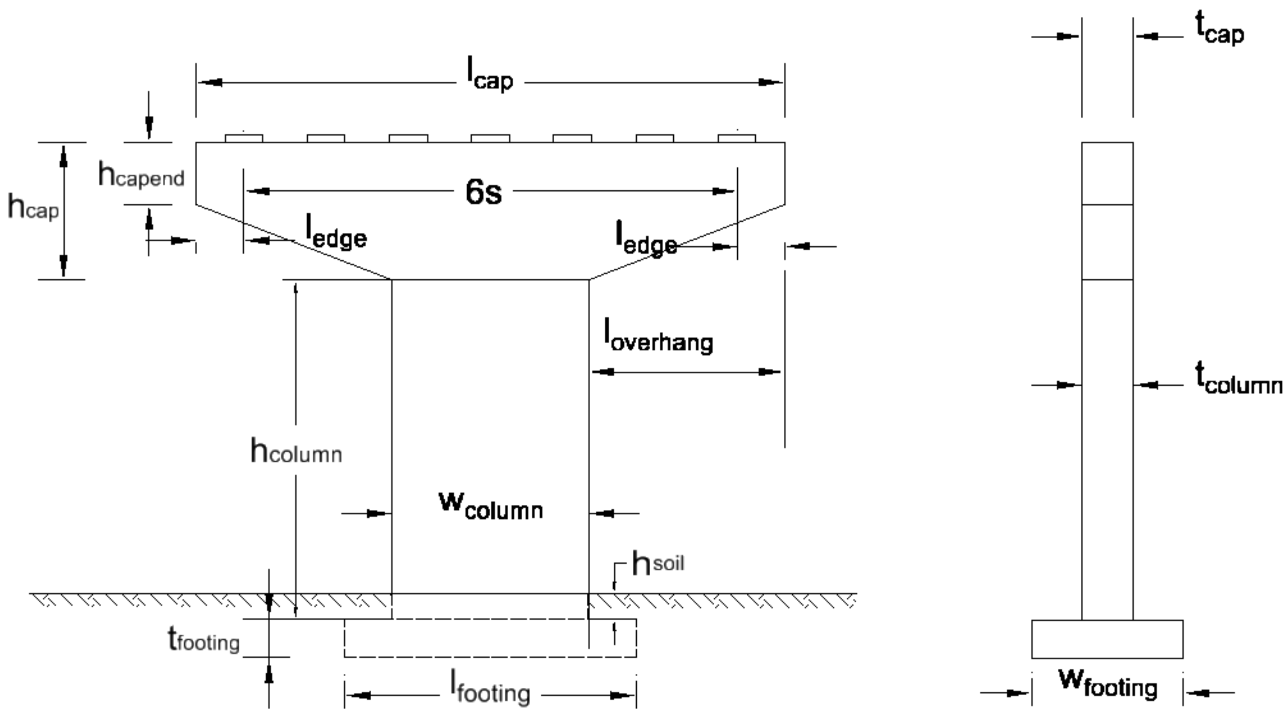
Description

This step presents the selected preliminary dimensions.

Refer to Section 2 of the *Design of Highway Bridge Abutments and Foundations Example* developed by Attanayake and Hu (2023) for the design criteria, bridge information, material properties, along with soil types and properties.

The preliminary dimensions are selected based on site-specific conditions, highway agency standards, and past experience.

The following figure shows the pier geometry and its associated dimensional variables:



The preliminary dimensions selected for this example are given below.

Pier cap length $l_{cap} := W_{deck} = 63.75\,ft$

Pier cap end height $h_{capend} := 5ft$

Pier cap height $h_{cap} := 11ft$

Pier cap thickness $t_{cap} := 4ft$

Length of the overhang $l_{overhang} := 21.25ft$

Column width $w_{column} := 21.25ft$

Column thickness $t_{column} := 4ft$

Column height $h_{column} := 14ft$

Footing length $l_{footing} := 27ft$

Footing thickness $t_{footing} := 3.5ft$

Footing width $w_{footing} := 11ft$

Depth of soil above the footing top $h_{soil} := 3ft$

Note: The depth from the ground level to the bottom of the footing needs to be maintained at a minimum of 4 ft. for frost depth. Typically, a 1-ft deep soil profile is maintained with normal grading when the pier is at a median. The depth of the soil may change to 2 to 3 ft based on the pavement profile when the pier is closer to the pavement.

Girder spacing $S = 9.719\,ft$

Distance from the exterior girder to the edge of the pier cap

$$l_{edge} := \frac{l_{cap} - S \cdot (N_{beams} - 1)}{2} = 2.719\,ft$$

Concrete Cover Requirements for Reinforcing Steel

Unless otherwise shown on the plans, the minimum concrete clear cover for reinforcement shall satisfy the following requirements: **BDM 8.02.N**

For concrete cast against earth: 3 in.
For all other cases unless shown on plans: 2 in.

The following concrete cover dimension is selected since it is greater than the required minimum. **BDG 5.16.01, 5.18.01, 5.22.01**

Cover for the top and side of footing $Cover_{ft} := 4in$

Since the concrete cover requirements for pier caps and columns are not provided in the BDM and BDG, the following dimensions are taken from the MDOT Sample Bridge Plans.

Cover for the pier cap $Cover_{cap} := 3.5in$

Cover for the pier column $Cover_{col} := 4in$

Step 8.2 Application of Dead Load

Description

This step describes the application of dead load on the pier.

Dead Load Girder Reactions

The superstructure dead load reactions per bearing are taken from the *Steel Plate Girder Design Example*. All the beam seats are assumed to be at the same elevation.

When calculating superstructure loads on the substructure, 75% of the barrier dead load should be applied with the fascia beam load. The remaining 25% of the barrier load should be applied with the first interior girder load. **BDM 7.01.04.J**

Note: The exterior and interior girder shear values presented in the *Steel Plate Girder Design Example* (Table 12 and 13) were calculated by equally distributing the barrier loads to all the girders. Therefore, the girder reactions over the pier due to barrier loads need to be recalculated as shown below.

Exterior Girders

Table 12 of the Steel Plate Girder Design Example

Reaction due to the weight of structural components and non-structural attachments (DC), including the stay-in-place formwork but excluding barrier weight

$$R_{DCEx\_noBarrier} := 161.4kip$$

Reaction due to 75% of the barrier weight (DB) on the exterior girder $R_{DCEx\_barrier} := 44kip$

Total exterior girder reaction due to DC $R_{DCEx} := R_{DCEx\_noBarrier} + R_{DCEx\_barrier} = 205.4 \cdot kip$

Reaction due to the weight of the future wearing surface (DW) $R_{DWEx} := 26.6kip$

First Interior Girder

Table 13 of the Steel Plate Girder Design Example

Reaction due to the weight of structural components and non-structural attachments (DC), including the stay-in-place formwork but excluding barrier weight

$$R_{DC1stIn\_noBarrier} := 190.4kip$$

Reaction due to 25% of the barrier weight (DB) on the first interior girder $R_{DC1stIn\_barrier} := 14.5kip$

Total first interior girder reaction due to DC $R_{DC1stIn} := R_{DC1stIn\_noBarrier} + R_{DC1stIn\_barrier} = 204.9 \cdot kip$

Reaction due to the future wearing surface weight (DW) $R_{DWIn} := 26.4kip$

Other Interior Girders

Table 13 of the Steel Plate Girder Design Example

Reaction due to the weight of structural components and non-structural attachments (DC), including the stay-in-place formwork but excluding barrier weight

$$R_{DCIn} := 190.4kip$$

Dead Load Calculation

Dead load of superstructure

Weight of structural components and non-structural attachments (DC)

$$DC_{Sup} := 2 \cdot R_{DCEx} + 2 \cdot R_{DC1stIn} + \left(N_{beams} - 4\right) \cdot R_{DCIn}$$

$$DC_{Sup} = 1.392 \times 10^{3} \cdot kip$$

Weight of the future wearing surface (DW) $DW_{Sup} := 2 \cdot R_{DWEx} + \left(N_{beams} - 2\right) \cdot R_{DWIn} = 185.2 \cdot kip$

Pier cap self-weight

$$DC_{cap} := W_c \cdot t_{cap} \cdot \left[2 \cdot \left(\frac{h_{capend} + h_{cap}}{2} \right) \cdot l_{overhang} + h_{cap} \cdot w_{column} \right] = 344.25 \cdot kip$$

Pier column self-weight

$$DC_{column} := W_c \cdot t_{column} \cdot h_{column} \cdot w_{column} = 178.5 \cdot kip$$

Pier footing self-weight

$$DC_{footing} := W_c \cdot w_{footing} \cdot t_{footing} \cdot l_{footing} = 155.925 \cdot kip$$

Step 8.3 Application of Live Load

Description

The live load application procedure and relevant calculations are described in Step 7.3.

Step 8.4 Application of Other Loads

Description

The application of other loads include braking force, wind load, temperature load, earth load, and vehicle collision load. They are discussed in Step 7.4. Ice load and centrifugal force are not applicable for this example. For illustrative purposes, the calculation of ice load and centrifugal force is given in Appendix 5.B and 5.C.

Step 8.5 Combined Load Effects

Description

This step presents the procedure of combining all load effects and calculates the total factored forces and moments acting on the pier cap, columns, base wall, and footing.

Since the combined loadings on the pier cap, columns, and base wall are identical to Step 7.5, only the calculation of combined load effects at the base of the footing is presented.

Strength I, Strength III, Strength V, and Service I limit states are considered for the analysis and design of the pier.

LRFD 3.4.1

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75LS + 0.5TU

Strength III = 1.25DC + 1.5DW + 1.5EH + 1.35EV + 1.0WS + 0.5TU

Strength V = 1.25DC + 1.5DW + 1.35LL + 1.35BR + 1.0WS + 1.0WL + 1.5EH + 1.35EV + 1.35LS + 0.5TU

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

BR	= vehicular braking force
DC	= dead load of structural components and nonstructural attachments
DW	= dead load of the future wearing surface and utilities
EH	= horizontal earth pressure load
EV	= vertical pressure from the earth fill
LL	= vehicular live load
LS	= live load surcharge
WL	= wind on live load
WS	= wind load on structure
TU	= force effect due to uniform temperature

Limit states that are not shown either do not control or are not applicable.

Note: These load combinations should include the maximum and minimum load factors; only the maximum factors are shown for clarity.

Forces and Moments at the Pier Footing

The bearing pressure distribution depends on the rigidity of the footing along with the soil type and condition. The pier footings are usually rigid, and the assumption q = (P/A) +/- (Mc/I) is valid. For an accurate calculation of bearing pressure distribution, the footing may be analyzed as a beam on an elastic foundation.

The braking force, wind load on the superstructure, and wind load acting on the live load are applied at the bearings.

The live load on all five lanes develops the critical load effects for the footing design.

Moment arm of Girder A and G reactions to the center of footing $\quad Arm_{AG} := 3S = 29.156\ ft$

Moment arm of Girder B and F reactions to the center of footing $\quad Arm_{BF} := 2S = 19.438\ ft$

Moment arm of Girder C and E reactions to the center of footing $\quad Arm_{CE} := S = 9.719\ ft$

For convenience, the x- and y- axes are defined as parallel to the longitudinal and transverse directions of the footing, respectively.

Strength I

Strength I = 1.25DC + 1.5DW + 1.75LL + 1.75BR + 1.5EH + 1.35EV + 1.75LS + 0.5TU

Factored vertical force

$$F_{VFtStrI} := 1.25\cdot\left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + 1.5DW_{Sup} \cdots$$
$$+ 1.75R_{LLFooting} + 1.35\cdot EV_{Ft}$$

$$F_{VFtStrI} = 3.707 \times 10^3 \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrI} := 0$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrI} := 1.75 \cdot BRK_{5L} = 56.875 \cdot kip$$

Factored moment about the longitudinal axis of the footing

$$M_{XFtStrI} := 1.75 \cdot BRK_{5L} \cdot \left(Arm_{col} + t_{footing}\right) = 1.621 \times 10^3 \cdot kip \cdot ft$$

Factored moment about the transverse axis of the footing

$$M_{YFtStrI} := 1.75 \cdot \left[\left(R_{GFt\_5L} - R_{AFt\_5L}\right) \cdot Arm_{AG} + \left(R_{FFt\_5L} - R_{BFt\_5L}\right) \cdot Arm_{BF} \cdots + \left(R_{EFt\_5L} - R_{CFt\_5L}\right) \cdot Arm_{CE}\right] = 894.546 \cdot kip \cdot ft$$

Strength III

Strength III = 1.25DC + 1.5DW + 1.5EH + 1.35EV + 1.0WS + 0.5TU

Factored vertical force

$$F_{VFtStrIII} := 1.25 \cdot \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + 1.5DW_{Sup} \cdots + 1.35 \cdot EV_{Ft}$$

$$F_{VFtStrIII} = 2.969 \times 10^3 \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrIII} := N_{beams} \cdot WS_{TStrIII} + WS_{SubT.StrIII} = 25.484 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrIII} := N_{beams} \cdot WS_{LStrIII} + WS_{SubL.StrIII} = 42.066 \cdot kip$$

Factored moment about the longitudinal axis of the footing

$$M_{XFtStrIII} := N_{beams} \cdot WS_{LStrIII} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubL.StrIII} \cdot \left(H_{WSSubL} + t_{footing}\right) = 946.685 \cdot kip \cdot ft$$

Factored moment about the transverse axis of the footing

$$M_{YFtStrIII} := N_{beams} \cdot WS_{TStrIII} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubT.StrIII} \cdot \left(H_{WSSubT} + t_{footing}\right)$$

$$M_{YFtStrIII} = 689.114 \cdot kip \cdot ft$$

Strength V

Strength V = 1.25DC + 1.5DW + 1.35LL + 1.35BR + 1.0WS + 1.0WL + 1.5EH + 1.35EV + 1.35 LS + 0.5TU

Factored vertical force

$$F_{VFtStrV} := 1.25 \cdot \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + 1.5DW_{Sup} \cdots + 1.35 \cdot R_{LLFooting} + 1.35 \cdot EV_{Ft}$$

$$F_{VFtStrV} = 3.538 \times 10^3 \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrV} := N_{beams} \cdot \left(WS_{TStrV} + WL_{TBearing}\right) + WS_{SubT.StrV} = 27.394 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrV} := 1.35 \cdot BRK_{5L} + N_{beams} \cdot \left(WS_{LStrV} + WL_{LBearing}\right) + WS_{SubL.StrV} = 80.587 \cdot kip$$

Factored moment about the longitudinal axis of the footing

$$M_{XFtStrV} := 1.35 \cdot BRK_{5L} \cdot \left(Arm_{col} + t_{footing}\right) + N_{beams} \cdot WS_{LStrV} \cdot \left(Arm_{col} + t_{footing}\right) \dots$$
$$+ N_{beams} \cdot WL_{LBearing} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubL.StrV} \cdot \left(H_{WSSubL} + t_{footing}\right)$$

$$M_{XFtStrV} = 2.125 \times 10^{3} \cdot kip \cdot ft$$

Factored moment about the transverse axis of the footing

$$M_{YFtStrV} := 1.35 \left[\left(R_{GFt\_5L} - R_{AFt\_5L}\right) \cdot Arm_{AG} + \left(R_{FFt\_5L} - R_{BFt\_5L}\right) \cdot Arm_{BF} \dots + \left(R_{EFt\_5L} - R_{CFt\_5L}\right) \cdot Arm_{CE}\right] \dots$$
$$+ N_{beams} \cdot WS_{TStrV} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubT.StrV} \cdot \left(H_{WSSubT} + t_{footing}\right) \dots$$
$$+ N_{beams} \cdot WL_{TBearing} \cdot \left(Arm_{col} + t_{footing}\right)$$

$$M_{YFtStrV} = 1.445 \times 10^{3} \cdot kip \cdot ft$$

Service I

Service I = 1.0DC + 1.0DW + 1.0LL + 1.0BR + 1.0WS + 1.0WL + 1.0EH + 1.0EV + 1.0LS + 1.0TU

Factored vertical force

$$F_{VFtSerI} := \left(DC_{Sup} + DC_{cap} + DC_{column} + DC_{footing}\right) + DW_{Sup} \dots$$
$$+ R_{LLFooting} + EV_{Ft}$$

$$F_{VFtSerI} = 2.754 \times 10^{3} \cdot kip$$

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtSerI} := N_{beams} \cdot \left(WS_{TSerI} + WL_{TBearing}\right) + WS_{SubT.SerI} = 23.317 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtSerI} := BRK_{5L} + N_{beams} \cdot \left(WS_{LSerI} + WL_{LBearing}\right) + WS_{SubL.SerI} = 62.482 \cdot kip$$

Factored moment about the longitudinal axis of the footing

$$M_{XFtSerI} := BRK_{5L} \cdot \left(Arm_{col} + t_{footing}\right) + N_{beams} \cdot WS_{LSerI} \cdot \left(Arm_{col} + t_{footing}\right) \dots$$
$$+ N_{beams} \cdot WL_{LBearing} \cdot \left(Arm_{col} + t_{footing}\right) + WS_{SubL.SerI} \cdot \left(H_{WSSubL} + t_{footing}\right)$$

$$M_{XFtSerI} = 1.649 \times 10^{3} \cdot kip \cdot ft$$

Factored moment about the transverse axis of the footing

$$M_{YFtSerI} := \left(R_{GFt\_5L} - R_{AFt\_5L}\right) \cdot Arm_{AG} + \left(R_{FFt\_5L} - R_{BFt\_5L}\right) \cdot Arm_{BF} \dots$$
$$+ \left(R_{EFt\_5L} - R_{CFt\_5L}\right) \cdot Arm_{CE} + N_{beams} \cdot WS_{TStrV} \cdot \left(Arm_{col} + t_{footing}\right) \dots$$
$$+ WS_{SubT.StrV} \cdot \left(H_{WSSubT} + t_{footing}\right) + N_{beams} \cdot WL_{TBearing} \cdot \left(Arm_{col} + t_{footing}\right)$$

$$M_{YFtSerI} = 1.267 \times 10^{3} \cdot kip \cdot ft$$

The design of the pier cap and column is presented in Steps 7.6, 7.7, and 7.8. The subsequent steps of this example present the design of piles and the footing.

Step 8.6 Pile Design

Description

This step presents the selection of pile type, the design of pile size and layout, and the evaluation of the lateral force resistance of piles.

Page Content

Pile Size and Layout Design

This example uses steel H piles since they are the most commonly used pile type in Michigan. Typically, pile type is selected after evaluating other possibilities, such as ground improvement techniques, other foundation types, and constructability.

BDM 7.03.09.A5

Pile embedment into the footing $Pile\_embd := 6in$

Note: A tremie seal is not used for this footing. If a tremie seal is used, the pile embedment into the footing is 1 ft. A tremie seal design is given in Appendix 4.A.

The following parameters are considered to determine the pile layout:

LRFD 10.7.1.2

1. Pile spacing: The depth of commonly used H-piles ranges from 10 to 14 inches. The minimum pile spacing is controlled by the greater of 30 inches or 2.5 times the pile diameter. As a practice, MDOT uses 3 times the pile diameter as the spacing.

   Select a trial section for the piles HP 14X73 $b_f := 14.6in$ $d_{pile} := 13.6in$

   Minimum spacing $Spacing_{min} := 3 \cdot b_f = 43.8 \cdot in$

BDM 7.03.09.A7

2. Edge distance: The typical minimum edge distance for piles is 18 inches.

   Pile edge distance $PileEdgeDist := 18in$

Start the design by assuming a number of pile rows and the number of piles in each row.

Number of pile rows $Pile_{row} := 3$

Number of piles in each row $PilesInEachRow := 6$

Total number of piles $N_{piles} := Pile_{row} \cdot PilesInEachRow = 18$

Pile spacing in the direction parallel to the x-axis $$Spacing_x := \frac{(l_{footing} - 2PileEdgeDist)}{PilesInEachRow - 1} = 57.6 \cdot in$$

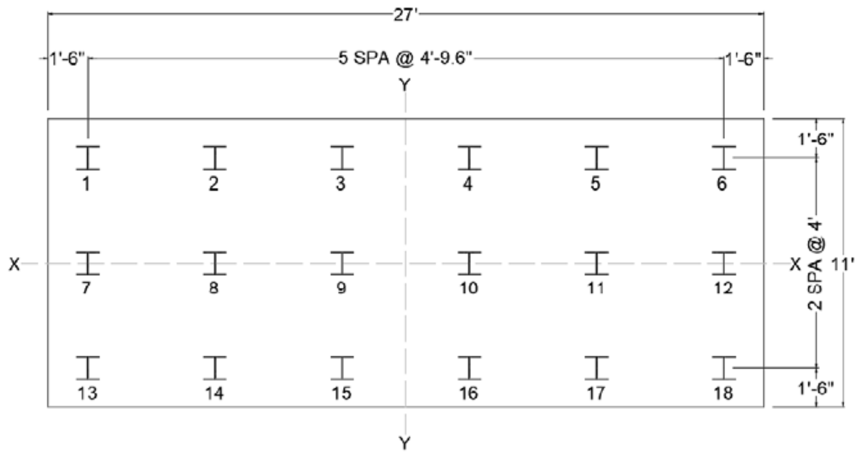
Pile spacing in the direction parallel to the y-axis $$Spacing_y := \frac{w_{footing} - 2PileEdgeDist}{Pile_{row} - 1} = 48 \cdot in$$

Check if the pile spacing in the x- and y-directions is greater than the required minimum spacing for the selected pile section.

$$Check := if\left(Spacing_{min} < Spacing_x, \text{"OK"}, \text{"Increase Spacing"}\right) = \text{"OK"}$$

$$Check := if\left(Spacing_{min} < Spacing_y, \text{"OK"}, \text{"Increase Spacing"}\right) = \text{"OK"}$$

The preliminary pile layout is shown below.



Section modulus of the pile group about x-axis

$$S_{XX} := 14 \cdot \frac{Spacing_y^{\;2}}{Spacing_y} = 56 \cdot ft$$

Section modulus of the pile group about y-axis

$$S_{YY} := 6 \cdot \frac{(0.5 Spacing_x)^2 + (1.5 \cdot Spacing_x)^2 + (2.5 \cdot Spacing_x)^2}{2.5 \cdot Spacing_x} = 100.8 \cdot ft$$

Strength I

$F_{VFtStrI} = 3.707 \times 10^3 \cdot kip$ $\quad M_{XFtStrI} = 1.621 \times 10^3 \cdot kip \cdot ft$ $\quad M_{YFtStrI} = 894.546 \cdot kip \cdot ft$

Maximum pile reaction

$$P_{uMax\_StrI} := \frac{F_{VFtStrI}}{N_{piles}} + \frac{M_{XFtStrI}}{S_{XX}} + \frac{M_{YFtStrI}}{S_{YY}} = 243.748 \cdot kip$$

Strength III

$F_{VFtStrIII} = 2.969 \times 10^3 \cdot kip$ $\quad M_{XFtStrIII} = 946.685 \cdot kip \cdot ft$ $\quad M_{YFtStrIII} = 689.114 \cdot kip \cdot ft$

Maximum pile reaction

$$P_{uMax\_StrIII} := \frac{F_{VFtStrIII}}{N_{piles}} + \frac{M_{XFtStrIII}}{S_{XX}} + \frac{M_{YFtStrIII}}{S_{YY}} = 188.682 \cdot kip$$

Strength V

$F_{VFtStrV} = 3.538 \times 10^3 \cdot kip$ $\quad M_{XFtStrV} = 2.125 \times 10^3 \cdot kip \cdot ft$ $\quad M_{YFtStrV} = 1.445 \times 10^3 \cdot kip \cdot ft$

Maximum pile reaction

$$P_{uMax\_StrV} := \frac{F_{VFtStrV}}{N_{piles}} + \frac{M_{XFtStrV}}{S_{XX}} + \frac{M_{YFtStrV}}{S_{YY}} = 248.838 \cdot kip$$

Service I

Pile reactions under the Service I limit state are needed in the flexural design of the footing.

$F_{VFtSerI} = 2.754 \times 10^3 \cdot kip$ $M_{XFtSerI} = 1.649 \times 10^3 \cdot kip \cdot ft$ $M_{YFtSerI} = 1.267 \times 10^3 \cdot kip \cdot ft$

Maximum pile reaction $$P_{uMax\_SerI} := \frac{F_{VFtSerI}}{N_{piles}} + \frac{M_{XFtSerI}}{S_{XX}} + \frac{M_{YFtSerI}}{S_{YY}} = 194.987 \cdot kip$$

The controlling maximum pile reaction $$P_{uMax} := \max\left(P_{uMax\_StrI}, P_{uMax\_StrIII}, P_{uMax\_StrV}\right) = 248.838 \cdot kip$$

Nominal pile resistance of commonly used steel H-piles **BDM 7.03.09.B.1**

HP 10X42	275 kips
HP 10X57	350 kips
HP 12X53	350 kips
HP 12X74	500 kips
HP 12X84	600 kips
HP 14X73	500 kips
HP 14X89	600 kips

The resistance factor for driven piles assuming that the nominal pile driving resistance is verified using the FHWA-modified Gates Dynamic Formula. **BDM 7.03.09.B2**

$$\varphi_{dyn} := 0.5$$

Required minimum nominal pile resistance $$R_{nReq} := \frac{P_{uMax}}{\varphi_{dyn}} = 497.676 \cdot kip$$

Nominal pile resistance recommended by the Geotechnical Services Section for the selected pile section $R_n := 500kip$ **BDM 7.03.09.B**

Resistance factor for driven piles $\varphi_{dyn} := 0.5$ **BDM 7.03.09.B2**

Factored nominal pile resistance $R_R := \varphi_{dyn} \cdot R_n = 250 \cdot kip$

Check if the factored nominal pile resistance > the maximum pile reaction $Check := if\left(R_R > P_{uMax}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Lateral Force Resistance of Piles

The lateral forces acting on the pier are assumed to be equally shared by the piles. Step 8.5 presents the lateral force calculations.

Note: Per MDOT practice, the typical lateral force resistance of a vertical pile is 12 kips. A pile bending (p-y) analysis may be performed by incorporating a soil-pile interaction to determine a more accurate lateral force resistance. Consult the Geotechnical Services Section for more information.

Lateral load resistance of a pile (from the Geotechnical Services Section) $P_{latProvided} := 12kip$

Strength I

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrI} = 0$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrI} = 56.875 \cdot kip$$

Required pile lateral force resistance parallel to the minor axis of the section (demand)

$$P_{ReqLatMinor\_StrI} := \frac{V_{TFtStrI}}{N_{piles}} = 0 \cdot kip$$

Check if the lateral force resistance > the demand

$$Check := if\left(P_{latProvided} > P_{ReqLatMinor\_StrI}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Required pile lateral force resistance parallel to the major axis of the section (demand)

$$P_{ReqLatMajor\_StrI} := \frac{V_{LFtStrI}}{N_{piles}} = 3.16 \cdot kip$$

Check if the pile lateral force resistance > the demand

$$Check := if\left(P_{latProvided} > P_{ReqLatMajor\_StrI}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Strength III

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrIII} = 25.484 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrIII} = 42.066 \cdot kip$$

Required pile lateral force resistance parallel to the minor axis of the section (demand)

$$P_{ReqLatMinor\_StrIII} := \frac{V_{TFtStrIII}}{N_{piles}} = 1.416 \cdot kip$$

Check if the lateral force resistance > the demand

$$Check := if\left(P_{latProvided} > P_{ReqLatMinor\_StrIII}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Required lateral force resistance parallel to the major axis of the section (demand)

$$P_{ReqLatMajor\_StrIII} := \frac{V_{LFtStrIII}}{N_{piles}} = 2.337 \cdot kip$$

Check if the lateral force resistance > the demand

$$Check := if\left(P_{latProvided} > P_{ReqLatMajor\_StrIII}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Strength V

Factored shear force parallel to the transverse axis of the bridge

$$V_{TFtStrV} = 27.394 \cdot kip$$

Factored shear force parallel to the longitudinal axis of the bridge

$$V_{LFtStrV} = 80.587 \cdot kip$$

Required lateral force resistance parallel to the minor axis of the section (demand)

$$P_{ReqLatMinor\_StrV} := \frac{V_{TFtStrV}}{N_{piles}} = 1.522 \cdot kip$$

Check if the lateral force resistance > the demand

$$Check := if\left(P_{latProvided} > P_{ReqLatMinor\_StrV}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Required lateral force resistance parallel to the major axis of the section (demand)

$$P_{ReqLatMajor\_StrV} := \frac{V_{LFtStrV}}{N_{piles}} = 4.477 \cdot kip$$

Check if the lateral force resistance > the demand

$$Check := if\left(P_{latProvided} > P_{ReqLatMajor\_StrV}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Step 8.7 Structural Design of the Footing

Description

This step presents the structural design of the pier footing.

Page Contents

Design of Transverse Details

The Strut-and-Tie Method (STM) is used for the design of deep footing and pile caps when the distance between the centers of applied load and the supporting reactions is less than two times the member depth.

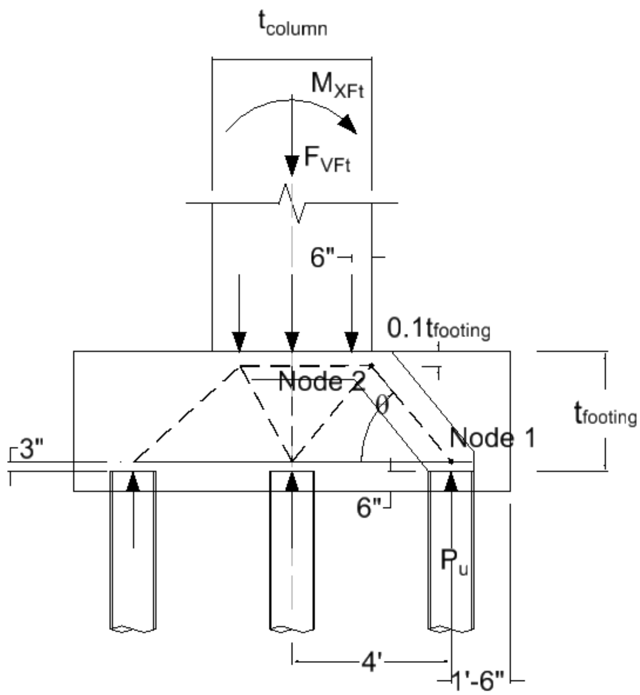
LRFD 5.8.2.1

Footing thickness $t_{footing} = 3.5\,ft$

Distance between the column vertical reaction and a row of piles $S_{center} := \left(\frac{w_{footing}}{2} - PileEdgeDist\right) = 4\,ft$

Check if the STM is a suitable model for this footing $Check := if\left(S_{center} < 2t_{footing}, \text{"Use STM"}, \text{"No"}\right) = \text{"Use STM"}$

The following figure shows the Strut-and-Tie Model selected for the design of the footing in the transverse direction.



The centroid of the top chord is assumed to be located at a distance of 1/10[th] the footing thickness below the top of the footing. There are several options that the designer may consider when placing the top chord. Please refer to the **FHWA-NHI-17-071** *Strut-and-Tie Model (STM) for Concrete Structures* for additional details. Also, Step 7.6 in this example provides more details on this topic.

The tension tie is located at the centroid of the reinforcement that carries the tensile force at the bottom of the footing. The tensile reinforcement is located at 3 in. above the top of the piles.

LRFD C5.8.2.2

Distance from the top of pile to the center of the transverse reinforcing steel bar $d_R := 3in$

Select a trial bar size $bar := 9$

Nominal diameter of a reinforcing steel bar $d_{bx} := Dia(bar) = 1.128 \cdot in$

Cross-section area of a reinforcing steel bar on the flexural tension side $A_{bar} := Area(bar) = 1 \cdot in^2$

Note: As shown in the following calculations, the footing design is based on the maximum pile reaction. Based on the direction of loads considered in this example, the pile at Node 1 is subjected to the maximum vertical force. Therefore, the analysis and design is performed considering the forces in (a) the strut between nodes 1 and 2 and (b) the tie connected to Node 1.

Projected horizontal length of the strut

$$l_a := \frac{(w_{footing} - t_{column})}{2} - PileEdgeDist + 6.in = 2.5\,ft$$

Projected vertical length of the strut

$$h_a := t_{footing} - Pile\_embd - 3in - 0.1 \cdot t_{footing} = 2.4\,ft$$

Angle between the strut and tension tie

$$\theta := atan\left(\frac{h_a}{l_a}\right) = 43.831 \cdot °$$

Tension Tie Reinforcement Design

The first step is to calculate the average pile reaction in a row under strength and service limit states.

Average reaction of a pile in a row, Strength I

$$P_{RowAvg\_StrI} := \frac{F_{VFtStrI}}{N_{piles}} + \frac{M_{XFtStrI}}{S_{XX}} = 234.874 \cdot kip$$

Average reaction of a pile in a row, Strength III

$$P_{RowAvg\_StrIII} := \frac{F_{VFtStrIII}}{N_{piles}} + \frac{M_{XFtStrIII}}{S_{XX}} = 181.845 \cdot kip$$

Average reaction of a pile in a row, Strength V

$$P_{RowAvg\_StrV} := \frac{F_{VFtStrV}}{N_{piles}} + \frac{M_{XFtStrV}}{S_{XX}} = 234.499 \cdot kip$$

Controlling average reaction of the piles in a row under strength limit states

$$P_{RowAvg\_Str} := max(P_{RowAvg\_StrI}, P_{RowAvg\_StrIII}, P_{RowAvg\_StrV}) = 234.874 \cdot kip$$

Since only Service Limit State I is considered, the controlling average reaction of the piles in a row under Service Limit States

$$P_{RowAvg\_SerI} := \frac{F_{VFtSerI}}{N_{piles}} + \frac{M_{XFtSerI}}{S_{XX}} = 182.423 \cdot kip$$

Tension force in the tension tie on a per-foot basis

$$T_h := \frac{P_{RowAvg\_StrI}}{Spacing_x} \cdot \frac{l_a}{h_a} = 50.971 \cdot \frac{kip}{ft}$$

Resistance factor for tension members

$$\phi_{tension} := 0.9$$

LRFD 5.5.4.2

Required reinforcing steel area on a per-foot basis

$$A_{s\_req} := \frac{T_h}{\phi_{tension} \cdot f_y} = 0.944 \cdot \frac{in^2}{ft}$$

LRFD Eq. 5.8.2.4-1

The spacing of the main reinforcing steel bars in walls and slabs shall not be greater than the lesser of 1.5 times the thickness of the member or 18 in. **LRFD 5.10.3.2**

The spacing of shrinkage and temperature reinforcement shall not exceed the following: **LRFD 5.10.6**

12 in. for walls and footings greater than 18 in.

For all other situations, 3 times the component thickness but not less than 18 in.

Note: MDOT limits reinforcement spacing to a maximum of 18 in. **BDG 5.22.01**

Select a spacing for reinforcing steel bars

$$s_{bar} := 12 \cdot in$$

Area of tension steel provided on a per-foot basis $A_{sProvided\_x} := \frac{A_{bar}}{s_{bar}} = 1 \cdot \frac{in^2}{ft}$

Check the adequacy of tension tie reinforcement

$$Check := if\left(A_{sProvided\_x} > A_{s\_req}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. Checking for crack control assures that the actual stress in the reinforcement does not exceed the service limit state stress.

The spacing requirement for the mild steel reinforcement in the layer closer to the tension face $s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$ **LRFD Eq. 5.6.7-1**

Exposure factor for the Class 1 exposure condition $\gamma_e := 1.00$

For large concrete covers, use a 2 in. clear cover. **LRFD C5.6.7**

Distance from extreme tension fiber to the center of the closest flexural reinforcement $d_c := 2in + \frac{1}{2}d_{bx} = 2.564 \cdot in$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face $\beta_s := 1 + \frac{d_c}{0.7\left(t_{footing} - d_c\right)} = 1.093$

Next, calculate the tensile stress in the reinforcement at the service limit state, f_{ss}.

Tensile force in the reinforcing steel due to the service limit state moment $T_{h\_SerI} := \frac{P_{RowAvg\_SerI}}{Spacing_x} \cdot \frac{l_a}{h_a} = 39.588 \cdot \frac{kip}{ft}$

Stress in the reinforcing steel due to the service limit state moment $f_{ss1} := \frac{T_{h\_SerI}}{\phi_{tension} \cdot A_{sProvided\_x}} = 43.987 \cdot ksi$

f_{ss} (not to exceed $0.6f_y$) $f_{ss} := \min\left(f_{ss1}, 0.6f_y\right) = 36 \cdot ksi$

Required reinforcing steel bar spacing $s_{barRequired} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 12.664 \cdot in$

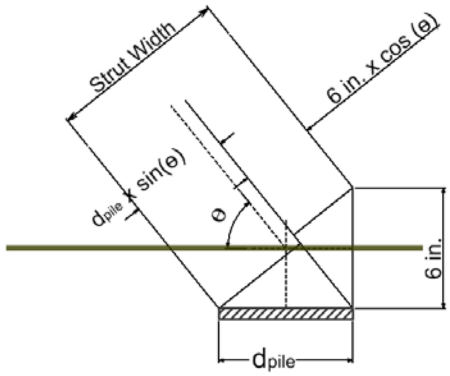
Check if the spacing provided < the required spacing $Check := if\left(s_{bar} < s_{barRequired}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Diagonal Strut Check

The compression force in the diagonal strut is calculated using static equilibrium.

$$P_{uStrut} := \frac{P_{RowAvg\_StrI}}{\sin(\theta)} = 339.152 \cdot kip$$

Depth of the selected pile section $d_{pile} = 13.6 \cdot in$



Node 1 is a CCT node at which a tie intersects only from one direction. The surface at which the diagonal strut meets the node is called the strut-to-node interface.

Modification factor to account for confinement $m := 1$ Conservatively taken as 1.0

Resistance factor for the strut $\phi_{strut} := 0.7$ **LRFD 5.5.4.2**

Concrete efficiency factor, assuming crack control reinforcement being present $v_{CCT} := 0.7$ **LRFD Table 5.8.2.5.3a-1**

Width of the strut $w_{strut} := d_{pile} \cdot \sin(\theta) + 6in \cdot \cos(\theta) = 13.747 \cdot in$

Compressive stress at the face of the node where the strut meets the node $f_{c\_strut} := \dfrac{P_{uStrut}}{\phi_{strut} \cdot w_{strut} \cdot Spacing_y} = 0.734 \cdot ksi$

Limiting compressive stress at the face of the node $f_{cu} := m \cdot v_{CCT} \cdot f_c = 2.1 \cdot ksi$ **LRFD Eq. 5.8.2.5.3a-1**

Check the adequacy of the strut $Check := if\left(f_{c\_strut} < f_{cu}, "OK", "Not OK"\right) = "OK"$

Shrinkage and Temperature Reinforcement Requirement

The following calculations check the adequacy of the flexural reinforcing steel to control shrinkage and temperature stresses in the footing. **LRFD 5.10.6**

Required minimum area of shrinkage and temperature reinforcement

$$A_{shrink.temp} := \min\left[\begin{bmatrix} \left(0.60 \dfrac{in^2}{ft}\right) \\ \max\left[\begin{bmatrix} \left(0.11 \dfrac{in^2}{ft}\right) \\ \left[\dfrac{1.3 \cdot w_{footing} \cdot t_{footing} \cdot \dfrac{kip}{in \cdot ft}}{2\left(w_{footing} + t_{footing}\right) \cdot f_y}\right] \end{bmatrix}\right] \end{bmatrix}\right] = 0.345 \cdot \frac{in^2}{ft}$$

Check if the provided area of steel > the required area of shrinkage and temperature steel $Check := if\left(A_{sProvided\_x} > A_{shrink.temp}, "OK", "Not OK"\right) = "OK"$

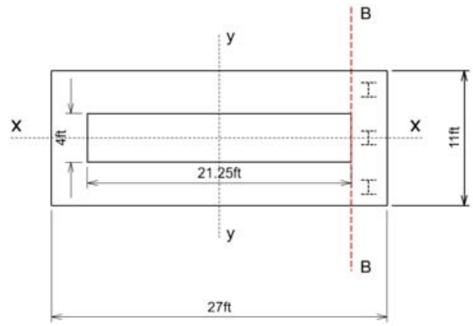
Therefore, the STM design requires the use of No. 9 bars at 12.0 in. spacing ($A_s = 1.0$ in.^{2}/ft) as the transverse reinforcement at the bottom of the footing.

Design of Longitudinal Details

The reinforcement design in the longitudinal direction uses the traditional section method.

Flexural Design

For flexural design of the reinforcement along the longitudinal direction of the footing, the critical section is located at section B-B (at the face of the column).



Moment at the Face of the Column (Section B-B)

Since the footing is designed using a 1-ft wide strip, the pile forces on a per-foot basis are calculated.

Note: As per the MDOT practice, the maximum reactions of the piles at the end column are conservatively assumed to be equal.

Maximum pile reaction under strength limit states $P_{uMax} = 248.838 \cdot kip$ From Step 8.6

Maximum pile reaction under service limit states $P_{uMax\_SerI} = 194.987 \cdot kip$ From Step 8.6

Applied factored load per-foot in the y-direction, strength limit state $R_{y\_Str} := \frac{3 \cdot P_{uMax}}{w_{footing}} = 67.865 \cdot \frac{kip}{ft}$

Applied factored load per-foot in the y-direction, service limit state $R_{y\_SerI} := \frac{3 \cdot P_{uMax\_SerI}}{w_{footing}} = 53.178 \cdot \frac{kip}{ft}$

Distance from the center of the piles in the end column to section B-B $Arm_y := \frac{(l_{footing} - w_{column})}{2} - PileEdgeDist = 1.375\ ft$

Moment about the y-axis on a per-foot basis at the critical section, strength limit state $M_{uy} := R_{y\_Str} \cdot Arm_y = 93.314 \cdot \frac{kip \cdot ft}{ft}$

Moment about the y-axis on a per-foot basis at the critical section, service limit state $M_{uy\_SerI} := R_{y\_SerI} \cdot Arm_y = 73.12 \cdot \frac{kip \cdot ft}{ft}$

Flexural Resistance

The design procedure consists of calculating the reinforcing steel area required to satisfy the moment demand and checking the selected steel area against the requirements and limitations for developing an adequate moment capacity, controlling crack width, and managing shrinkage and temperature stresses. **LRFD 5.6.3.2**

Select a trial bar size $bar := 8$

Nominal diameter of a reinforcing steel bar	$d_{by} := Dia(bar) = 1\cdot in$	
Cross-section area of a bar on the flexural tension side	$A_{bar} := Area(bar) = 0.79\cdot in^2$	

The spacing of the main reinforcing steel bars in walls and slabs shall not be greater than the lesser of 1.5 times the thickness of the member or 18 in. **LRFD 5.10.3.2**

The spacing of shrinkage and temperature reinforcement shall not exceed the following: **LRFD 5.10.6**

12 in. for walls and footings greater than 18 in.

For all other situations, 3 times the component thickness but not less than 18 in.

Note: MDOT limits reinforcement spacing to a maximum of 18 in. **BDG 5.22.01**

Footing thickness $t_{footing} = 3.5\,ft$

Select a spacing for the reinforcing steel bars $s_{bar} := 10in$

Provided area of tension steel in a 1-ft wide section $A_{sProvided\_y} := \frac{A_{bar}\cdot 12in}{s_{bar}} = 0.948\cdot in^2$

The reinforcing bars in footing length direction are placed on top of the reinforcing bars in the footing width direction. Therefore,

Effective depth $d_{ey} := t_{footing} - Pile\_embd - 3in - \frac{d_{bx}}{2} - \frac{d_{by}}{2} = 31.936\cdot in$

Resistance factor for flexure $\phi_f := 0.9$ **LRFD 5.5.4.2**

Select a 1-ft wide strip for the design.

Width of the compression face of the section $b := 12in$

Stress block factor $\beta_1 = 0.85$

Solve the following equation of A_s to calculate the required area of steel to satisfy the moment demand. Use an assumed initial A_s value to solve the equation.

Initial assumption $A_s := 1in^2$

Given $$M_{uy}\cdot ft = \phi_f\cdot A_s\cdot f_y\cdot\left[d_{ey} - \frac{1}{2}\cdot\left(\frac{A_s\cdot f_y}{0.85\cdot f_c\cdot b}\right)\right]$$

Required area of steel $A_{sRequired\_y} := Find(A_s) = 0.663\cdot in^2$

Check if $A_{sProvided} > A_{sRequired}$ $Check := if(A_{sProvided\_y} > A_{sRequired\_y}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Moment capacity of the section with the provided steel

$$M_{Provided} := \phi_f\cdot A_{sProvided\_y}\cdot f_y\cdot\frac{\left[d_{ey} - \frac{1}{2}\cdot\left(\frac{A_{sProvided\_y}\cdot f_y}{0.85\cdot f_c\cdot b}\right)\right]}{ft}$$

$$M_{Provided} = 132.274\cdot\frac{kip\cdot ft}{ft}$$

Distance from the extreme compression fiber to the neutral axis $c := \frac{A_{sProvided\_y}\cdot f_y}{0.85\cdot f_c\cdot\beta_1\cdot b} = 2.19\cdot in$

Check the validity of assumption, $f_s = f_y$ $\qquad$ $Check\_f_s := \text{if}\left(\frac{c}{d_{ey}} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Limits for Reinforcement

LRFD 5.6.3.3

The tensile reinforcement provided must be adequate to develop a factored flexural resistance at least equal to the lesser of the cracking moment or 1.33 times the factored moment from the applicable strength limit state load combinations.

Flexural cracking variability factor $\qquad$ $\gamma_1 := 1.6$ $\quad$ For concrete structures that are not precast segmental

Ratio of specified minimum yield strength to ultimate tensile strength of the nonprestressed reinforcement $\qquad$ $\gamma_3 := 0.67$ $\quad$ For ASTM A615 Grade 60 reinforcement

Section modulus $\qquad$ $S_c := \frac{1}{6} \cdot b \cdot t_{footing}^{\;2} = 3.528 \times 10^3 \cdot in^3$

Cracking moment $\qquad$ $M_{cr} := \frac{\gamma_3 \cdot \gamma_1 \cdot f_r \cdot S_c}{ft} = 131.013 \cdot \frac{kip \cdot ft}{ft}$

1.33 times the factored moment demand $\qquad$ $1.33 \cdot M_{uy} = 124.108 \cdot \frac{kip \cdot ft}{ft}$

The factored flexural resistance required to satisfy the minimum reinforcement requirement $\qquad$ $M_{req} := \min\left(1.33 M_{uy}, M_{cr}\right) = 124.108 \cdot \frac{kip \cdot ft}{ft}$

Check the adequacy of the section capacity $\qquad$ $Check := \text{if}\left(M_{Provided} > M_{req}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Control of Cracking by Distribution of Reinforcement

LRFD 5.6.7

Limiting the width of expected cracks under service conditions extends the service life. The width of potential cracks can be minimized through proper placement of the reinforcement. Checking for crack control assures that the actual stress in the reinforcement does not exceed the service limit state stress.

The spacing requirement for the mild steel reinforcement in the layer closer to the tension face $\qquad$ $s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$ $\qquad$ **LRFD Eq. 5.6.7-1**

Exposure factor for the Class 1 exposure condition $\qquad$ $\gamma_e := 1.00$

LRFD C5.6.7

For large concrete cover, 2 in. is recommended.

Distance from the extreme tension fiber to the center of the closest flexural reinforcement $\qquad$ $d_c := 2in + \frac{1}{2} d_{by} = 2.5 \cdot in$

Ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer closest to the tension face $\qquad$ $\beta_s := 1 + \frac{d_c}{0.7\left(t_{footing} - d_c\right)} = 1.09$

The calculation of tensile stress in nonprestressed reinforcement at the service limit state, f_{ss}, requires establishing the neutral axis location and the moment demand at the critical section.

The position of the section's neutral axis is determined through an iterative process to calculate the actual stress in the reinforcement. This process starts with an assumed position of the neutral axis.

Assumed distance from the extreme compression fiber to the neutral axis

$x := 5 \cdot in$

Given $\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_c} \cdot A_{sProvided\_y} \cdot (d_{ey} - x)$

Position of the neutral axis

$x_{na} := Find(x) = 5.753 \cdot in$

Tensile force in the reinforcing steel due to the service limit state moment

$$T_s := \frac{M_{uy\_SerI}}{d_{ey} - \frac{x_{na}}{3}} \cdot ft = 29.2 \cdot kip$$

Stress in the reinforcing steel due to the service limit state moment

$$f_{ss1} := \frac{T_s}{A_{sProvided\_y}} = 30.833 \cdot ksi$$

f_{ss} (not to exceed $0.6f_y$)

$f_{ss} := \min(f_{ss1}, 0.6f_y) = 30.833 \cdot ksi$

Required reinforcement bar spacing

$$s_{barRequired} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{ss}} - 2 \cdot d_c = 15.82 \cdot in$$

Check if the spacing provided < the required spacing

$Check := if(s_{bar} < s_{barRequired}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Shrinkage and Temperature Reinforcement Requirement

LRFD 5.10.6

The required minimum area of shrinkage and temperature reinforcement

$A_{shrink.temp} = 0.345 \cdot \frac{in^2}{ft}$ From transverse reinforcement design

Check if the provided area of steel > the required area of shrinkage and temperature steel

$$Check := if\left(\frac{A_{sProvided\_y}}{ft} > A_{shrink.temp}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

Therefore, the flexural design requires the use of No. 8 bars at 10.0 in. spacing ($A_s = 0.948$ in.^{2}/ft) as the transverse reinforcement at the bottom of the footing.

Design for Shear

One-Way Shear

Since the STM was used for the design of transverse direction details, the one-way shear design is not required in the transverse direction. The following calculations present the one-way shear design in the longitudinal direction.

In the longitudinal direction of the footing, the factored shear force at the critical section is computed by calculating the total pile reaction force acting on the footing base that is outside of the critical section.

Depth of equivalent rectangular stress block

$$a := \frac{A_{sProvided\_y} \cdot f_y}{0.85 \cdot f_c \cdot b} = 1.859 \cdot in$$

Effective shear depth

$$d_{vy} := \max\left(d_{ey} - \frac{a}{2}, 0.9 \cdot d_{ey}, 0.72 \cdot t_{footing}\right) = 31.007 \cdot in$$

LRFD 5.7.2.8

Since the piles are located inside the critical sections, there is no need to check one-way shear in the footing length directions.

Two-Way Shear

Two-way shear (punching shear) in the footing is checked at critical perimeters around the column and a pile.

Critical Perimeter around the Pier Column

The critical perimeter around the pier column, b_o, is located at a minimum of $0.5d_v$ from the perimeter of the column. **LRFD 5.12.8.6.3**

Note: An average effective shear depth, d_v, should be used since the two-way shear area includes both x- and y- directions of the footing.

Effective depth in the transverse direction of the footing

$$d_{ex} := t_{footing} - Pile\_embd - 3in = 33 \cdot in$$

Depth of equivalent rectangular stress block in the transverse direction

$$a_x := \frac{A_{sProvided\_x} \cdot f_y}{0.85 \cdot f_c} = 1.961 \cdot in$$

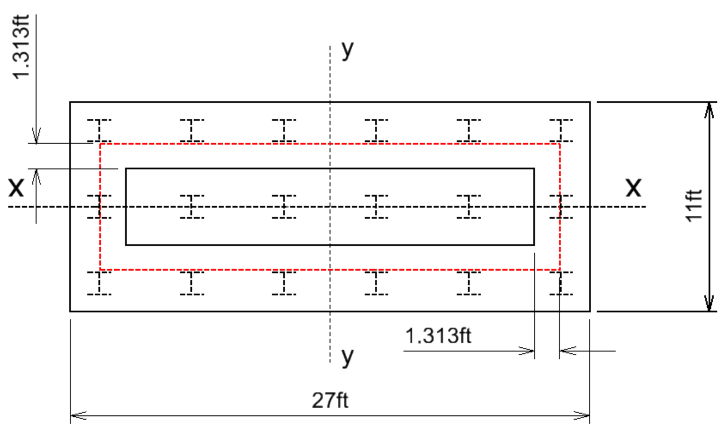
Effective shear depth in the transverse direction

$$d_{vx} := \max\left(d_{ex} - \frac{a_x}{2}, 0.9 \cdot d_{ex}, 0.72 \cdot t_{footing}\right) = 32.02 \cdot in$$

Average effective shear depth

$$d_{v\_avg} := \frac{\left(d_{vx} + d_{vy}\right)}{2} = 2.626\,ft$$

As shown in the following figure, the 1st and 3rd rows of piles are located outside the critical perimeter. The two piles in the end of the 2nd row are partially located outside the critical perimeter. They are conservatively considered to be outside the critical perimeter.



Critical perimeter

$$b_0 := 2 \cdot \left(w_{column} + d_{v\_avg}\right) + 2 \cdot \left(t_{column} + d_{v\_avg}\right) = 61.004\,ft$$

Ratio of long to short side of the critical perimeter

$$\beta_c := \frac{w_{column}}{t_{column}} = 5.313$$

Nominal shear resistance

$$V_{n1\_2way} := \left(0.063 + \frac{0.126}{\beta_c}\right) \cdot \sqrt{f_c \cdot ksi} \cdot b_0 \cdot d_{v\_avg} = 3.465 \times 10^3 \cdot kip$$

$$V_{n2\_2way} := 0.126 \cdot \left(\sqrt{f_c \cdot ksi} \cdot b_0 \cdot d_{v\_avg}\right) = 5.035 \times 10^3 \cdot kip$$

LRFD Eq. 5.12.8.6.3-1

$$V_{n\_2way} := \min\left(V_{n1\_2way}, V_{n2\_2way}\right) = 3.465 \times 10^3 \cdot kip$$

Factored shear resistance (Capacity) $V_{r\_2way} := \phi_v \cdot V_{n\_2way} = 3.118 \times 10^3 \cdot kip$

The average pile reaction of the piles outside the critical perimeter (Strength I limit states governs.)

$$P_{avg} := \frac{F_{VFtStrI}}{N_{piles}} = 205.928 \cdot kip$$

Number of piles outside the critical perimeter $N_{piles\_outside} := 14$

Note: For piles located partially outside the critical perimeter, conservatively assume that they are completely outside the critical perimeter.

Resultant force acting on the area outside of the critical perimeter (Demand) $V_{u\_2way} := N_{piles\_outside} \cdot P_{avg} = 2.883 \times 10^3 \cdot kip$

Check if the shear resistance > the demand $Check := if\left(V_{r\_2way} > V_{u\_2way}, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

Critical Perimeter around a Pile

The critical perimeter around a pile, b_o, is located at a minimum of $0.5d_v$ from the perimeter of the pile. When portions of the critical perimeter are located off the footing, the critical perimeter is limited by the footing edge.

LRFD 5.12.8.6.3

Flange width and depth of the selected pile section $b_f = 14.6 \cdot in$ $\quad d_{pile} = 13.6 \cdot in$

Check if the critical perimeter is off the footing in y-axis direction

$$OffFooting\_y := if\left(\frac{d_{pile}}{2} + \frac{d_{v\_avg}}{2} > PileEdgeDist, \text{"Yes"}, \text{"No"}\right)$$

$$OffFooting\_y = \text{"Yes"}$$

Check if the critical perimeter is off the footing in x-axis direction

$$OffFooting\_x := if\left(\frac{b_f}{2} + \frac{d_{v\_avg}}{2} > PileEdgeDist, \text{"Yes"}, \text{"No"}\right)$$

$$OffFooting\_x = \text{"Yes"}$$

Side length of the critical perimeter parallel to y-axis

$$b_{0y} := if\left(OffFooting\_y = \text{"Yes"}, \frac{d_{pile}}{2} + \frac{d_{v\_avg}}{2} + PileEdgeDist, d_{pile} + d_{v\_avg}\right)$$

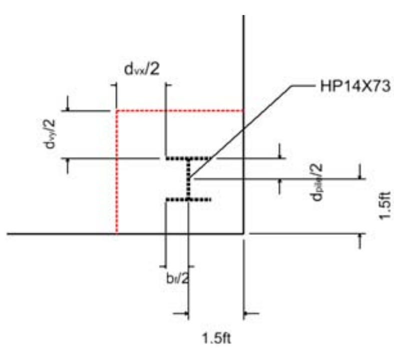
$$b_{0y} = 3.38\ ft$$

Side length of the critical perimeter parallel to x-axis

$$b_{0x} := if\left(OffFooting\_x = \text{"Yes"}, \frac{b_f}{2} + \frac{d_{v\_avg}}{2} + PileEdgeDist, d_{pile} + d_{v\_avg}\right)$$

$$b_{0x} = 3.421\ ft$$

Critical perimeter $b_0 := b_{0x} + b_{0y} = 6.801\ ft$



Ratio of long-to-short sides of the critical perimeter

$$\beta_c := \frac{\max(b_{0x}, b_{0y})}{\min(b_{0x}, b_{0y})} = 1.012$$

Nominal shear resistance

$$V_{n1\_2way} := \left(0.063 + \frac{0.126}{\beta_c}\right)\cdot\sqrt{f_c\cdot ksi}\cdot b_0\cdot d_{v\_avg} = 835.089\cdot kip$$

$$V_{n2\_2way} := 0.126\cdot\left(\sqrt{f_c\cdot ksi}\cdot b_0\cdot d_{v\_avg}\right) = 561.283\cdot kip$$

LRFD Eq. 5.12.8.6.3-1

$$V_{n\_2way} := \min(V_{n1\_2way}, V_{n2\_2way}) = 561.283\cdot kip$$

Factored shear resistance (Capacity) $V_{r\_2way} := \phi_v\cdot V_{n\_2way} = 505.155\cdot kip$

Maximum pile reaction (Demand) $P_{uMax} = 248.838\cdot kip$

Check if the capacity > the demand $Check := if(V_{r\_2way} > P_{uMax}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Development Length of Reinforcement

The flexural reinforcing steel must be developed on each side of the critical section for its full development length. **LRFD 5.10.8.1.2**

Longitudinal Direction of the Footing

Available development length

$$l_{dy\_avail} := \frac{l_{footing} - w_{column}}{2} - Cover_{ft} = 30.5\cdot in$$

From flexural design

Longitudinal reinforcing steel bar size No. 8

Bar spacing 12 in

Assuming that the bars are at high stress, the required bar development length $l_{dy.req} := 28in$ **BDG 7.14.01**

Check if $l_{dy.avail} > l_{dy.req}$ $Check := if(l_{dy\_avail} > l_{dy.req}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Transverse Direction of the Footing

Available development length

$$l_{dx\_avail} := \frac{w_{footing} - t_{column}}{2} - Cover_{ft} = 38\cdot in$$

From flexural design

Longitudinal reinforcing steel bar size No. 8

Bar spacing 8 in

Assuming that the bars are at high stress, the required bar development length $l_{dx.req} := 28in$ **BDG 7.14.01**

Check if $l_{dx.avail} > l_{dx.req}$ $Check := if(l_{dx\_avail} > l_{dx.req}, \text{"OK"}, \text{"Not OK"}) = \text{"OK"}$

Shrinkage and Temperature Reinforcement Requirement

This requirement for the steel at the bottom of the footing was already checked and satisfied.

The reinforcement along the longitudinal and transverse directions of the footing at the top should satisfy the shrinkage and temperature reinforcement requirement. **LRFD 5.10.6**

The spacing of shrinkage and temperature reinforcement shall not exceed the following: **LRFD 5.10.6**

12 in. for walls and footings greater than 18 in.

For all other situations, 3 times the component thickness but not less than 18 in.

Note: MDOT limits reinforcement spacing to a maximum of 18 in. **BDG 5.22.01**

Select a trial bar size $bar := 6$

Nominal diameter of a reinforcing steel bar $d_{bx} := Dia(bar) = 0.75 \cdot in$

Cross-section area of a reinforcing steel bar $A_{bar} := Area(bar) = 0.44 \cdot in^2$

Select a spacing for reinforcing steel bars $s_{barST} := 12 \cdot in$

Provided horizontal reinforcement area $A_{sProvidedST} := \frac{A_{bar}}{s_{barST}} = 0.44 \cdot \frac{in^2}{ft}$

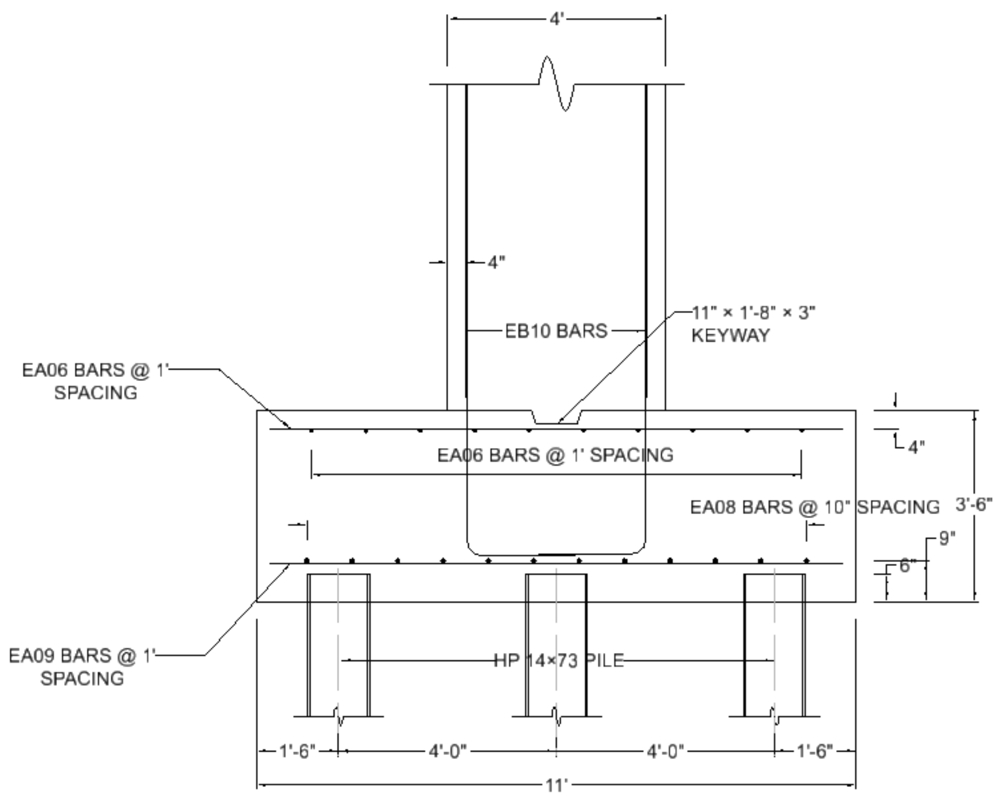
Required shrinkage and temperature steel area (calculated during flexural design) $A_{shrink.temp} = 0.345 \cdot \frac{in^2}{ft}$

Check if the provided area of steel > the required area of shrinkage and temperature steel $Check := if\left(A_{sProvidedST} > A_{shrink.temp}, "OK", "Not OK"\right) = "OK"$

Therefore, the design requires the use of No. 6 bars at 12.0 in. spacing ($A_s = 0.44$ in.^{2}/ft) as the shrinkage and temperature reinforcement at the top of the footing in both longitudinal and transverse directions.

The footing design presented in this step provides the following details:

- No. 9 bars @ 12.0 in. spacing (A_s = 1.0 in.^{2}/ft) as the transverse flexural reinforcement at the bottom of the footing
- No. 8 bars @ 10.0 in. spacing (A_s = 0.948 in.^{2}/ft) as the longitudinal flexural reinforcement at the bottom of the footing
- No. 6 bars @ 12.0 in. spacing (A_s = 0.44 in.^{2}/ft) as the shrinkage and temperature reinforcement at the top of the footing in both longitudinal and transverse directions.



Note: Certain details are not shown in this drawing for clarity of main reinforcement.
Refer to MDOT Bridge Design Guides for additional details.