EVALUATION OF A QUASI-ELASTIC MODULUS OF GRANULAR BASE MATERIAL

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INTRODUCTION

The project for developing equivalency factors of asphalt treated bases includes three major phases of study: first, develop a testing procedure to measure the elastic moduli of base materials; second, theoretically establish the equivalency factors; and the third and final phase, field verification of the findings of the first two phases. This report covers results of the first phase of the study and includes a detailed description of the testing procedure developed for measuring the elastic modulus of a base material within a specific range of stresses. Also included is a summary of the measured quasi-elastic moduli of the various types of granular base materials tested in this study.

In general, the behavior of pavement systems is elastic in the sense that deformations are recoverable; however, they are not necessarily proportional to stress or instantaneous (1). Burmister (2) in 1943, gave the first analysis that took into account the different elastic properties of the various layers. Although the pavement system usually consists of more than three layers, solutions are available for three-layer systems only (3, 4). The basic assumptions are that each layer is an isotropic material whose behavior is completely elastic within the stress range encountered; that the lower layer is infinite in depth; and that the load is applied uniformly over a circular area. The physical behavior of highway materials when subjected to moving loads is so close to being completely elastic that the assumptions are reasonable and valid (5).

At the present time it would appear that the development of elastic theory is more advanced than the property evaluations which must accompany it. To use the solution of a three-layer elastic system, it is essential that the elastic moduli of the layers forming the pavement be developed under field loading conditions.

This study describes an effort to determine a reasonably accurate and practical procedure for evaluating an equivalent elastic modulus of a highway base material, using a conventional triaxial testing apparatus.

Assuming an elastic, homogeneous, and isotropic material, the strain ϵ_1, in the axial or σ_1, direction is

$$\epsilon_1 = \frac{1}{E} [\sigma_1 - \mu (\sigma_2 + \sigma_3)] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where: ϵ_1 is the strain in the direction of σ_1 .

σ_1, σ_2, σ_3; are respectively the major, intermediate, and minor principal stresses.

E: Young's Modulus

μ: Poisson's Ratio

In the standard triaxial compression test $\sigma_2 = \sigma_3$, and assuming $\mu = 0.5$[(1)], Equation (1) becomes:

$$\epsilon_1 = \frac{1}{E} (\sigma_1 - \sigma_3)$$

or

$$E = \frac{(\sigma_1 - \sigma_3)}{\epsilon_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

In this study a Quasi-Elastic Modulus, E*, is defined as follows:

$$E^* = \frac{D}{\epsilon_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where D is the principal stress difference, $(\sigma_1 - \sigma_3)$, and ϵ_1 is the rebound or recoverable strain in the axial or σ_1 direction.

Stresses in Base Layer

To utilize the results of triaxial compression tests for obtaining an elastic modulus, the stresses within the pavement must first be estimated. These stresses should then be duplicated in laboratory tests in order to obtain an equivalent elastic modulus of the base material, corresponding to that range of stresses. Since the modulus is stress-dependent, particularly on lateral stress (1), the lateral stresses due to the applied load and

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(1) In the elastic solution, Poisson's ratio has no effect on the vertical stress in the single-layer case, and only a small effect in the case of a multi-layer system. However, the lateral stress, σ_h, is dependent on Poisson's ratio. The value of μ affects vertical stresses and displacements to a much smaller degree than does the value of E. Theoretically, the upper limit of μ is 0.5, corresponding to an incompressible material. This value appears appropriate for saturated clay loaded under undrained conditions, sands are usually assigned a value in the range 0.3 to 0.4 and well graded gravel and asphaltic concrete values somewhat higher (6). Since available solutions of elastic layered systems assume a value of 0.5 for Poisson's ratio, a similar value of μ is used in this study in order to utilize these solutions.

the weight of the pavement must be determined. To estimate the stresses in a layered pavement. Boussinesq's analysis is not adequate (1) and Burmister's analysis of elastic layered systems should be used. In determining stresses in the pavement using the solution of the three-layer elastic system (3), the values of elastic moduli of the materials forming the layers are required. In order to determine an initial value of the elastic modulus of base layer material, Boussinesq's solution should be used to estimate an initial value of the stresses in the base then, by inducing these stresses in a base material sample in a triaxial test, the results could be used to calculate the modulus value. The initial value of the elastic modulus would then be substituted in the solution of the three-layer elastic system to determine more accurate values of stresses in the base layer.

Assuming that the vertical and lateral stresses, due to the applied load, approximate those of a homogeneous isotropic elastic solid with a uniform surface load on a circular area (Boussinesq's analysis), and that the lateral stress due to the weight of the material above is equal to the earth pressure at rest with $K_o = 0.5$, then according to Boussinesq's solution and using tables prepared by Ahlvin and Ulery (7) the stresses at point O, at mid-height of base layer, in Figure 1 are:

a) Vertical Stress, σ_ν:

$$\sigma_\nu = \sigma_o \ (A + B)$$

where A = 0.2041, B = 0.28935 (from Tables in Ref. 5)

and σ_o is surface contact pressure

$\sigma_\nu = 39.47$ psi

b) Horizontal Stress, σ_h:

1. due to surface loading

$$\sigma_{h_1} = \sigma_o \ [\ 2\mu \ A + C + (1 - 2\mu) \ F\ ]$$

A = 0.20410, C = -0.14468, F = 0.10207 (from Tables in Ref. 5)

assuming $\mu = 0.5$

$\sigma_{h_1} = 4.75$ psi

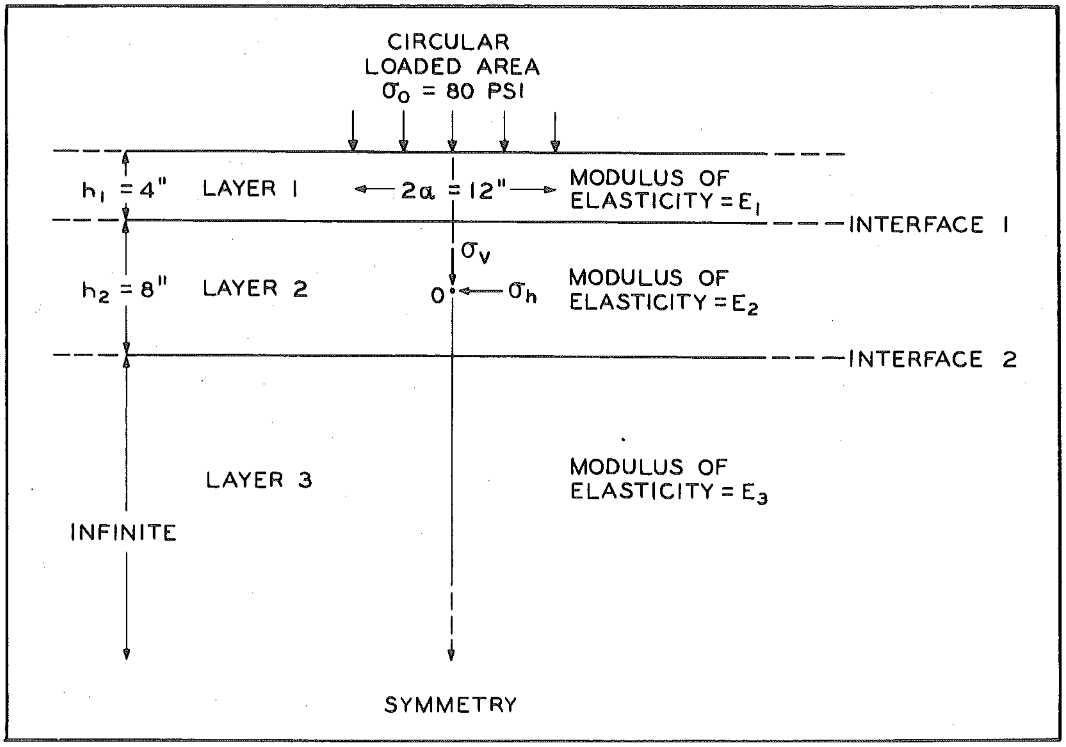

Figure 1. Structure of layered system.

2. due to overburden material

$$\sigma_{h_2} = \nu . h . K_o$$

assuming ν = 144 pcf

$$\sigma_{h_2} = 0.33 \text{ psi}$$

Total horizontal stress, $\sigma_h = \sigma_{h_1} + \sigma_{h_2}$

$$\sigma_h = 5.08 \text{ psi}$$

Several preliminary triaxial tests were conducted on samples of 22A gravel, which were compacted to maximum Proctor density (AASHO: T-180) at optimum moisture content. Triaxial testing was conducted with a confining pressure equal to the calculated lateral stress (σ_3 = 5 psi) and the major principal stress was increased from 1 psi to a level equal to the calculated vertical stress (σ_1 = 40 psi). The test results were used to evaluate an initial value of elastic modulus. Initial values of E ranged between 17,000 and 24,000 psi. By assuming an average value of 20,000 psi as the modulus for base material (22A gravel), and assuming a range of values of the surface layer material and subgrade soils, a computer program for the solution of three-layer elastic systems was used to evaluate the vertical and lateral stresses at mid-height of the base layer and below the center of the loaded area. The results are listed in Table 1.

Table 1 shows that the lateral stresses due to surface load ranged between 4.02 and 5.91 psi, and the vertical stresses ranged between 17.85 and 30.16 psi. Lateral stress due to overburden material was estimated previously to be 0.33 psi. When the pavement is not loaded, the only stresses in the base are due to the weight of the overburden material. When the pavement is loaded with 80 psi pressure, distributed on a 6-in radius circular area (18 K axle load), the vertical stress at point O in Figure 1 would be up to 30 psi and the lateral stress would be up to 5.9 psi. The developed lateral stress is due to surface load and weight of the material above. In order to duplicate field loading conditions on base material samples used in the triaxial test, the test sample prior to axial loading, should be subjected to a confining pressure equal to lateral earth pressure at rest.

TABLE 1
VERTICAL AND LATERAL STRESSES
AT MID-HEIGHT OF THE BASE LAYER
IN A THREE-LAYER ELASTIC SYSTEM

psi	E_2 = 20,000 psi	
	Lateral Stress ($\sigma\nu$), psi	Vertical Stress (σ_h), psi
E_1 = 200,000 E_3 = 15,000	24.95	4.05
E_1 = 200,000 E_3 = 5,000	17.85	5.38
E_1 = 100,000 E_3 = 15,000	30.16	4.02
E_1 = 100,000 E_3 = 5,000	22.36	5.91
E_1 = 150,000 E_3 = 5,000	20.47	5.66
E_1 = 150,000 E_3 = 15,000	27.20	4.06

The axial load and confining pressure should then be increased, at a rapid rate, up to stress levels equal to the calculated values of vertical and lateral stresses, respectively. For practical purposes and to simplify the procedure, the sample is exposed to a confining pressure of 1 psi (σ_3 =1 psi). When the axial stress (σ_1) is increased to 30 psi, and at the same time the confining pressure (σ_3) is increased to 5 psi, the principal stress difference measured in this test would be 25 psi, when the lateral and vertical stresses are at their maximum values. The loading sequence was repeated for many cycles in order to study the effect of load repetition. This procedure simulates field loading conditions, except for the loading rate, which could not be accomplished without a repetitive loading device capable of producing rapid loading cycles simulating traffic loading.

TESTING PROCEDURE

Aggregate Used

Four gravel samples meeting specification requirements for Michigan's 22A gravel were used in this study. The first three samples were taken from the Holt, Ann Arbor, and Maple Rapids areas, and are designated in this study by the name of their site of origin. The fourth material was prepared in the Laboratory and designated as "straight line gradation" gravel in order to be able to reproduce samples with identical gradation. Figure 2 shows the grain size distribution of the four gravel samples. Maximum dry

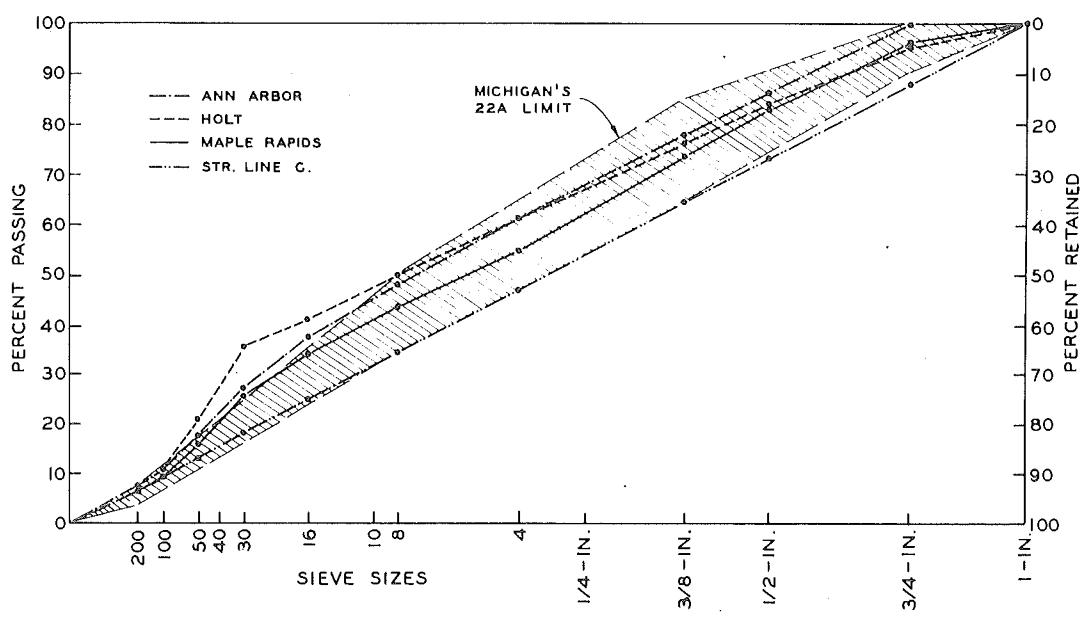

Figure 2. Grain size distribution of the gravel samples.

densities and corresponding optimum moisture contents of the four samples are listed in Table 2.

Sample Preparation

The samples were mixed at a moisture content slightly less than their optimum values and compacted in a 6-in. diameter, 12-in. high split mold. The sample was compacted in five layers with a 10 lb hammer to a density value close to the maximum density of the material. By changing the compactive effort, a range of densities higher and lower than maximum density were obtained. Compacted samples were then placed on the pedestal of the triaxial cell and covered with a rubber membrane. A loading disc was placed on the top of the sample and rubber O-rings were placed around the upper disc and the pedestal to prevent water from leaking into the sample. The top of the triaxial cell was placed in position, the cell filled with water, and a confining pressure of 1 psi applied to the sample. Figure 3 shows a test sample within the triaxial cell.

Equipment

Standard triaxial apparatus was used in this study. Two major changes in the apparatus were made; the proving ring was replaced by a load cell embedded in the base of the triaxial cell to eliminate the effect of piston friction on the load measurement and a linear differential transformer was used to measure the axial deformation of the sample instead of the regular dial gauge. Figure 4 shows the arrangements of the deflection and load measuring devices. Both the load transducer and the linear differential transformer were connected to a two-channel recorder with which the axial load and deformation were recorded continuously throughout the test. The confining pressure in this apparatus was supplied through a self-compensating column of mercury. Figure 4 shows a general view of the testing apparatus.

TABLE 2
MAXIMUM DENSITY AND OPTIMUM MOISTURE CONTENTS OF MATERIALS TESTED (AASHO T-180)

Material (22A)	Maximum Density, pcf	Optimum Moisture Content, percent
straight line	146.0	5.5
Maple Rapids	146.5	5.0
Holt	140.5	6.0
Ann Arbor	143.5	5.5

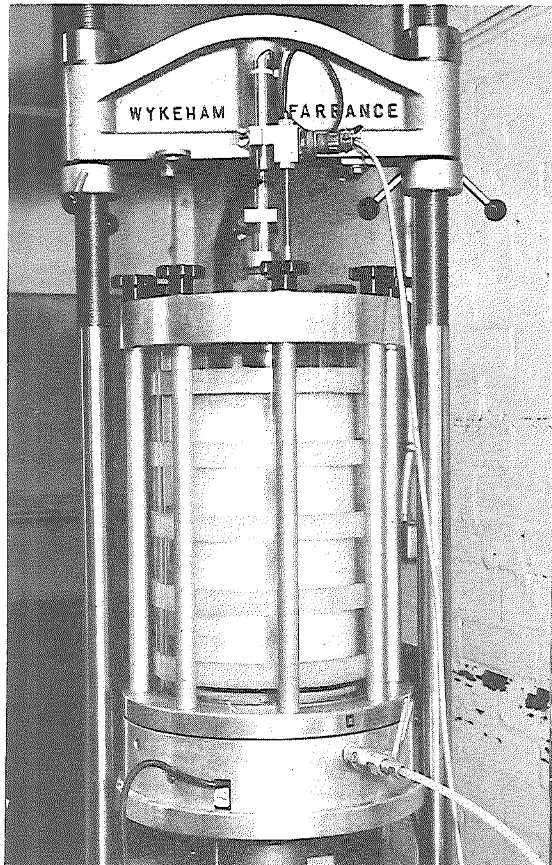

Figure 3. A test sample within the triaxial cell.

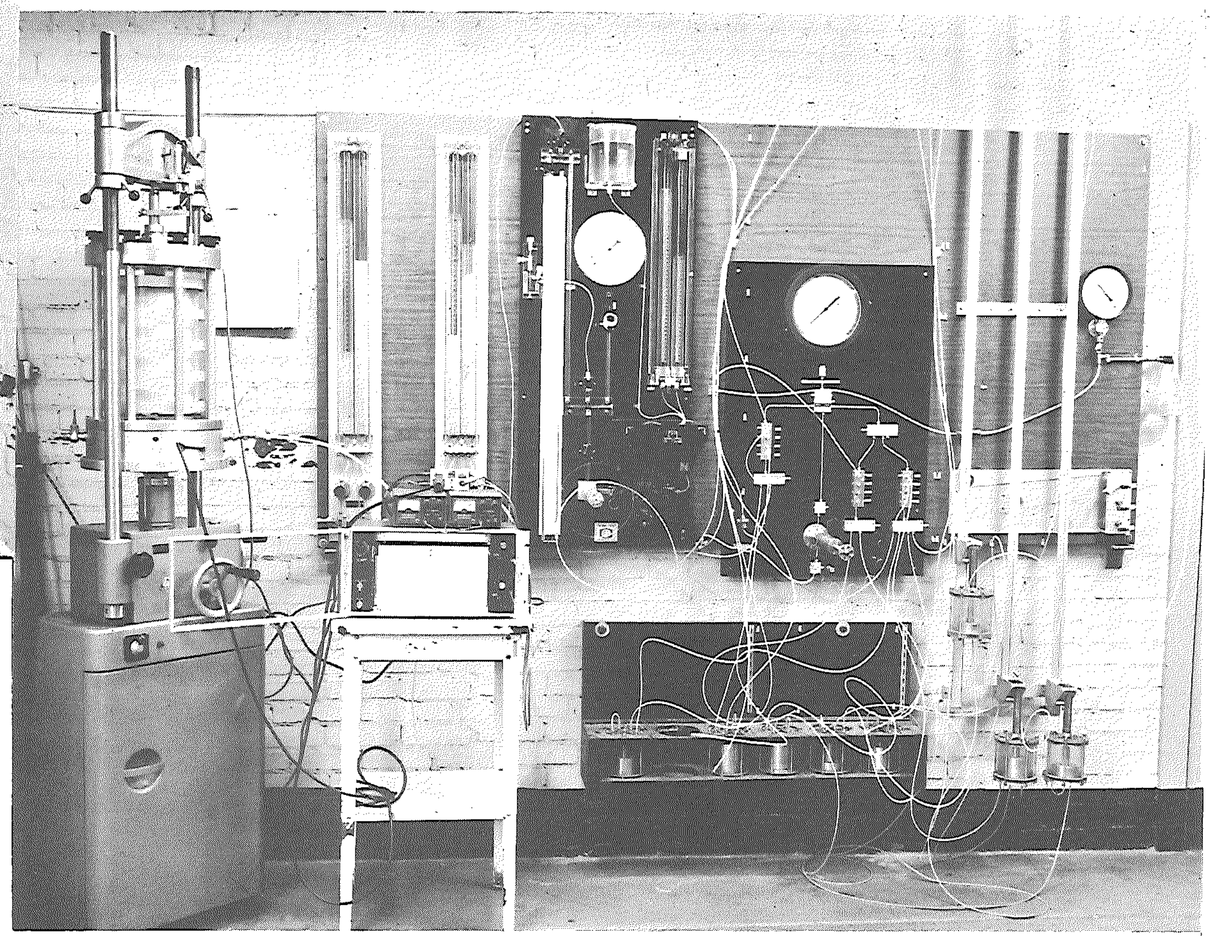

Figure 4. General view of the testing apparatus.

Test Procedure

With the confining pressure at 1 psi, the axial load was applied to the sample at a constant rate of 0.004 in. per minute. The load was increased to a principal stress difference equal to 25 psi. At the beginning of the axial loading process, the confining pressure was increased, linearly with time, to 5 psi. The confining pressure reached 5 psi at or within a short interval prior to the instant the principal stress difference reached 25 psi. The principal stress difference and the axial deformation were recorded simultaneously on the two-channel recorder. The loading cycle was repeated up to 20 times, or until the rebound deflection reached a constant value in consecutive cycles.

EXPERIMENTAL RESULTS

A total of 18 tests have been conducted on the four 22A gravels. Figure 5 shows typical results of a test conducted on 22A gravel of straight line gradation. The axial deformation and the principal stress difference were recorded simultaneously on the two-channel recorder for 20 cycles of loading. A stress-strain relationship was established by calculating the strain from the data supplied by the axial deformation-time curve, and the corresponding stress from the stress-time curve. Figure 6 shows a curve representing the axial strain versus the principal stress difference for the test data shown in Figure 5.

From the stress-strain curve, the quasi-elastic moduli were calculated by dividing the principal stress difference by the rebound axial strain increment for every load cycle (Equation 1). Table 3 shows the calculated quasi-elastic moduli for every loading cycle of the test conducted on 22A straight line gradation gravel.

It was observed, in all tests, that the value of the moduli changed very little after the tenth load cycle; and in most tests, the value did not change appreciably in the last five load cycles. Thus, the average value of the quasi-elastic modulus was assumed to be equal to the average of the calculated modulus of the last five loading cycles. Table 4 lists the values of the quasi-elastic moduli of all 22A samples tested.

From Table 4, it appears that the modulus of a specific material increases with density. Figure 7 shows a plot of the quasi-elastic modulus

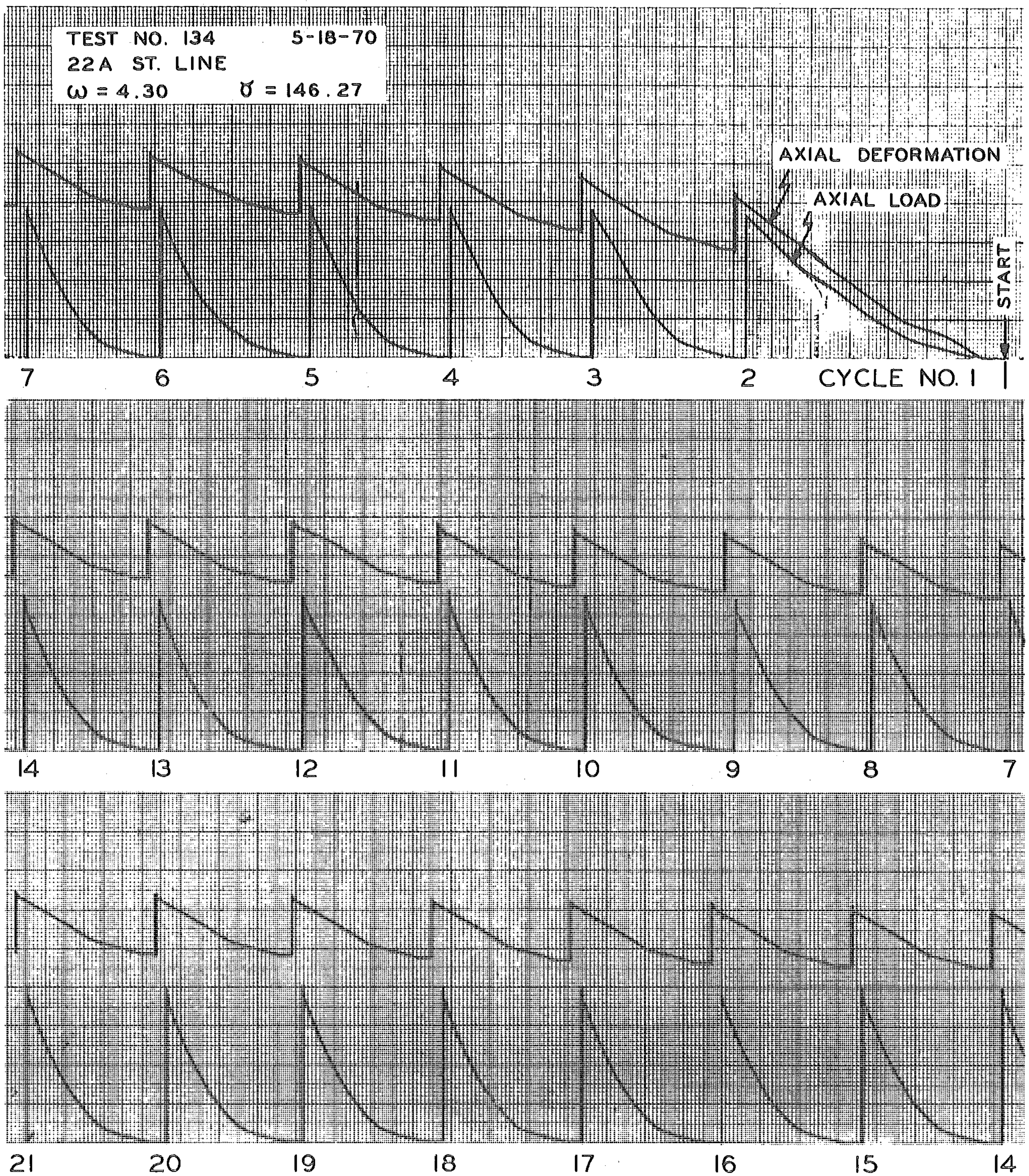

Figure 5. Typical test results for a 22A straight line gravel sample.

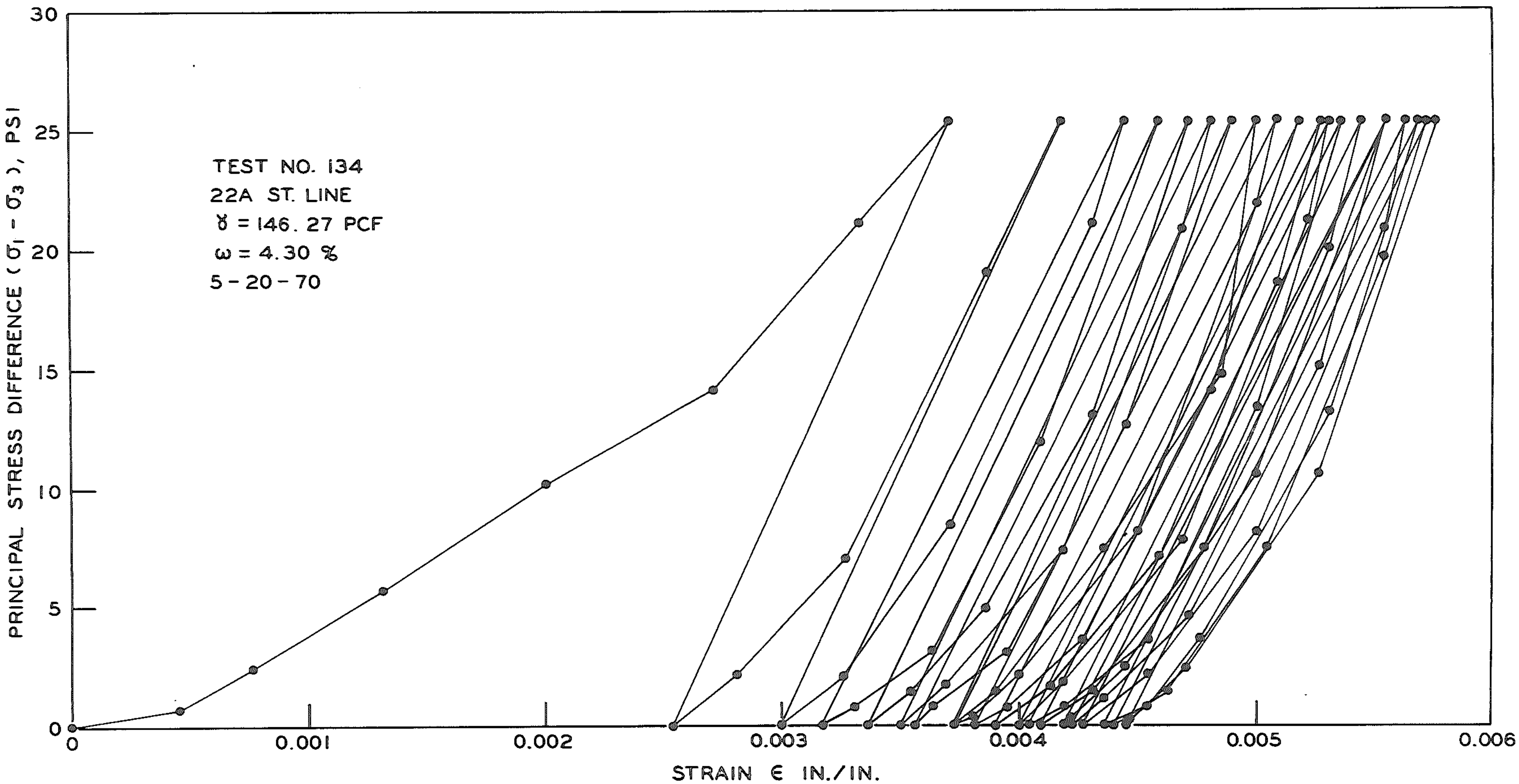

Figure 6. Principal stress difference vs. strain for test number 134.

versus density for tests conducted on the same material (22A - straight line gradation). This plot could be used to obtain the modulus value at optimum density or for any density within the range tested. The modulus-density relationship can be approximated by a straight line for the range of stresses and densities encountered in this testing program. Similar relationships could be developed for other types of 22A gravels.

TABLE 3
QUASI-ELASTIC MODULI, E*
OF STRAIGHT LINE GRADATION GRAVEL

Cycle	Test No. 130 = 143.72 pcf	Test No. 131 = 144.31 pcf	Test No. 132 = 142.99 pcf	Test No. 134 = 146.27 pcf	Test No. 133 = 145.13 pcf
1	19.94	19.22	18.62	20.68	18.62
2	18.60	19.24	18.61	21.47	21.48
3	18.59	19.24	18.61	19.93	19.25
4	18.58	19.23	18.60	20.66	18.61
5	17.41	18.59	18.00	20.66	18.61
6	17.41	17.98	18.60	19.92	17.44
7	17.40	18.59	17.43	21.45	20.67
8	16.87	17.98	17.99	19.91	19.92
9	16.37	18.58	17.43	21.61	19.24
10	16.37	17.97	17.43	19.91	19.92
11	17.39	17.97	18.59	19.91	19.24
12	16.86	17.97	18.58	19.91	20.66
13	16.36	18.57	18.58	19.91	18.59
14	16.85	19.20	18.57	19.91	17.99
15	16.35	17.40	18.57	19.91	17.43
16	17.37	17.40	18.57	19.91	17.99
17	17.37	17.96	18.57	19.91	18.59
18	16.35	17.96	19.21	19.90	18.58
19	16.34	18.55	17.97	19.90	17.98
20	---	---	---	---	18.58
Average last five	16.76	17.85	18.58	19.91	18.34

TABLE 4
QUASI-ELASTIC MODULI
OF ALL TESTED BASE MATERIALS

Test No.	Base Material (22A)	Dry Density, pcf	Quasi-Elastic Modulus, E*, psi
116	Ann Arbor	140.4	16.77×10^3
117	"	142.4	16.97×10^3
118	"	142.75	17.40×10^3
119	"	143.46	17.06×10^3
120	"	146.15	19.90×10^3
122	Holt	139.96	17.95×10^3
123	"	141.63	19.48×10^3
124	"	142.11	22.90×10^3
125	Maple Rapids	143.08	19.1×10^3
126	"	143.53	16.99×10^3
127	"	143.76	17.65×10^3
128	"	144.79	18.80×10^3
129	"	145.70	17.86×10^3
130	Str. Line Grad.	143.72	16.76×10^3
131	"	144.31	17.85×10^3
132	"	142.99	18.58×10^3
133	"	145.13	18.34×10^3
134	"	146.27	19.91×10^3

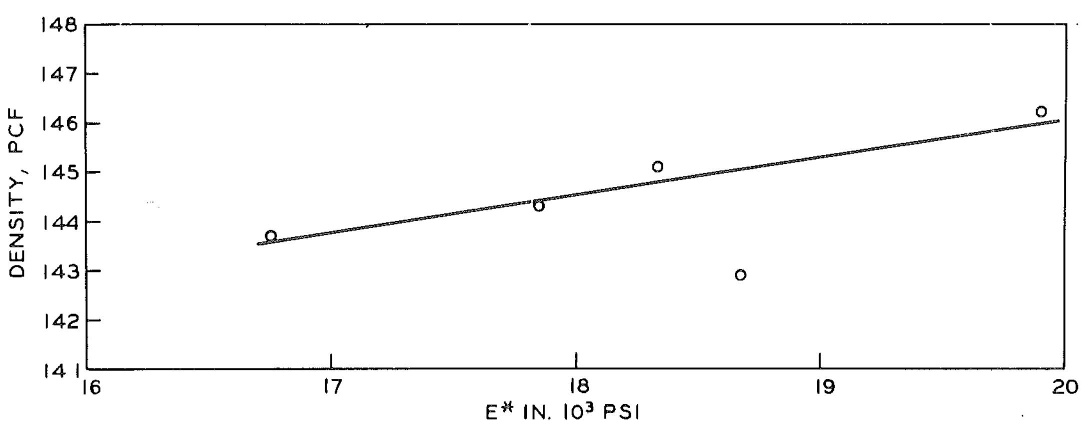

Figure 7. Quasi-elastic modulus vs. Density, 22A straight line gradation gravel.

DISCUSSION

For analysis of three-layer systems, consisting of asphalt concrete surface, untreated aggregate base, and subgrade, the layered-system theory predicted by Burmister appears most suitable, as long as it is recognized that the moduli of untreated granular material and fine grained soils are stress-dependent. Because the vertical and horizontal stresses induced in a pavement by a wheel load vary both vertically and horizontally, the elastic modulus within the base will also vary. However, at the present time, no workable theoretical solution of pavement stresses and deflections is available that takes into account variations of modulus with respect to the horizontal and vertical directions. Recognizing these limitations, the test procedure developed in this study could be used to establish a form of stress-strain relationship for highway materials. This relationship is described approximately by the quasi-elastic modulus, E*, which is a constant quantity representing the mechanical behavior for the particular range of stresses used and the type of materials tested. The quasi-elastic modulus, E*, describing the stiffness characteristics of a base material, could be used in a readily available solution of three-layer elastic systems to evaluate pavement stresses and deflections.

In general, the stress conditions for which E* was measured duplicate field loading conditions, except that the frequency and duration of stress application is different. The effect of change in frequency or duration of stress application on the measured value of E* might be considerable and should be studied. This type of study would require a laboratory repetitive loading device capable of varying frequency and duration of stress application in order to duplicate field loading conditions more accurately.

The range of values for the elastic modulus of granular materials, as reported in the literature, is between 5,000 psi and 30,000 psi (1, 5). Values of E* for granular base samples tested in this study were within this range.

CONCLUSIONS

1. Within the range of stresses developed in the base layer of a pavement, it is possible to approximately describe the stiffness characteristics of a granular base material by the constant quasi-elastic modulus developed in this study.

2. The test procedure developed is relatively simple, and utilizes conventional triaxial testing equipment.

3. The values of the quasi-elastic modulsu, E*, of granular base material ranged between 16.76 x 10^3 psi (for 22A straight line gradation gravel, compacted at a dry density of 143.72 pcf) and 22.90 x 10^3 psi (for 22A Holt gravel compacted at 142.11 pcf).

4. For the same material, there seems to be a correlation between the quasi-elastic modulus and density, with increasing density generally resulting in an increase in the modulus.

5. The quasi-elastic modulus, E*, describing the stiffness characteristics of the base material can be used in readily available solutions of three-layer elastic systems to evaluate pavement stresses and deflection required in the second phase of this project.

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